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Research Article

Error Signal Differential Term Feedback Enhanced Variable Step Size FxLMS Algorithm for Piezoelectric Active Vibration Control

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FxLMS (Filtered-x Least Mean Square) algorithm is widely used in the field of AVC (active vibration control) for its good convergence and strong adaptability. However, the convergence rate and steady-state error are mutually restricted for the fixed step FxLMS algorithm. Increasing step size μ to accelerate the convergence rate will result in larger steady-state error and even cause control divergence. In this paper, a new DVSFxLMS (error signal Differential term feedback Variable Step size FxLMS) algorithm is proposed by establishing nonlinear function between μ and error signal, while using differential term of the error signal as the feedback control function. Subsequently, a DVSFxLMS controller is designed to carry out the AVC simulation and experiments on cantilever beam with PSA (piezoelectric stack actuator). Simulation and experimental results show that the proposed DVSFxLMS algorithm has faster convergence rate and smaller steady-state error than the traditional FxLMS algorithm, which also has strong antinoise ability and adaptive control ability to quickly track the variable external disturbance.

1. Introduction

Adaptive filter technology has wide range applications in the field of digital signal processing. Compared with a conventional filter, an adaptive filter can adjust characteristics of the filter online according to adaptive filter technology, and obtain the best performance filter by finding the appropriate weight coefficients [1]. The adaptive filter is often divided into two separate parts: one part is a digital filter and the other part is an adaptive algorithm. The digital filter adjusts weight coefficients through the adaptive algorithm to improve its signal processing performance. The schematic diagram of adaptive filtering is shown in Figure 1.

When applying adaptive filtering technology to the field of AVC, a FIR (finite impulse response) filter is usually used as a feedforward controller, called the adaptive controller. The weight coefficients of the adaptive controller are adjusted according to the adaptive algorithm that usually is LMS algorithm, so that the mean square value of the vibration response measured by sensor converges to the minimum direction. The developed FxLMS vibration control algorithm has advantages of good convergence and strong adaptability, and does not depend on the accurate model of controlled structure, which has become one of the hotspots of AVC algorithm research [2–5]. However, in the FxLMS control algorithm, the convergence rate and steady-state error of LMS algorithm are greatly affected by the step size μ. Generally, the larger the step size, the faster the convergence rate, but the steady-state error will also increase; reducing step size can reduce the steady-state error, thereby improving the convergence accuracy, but a smaller step size will significantly reduce the convergence and tracking rate of LMS algorithm [6]. Therefore, the traditional fixed step LMS algorithm contradicts the adjustment of step size in terms of convergence rate, tracking rate, and convergence accuracy. In order to solve this contradiction, it is necessary to adjust the step size μ of LMS algorithm in real time during control process, which is called the variable step size LMS algorithm [7–9]. Gitlin et al. [10] proposed to reduce step size with the increase in number of the algorithm iterations, so as to achieve the purpose of variable step size, but this adjustment rule is only applicable to the time-invariant systems. Qin and Ouyang [11] proposed a variable step size SVSLMS (Sigmoid function Variable Step size LMS)

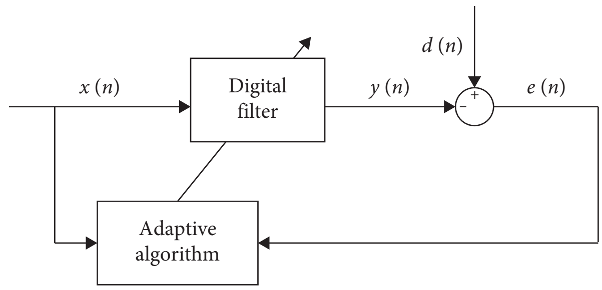

FIGURE 1: The schematic diagram of adaptive filtering where $x(n)$ is the input signal of adaptive filter; $y(n)$ and $d(n)$ are the output signal and the desired signal of the filter; $e(n)$ is the error signal.

algorithm based on the sigmoid function. By establishing the nonlinear relationship between the step size and error signal, which effectively compensated for the shortcomings in [10]. However, with continuous convergence of the SVSLMS algorithm, the step size changes more and more drastically, which greatly affects the steady-state performance of the SVSLMS algorithm. Gao and Xie [12] proposed a simpler variable step size algorithm $\mu(n) = \beta(1 - \exp(-\alpha|e(n)^2|))$, which overcomes the shortcomings of the sigmoid function in steady-state step adjustment process and also achieves faster convergence rate and smaller steady-state error. However, the current step size of this algorithm is only related to the current error signal, ignoring the effect of previous iteration error signal on the current step size, so it has a certain negative impact on the steady-state error and the convergence rate [13–15]. At the same time, under low SNR (signal noise ratio) environment, the convergence effect of this algorithm is not very ideal, which greatly restricts its application range [16]. In addition, most of the existing variable step size LMS algorithms are mainly used in the fields of system identification [17, 18] and active noise control [19, 20], which are different from their application in the active control of structural vibration [21, 22]. In the field of AVC, the system may be interfered by strong external noise, which makes the algorithm generate larger step size and output too large control signals, causing system instability or even damage. Therefore, when the variable step size FxLMS algorithm is applied to the active control of structural vibration, the robustness to noise interference needs to be considered [23].

In this study, a new variable step size FxLMS algorithm is developed which is called DVSFxLMS algorithm, by establishing nonlinear function between the step size μ and the error signal, while the differential term of the error signal $|e(n) - e(n-1)|$ is adopted as feedback control function in original algorithm, so that the current step size of the proposed algorithm is related to error signal rate. At the same time, the correlation value of the error signal $|e(n)e(n-1)|$ is used instead of the square of the error signal $|e(n)|^2$ to adjust step size. Subsequently, the DVSFxLMS controller is designed to actively control the vibration response of cantilever beam with PSA. The AVC simulation and experiments of piezoelectric cantilever beam under harmonic excitation and harmonic excitation with superimposed noise are carried out, and the adaptability of the DVSFxLMS algorithm is also studied. Simulation and experimental results show that compared with the traditional fixed step FxLMS algorithm, the proposed DVSFxLMS algorithm has faster convergence rate, smaller steady-state error, which also has strong antinoise ability and adaptive control ability to quickly track the variable external disturbances. The block diagram of the proposed research is shown in Figure 2.

2. Design of DVSFxLMS Controller

The LMS algorithm proposed by Widrow and Hoff [24] in 1960 is widely used in the fields of system identification [25], signal processing [26], and adaptive control [27] because of its advantages such as small calculation, easy implementation, and great stability. The LMS algorithm based on the steepest descent method can be summarized as the following iterative process:

$$\begin{cases} e(n) = \mathrm{d}(n) - \widehat{\mathbf{X}}^{\mathrm{T}}(\mathbf{n})\mathbf{W}(\mathbf{n}), \\ \mathbf{W}(\mathbf{n+1}) = \mathbf{W}(\mathbf{n}) + 2\mu(n)e(n)\widehat{\mathbf{X}}(\mathbf{n}), \end{cases} \tag{1}$$

where $\widehat{\mathbf{X}}(\mathbf{n})$ is the filter input signal vector of length L; $\mathbf{W}(\mathbf{n})$ is the N weight coefficients of the filter at time n; μ is the step size, which determines the steady-state performance and convergence rate of the LMS algorithm. The convergence condition of the LMS algorithm is

$$0 < \mu < \frac{1}{\lambda_{\max}}, \tag{2}$$

where $\lambda_{\max}$ is the maximum eigenvalue of the autocorrelation matrix of $\widehat{\mathbf{X}}(\mathbf{n})$.

The variable step size algorithm can solve the contradiction between step size, convergence rate, and convergence accuracy in the traditional fixed step LMS algorithm. The principle of the variable step size LMS algorithm is to use a larger step size to obtain a faster convergence rate at the initial stage of the convergence or the system changes suddenly; when the algorithm converges to steady state, a smaller step size is used to reduce the steady-state error. At the same time, the calculation amount of the algorithm should be as small as possible, and the parameters that need to be adjusted should be as few as possible to enhance the practicality of the algorithm. Therefore, based on the algorithm in [13], and combining the advantages of the algorithm in [16], while a feedback control function inversely proportional to the differential term of error signal $\Delta e(n) = |e(n) - e(n-1)|$ is introduced, a new variable step size LMS algorithm is proposed in this paper:

$$\begin{cases} \Delta e(n) & = |e(n) - e(n-1)|, \\ \alpha(n) & = p\left(\dfrac{e(n)}{e(n-1)}\right)^2, \\ \mu(n) & = \beta\left(1 - \exp\left(-\alpha(n)\left|\dfrac{e(n)e(n-1)}{\Delta e}\right|\right)\right). \end{cases} \tag{3}$$

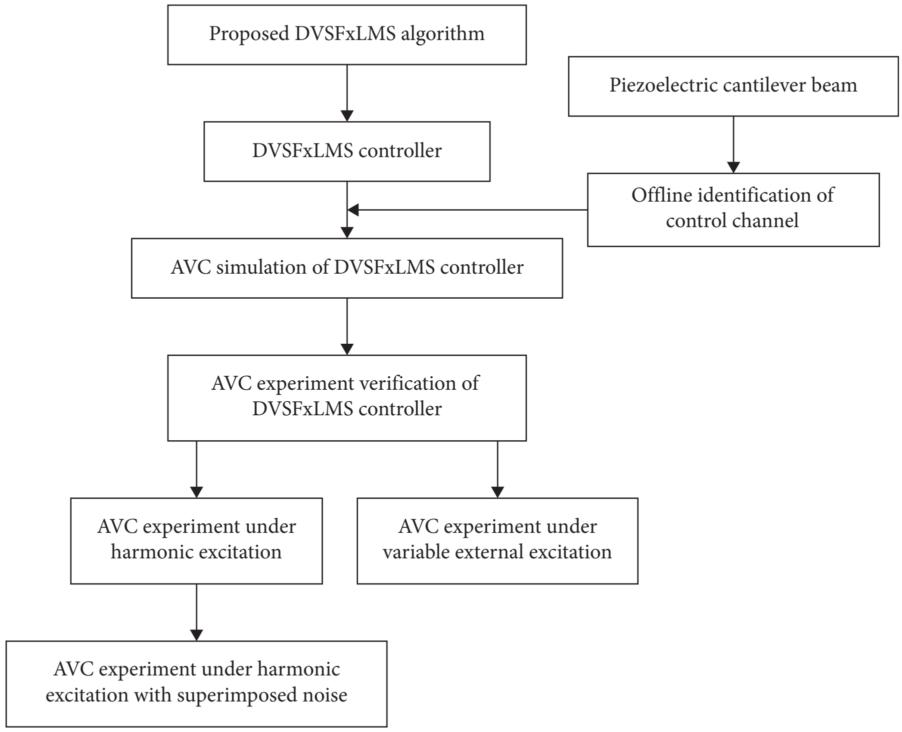

Figure 2: The block diagram of the proposed research.

The proposed algorithm contains two parameters p and β. Among them, the parameter $p > 0$, which controls the shape of the variable step size function; the parameter $\beta > 0$, which controls the value range of the variable step size function. From equation (2), $0 < \mu(n) < 1/\lambda_{max}$; therefore, $\beta < 1/\lambda_{max}$. Compared with the original algorithm, equation (3) considers the difference term of the error signal, which reflects the correlation between the step size and the error signal rate. At the same time, the correlation value of the error signal $|e(n)e(n-1)|$ is used instead of the square of the error signal $|e(n)|^2$ to adjust step size, which can further improve the antinoise ability of the variable step size algorithm [16].

The DVSFxLMS vibration control algorithm thus constructed takes the vibration response of the controlled structure caused by external disturbance as starting point, and requires the control signal to drive the actuator to generate control force or moment on the controlled structure, so that the control response will cancel out the response caused by external disturbances at observation points, so as to achieve the purpose of eliminating or reducing the vibration level of the controlled structure. Figure 3 shows the structural diagram of the DVSFxLMS controller.

In Figure 3, $d(n)$ is the structural vibration response at time n when no control signal is applied. The channel $H(z)$ from actuator to sensor is called the control channel, and $\widehat{H}(z)$ is the model obtained from the offline identification of the control channel. $\widehat{y}(n)$ is the structural vibration response caused by the control signal $y(n)$ through the control channel model $\widehat{H}(z)$. $e(n)$ is the structural vibration response error signal.

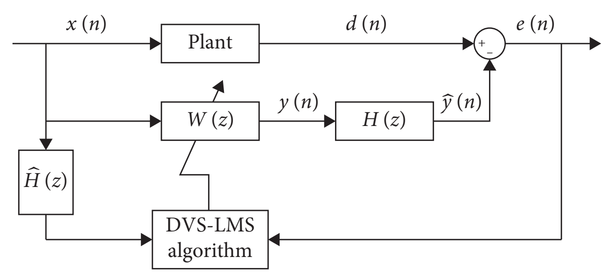

Figure 3: The structural diagram of the DVSFxLMS controller.

In summary, the iterative process of the DVSFxLMS vibration control algorithm is obtained as follows:

$$\begin{cases} y(n) = \widehat{\mathbf{X}}^{\mathrm{T}}(\mathbf{n})\mathbf{W}(\mathbf{n}), \\ e(n) = d(n) - \widehat{y}(n), \\ \Delta e(n) = |e(n) - e(n-1)|, \\ \alpha(n) = p\left(\dfrac{e(n)}{e(n-1)}\right)^2, \\ \mu(n) = \beta\left(1 - \exp\left(-\alpha(n)\left|\dfrac{e(n)e(n-1)}{\Delta e}\right|\right)\right), \\ \mathbf{W}(\mathbf{n}+\mathbf{1}) = \mathbf{W}(\mathbf{n}) + 2\mu(n)e(n)\widehat{\mathbf{X}}(\mathbf{n}). \end{cases} \tag{4}$$

3. AVC Simulation of DVSFxLMS Algorithm

3.1. Piezoelectric Cantilever Beam. In the current simulation, an aluminum alloy cantilever beam is controlled structure, and its dimension is 900 mm × 20 mm × 5 mm. As shown in Figure 4, based on the maximum modal strain energy criterion, the PSA is installed at the root of the cantilever beam to establish vibration control simulation system, in which the observation point A is at the tip of the cantilever beam.

The dimension of PSA is 170 mm × 20 mm × 37 mm, and its total weight is 190 g. The piezoelectric stack used therein is PI™ PICMA® P-840.60, and the parameters of this piezoelectric stack are shown in Table 1 [28]. The PSA is glued to controlled structure through epoxy resin to realize the axial actuation of the piezoelectric stack into a pair of actuating bending moments on the controlled structure.

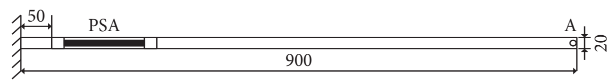

Figure 4: The dimension of the piezoelectric cantilever beam (unit: mm).

Table 1: Parameters of piezoelectric stack.

Parameter	Value
Length × width × height	12 mm × 12 mm × 122 mm
Maximum dynamic displacement	90 μm
Maximum output force	1000 N
Maximum operation voltage	100 V
Elastic compliance coefficient	16.1×10^{-12} m^2/N
Piezoelectric charge coefficient	4×10^{-10} C/N

3.2. Offline Identification of Control Channel. Figure 5 shows the schematic diagram of the control channel offline identification. Applying the excitation signal $x(n)$ to the unknown control channel, then generating an output response $d(n)$, and applying the same excitation signal to the adaptive filter, the filter output is $y(n)$. Then $d(n)$ and $y(n)$ are subtracted to get the identification error signal $e(n)$. The LMS algorithm adjusts the weight coefficients of the filter according to the error signal, and finally makes the output $y(n)$ of the filter close to the output response $d(n)$ of the control channel. At this time, the characteristic of the adaptive filter can be used as an estimate of the control channel.

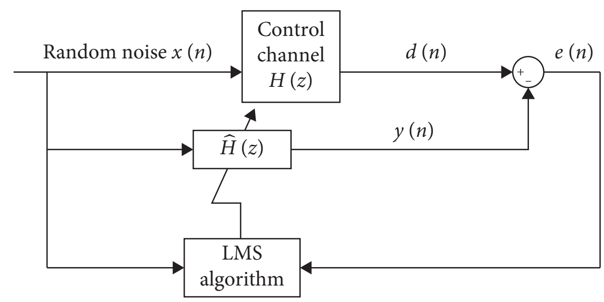

Figure 5: Schematic diagram of the control channel offline identification.

After the input and output data are obtained, the least square method is used to identify the FIR model for control channel. Whether the results of the model identification meet the needs can be qualitatively evaluated from following two requirements: firstly, comparing the frequency response function of the experimental model and the identification model, which requires the amplitude and phase as consistent as possible; secondly, the iterative curves of weight coefficient are required to smoothly converge during the identification process, and the identification error gradually decreases. If the above requirements are not met, it is necessary to readjust the step size or the order of the FIR model in LMS identification algorithm.

In the control channel offline identification experiment, 28–32 Hz narrow-band random signal is input to the control channel, the output signal of control channel is measured by the acceleration sensor A, and the step size is set to $5E-9$; then, a 1500 order FIR filter model is obtained by the LMS algorithm identification. The identification results are shown in Figure 6. It can be seen from the figure that the frequency response function of the control channel model obtained by offline identification is in good agreement with the experimental curve, and the weight coefficients of the adaptive filter converge smoothly, while the MSE (mean square error) of the identification model gradually approaches zero, which indicates that the FIR filter model identified in the control frequency band can truly reflect the dynamic characteristics of the control channel.

3.3. Simulation of DVSFxLMS Control Algorithm. In order to verify the effectiveness of the proposed DVSFxLMS control algorithm (DVSFxLMS in legends), the traditional fixed step FxLMS control algorithm (fixed FxLMS in legends) is used as a comparison group to compare and study its vibration suppression effect in the active control of piezoelectric cantilever beam harmonic vibration. The control channel model is obtained by offline identification process in Section 3.2. The simulation results are shown in Figure 7. It can be seen from Figure 7 that the DVSFxLMS control algorithm has better vibration suppression performance, compared with the FxLMS control algorithm, it has faster convergence rate and smaller steady-state error.

4. AVC Experiment Verification of DVSFxLMS Algorithm

The AVC experimental diagram of the piezoelectric cantilever beam is shown in Figure 8. The external excitation signal is generated by the Quanser real-time system, and the external excitation signal passes through the output board and amplified by the power amplifier (MB YE5872A), and then input into the electromagnetic exciter (MB Dynamics) to excite the cantilever beam. Structural vibration response is collected by the acceleration sensor (PCB 333B30, sensitivity: 100 mV/g), enters the DVSFxLMS controller through the acceleration signal conditioner (PCB 482C) and Quanser input board, then the control voltage calculated by the DVSFxLMS controller passes through Quanser output

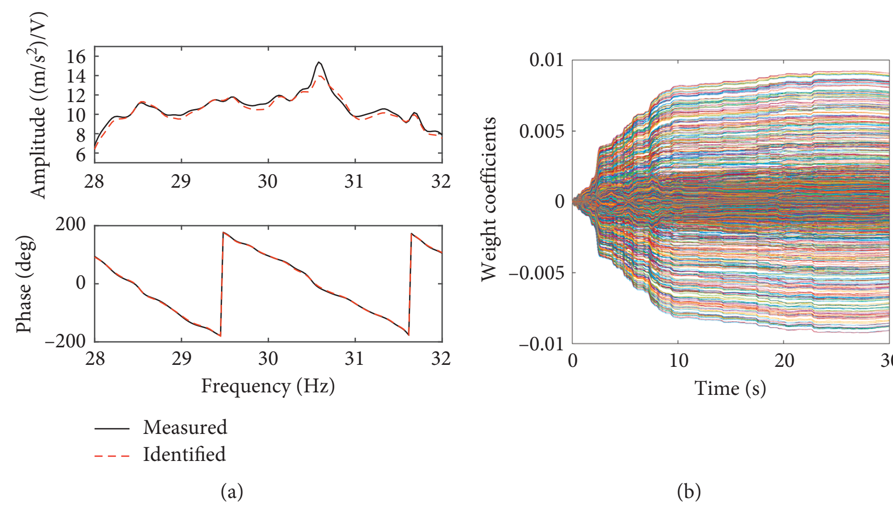

Figure 6: Offline identification results of control channel: (a) comparison of frequency response functions; (b) adaptive filter weight coefficient iteration curves; (c) MSE of the identification model.

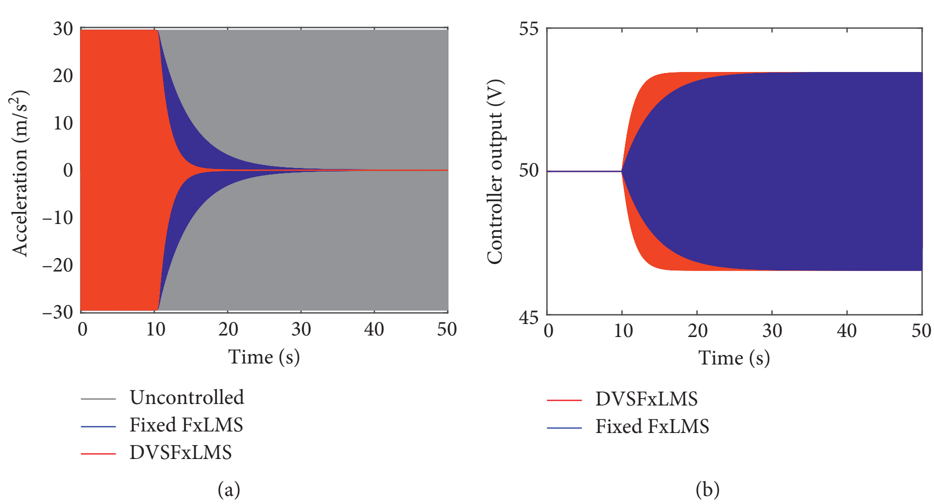

Figure 7: Comparison of simulation results of the DVSFxLMS controller: (a) acceleration response at point A; (b) output voltage.

board, which is amplified by the power amplifier (PA-V-M4) and drive the PSA to control the cantilever beam, thereby achieving active control of the cantilever beam vibration response. The setup of AVC experiment using PSA is shown in Figure 9.

4.1. AVC Experiment under Harmonic Excitation. The external excitation is selected near the natural frequency of the second-order bending mode of the cantilever beam, which is 30 Hz, and the control channel model is obtained by offline identification process described in Section 3.2. After the

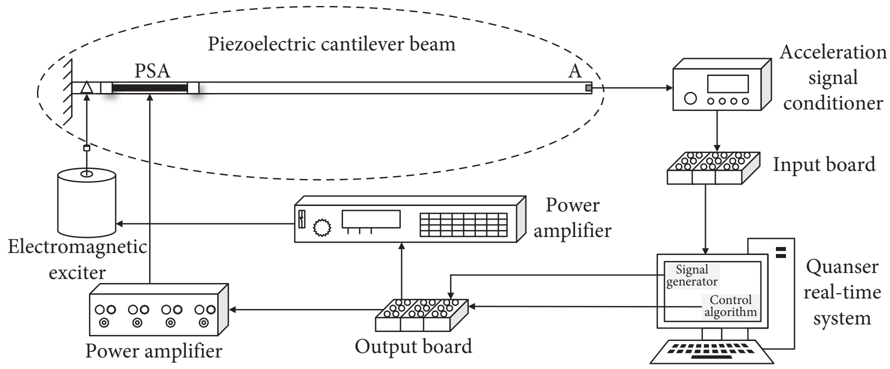

Figure 8: AVC experimental diagram of the piezoelectric cantilever beam.

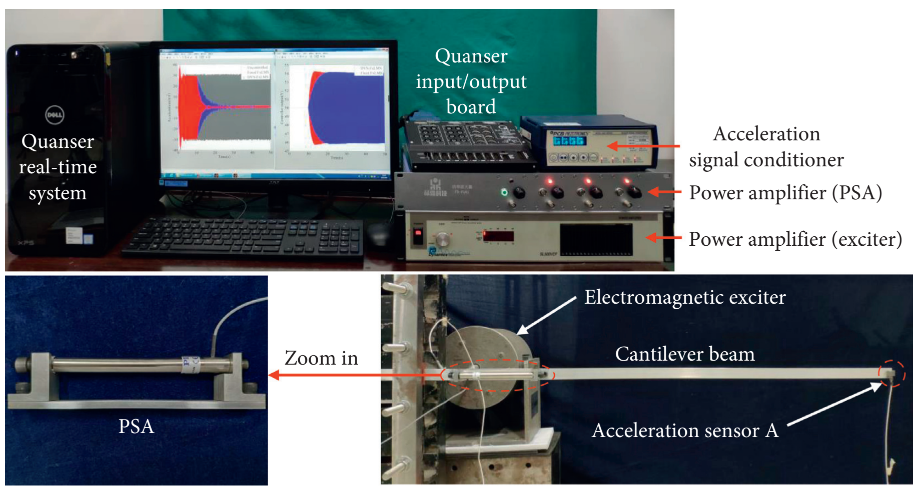

Figure 9: The setup of AVC experiment using PSA.

vibration response of the cantilever beam reaches steady state, the controller is turned on at 10 s, and the experiment lasts for 50 s. Among them, the adaptive filter order of the traditional fixed step FxLMS controller is set to 32, and the step size in the adaptive algorithm is adjusted to $2E-6$. Meanwhile, the adaptive filter order of the DVSFxLMS controller is set to 32, and the step size parameters are adjusted to $\beta = 5E-6$ and $p = 100$. The superiority of the DVSFxLMS control algorithm proposed over the traditional fixed step FxLMS control algorithm is compared and studied.

The experimental results are shown in Figure 10. From Figures 10(a) and 10(c), it can be seen that the convergence time of the fixed step FxLMS controller is 19.5 s. After the structural vibration reaches steady state, the peak acceleration at the tip of the cantilever beam is decreased by 93.7%. At the same time, the convergence time of the DVSFxLMS controller designed is 8.1 s, and the control effect reaches 95.1%, which is better than fixed step FxLMS controller. As shown in Figure 10(b), at the initial period, the output voltage of the DVSFxLMS controller is higher than that of the fixed step FxLMS controller, and then quickly reaches the optimal value to ensure high level of the vibration suppression performance. Figure 10(d) shows the iterative curves of the adaptive filter weight coefficients of the DVSFxLMS controller, and the filter weight coefficients converge quickly and smoothly. In summary, compared with the fixed step FxLMS control algorithm, the DVSFxLMS control algorithm proposed has the characteristics of fast convergence rate and good steady-state vibration suppression performance, which effectively solve the constraints of the convergence rate, steady-state error and step size of the fixed step FxLMS control algorithm.

4.2. AVC Experiment under Harmonic Excitation with Superimposed Noise. In order to verify the ability of the DVSFxLMS control algorithm proposed to resist noise interference, this section conducts experimental research on AVC under the harmonic excitation with superimposed noise.

Excitation signal consists of 30 Hz sine signal superimposed 28–32 Hz zero-mean Gaussian white noise, and the variance of the superimposed noise σ^2 is set to 1, 0.25, and 0.01, respectively. The controller parameter settings are the same as described in Section 4.1. Turn on the controller at 10 s, and the experiment lasts for 50 s. The experimental results of AVC under the harmonic excitation with superimposed noise are shown in Figures 11 and 12.

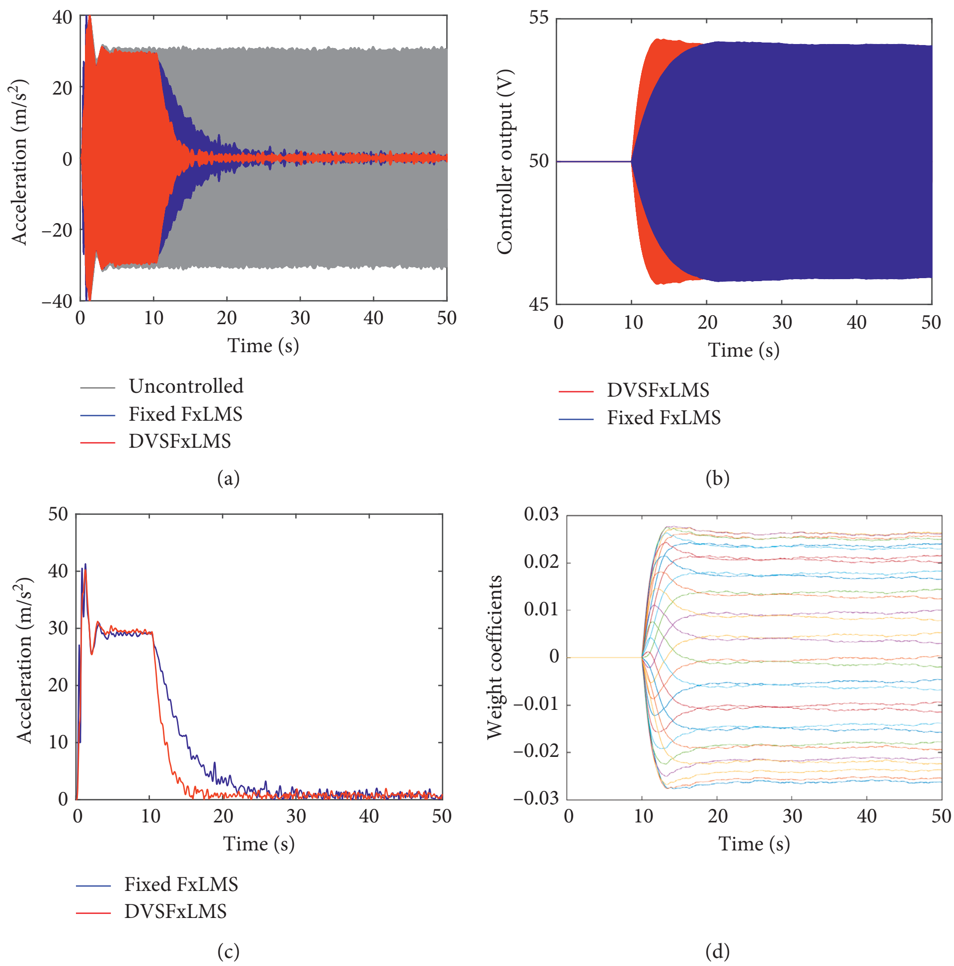

FIGURE 10: Comparison of experimental results of the DVSFxLMS controller: (a) acceleration response at point A; (b) output voltage; (c) peak acceleration response at point A; (d) adaptive filter weight coefficient iteration curves of the DVSFxLMS controller.

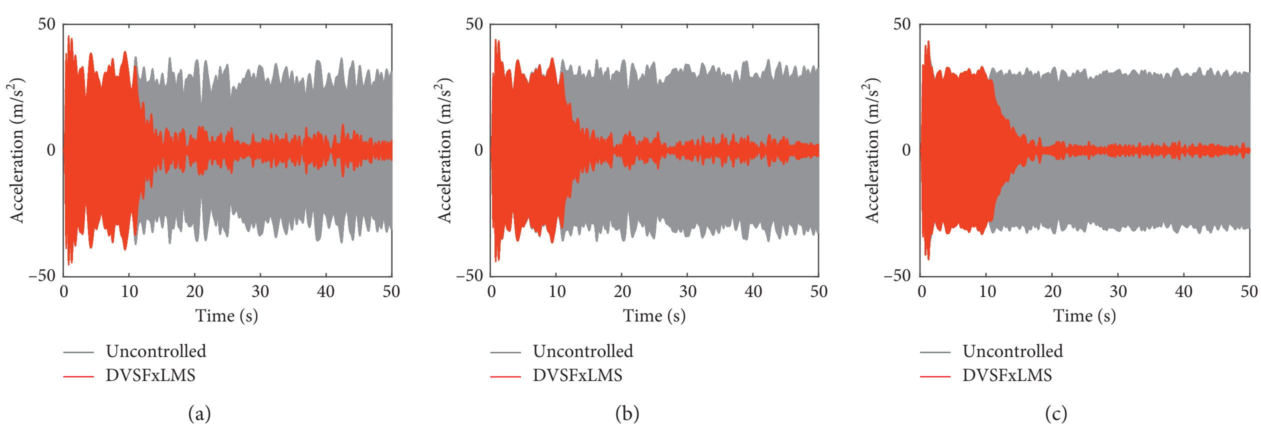

FIGURE 11: Experimental results of the DVSFxLMS controller under harmonic excitation with superimposed noise: acceleration response at point A. (a) $\sigma^2 = 1$, (b) $\sigma^2 = 0.25$, and (c) $\sigma^2 = 0.01$.

It can be seen from Figure 11 that under the harmonic excitation with superimposed noise, the DVSFxLMS controller can all converge to steady state in about 8 s. During the steady state, the DVSFxLMS controller can also adaptively adjust the filter parameters according to the changes of the error signal to suppress vibration response of the controlled structure, which shows the ability to resist noise interference.

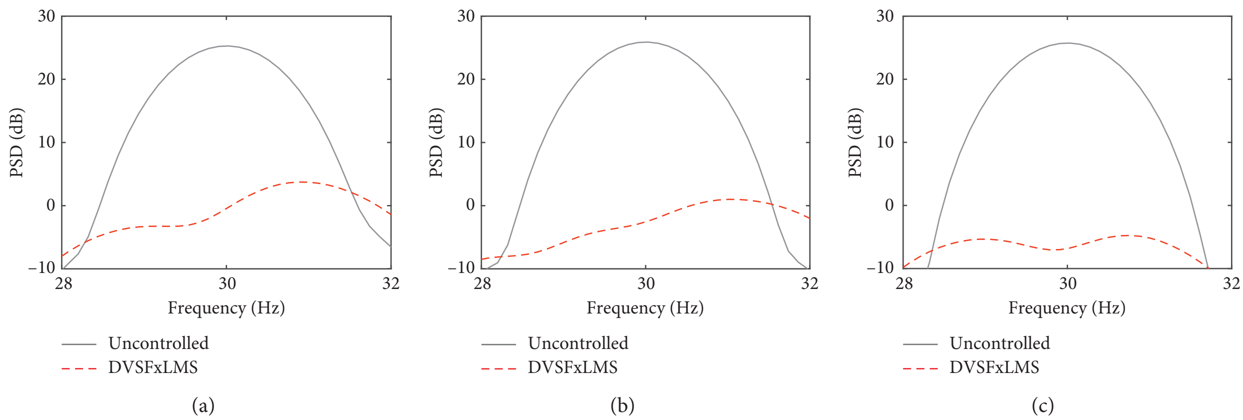

FIGURE 12: Acceleration response PSD contrast at point A (open/closed loop). (a) $\sigma^2 = 1$, (b) $\sigma^2 = 0.25$, and (c) $\sigma^2 = 0.01$.

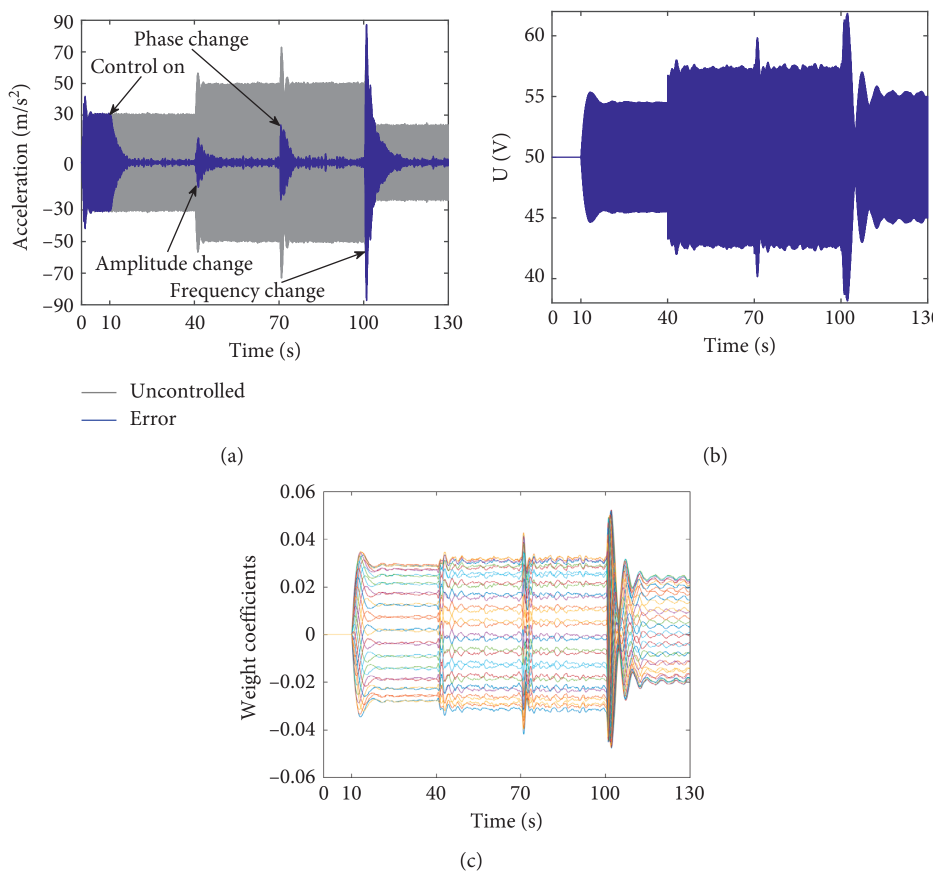

FIGURE 13: Adaptive control experimental results of the DVSFxLMS controller: (a) acceleration response at point A; (b) output voltage; (c) adaptive filter weight coefficient iteration curves.

The acceleration response PSD (power spectral density) contrast at point A in the open/closed loop state of the control system is shown in Figure 12. Under the harmonic excitation of superimposed noise signals with variances of 1, 0.25, and 0.01, the DVSFxLMS controller can effectively reduce vibration level of the controlled structure. The lower the variance of the superimposed noise, the better effect of the active structural vibration control. Finally, the peak value of the PSD spectrum is decreased by 21.5 dB, 24.9 dB, and 30.5 dB, and the RMS (root mean square) of the acceleration response is decreased by 89.5%, 92.2%, and 95.5%, respectively, which verifies that the proposed DVSFxLMS control algorithm has strong antinoise ability.

4.3. AVC Experiment under Variable External Excitation. In order to investigate the adaptability of the DVSFxLMS control algorithm to variable external disturbance, this section conducts an adaptive control experiment on the changes of the excitation signal amplitude, phase, and frequency. The control system starts to use 30 Hz sine signal as the excitation, and the controller is turned on at 10 s. After vibration response of the structure is attenuated and stabilized, the excitation signal amplitude increases to 1.5 times of the original amplitude at 40 s, the phase of excitation signal changes $\pi/4$ at 70 s, and the frequency of excitation signal increases by 5% at 100 s; that is, it becomes 31.5 Hz. The experiment time lasts for 150 s, and the controller parameters are consistent with Section 4.1.

Figure 13(a) shows the acceleration response at point A, and Figure 13(b) shows the output voltage of the DVSFxLMS controller. It can be seen that after the controller is turned on, the structural vibration response quickly decays to steady state. When the excitation signal amplitude, phase, or frequency changes, the structural vibration response rapidly increases. Then, the DVSFxLMS controller adjusts the output voltage according to the changes of the structural vibration response, and drives the PSA to generate actuating bending moment, so that the structural vibration response quickly decays to steady state again. The adjustment process takes 9.3 s, 5.1 s, and 13.4 s, respectively, which realizes the adaptive control of the structural vibration response when the external excitation changes. It can be seen from Figure 13(c) that when the excitation signal parameters change, the DVSFxLMS controller can quickly adjusts the weight coefficients of the adaptive filter according to the DVSFxLMS algorithm to reach steady state, which ensures good vibration suppression performance. The experimental results of adaptive vibration control show that the DVSFxLMS control algorithm proposed has fast tracking rate and strong adaptive control ability to variable external disturbance.

5. Conclusion

In the present study, a new variable step size FxLMS algorithm (DVSFxLMS algorithm) is proposed by establishing nonlinear function between the step size μ and the error signal, while using the difference term of error signal $\Delta e(n) = |e(n) - e(n-1)|$ into the original algorithm as a feedback control function. At the same time, the correlation value of the error signal $|e(n)e(n-1)|$ is used instead of the square of the error signal $|e(n)|^2$ to adjust step size. Subsequently, DVSFxLMS controller is designed, and PSA is used to conduct AVC simulation and experiments on a cantilever beam. Results show that the proposed DVSFxLMS control algorithm can effectively suppress the vibration response of the cantilever beam. The control effect of single frequency harmonic excitation is 95.1%, and the convergence time is 8.1 s, while that of the fixed step FxLMS controller is 19.5 s. Compared with the traditional fixed step FxLMS control algorithm, the convergence rate is faster, and the steady-state vibration suppression performance is better. The DVSFxLMS controller can still achieve good control effects under the harmonic excitation with superimposed noise. The vibration response of the cantilever beam converges to steady state in about 8 s. Under the harmonic excitation of superimposed noise signals with variances of 1, 0.25, and 0.01, the peak value of the PSD spectrum of the acceleration response at observation point is decreased by 21.5 dB, 24.9 dB, and 30.5 dB, and the RMS is reduced by 89.5%, 92.2%, and 95.5%, respectively, which shows that the DVSFxLMS control algorithm has strong antinoise ability. When the amplitude, phase, or frequency of the excitation signal changes, the DVSFxLMS controller can adjust the control voltage accordingly, quickly suppress the structural vibration response in about 8 s, and achieve good control effect, which indicates that the proposed DVSFxLMS control algorithm has strong adaptive control ability to quickly track the variable external disturbance.

A new DVSFxLMS control algorithm is proposed in this paper, and the DVSFxLMS controller is designed to effectively suppress the vibration response of the piezoelectric cantilever beam. Considering that the physical characteristics and system characteristics of the controlled structure used in this paper are relatively stable, and therefore, a reliable identification result is obtained by using the offline identification strategy of control channel based on the LMS algorithm. In order to further improve the practicability and applicability of the algorithm proposed, and make it also suitable for time-varying structures, an adaptive controller with online identification function of control channel can be studied based on the variable step size LMS algorithm [3, 29]. In addition, when faced with large and complex controlled structures, the single-channel controller has certain limitations. The multichannel FxLMS controller can be designed based on the variable step size LMS algorithm [30, 31], while considering the control channel coupling phenomenon to further optimize the structure of variable step size FxLMS controller to be applied to actual engineering structures.

Data Availability

The MATLAB .mat data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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