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Review Article

Classifying, Predicting, and Reducing Strategies of the Mesh Excitations of Gear Whine Noise: A Survey

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Gear whine noise has attracted increasing attention from researchers in both the academe and the industry over the past two decades. The wide range of research topics demonstrates that there is a huge technical challenge in understanding the source-path-receiver mechanisms deeply and predicting the gear whine noise precisely. Thoroughly understanding the sources of gear whine noise is the first step to solving this issue. In this paper, the authors summarize a certain number of published articles regarding the sources of gear whine noise. The excitations of gear whine noise are classified into three groups: transmission error along the line of action direction, frictional excitations along the off-line of action direction, and shuttling excitation along the axial direction. The mechanisms, characteristics, predicting approaches, measuring methods, and decreasing strategies for these excitations are summarized. Current research characteristics and future research recommendations are presented at the end.

1. Introduction

With the rapid development of technology in the automotive industry, electric vehicles fiercely shock the automobile market and are expected to occupy the market in a few years. The increasingly intense competition in the vehicle market and the rapid emergence of electric vehicles pose a series of challenges in improving interior acoustic comfort. Internal combustion engine noise, exhaust noise, and gearbox noise are major interior noise sources in traditional vehicles. However, for electric vehicles, gearbox noise can be perceived more easily, given the absence of the masking effect from the internal combustion engine and exhaust noise. Therefore, the key to reduce interior noise and improve interior acoustic comfort for electric vehicles is to reduce the noise from the gearbox.

According to the literature reviewed, there are two kinds of noise emitted from the automotive gearbox, namely, gear rattle noise and gear whine noise. Gear rattle noise is generally caused by fluctuations in the engine torque and speed [1–4], which usually lead to contact loss and impacts between lightly loaded mating gears. Gear rattle noise also has a close relationship with lubricant conditions [5–7]. Details of gear rattle noise for traditional and hybrid vehicles can be found in publications by Singh et al. [8] and Zhang et al. [9], respectively. Mesh excitations of gear rattle noise are beyond the scope of this survey. However, gear whine noise, with its pure tonal characteristic and high frequency, can be much more annoying in electric vehicles, where the mask effect of the engine noise is weak. Therefore, gear whine noise represents the main concern for acoustic comfort in an electric vehicle. This article focuses mainly on publications related to gear whine noise; limited space does not allow for publications related to gear rattle noise to be included in this paper. The objectives are limited to traditional cylinder involute gears. The objective gears discussed in this survey are restricted to those in parallel gearboxes.

In geared systems, if the loads transmitted by the gears were constant, the geometry of the gears were perfect, and the motions of the gears were smooth, there would not be any vibration. However, in real conditions, the profiles of gears are imperfect because of manufacturing errors and intentional modifications. In addition, the teeth will deflect significantly when they are subjected to transmitted torque. Moreover, supporting structures of gears are also deformable when subjected to operation loads, which induces inevitable misalignments. Deviations of the real gear profile from the ideal involute one, deflections of teeth under operating conditions, and inevitable misalignments all contribute to periodic displacement, which induces varying meshing forces along the line of action (LOA). The friction forces between gear surfaces, which act along the off-line of action (OLOA), change directions before and after the teeth passes through the pitch point. For helical gears and spur gears with severe misalignments, the centroid of the meshing forces shifts back and forth axially along the tooth face width. The displacement along the LOA and the friction forces along the OLOA, together with the axially back-and-forth shifts of the centroid of the meshing forces, change the amplitudes, action positions, and directions of the meshing forces between mating gears. The oscillating meshing forces are transmitted to supporting bearings by shafts, resulting in varying bearing forces. The varying bearing forces vibrate gearbox-housing plates, which finally radiate undesired gear whine noise.

The complexity of gear whine noise has inspired a large number of researchers to study this issue. Dating back to 1958, Harris [10] noticed the gear whine noise phenomenon and investigated the effect of the transmission error (TE) on gear whine noise. Because of computer technology breakthroughs in 1980s, a large number of studies on topics related to gear whine noise were conducted. Coming into the 21st century, the number of the publications in this area shows an exponential growth trend, as shown in Figure 1. Many outstanding researchers published review papers related to gear vibration and noise, such as gear system dynamic models [11], nonlinear dynamics of gear-driven systems [12], condition monitoring and fault diagnosis [13, 14], and planetary dynamics and vibration [15], respectively. Conversely, so far no one has made a comprehensive summary on the sources of gear whine noise. Therefore, there is an urgent need for a systematic literature review on the sources of gear whine noise.

During the generation of gear whine noise, excitations of gear meshes act as sources, shafts and bearings as vibration transfer paths, and the gearbox-housing plates as the receiver. The reduction of gear whine noise can be reached only via the reduction of the amplitudes of the housing plate vibrations. Both the strength of the sources and the propagation property of the transfer paths influence the magnitudes of the vibrations of the housing plates. In this review, the authors focus on the sources of gear whine noise.

This paper aims to summarize the excitations of gear whine noise from the literature, classify these excitations according to the three action directions of the meshing forces, present methods for predicting these excitations,

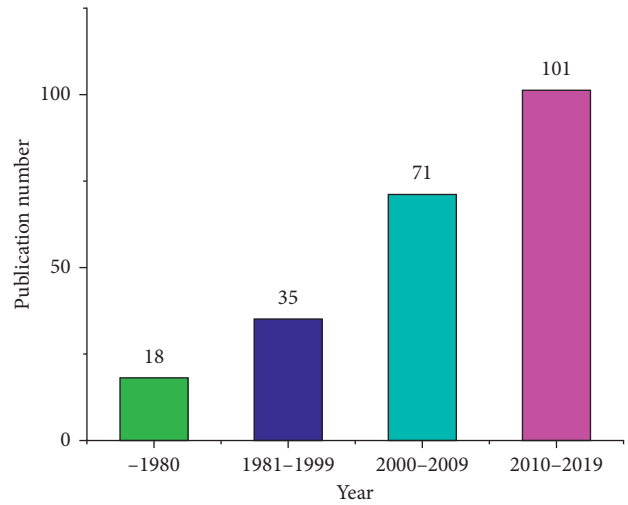

Figure 1: Histogram of research papers on gear vibration and noise.

conclude strategies to reduce these excitations, and propose new possibilities for gear whine noise reduction at its source. The remaining part of this paper is organized as follows. In Section 2, the excitation along the LOA direction, namely, TE, is summarized; the definition of TE, harmonic contributions from TE, strategies to calculate and measure TE, and approaches to reduce TE are also described. In Section 3, excitations along the OLOA are reviewed. The approaches to evaluate the frictional excitations and strategies to reduce them are presented. In Section 4, shuttling excitations in the axial direction and the properties of shuttling excitation are summarized. The potential of vibration reduction for lightweight gears is discussed in Section 5. Section 6 gives a summary. The authors also point out essential existing problems in reported research work and describe prospects of future research directions regarding excitations of gear whine noise.

2. Transmission Error

If the profiles of gears were geometrically perfect, the teeth on gears were perfectly rigid and correctly spaced, and the supporting structures were rigid and accurately installed, there would be no variance in meshing forces when meshing, which results in no vibration being generated. In reality, this ideal scenario does not occur for a variety of reasons such as manufacturing imperfections, intentional modifications, and inevitable teeth deflections. All of these imperfections contribute to a periodic displacement, namely, TE, which was first defined by Harris [10]. As stated by Munro [16], the periodic variation of TE induces periodic advancement and retardation of the driven gear while the gears are rotating. In addition, if the rotation speed coincides with the frequency of one component of TE, then resonance will occur. The periodic motions of the driven gear and the potential resonance may give rise to large dynamic meshing loads and high noise levels. Transmission error is therefore a primary source of gear noise and vibration. Since these two studies,

gear designers have gradually appreciated the importance of TE and tried to explore the relationship between TE and gear whine noise. Few researchers [17–20] showed that directionally reducing the transmission error should reduce noise as well. However, the direct relationship between TE and the level of gear whine noise remains unrevealed.

In this section, a detailed description of TE is presented. For researchers who have just started their studies in this area, this section will help them to quickly understand the relationship between TE and gear whine noise and the basic definition, components, prediction methods, and reduction strategies of TE.

2.1. The Definition of Static Transmission Error. Harris [10] proved that the periodic variations in the velocity ratio and the variance and nonlinearity of mesh stiffness, which all contribute to static measured relative displacement, were the main internal sources of vibration for spur gears. The statically measured relative displacement in that paper was the well-known TE that was subsequently defined [21], for any instantaneous angular position of one gear, as the angular displacement (given by equation (1)) of the mating gear from the position it would occupy if the teeth were perfect. To the authors' knowledge, Harris was the first who defined TE and plotted the TE curves under different loads for rotating gears. These curves were known as the Harris map and were of great importance in understanding gear motions. Transmission error can also be described as linear displacement, as in equation (2), along the LOA direction. It is much more convenient to describe TE as a linear displacement than as an angular difference because the linear displacement of gear pairs along LOA closely relates to and triggers angular vibrations:

$$\mathrm{TE}(\theta) = \theta^g - \frac{R_{bp}}{R_{bg}}\theta^p, \tag{1}$$

$$\mathrm{TE}(d) = R_{bg}\theta^g - R_{bp}\theta^p, \tag{2}$$

in which θ^i is the rotating angular of gear i as in Figure 2 and R_{bi} is the base radius of gear i, $i = p, g$, in which p and g represent the pinion and the gear, respectively.

Transmission error is widely recognized as the dominant excitation of gear whine noise [10, 11, 21–28]. A thorough research in TE is beneficial for deep understanding of gear whine noise. From the publications of Mark [29–32], TE can be expressed accurately using equation (3a) and (3b) as follows:

$$\mathrm{TE}(l) = \frac{W}{K_M(l)} + \frac{\sum_j \eta_{Kj}^{(p)}(l) + \eta_{Kj}^{(g)}(l)}{K_M(l)}, \tag{3a}$$

$$K_M(l) = K_M^- \times \left(1 + \frac{\delta K_M(l)}{K_M^-}\right) \approx K_M^- \times \left(1 - \frac{\delta K_M(l)}{K_M^-}\right)^{-1}, \tag{3b}$$

in which l is the rotating position, $K_M(l)$ is the total time-varying mesh stiffness, K_M^- is the mean component of mesh stiffness, $\delta K_M(l)$ is the varying component of mesh stiffness, W is the transmitted torque, j is the tooth number, $i = p, g$, and $\eta_{Kj}^{(i)}(l)$ is the deviation of the tooth surface for gear i.

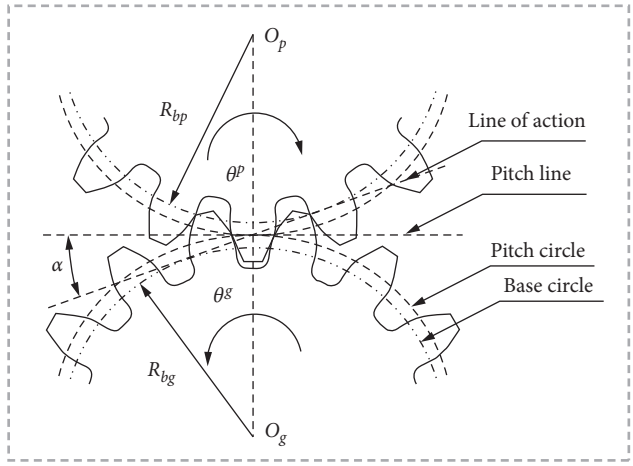

Figure 2: Gear motions along the LOA.

The first term in the right-hand side of equation (3a) is the elastic deformation, which also contains the mesh stiffness variation. The second term consists of deviations from perfect involute surfaces; these deviations include manufacturing errors, intentionally designed microgeometry modifications, and supporting misalignments. This analytical equation is suitable for both helical and spur-gear pairs. In a word, there are four primary sources of TE for a gear pair: gear teeth elastic deformation [33, 34] and stiffness variance [35–47], manufacturing errors [30, 48–50], misalignments [51–58] due to supporting structures, and intentional profile modifications [33, 53, 59–66]. In addition, gear surface roughness [67–70] also contributes to TE. It is worth mentioning that gear microgeometry correction is a significant approach for minimizing TE excitation; the details of the approach are described in the subsequent sections.

2.2. Harmonic Contributions for Transmission Error. Transmission error is generally analysed in the frequency domain. There are four sets of harmonics generated by the TE excitation of a meshing gear pair: tooth mesh harmonics, two sets of rotational harmonics, and the fundamental harmonics of the meshing gear pair. As described by Mark [31, 32], the mean deviations of the tooth working surfaces from equally spaced perfect involute surfaces include the average elastic deflections, mesh stiffness, mean component of manufacturing errors [71], and intentional modifications, all of which contribute to gear mesh harmonics. The tooth-to-tooth variations such as individual tooth manufacturing errors [72] contribute to rotational harmonics. In addition, shaft misalignment [73], gear plastic deformation [74, 75], and gear damages [76–79] also provide contributions to TE rotational harmonics. Variations associated with tooth surface contact points contribute to the fundamental harmonics of the gear pair.

Among the four sets mentioned above, the tooth meshing harmonics are the strongest excitation, the

rotational ones are the second strongest, and the one associated with fundamental harmonics of the gear pair is the weakest. Because of frequency modulation, there are sidebands around each harmonic of the mesh frequency, as shown in Figure 3. For the analysis of gear whine noise, it is useful to examine how various TE sources are exhibiting in the frequency domain.

2.3. Transmission Error Calculation Methods. As we know, TE is the primary source of gear whine noise and serves as the forcing term in gear system dynamic analysis [80, 81]. Moreover, at the design stage, TE is the guideline to determine optimal tooth modifications [82]. TE analysis is also an important tool for gear fault detection [74–79] at the early stage. Thus, the ability to accurately predict the TE of the target gears is essential in evaluating and minimizing gear whine noise. As described in equations (3a) and (3b), there are two parts of TE: tooth elastic deflections and surface deviations, respectively. Surface deviations, which consist of manufacturing errors determined by gear accuracy and intentional modifications designed by engineers [83], can be measured accurately. Therefore, TE calculation mainly focuses on the prediction of gear teeth elastic deflection and the varying mesh stiffness.

2.3.1. Analytical Method. When spur-gear teeth are considered as nonuniform cantilever beams, bending, shearing, and contact deformations all contribute to the teeth deflections. Mesh stiffness is the transmitted force divided by the elastic deformations. In 1963, Gregory et al. [21] developed a simple approximate TE formula, which was based on Weber's deflection equation [34] and varied as a sinusoidal function. Based on the publications by Cornell [84] and Lin et al. [85, 86], there are three components in gear tooth deflection, which are bending and shearing, contact deformation, and deflections related to foundational effects. Subsequent TE calculations [87–90] considered corner contact and proposed an additional approximate equation for TE outside the normal path of contact. Another method for calculating the deflections of gear teeth [91, 92] was based on conformal mapping of complex variables in plane elasticity.

The teeth of helical gear pairs are considered as infinite cantilever plates when their deformations are calculated [93, 94]. For helical gears, the calculation of elastic deformation is more difficult than for spur gears because the geometry and load distribution are more complicated. Based on the cantilever plate theory and the mathematic programming approach, Conry and Seireg [95, 96] developed a load distribution program to determine the load distribution and elastic deformation of the gear tooth for both helical and spur gears. Many recent research studies [59, 97–100] related to TE prediction apply this computer program.

2.3.2. Numerical Method. Because of the vast assumptions and empirical parameters, the analytical equations for TE were not accurate enough to perform exact calculations.

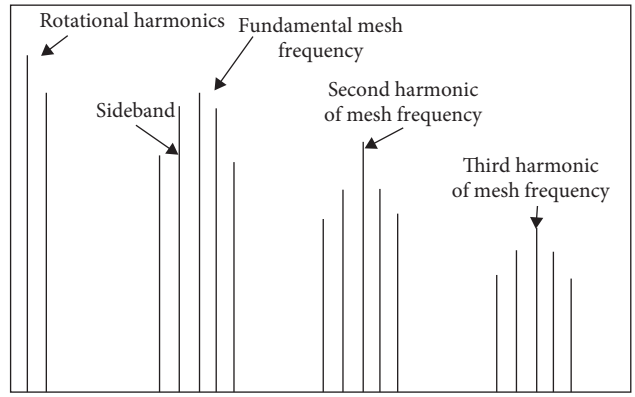

Figure 3: Harmonic contribution for the transmission error.

With recent computer hardware and software improvements, numerical simulation becomes increasingly popular in calculating TE. As long as the detailed 3D model of the teeth is established and the tooth surface is described precisely by high-quality elements, given the material property and boundary conditions, the finite element (FE) model can calculate the TE for a pair of gears accurately.

Finite element method is widely applied in calculating tooth deformation and mesh stiffness for spur [101, 102] and helical gears [103, 104]. The challenge is the modelling of contact deformation with its nonlinearity. For the improvement of computational efficiency, a properly simplified 2D FE model [105, 106] is popular for calculating TE for spur gears. When misalignment and nonuniform load distribution along contact lines are considered, a 3D FE model [107–111] of the spur gear should be applied. However, for helical gears [112, 113], a 3D FE model is essential for the complex geometry. Generally, the FE model of a spur or helical gear is specific for one gear pair, so parameter analysis based on FE models seems impractical. However, parametric FE modelling approaches [114–119] overcome this difficulty and offer an opportunity to study the effects of microgeometry parameters on TE.

For both spur and helical gears, the contact between two surfaces is localized in a very small region, so there is an urgent need for refined mesh [120] in the contact area. Accordingly, preparing gear element models with high quality is a time-consuming burden requiring high levels of skill. The microgeometry complexities of the tooth profile and tooth lead make it more difficult to model every detail on the tooth surface. In addition, too many details in FE models slow down the numerical simulation.

2.3.3. Analytical-Numerical Method. Due to too many assumptions, the analytical models are efficient but not accurate. Although the results of FE simulation are very accurate, its efficiency is not satisfactory. Therefore, researchers need to find a way to balance the efficiency and accuracy of the methodologies for calculating TE. A combination of analytical and FE methods is a compromise method offering quick simulation and high accuracy simultaneously.

For the gear teeth, the bending and shearing deflections are easy to calculate using the FE method, while the calculation of Hertzian contact deformations is more complicated due to the nonlinearity of the deformations. Therefore, Umeyama et al. [130] and Rincon et al. [40] estimated the deformations of a helical gear tooth by analytical Hertzian formulae for the contact deformation and by the FE method for the bending deflections. Simon [122, 123] calculated the tooth deflection of helical gears using analytical equations, which were obtained via regression analysis and interpolation of the results of the 3D FE model. Rao and Yoon [131] subsequently adopted the formulae proposed by Simon to calculate the deformations of helical gears. Another combined method is the integral equation method [62, 124–126, 129] consisting of the load distribution coefficient and mesh stiffness. The integral equations can calculate TE faster than the normal FE method. Normally, the mesh stiffness in the integral equations is obtained via the FE method, while the load distribution coefficient is determined analytically [127, 128]. Li [52, 53, 121] combined the mathematical approach with the FE method to calculate tooth load distribution and TE. Based on the surface response approach [132], the peak-to-peak TE derived from the FE model is represented and optimized via a mathematical formula. Many papers related to TE reduction via gear microgeometry modification adopt combined analytical-FE models. A summary of the analytical equations adopted in the reviewed publications is presented in Table 1.

2.4. Transmission Error Measurement. As described in equations (3a) and (3b), tooth surface deviations are the primary source of TE. Transmission error predictive approaches cannot accurately calculate the deviations due to manufacturing imperfections, which have a significant influence on the gear whine noise. Therefore, the prediction of TE mainly focuses on the calculations of deflections and deformations of gear teeth. The measurement strategies for TE make these calculations possible and can obtain tooth surface deviations and deflections simultaneously.

Transmission error can be measured statically or dynamically (under low or high speed) as described by Åkerblom [133]. Gregory et al. [134] proposed the first static TE measurement equipment, but it is only suitable for 1 : 1 ratio gear pairs. Houser and Blankenship [135] summarized four methods for measuring static and dynamic TE. Transmission error measurements in the early stage were usually limited to isolated gears tested under extremely light-load and low-speed condition using the single-flank test rig. Traditionally, the measurements from single-flank tests were mainly for verifying gear manufacturing accuracy. More recently, measurements are conducted under quasistatic loaded conditions [136–141]. The measurements of loaded gears are the interactions of microgeometry deviations and tooth deflections. These experiments are generally conducted on a noncirculating power test rig [141], or on a power-circulating test rig [142]. Optical encoders with proper resolution are generally installed at the shaft ends to record the rotating displacements of the gears. The challenge with TE measurement under quasistatic loaded conditions is that the magnitude of TE is extremely small and often in the micron order. Thus, selection of encoders with proper resolution is an important step in measuring TE.

Nowadays, the need for dynamic TE measurement grows faster because dynamic TE has a closer relationship with gear whine noise. There are three methods suitable for measuring dynamic TE: one based on optical encoders [143, 144], another based on vibrometers [145], and the last based on accelerations [142, 146–148].

Methods based on encoders are conducted on a power-circulating test rig under high-speed conditions. The challenge with this method is that it is difficult to eliminate the dynamic effects of the slave gearbox. In addition, the tested gears are generally mounted on two isolated shafts or in a specially designed gearbox. The boundary conditions of the gears are extremely different from the ones that gears are subject to in a real gearbox. Thus, there is an urgent need for technology that can successfully measure the static and dynamic TE in a real gearbox. Methods based on vibrometers provide a chance to measure the dynamic TE in a real gearbox [145]. However, the outputs of this method are not as good as those produced by encoders and require an integration to obtain the dynamic TE. In addition, there should be several holes on the housing plates for laser access. Thus, the lubricant of the gearbox may be a great challenge for accurate measurement. Methods based on accelerations are conducted on a power-circulating test rig, and the accelerometers are installed either on the walls of the gearbox [142] or on the flanks of the gears [148]. The outputs of this method are the second derivative of the TE, which means that there would be a large error when calculating TE. When the accelerations are measured, precise calibration, tight misalignment, and adequate mounting space are necessary conditions.

2.5. Methods to Reduce Transmission Error. As mentioned before, the meshing variations during the meshing process are the primary excitations of gear whine noise. A reduction in the level of excitations directly results in the decrease in the gear whine noise. One-micrometre reduction of TE results in 5 dB reduction of the gear whine noise [149]. Gear tooth modification is such an approach for gear whine noise reduction by decreasing the meshing excitations. A smooth load transfer is crucial for reducing the gear whine noise and can be achieved by intentional tooth surface modifications. Microgeometries should be optimized carefully to obtain a quiet gearbox design.

For uniform transfer of motions, tip and root relieves are frequently implemented on the gear tooth profile. In the 1930s, Walker [33] first proposed the gear tip relief theory. Harris [10] observed that the amount of tip relief could be appropriately chosen so that TE can be constant at the design load. Since then, many researchers devote their efforts in clarifying the effects of linear [21, 59, 150, 151], parabolic [106, 152–156], cubic [88], and other complicated types [157, 158] of relief on TE reduction. For helical gears with bias modification [159], the TE is less sensitive to the

Table 1: Formulae for transmission error calculation.

Formula	Reference
$x = \varepsilon \cos(N\Omega t)$	[21]
$x = x_p + x_g;$ $x_j = x_{bj} + x_{fj} + x_{cj}, j = p, g;$	[84–86]
$x_{o.p.c} \approx (c^2/2)((1/r_1) + (1/r_2));$	[89, 90]
$x = c\varepsilon - \sum_{i=1}^{7}(c_i/(a_i + \varepsilon))$	[91, 92]
$x = w_k + \varepsilon_k + P;$ $w_k = \sum_{i=1}^{N} a_{ki}F_i;$ $a_{ki} = w_{kbi} + w_{kTi} + w_{kpi} + w_{kHi}$	[94–96, 121]
$x = (1515.37F_n/Em)nf_1f_2f_3nz^{-1.0622}n(\alpha_0/20)^{-0.3879}n(1 + (\beta_0/10))^{0.08219}n(1 + x_p)^{-0.2165}$ $n(h_f/m)^{0.5563}n(h_k/m)^{0.6971}n(r_{\text{fil}}/m)^{0.00043}n(b_f/m)^{-0.604}$	[122, 123]
$x = \int K_b(x_L, \zeta_L)P(\zeta_L)\mathrm{d}\zeta_L + \int K_c(x_L, \zeta_L)P(\zeta_L)\mathrm{d}\zeta_L;$ $F = \int P(\zeta_L)\cos \quad \beta \mathrm{d}\zeta_L$	[124–128]
$x(\tau) = \delta_m n((1 - \int \widehat{K}(M)n(e(M)/\delta_m)\mathrm{d}M)/\cos \quad \beta_b n\widehat{k}(\tau, x_s))$	[129]

x is the TE, and the definition and value of other coefficients in the formula can be found in the corresponding reference.

transmitted torque. The common function of lead and profile crowning is the reduction of contact and root stress [160, 161]. To understand the influence of the combined profile and lead crown modifications on TE and contact stress for helical gears, Zhang et al. [98] conducted a numerical investigation, from which one could learn that each pair of modification parameters gives an optimum peak-to-peak TE at different torques.

These investigations mentioned above are limited to case studies and numerical analysis for specific gear applications. Although the outputs of the analysis are rather accurate, the global trend of TE when changing modification parameters is difficult to illustrate. The optimal modifications mentioned above are valid solely at the design load, while gears usually work under multiple load conditions. There is an urgent need for a robust modification optimization that can guarantee a relatively small excitation level under the whole operating range. Analytical methods seem to have more advantages in finding robust modification designs [82, 162] and allow for parameter analysis [129, 163–165]. Minimizing TE is the primary objective for gear-form modifications. Meanwhile, optimal TE achieves good performance only under static conditions. Reduction of meshing forces and bearing forces for gear systems is of great importance in the reduction of gear whine noise. Therefore, researchers should consider dynamic criteria [166–169] when the setup entails gear profile modifications.

3. Frictional Excitations

During the process of meshing, the teeth undergo pure rolling at the pitch point. The motions before and after the pitch point are known as approaching and recessing, respectively. The directions of the friction forces are opposite in these two periods. The sudden direction reversal of the friction forces near the pitch point has a significant influence on dynamic meshing forces and induces varying bearing forces. In addition, the arms of friction moments vary as the gears rotate. Time-varying sliding friction forces and friction moments between meshing teeth are significant excitations of gear whine noise. Friction forces and moments are usually referred as the secondary excitation when compared to TE.

3.1. Frictional Excitations. To the knowledge of the authors, Iida et al. [170] were the first researchers who investigated the vibrational characteristics of gears due to friction forces. Borner and Houser [171] quantitatively evaluated the influences of friction forces and reached the conclusion that friction forces should be considered as an excitation of gear whine noise when TE is low. Hochmann [172] proved via experimentation that friction force is a potential excitation. Vedmar and Henriksson [101] highlighted the importance of the motions along the OLOA direction on dynamic meshing forces. Velex and Cahouet [173] revealed that friction forces have a significant contribution on gear vibration and noise at low and medium speeds under high torque levels. Houser et al. [174] conducted a series of experiments to identify frictional excitation and to study the influences of tribology parameters. From the experimental results, one can learn that friction force is a dominant excitation for gear whine noise in higher harmonics of mesh frequencies. Vaishya and Singh [175, 176] proposed an analytical model to evaluate the contribution of friction torque, as an external excitation, on TE. Results show that frictional excitations have limited influence on lower harmonics of mesh frequency but have significant influence on higher harmonics. Velex and Sainsot [177] concluded that translational responses were sensitive to frictional excitations and that torsional responses were less sensitive to them. According to Gunda and Singh's [178] observation, frictional excitations could not only change the shape of the TE curves but also increase the amplitude of the second harmonic of the mesh frequency. In Lundvall et al.'s investigation [179], the friction forces increased the peak-to-peak TE. In He et al.'s [180] study, friction forces induced large oscillations in bearing forces along the OLOA direction. The authors also emphasized the importance of friction forces when TE was small. He et al. [181] then proposed a 12-degree-of-freedom (DOF) helical gear model, which coupled rotation motions, translation motions, and axial motions. Simulation showed that friction forces are a potential

excitation in the LOA direction, significant excitation in the OLOA direction, and insignificant excitation in the axial direction. According to another publication of He and Singh [182], motions along the LOA are insensitive to friction forces. Singh et al. [183] assessed the contributions of TE and friction forces on gear vibration and noise. The bearing force curves indicated that besides the excitation role along the OLOA, friction forces might influence the forces along the LOA significantly. He and Singh [184] stated that the coefficient of the friction forces affected only the first two harmonics of TE. Kahraman et al. [185] concluded that the motions along the OLOA were sensitive to friction forces, while the motions along the LOA were insensitive to these friction forces. He et al. [186] observed that the gear whine noise induced by friction forces was comparable to that induced by optimized TE. Liu and Parker [187] reported that tooth bending, induced by friction moments and forces, affected system dynamics significantly. Chen et al. [188] suggested that friction force might reduce the dynamic responses of gear systems in high-frequency and low-speed conditions. Wang et al. [189] stated that friction forces could change the motion stability of a gear system. Brethee et al. [190] discovered that the motions along the OLOA were sensitive to the variance of friction forces.

All the research studies mentioned above reach an agreement that friction forces and moments are indeed significant excitations of gear vibration and noise along the OLOA direction. Some investigators [171, 180, 186] emphasize the importance of frictional excitations when TE is low. However, there is no consistent conclusion about the role of frictional excitations in the LOA direction. A small number of scholars [175–177, 182, 185] believe that in the LOA direction, the influence of friction forces is ignorable. From other studies, the effects of frictional excitations on the amplitudes [179, 184] and curve shape [178] of TE, dynamic meshing forces [183, 188, 189], and gear bending [187] cannot be easily concluded. To the authors' knowledge, He et al. [181] were the only researchers who studied the influence of frictional excitations on motions along the axial direction, indicating that this area needs further research. Frictional excitations have a significant influence on gear dynamic response, a fact that is recognized by many scholars. Meanwhile, Jiang et al. [191] reported that frictional excitations and gear vibration were interactive and coupled. Vibration affected the sliding velocities and directions of the frictional excitations and hence magnified vibrations along the OLOA direction.

3.2. Friction Force Prediction. Vaishya and Singh [192] summarized various strategies calculating friction forces and moments. Kar and Mohanty [193] proposed formulae of friction forces and moments based on time-varying contact length for helical gear pairs. The general expressions for frictional excitation are shown as follows:

$$F_f = \mu n F_{\text{mesh}} n \varepsilon, \tag{4a}$$

$$M_f = \mu n F_{\text{mesh}} n \varepsilon n \zeta, \tag{4b}$$

in which μ is the friction coefficient; F_{mesh} is the normal tooth meshing load; ε is the sign function; ζ is the arm of the friction moment.

The normal tooth load is the contact force along the LOA, which has a close relationship with the stiffness and deflections of the mating teeth. The general expression of the normal force in Ref [178] is illustrated by

$$F_{\text{mesh}} = k_i(t) n \left(\delta(t) - \varepsilon_i(t)\right) + C_i(t) n \left(\dot{\delta}(t) - \dot{\varepsilon}_i(t)\right), \tag{5}$$

in which $k_i(t)$ is the varying mesh stiffness; $C_i(t)$ is the varying damping coefficient; $\delta(t) - \varepsilon_i(t)$ is the displacement along the LOA direction; $\dot{\delta}(t) - \dot{\varepsilon}_i(t)$ is the first derivation of the displacement.

The direction reversal is a nonlinear factor and of great importance in predicting frictional excitations. The formulae determining this nonlinear factor are summarized in Table 2. As can be seen from equations (4a) and (4b), besides the direction reversal, friction forces are the product of the normal tooth load and frictional coefficient. Friction moments are the product of friction forces and moment arms. Formulae for moment arms based on the gear geometry are described in Table 3. The coefficient of the frictional excitation is complicated and discussed in the following paragraph in detail.

As reviewed by Martin [194], the coefficient of friction force was mainly estimated empirically before the year of 2000, such as by Benedict and Kelley [195], Kelly and Lemanski [196], Johnson and Spence [197], and Rebbechi et al. [198]. Four different frictional coefficient formulae, corresponding to the Coulomb friction [180], empirically estimated friction [195], smoothed Coulomb friction [199], and thermal non-Newtonian elastohydrodynamic lubricant (EHL) [200, 201], were then adopted into a spur-gear dynamic model by He et al. [202]. The difference between motions along the OLOA and TE curves predicted by these models is not significant. A mixed EHL frictional coefficient formula [203] is accurate enough to predict the frictional excitation. Han et al. [204] proposed helical gear friction forces and torque formulae, considering nonuniform load distribution and the time-varying friction coefficient. Predictions indicated that effects of nonuniform load distribution were more significant than the variance of the friction coefficient. In addition, it has been proved that [173, 177] the exact form of the friction law is of secondary importance and the dominant factor is the reversal of the sliding directions at the pitch point. Afterwards, researchers understand that a constant friction coefficient is acceptable in gear dynamic analysis and whine noise prediction. Most scholars adopt the Coulomb friction model [205–210] and a user-defined constant friction coefficient. However, Liu et al. [211] observed that the variance of the frictional coefficient should not be neglected.

Among the reviewed frictional research studies, a vast majority of researchers adopted a Coulomb friction model, assumed a uniform distributed friction coefficient along the OLOA, and changed its value within an acceptable range to study the effects of the frictional excitations on gear vibration and noise. Some investigators utilized Benedict and

Table 2: Sign functions for frictional excitations.

Sign functions	Reference
$\varepsilon_i = (-\overrightarrow{V_1^2}(M_i)n\vec{y}/\overrightarrow{V_1^2}(M_i)) = \pm\vec{y}$	[173, 177]
$\overrightarrow{V_1^2}(M_i) = [\Omega_1^0 n l_{1i} + \Omega_2^0 n l_{2i} + (\Omega_2^0 + \Omega_1^0) n \eta_i n \sin\beta_b]$	
$\mathrm{sgn}[AP - \Omega_{pn} r_{bpt} + \lambda n \,\mathrm{floor}(\Omega_{pn} r_{bpt}/\lambda)]$	[178]
$\mathrm{sgn}(f_1) = \tan^{-1}((\alpha_{pi} n R_p n \Omega_p - \alpha_{gi} n R_g n \Omega_g)/V_R)$	
$\mathrm{sgn}(f_2) = \mathrm{sgn}[\zeta_g(t) n(\Omega_g + \dot{\theta}_g(t)) - \zeta_p(t) n(\Omega_p + \dot{\theta}_p(t))]$	[175, 176, 189]
$\mathrm{sgn}[-x n \Omega_p + (x_g - x) n \Omega_g]$	[181]
$\mathrm{sgn}[\omega_p n \rho_{pi}(t) - \omega_g n \rho_{gi}(t)]$	[190]

The definition and value of each coefficient in the formula can be found in the corresponding reference.

Table 3: Formula of frictional moment arms.

Formula of frictional moment arms	Reference
$\zeta_p = l_{1i} - \eta_i n \sin \beta_b$	[173]
$\zeta_g = (R_{b1} + R_{b2}) n \tan \alpha_p$	
$x_{pi}(t) = L_{XA} + (n - i)n\lambda + \Omega_{pn} r_{bp} t - \lambda n \,\mathrm{floor}(\Omega_{pn} r_{bp} t/\lambda)$	[178, 180, 186, 190]
$x_{gi}(t) = L_{YC} + in\lambda - \Omega_{pn} r_{bp} t + \lambda n \,\mathrm{floor}(\Omega_{pn} r_{bpt}/\lambda)$	
$\zeta_1(t) = (R_{b1} + R_{b2}) n \tan \alpha - \sqrt{R_{a1}^2 - R_{b2}^2} + R_{b1} n \omega_1 n t_1 + R_{b1} n \theta_1$	[189]
$\zeta_2(t) = (R_{b1} + R_{b2}) n \tan \alpha - \zeta_1(t)$	

The definition and value of each coefficient in the formula can be found in the corresponding reference.

Kelly's empirical formulae to calculate the coefficient of frictional excitation. The empirical formulae indicate that the coefficient of frictional excitation is a function of many parameters such as sliding and rolling velocities, surface roughness, and load distribution. These formulae are more accurate than the constant coefficient, but being based on experimental results, they are expensive and not general. Based on experimentally validated simulations, Xu's EHL formulae are accurate, general, and convenient. However, these formulae are widely used only in the area of gear system efficiency; the application of these formulae in gear dynamic, vibration, and noise areas should be more extensive in the future. Meanwhile, a mixed EHL formula is widely utilized in gear dynamic analysis. The frictional coefficient formulae from the abovementioned publications can be classified into four groups (listed in Table 4): Coulomb formula, empirical formula, EHL formula, and mixed EHL formula. Here, the ISO formula is cited for comparison.

3.3. Strategies to Reduce Frictional Excitations. Although the frictional coefficient is of secondary importance when calculating the frictional excitations, it is the most important term for reducing frictional excitation. From Table 4, one can learn that many parameters, such as gear surface roughness and property of the lubricant oil, have great influence on the coefficient of frictional excitation. This is because the gear whine noise increases with the surface roughness [214, 215]. Moreover, as described by Huang et al. [212], a smoother surface leads to a smaller frictional excitation. Therefore, by improving the surface finish of the gear teeth, the level of the gear whine noise induced by frictional excitations would be reduced significantly [174]. Frictional excitation also has a close relationship with the viscosity of the lubricant oil. With a decrease in the viscosity of the lubricant oil, the frictional excitations decrease significantly [174]. Sufficient lubricant oil would decrease the viscosity of the oil and hence reduce the frictional excitations for the gear whine noise. However, a possible alternative cause of noise in a spur gearbox is an overgenerous oil supply, if oil is trapped in the roots of the meshing teeth [5–7]. If the oil cannot escape fast through the backlash gap, it will be expelled forcibly axially from the tooth roots and, at once-per-tooth frequency, can affect the end walls of the gear case. This phenomenon induces another well-known annoying noise, gear rattle noise.

4. Shuttling Excitation

In the meshing process of helical gears, the centroid of the meshing force axially shifts back and forth along the tooth face width [124], which induces time-varying bearing forces and dynamic moments. In addition, angular misalignments may move the action position of the meshing force to one end of the face. The angular misalignments are also load dependent, which means that the action position of the meshing force would change as the load varies. Shuttling effect due to the helical angle acts along the LOA direction with a frequency equal to the gear mesh frequency. Shuttling effect due to angular misalignments results in a twisting moment that may push the gears to an aligned position.

Borner and Houser [171] quantitatively evaluated the influence of shuttling excitation and reported that the amplitude of the bearing forces induced by shuttling excitation might be three times as those induced by TE. Houser et al. [221] suggested that shuttling was one kind of excitation for gear whine noise. Nishino [124] proposed an excitation model in which both TE and shuttling excitation

Table 4: Formula of the friction force coefficient.

Formulae	Model type	Reference
$\mu = \overline{\mu} n \operatorname{sgn}(V_s)$	Coulomb	[176–184, 187–190, 192, 193, 199, 205–210]
$\mu = 0.0127[50/(50-S)]\log_{10}^{[3.17(10)^8 \dot{W}/\nu V_s V_r^2]}$	Empirical	[191, 195]
$\mu = 0.0099n(1/((1-S)/45))\log[(3.5\times 10^8 w)/\eta_0 V_S V_T^2 (R_p+R_g)^2]$	Mix lubricant condition (experimentally)	[174, 196]
$\mu = e^{f(SR,P_h,v_0,S)} P_h^{b_2} \lvert SR\rvert^{b_3} V_e^{b_6} V_0^{b_7} R^{b_8}$ $f(SR,P_h,v_0,S) = b_1 + b_4 \lvert SR\rvert P_h \log_{10}^{(v_0)} + b_5 e^{-\lvert SR\rvert P_h \log_{10}^{(v_0)}} + b_9 e^S$	EHL	[185, 200, 201, 211, 212]
$\mu_{ML} = (1-f_a)n\mu_{BL} + f_a n \mu_{EL}$ $\mu_{BL} = 0.08 \sim 0.13$ $\mu_{EL} = \mu$ by Xu Hai	Mixed EHL condition (analytically)	[203]
$\mu = 0.325[V_s V_r v_k]^{-0.25}$	(ISO)	[213]

The definition and value of each coefficient in the formula can be found in the corresponding reference.

were considered. After evaluating the individual contributions of TE and shuttling excitation, the authors reached the conclusion that shuttling excitation had a significant influence on the total response of the housing plates. From the publications by Eritenel and Parker [216, 217], one can learn that shuttling motions can excite the twist mode of the geared system. According to the simulation by Palermo et al. [218, 219], shuttling excitation due to angular misalignment and helix angle both contribute to varying bearing forces that finally induce gear whine noise. Teja et al. [220] calculated the bearing forces induced by shuttling forces and TE for helical gears. The authors concluded that when TE was minimized by microgeometry modification, shuttling excitation becomes the dominant excitation for the gear whine noise.

Shuttling excitation is inherent for helical gear pairs with wide faces. As described by Eritenel and Parker [216, 217], for spur gears with misalignments and aligned helical gears, the shuttling excitation generates fluctuating moments with a mean value. For helical gears with misalignments, the fluctuating moments for gear and pinion are quite different. Helical angle and amplitude of misalignment both influence the strength of the shuttling excitation. The distance between supporting bearings also has a significant influence on the strength of the shuttling excitation [220]: the longer the distance, the weaker the excitation. In addition, unequal length on either side of the gear increases the shuttling excitation. Teja et al. [220] also observed that helical gears with a higher contact ratio introduce stronger shuttling excitation.

Despite the fact that shuttling may be a nonnegligible excitation for the gear whine noise in the axial direction, there are a limited number of research studies focusing on shuttling excitation and its effect on gear whine noise. Shuttling is a 3D issue, so two-dimensional contact models of gear pairs cannot calculate the shuttling excitation. Detailed 3D contact models for gear pairs enable the inclusion of nonuniform load distribution and misalignments, and thus are suitable for predicting shuttling excitation. Analytical formulae for shuttling forces and moments in reviewed articles are summarized in Table 5. Meanwhile, strategies for decreasing the shuttling excitations are difficult to illustrate. Modelling strategies and the influence of shuttling excitation on gear whine noise should be investigated further. Approaches reducing shuttling excitations should also be proposed and verified as soon as possible. The relationship between shuttling excitation, TE, and frictional excitation should be studied further.

5. Lightweight Gears

Most of the strategies relating to gear whine noise reduction are gear tooth micromodification as mentioned in Section 2. However, with a precisely designed optimum profile, tooth meshing remains a vibration generator. Using lightweight gears originated in the aerospace industry and is recently extending to the vehicle industry. Because of the stricter regulation on emissions in common fuel vehicles and the population of the electric vehicles, lightweight gears are prior to solid steel gears in the gearbox. Therefore, the modern gear design should meet the need of weight reduction without compromising the requirements of NVH performance and reliability. From this point of view, several strategies have been proposed, which include the thin-rimmed gears, the material removal through holes, and the adoption of new materials.

Common lightweight strategies are based on material removal such as a thinner rim or manufacturing holes and sluts in the gear body. As mentioned by Li [222], although there is an increasing trend of adopting lightweight gears, design problems related to the dynamic strength and vibration of such gears are not fully solved yet. From the survey by Li, one can learn that the thickness of the rim and web significantly influences the bending modes and natural vibration behavior of the lightweight gears. Researchers [223–225] point out that, due to the nonuniform mass and stiffness distribution along the gear blank, the lightweight gears with holes produce additional harmonic components at a lower frequency range. The additional order of the static TE would increase the risk of exciting the rest of the system. System responses to the excitation due to the holes in the web are comparable to that of the mesh excitation. The lightweight gears with holes are more compliant than the solid gears, so the contact loss phenomenon would be eased. However, as described by Shweiki et al. [224], the peak-to-peak magnitude of the static TE and the magnitude of dynamic TE for the lightweight gears are larger than that of solid ones. The modelling strategies of the thin-rimmed gears and lightweight gears with holes are complicated than common solid gears as described by Guilbert et al. [226, 227]

Table 5: Formulae for shuttling forces and moments.

Formulae	Reference
$M_{\text{res}} = \sum_{i=1}^{n} M_i$ $M_i = q_m n \int_{x_1^i}^{x_2^i} xn(1 + an(x/b))\mathrm{d}x$	[171]
$M_\theta = \sum_{q=1}^{m} E_q n \exp(jn\varphi_q) n Z_q n \cos\beta_b$ $M_p = K_t n\gamma + Fn\overline{A}_p$	[124]
$\overline{M}_g = K_t n\gamma + Fn\overline{A}_g$ $\overline{A}_p = (\overline{c} - te_p)n\cos\varphi + \overline{b}n\sin\varphi$ $\overline{A}_g = -(\overline{c} - te_g)n\cos\varphi + (B - \overline{b})n\sin\varphi$	[216, 217]
$sp_1 = \frac{F_{CFWa}}{F_C}$ $sp_2 = \begin{cases} n0.1, & \alpha > \alpha_{0.1}, \\ n0.4((\alpha - \alpha_{0.1})/(\alpha_{0.5} - \alpha_{0.1})) + 0.1, & \alpha_{0.1} \geq \alpha > \alpha_{0.5}, \\ n0.4((\alpha - \alpha_{0.5})/(\alpha_{0.9} - \alpha_{0.5})) + 0.5, & \alpha_{0.5} \geq \alpha > \alpha_{0.9}, \\ n0.9, & \alpha \leq \alpha_{0.9} \end{cases}$	[218, 219]
$M_z = M_{bz} + M_{az}$ $M_{bz} = ([\sum_i F_{bi} * x_i + \sum_i F_{bi} * (fw - x_i)]/2)$ $M_{az} = ([\sum_i F_{ai} * y_i + \sum_i F_{ai} * (L_a - y_i)]/2)$	[220]

The definition and value of each coefficient in the formula can be found in the corresponding reference.

and Shweiki et al. [225], respectively. Only continuous flexible models can capture the particular dynamic properties of lightweight gears. Guilbert et al. [227] developed a mortar-based methodology that can connect 1D grids and 3D finite elements. Based on this approach, thin-rimmed gears can be precisely modelled in dynamic models. In order to calculate the load distribution and static TE of the lightweight gears precisely, a complex numerical model is required as demonstrated by Shweiki et al. [225]. The remaining issue is the development of simulation models able to predict the impact of different design parameters on gear vibration behavior. Hou et al. [228] numerically and quantitatively investigated the impact of the rim and web thickness on static TE, meshing forces, bearing forces, and housing vibration. Gear rim and web thickness have a great influence on both the static and dynamic responses of the geared system of an electric vehicle. Optimization design of lightweight gears would at most reduce 68.5 percentage of the dynamic meshing force and 66.7 percentage of the housing vibration compared to solid gears.

Lightweight gears by material removal have many drawbacks such as the introduction of additional mechanical excitations, more harmonic components, greater fluctuations of mesh stiffness and static TE, severe stress concentration, lower load capacity, and higher magnitude of deformation. Meanwhile, thinner walls and induced holes simply do not reduce the vibration transmission from the toothed ring to other parts of the drive. Therefore, the dynamic responses and vibration level of these geared systems show no significant improvement compared to the solid ones.

Researchers develop innovative lightweight strategies that can maintain the load-carrying capability of the gears and improve the dynamic performance by the exploitation of new materials. Bimetallic gears and hybrid composite-metal gears are the most popular approaches. With steel ring and aluminum on the hub region, bimetallic gears [229] can provide up to 40 percentage of weight reduction without a decrease in the performance of the gears. Ring thickness has a great influence on the static performance of gears such as root stress, time-varying mesh stiffness, and static TE but has an ignorable influence on dynamic responses. However, in another publication [230], a hybrid gear with a carbon fiber-reinforced polymer web is superior to that with an aluminum alloy web. Composite materials are initially used in rotorcraft transmission and then are widely used in vehicles in recent years [231] for their outstanding damping property. Incorporating composite materials with metals into the structure of a gear has many promising benefits such as weight reduction and vibration dispassion. Carbon fiber-reinforced polymer material [232] is attractive for its capability of reducing vibrations and high damping capacity. Both experimental [233–235] and numerical [236] surveys showed that there are significant improvements in both weight reduction and NVH performance when adopting hybrid composite-metal gears. From the experiments conducted by Handschuh et al. [233], the hybrid gear pairs generally reduce both the vibration level and the sound pressure level of the gearbox due to their higher damping properties. From the report by Handschuh et al. [234], a hybrid-hybrid gear pair exhibits the lowest vibrations and noise level at high-speed ranges, while the hybrid gear driving the steel gear pair exhibits the highest vibrations. From the experiments conducted by Laberge et al. [235], the vibration and orbit magnitudes are comparable to steel gears. Catera et al. [236] compared the numerically calculated static TE of the thin-rimmed steel gear pair and the composite-steel hybrid one. The peak-to-peak and mean value of the static TE of the hybrid gear pair are lower than the thin-rimmed steel one, which indicates an enhancement in NVH performance for hybrid gears. Innovative web structures alter the vibration propagation path, which starts from the gear mesh through the gear body and shaft to the housing. Filling powders [237] and particle damper [238] with optimum diameter to the holes of the lightweight gears can effectively reduce the vibration since the holes are a vital

place of vibration transfer. As described by Xiao et al. [237], tungsten alloy powder is the best, and the steel alloy and lead alloy are much better than that of magnesium alloy and aluminum alloy powder. Ramadani et al. [239] replaced the solid structure of the gear body by a lattice structure to reduce the total weight of the gear and to reduce the propagation of the vibration generated by the gear teeth. The addition of the polymer matrix to the cellular lattice structure may reduce the vibration significantly. Generally, the hybrid gear has three parts: steel rim, composite web, and steel hub, respectively. There are two composite-steel interfaces in a common hybrid gear, which increase the opportunity of stress concentration. To minimize the stress concentration phenomenon, Gauntt et al. [240] adopted composite shafts and a sinusoidal interface. Meanwhile, the accuracy of the FE model of the hybrid composite-metal gears strongly relies on the property estimations for interfaces and composite materials. The complicated property of the composites and interfaces poses difficulty in establishing predictive models for hybrid gears [236, 241–243]. There are two different joining technologies for metal-composite gears, namely, interface fitting and adhesive bonding, respectively. Cooley and Parker [15] established FE models with ply-by-ply and homogenized webs for hybrid composite-steel gears, in which the interface is an adhesive bounding one. Catera et al. [242] adopted an adhesive bounding interface in their multiscale model of the hybrid metal-composite gears. The FE models [243] of the hybrid gear with an adhesive bounding interface do not accurately predict the higher orders of the modes. Catera et al. [244] experimentally and numerically investigated two different joining technologies for metal-composite gears, namely, interface fitting and adhesive bonding. Both techniques are prior to the lightweight steel gears in terms of gear natural frequencies and damping properties. A hybrid metal-composite gear with the variable thickness web [245] is proposed to explore its dynamic properties, in which a film adhesive bounding method is adopted at the metal-composite interfaces.

Both material removal and composite-metal gears meet the requirement of weight reduction. However, the performance of lightweight gears by material removal in terms of load-carrying capacity and static and dynamic responses is not satisfactory. Hybrid composite-metal gears can reduce vibration not only by improving the gear body damping property but also by modifying the vibration propagation path. However, there are still several challenges to be faced. The manufacturing procedure of the hybrid gears is more complicated than solid and material removal gears. The properties of composite materials and hybrid interfaces are key parameters in establishing predictive models, which need further investigation. The modelling of the composite materials is at the microscale, while the modelling of the steel parts of the gear is at the macroscale, leaving the overall model of the hybrid gear at multiscales. Further effort should be devoted to the faster and more accurate modelling strategies of the hybrid gears. The author believes that innovative web structures in terms of modifying vibration propagation would be popular in the coming years.

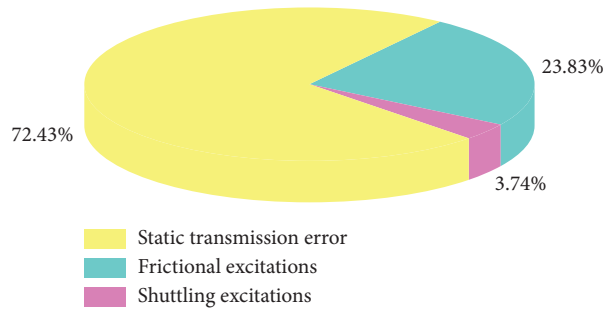

Figure 4: Gear whine noise excitation distribution.

6. Summary and Prospects

6.1. Summary. The authors reviewed more than 200 published papers related to the excitations of gear whine noise. The proportion of each excitation in the reviewed articles is calculated as can be seen in Figure 4. A considerable number of investigators considered TE as the primary excitation. Therefore, there are numerous papers focusing on prediction, measurement, and optimization of TE. A few researchers have mentioned that frictional excitation between mating gears is the secondary excitation of gear whine noise. Limited studies are related to shuttling excitation.

The main content of this article is summarized as follows.

Three kinds of excitations for gear whine noise are summarized: transmission error along the LOA direction, frictional excitations along the OLOA direction, and shuttling excitations along the axial direction. Transmission error is the primary excitation of gear whine noise, frictional excitations are the secondary excitations, and shuttling excitation is less important than the aforementioned two kinds of excitations.

Transmission error consists of two parts: deflections and deviations. Gear teeth deflections and variance of meshing stiffness contribute to the deflection part of TE. Manufacturing errors, intentional modifications, and misalignments due to supporting structures contribute to the deviation part of TE.

There are four sets of harmonics for TE in the frequency domain: mesh frequency harmonics, two sets of rotational harmonics related to the shaft rotating frequency, and fundamental harmonic of the meshing gear pairs. Among the four sets mentioned above, the tooth meshing harmonics are the strongest excitation, the rotational ones are the second strongest, and the one associated with fundamental harmonics is the weakest.

The deflection part of TE is generally predicted using analytical, numerical, or combined analytical and numerical methods. The advantages and disadvantages of these three methods are discussed. Among these methods, the analytical methods requiring many assumptions are efficient but not accurate, the numerical approach is precise but not efficient, and the combined methods offer a quick and accurate solution. The analytical formulae for predicting TE are summarized in Table 1. From this table, researchers who are new to this field can learn about many TE formulae without reading so many articles.

The deviation part of TE is generally measured using a specially designed test rig. Transmission error measured under light-load and slow-speed conditions mainly consists of the manufacturing error and intentional modifications. Transmission error measured under low-speed and varying-load conditions consists of deflections and deviations. Dynamic TE measured under varying speed and load conditions also consists of dynamic effects. The test rigs and instruments measuring TE under different operating conditions are summarized. Benefits and drawbacks of each measurement approaches are discussed. Methods based on encoders are accurate, while the boundary condition of the tested gears is quite different from that in a real gearbox. Methods based on vibrometers offer an opportunity to measure TE in a real gearbox. However, there should be holes in the housing plates for laser access, which poses a challenge for gearbox lubrication. Approaches based on accelerations require precise calibration, tight misalignment, and adequate mounting space. In spite of that, the output still requires a second integration to obtain the TE.

A reduction in the primary excitation directly leads to a reduction in the level of the gear whine noise. The most popular way to reduce TE is through microgeometry modifications. Tip and root relieves are widely applied to spur-gear pairs to reduce TE. For helical gears, bias modification may be beneficial.

For frictional excitations, it is clear that the friction forces are the main excitation of the gear whine noise along the OLOA direction. The friction forces and moments may become dominant excitations of the gear whine noise when the TE is minimized via microgeometry modifications. Meanwhile, the influences of frictional excitations on motions along LOA and axial directions are difficult to illustrate.

Frictional excitations are, in general, predicted analytically. Normal meshing forces, sign function, frictional coefficients, and moment arms are essential parameters for predicting frictional excitations. Among them, the sign function is the dominant factor, and the coefficient is of secondary importance. Analytical formulae for sign function, frictional coefficient, and moment arms are summarized in Tables 2–4.

Improving surface roughness and providing sufficient lubricant oil can reduce frictional excitations. Meanwhile, an overgenerous supply of lubricant oil will increase the chance of impacts and induce rattle noise.

Shuttling is an inherent excitation for helical gears; spur gears with misalignment should take shuttling into consideration as well. Parameters influencing the strength of shuttling excitation are summarized. Helical angle and amplitude of misalignment both have a close relationship with shuttling excitation. The distance between gears and bearings has a significant influence on the strength of the shuttling excitation. Analytical formulae for predicting the shuttling excitations are summarized in Table 5.

Lightweight gears based on material removal and adoption of new materials are popular in automotive transmission lately. The advantages and disadvantages of these gears are summarized. The author also points out the remaining challenges for lightweight gears in terms of manufacturing and modelling strategies.

6.2. Prospects. Based on the topics covered in this review paper, some essential existing problems in the reported research work are presented and some prospects for future research directions are suggested.

Current TE prediction methods model two isolated gears under quasistatic condition, assuming rigid shaft and bearings and ignoring housing plates. A system-level model, containing multiple pairs of gears, flexible shafts, bearings, and housing plates, should be adopted in predicting TE and other kinds of excitations.

Once manufactured and mounted, any individual gear pair has a unique TE. When measuring TE, the tested gear pairs are mounted on two isolated shafts. The boundary conditions of the gears are extremely different from the ones the gear pairs are subject to in a real gearbox. Thus, the measured TE is quite different from the TE of the gear pair in the gearbox.

Microgeometry modification does reduce the gear whine noise to a certain extent. However, the amounts and the effects of modification both have a limitation especially for gears with a small module. Therefore, it is necessary to propose new strategies to reduce TE and hence gear whine noise. Using magnetic gears, gears with a small module, or gears with thin and tall teeth might be an effective strategy.

The effect of frictional excitations on motions along the OLOA is straightforward, while its influence on motions along LOA and axial directions is difficult to illustrate, which requires further study.

There are two strategies to reduce the frictional excitations: improving the surface roughness and providing sufficient lubricant oil. However, improving the surface roughness is expensive and providing sufficient lubricant oil may induce gear rattle noise. Other approaches to decrease the frictional excitations should be proposed.

A limited number of papers focus on the prediction and reduction of shuttling excitation. This does not mean shuttling excitation is not important; on the contrary, shuttling excitation may be dominant when TE and frictional excitations are minimized. Strategies for minimizing shuttling excitation should be proposed as soon as possible.

Reducing excitations is not the only way to minimize gear whine noise. Modifying vibration transmission paths and optimizing housing plates are also effective means to reduce radiated noise. Hybrid composite-metal gears have a great potential on vibration and noise reduction of the gearbox.

Conflicts of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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