Research Article

Loading-Rate Dependency of Young's Modulus for Class I and Class II Rocks

Hailong Zhang ^{1,2,3}, Seisuke Okubo,¹ Cancan Chen,⁴ Yang Tang ^{1,3} and Jiang Xu⁴

¹School of Civil Engineering, Chongqing University of Arts and Sciences, Chongqing 402160, China

²China Construction Seventh Eng. Division Co. Ltd., Zhengzhou 450004, China

³School of Civil Engineering, Chongqing University, Chongqing 400044, China

⁴State Key Laboratory of Coal Mine Disaster Dynamics and Control, Chongqing University, Chongqing 400044, China

Correspondence should be addressed to Yang Tang; tangyangchina@cqu.edu.cn

Received 8 May 2021; Revised 21 August 2021; Accepted 10 September 2021; Published 30 September 2021

Academic Editor: FuRen Ming

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Understanding the time-dependent behavior of rocks is important for ensuring the long-term stability of underground structures. Aspects of such a time-dependent behavior include the loading-rate dependency of Young's modulus, strength, creep, and relaxation. In particular, the loading-rate dependency of Young's modulus of rocks has not been fully clarified. In this study, four different types of rocks were tested, and the results were used to analyze the loading-rate dependency of Young's modulus and explain the underlying mechanism. For all four rocks, Young's modulus increased linearly with a tenfold increase in the loading rate. The rocks showed the same loading-rate dependency of Young's modulus. A variable-compliance constitutive equation was proposed for the loading-rate dependency of Young's modulus, and the calculated results agreed well with measured values. Irrecoverable and recoverable strains were separated by loading-unloading-reloading tests at preset stress levels. The constitutive equations showed that the rate of increase in Young's modulus increased with the irrecoverable strain and decreased with increasing stress. The increase in the irrecoverable strain was delayed at high loading rates, which was concluded to be the main reason for the increase in Young's modulus with an increasing loading rate.

1. Introduction

For rocks, the strength is known to increase with the loading rate (i.e., loading-rate dependency) [1–4], the strain increases under constant stress (i.e., creep), and stress decreases under a constant strain (i.e., relaxation) [5, 6]. In various tests, the displacement rate affects the measured strength, elastic modulus, cohesive force, internal friction angle, and fracture mode [7, 8]. The loading-rate dependency of the strength has been comprehensively studied through uniaxial compression and indirect tensile tests [9] and shear and triaxial compression tests [10]. In particular, a 15-year creep test was performed on Tage tuff at a stress level of 30% [11]. These studies showed that understanding the time-dependent behavior of rocks is important for ensuring the long-term stability estimating the lifetime of underground structures [12]. The basic characteristics and theory of relaxation have been used to explain strain hardening and

softening [13]. Some scholars have suggested that creep and relaxation are not fully independent; Fukui et al. [14] analyzed the coupling characteristics of creep and relaxation to develop the concept of generalized relaxation.

However, the loading-rate dependency of Young's modulus of rocks has not been fully clarified. In a previous study, the data scatter for the loading-rate dependency of Young's modulus was reduced by normalizing Young's modulus at 50% of the peak strength against Young's modulus at 10% of the peak strength, which was first obtained at a predetermined loading rate. For Sanjome andesite, Young's modulus was shown to increase by 2% with a tenfold increase in the loading rate; meanwhile, Young's moduli of marble, granite, and sandstone decreased slightly with an increasing loading rate [15]. Only a few studies have explained the microcrack evolution, associated deformation, and strength properties of rocks with various strain rates [16]. Xu and Dai [17] showed that Young's and shear moduli exhibited some loading-path

dependency under quasistatic loading but were insensitive to the loading path at high loading rates. Both the strength and Young's modulus of the rock-like specimens strongly depended on the strain rate. Yan et al. [18] numerically studied the rate-dependent cracking behavior of single-flawed rock specimens and found that the initiation angle and fracture process zone are significantly affected by the strain rate.

In this study, four rocks were tested under uniaxial compression, direct tension, and indirect tension to evaluate Young's moduli at different loading rates. The values of Young's moduli from loading-unloading tests in literature [15] were statistically analyzed, and the irrecoverable and recoverable strains were separated [19]. The results were used to develop constitutive equations that represent the loading-rate dependency of Young's modulus and clarify the underlying mechanism.

2. Materials and Methods

2.1. Specimen Description. Wawersik and Fairhurst [20] divided rocks into two classes based on the complete stress-strain curve in the postfailure region, as shown in Figure 1. Class I rocks always have a negative slope in the postfailure region, whereas Class II rocks always have a positive slope in the postfailure region.

Four types of rocks were tested: Tase tuff (Class I), Ogino tuff (Class I), Emochi andesite (Class II), and Jingkou sandstone (Class II). Tase tuff was obtained from Tochigi Prefecture in Japan and mainly comprised albite, feldspar, and small amounts of calcite. Ogino tuff was obtained from Fukushima Prefecture in Japan and mainly comprised zeolite, mullite, clay, and small amounts of plagioclase and biotite. Emochi andesite was obtained from Fukushima Prefecture in Japan and mainly comprised plagioclase, pyroxene, and a small amount of biotite. Jingkou sandstone was obtained from Chongqing in China and mainly comprised quartz, feldspar, vermiculite, and muscovite. Table 1 lists the basic physical and mechanical parameters of the rocks. Tase tuff and Ogino tuff had a higher saturated water content than Emochi andesite and Jingkou sandstone but a lower peak strength.

The specimens were obtained by drilling blocks of rock that were then cut into cylinders with diameters of 25 mm and heights of 50 mm for the uniaxial compressive and uniaxial direct tensile tests. The specimens for the indirect tensile tests (i.e., Brazilian splitting tests) had a diameter of 25 mm and a height of 12.5 mm. To ensure accurate data, the specimens were polished by a 55-C0201/C polishing machine (Controls Group, Milan, Italy) according to International Society for Rock Mechanics and Rock Engineering (ISRM) standards to ensure flatness, verticality, and parallelism. The surfaces of the polished specimens had a flatness of ≤ 0.01 mm. More than eight specimens of each rock type were employed for each testing condition. The scatter in the test data was addressed by calculating the average value from the eight specimens.

2.2. Control Method. The strain rate-controlled method was used to test Class I rocks (i.e., Tase tuff and Ogino tuff), as shown in Figure 1. The stress-feedback method [21] was used

to test the Class II rocks (i.e., Emochi andesite and Jingkou sandstone). The control formula is given by

$$\varepsilon - \alpha \cdot \frac{\sigma}{E} = C \cdot t = f(t), \quad (1)$$

where t is the time, $f(t)$ is a function of time, C is the loading rate, E is Young's modulus of the rock, ε and σ are the strain and stress, respectively, and α is the ratio of the stress feedback. The Class II rocks were tested at $\alpha = 0.3$ with variable-resistance technology, in which a loading cell and linear variable differential transformer were used to collect stress and strain signals in the servo-controller. The values of the strain and stress were controlled by the variable resistances VR_1 and VR_2 , respectively. The ratio between VR_1 and VR_2 is represented by β . α is the ratio of the stress feedback and is expressed as $\alpha = k \cdot \beta$, where k is a constant. The stress signal value is $\alpha \cdot \beta / E$, and the strain signal value is ε . A constant linear combination of the stress and strain (i.e., $\varepsilon - \alpha \cdot \beta / E$) was used in the Add/Sub amplifier as the feedback signal in a closed loop. This allows the stress-feedback method to obtain a precise value for Young's modulus.

All stress-strain curves were obtained at loading rates of $C = 1 \times 10^{-6}$, 1×10^{-5} , 1×10^{-4} , and $1 \times 10^{-3} \text{ s}^{-1}$ for uniaxial compression; 1×10^{-5} , 1×10^{-6} , and $1 \times 10^{-7} \text{ s}^{-1}$ for direct tension; and 5×10^{-6} , 5×10^{-5} , 5×10^{-4} , and $5 \times 10^{-3} \text{ mm/s}$ for indirect tension. For the uniaxial direct tensile tests, the specimens were prepared as follows: the top end of the specimen was liberally coated with epoxy resin, and the specimen was affixed to the surface of the upper plate and allowed to set for 24 h. The bolt portion of the plate was then screwed into the ram head, and the bottom end of the specimen was affixed to a similar lower plate. Then, a slight load was placed on the specimen for 24 h prior to the tensile test.

2.3. Modulus Calculation Method. For uniaxial compressive tests, the tangent modulus E was used to represent Young's modulus. For the uniaxial direct tensile tests, the initial modulus E_{DT} was used to represent Young's modulus; this is the slope of the line passing through the origin point and tangent to the prefailure curve. For the uniaxial indirect tensile tests (Brazilian splitting tests), the splitting modulus E_{TT} was obtained as follows:

$$E_{TT} = \frac{F/S_{ABCD}}{D_v/D_d}, \quad (2)$$

where E_{TT} is the splitting modulus, F is the axial force, S_{ABCD} is the area of the meridional plane, D_v is the displacement, and D_d is the specimen diameter. Figure 2 shows the meridional plane of the indirect tensile test specimens.

3. Experimental Results

3.1. Stress-Strain Curves under Different Loading Conditions. Figure 3 shows the stress-strain curves of the Class I rocks under uniaxial compression, and Figure 4 shows the corresponding stress-strain curves of the Class II rocks. The Class I and Class II rocks showed the same loading-rate

FIGURE 1: Stress-feedback method: (a) Class I and (b) Class II rocks.

TABLE 1: Mechanical properties of rocks.

Rocks	Density (g/cm ³)	Saturated water content (%)	UCS* (MPa)	E^* (GPa)	ν^*
Tage tuff	1.74	15.04	22.26	3.83	0.32
Ogino tuff	1.81	10.53	27.35	4.02	0.31
Emochi andesite	2.11	6.98	77.50	10.05	0.25
Jingkou sandstone	2.29	4.81	67.49	11.89	0.24

*UCS: uniaxial compression strength; E : Young's modulus; ν : Poisson's ratio.

FIGURE 2: Meridional plane of specimens for the indirect tensile test.

dependency. Figure 5 shows the stress-strain curves and stress-irrecoverable strain curves of Tage tuff (Class I) under uniaxial direct tension. The irrecoverable strain was separated from the slope of the unloading curve. These curves showed a clear loading-rate dependency. Figure 6 shows the force-displacement curves of Tage tuff (Class I) and Jingkou sandstone (Class II) under indirect tension. The load sharply dropped at the peak, but a loading-rate dependency can clearly be observed. The stress-strain curves of the four rocks at different loading rates were used to analyze further the loading-rate dependency of Young's modulus.

3.2. Loading-Rate Dependency of Young's Modulus under Uniaxial Compression. Figure 7 shows double-logarithmic plots of the loading-rate dependency of Young's modulus for

the four rocks at stress levels of 30%, 50%, and 70%. For the Class I rocks, Young's modulus increased with the loading rate. For Tage tuff, the increase was greatest at a stress level of 50% and least at 70%. For Ogino tuff, the increase was greatest at 50% and least at 30%. For the Class II rocks, Young's modulus also increased with the loading rate. For Eموchi andesite, the increase was greatest at 70% and least at 30%. The increases only slightly differed at stress values of 50% and 70%, and the two curves almost coincided. Linear relaxation was observed between Young's modulus and loading rate for all four rock types. The maximal correlation coefficient was 0.9947 for E_{70} of Ogino tuff, and the minimal coefficient was 0.6385 for E_{70} of Eموchi andesite. Figure 8 shows the relationship between Young's modulus and loading rate for the four rock types. For Tage tuff, Young's modulus had average values of 4.40, 4.55, and 4.12 GPa at 30%, 50%, and 70%, respectively, of the maximum stress. This corresponds to increases of 1.57%, 1.38%, and 1.92%, respectively, in Young's modulus for each tenfold increase in the loading rate. For Ogino tuff, the average values of Young's modulus were 4.18, 4.56, and 4.33 GPa, respectively, at the three stress levels. This corresponds to increases of 1.57%, 1.38%, and 1.92%, respectively, for each tenfold increase in the loading rate. For Eموchi andesite, the average values of Young's moduli were 8.16, 8.80, and 8.82 GPa, respectively, which correspond to increases of 3.17%, 3.26%, and 2.95%, respectively, for each tenfold increase in the loading rate. For Jingkou sandstone, the average values of Young's modulus were 10.80, 12.42, and 12.32 GPa, respectively, which correspond to increases of 1.88%, 1.76%, and 1.78%, respectively, for each tenfold increase in the loading rate.

These results show that Young's modulus increased linearly with each tenfold increase in the loading rate for both Class I and Class II rocks, and the same loading-rate

FIGURE 3: Uniaxial compressive stress-strain curves for Class I rocks at different strain rates: (a) Tage tuff and (b) Ogino tuff.

FIGURE 4: Uniaxial compressive stress-strain curves for Class II rocks at different strain rates: (a) Emochi andesite and (b) Jingkou sandstone.

FIGURE 5: (a) Stress-strain and (b) stress-irrecoverable strain curves for Tage tuff (Class I) at different loading rates for the uniaxial direct tensile test.

FIGURE 6: Load-displacement curves for Class I and Class II rocks at different loading rates in the indirect tensile test: (a) Tage tuff and (b) Jingkou sandstone.

dependency was observed for Young's modulus of all rocks. However, Class II rocks showed greater increases in Young's modulus than Class I rocks. This may be related to rock strength, rock grades, and weak internal surfaces.

3.3. Loading-Rate Dependency of Young's Modulus under Uniaxial Tension. Figure 9 shows the loading-rate dependency of Young's modulus for Tage tuff (Class I) in direct tension. As noted in Section 2.3, the initial modulus E_{DT} was taken as Young's modulus. The relationship between Young's modulus and loading rate is nearly linear on the double-logarithmic plot with a correlation coefficient of 0.9959. Figure 9 also shows the loading-rate dependency of Young's modulus in indirect tension, which is represented by the splitting modulus E_{TT} from equation (2). The splitting modulus and loading rate showed a linear relationship in the double-logarithmic plot with a correlation coefficient of 0.9309. Thus, Young's modulus of Class I rocks under uniaxial direct and indirect tension increased linearly with a tenfold increase in the loading rate. For Tage tuff, the ratio K_1 between Young's modulus and initial modulus was 1.086, and the ratio K_2 between Young's modulus and splitting modulus was 7.983. The ratio K can be used to quantitatively analyze the loading-rate dependency of Young's modulus under different loading conditions and is important for predicting the engineering properties of rock masses.

4. Constitutive Equation and Calculated Results

4.1. Variable-Compliance Constitutive Equation. Figure 10 shows the nonlinear Maxwell model that was used to simulate the loading-rate dependency of Young's modulus. The following constitutive equations were proposed:

$$\varepsilon = \varepsilon_1 + \varepsilon_3, \quad (3)$$

$$\frac{d\varepsilon_1}{dt} = \alpha_1 \varepsilon_1^{-m_1} \sigma^{n_1}, \quad (4)$$

$$\varepsilon_3 = \lambda \sigma, \quad (5)$$

$$\frac{d\lambda}{dt} = \alpha_3 \lambda^{m_3} \sigma^{n_3}, \quad (6)$$

where λ is the variable compliance. Its initial value is the reciprocal of the spring modulus with the parameter ranges of $\alpha_1 > 0$, $\alpha_3 > 0$, $+\infty > m_1 > -\infty$, $+\infty > m_3 > -\infty$, $n_1 \geq 1$, and $n_3 \geq 1$. When $n_1 = n_3 = 1$, the model reflects the behavior of materials with Newtonian viscosity. When $n_1 \geq 1$ and $n_3 \geq 1$, the model is accurate for materials with ordinary viscosity. In addition, t is the time; ε and σ are the strain and stress, respectively; and ε_1 and ε_3 are the irrecoverable and recoverable strains, respectively.

Equation (7) is obtained from equations (3)–(6) to give Young's modulus at a preset stress level:

$$E = \frac{d\sigma}{d\varepsilon} = \frac{1/\lambda_0}{1 + (A'/\lambda_0) \cdot C^{n'}}, \quad (7)$$

$$A' = a_1^{1/m_1+1} \left(\frac{n_1+1}{m_1+1} \right)^{m_1/m_1+1} \cdot \left\{ \left[\frac{n_3+1}{a_3(m_3-1) \cdot \lambda_0^{m_3-1}} \right]^{1/n_3+1} \right\}^{n_1-m_1/m_1+1}, \quad (8)$$

$$n' = \frac{n_1 - m_1 - n_3 - 1}{(n_3 + 1)(m_1 + 1)}, \quad (9)$$

where C is the loading rate, A' and n' are constants, and σ_{psl} is the preset stress level.

4.2. Calculated Parameters. In previous studies [10, 22], a method was introduced for calculating the parameters α_1 , α_3 , m_1 , m_3 , n_1 , and n_3 , where the value of n_3 is obtained in tests at a constant or alternating loading rate. Then, $n_1 = n_3$ was assumed for numerical simulations, and the loading-rate dependency of the strength for porous rocks and the generalized relaxation behavior of Kawazu tuff were simulated. Here, a new method is proposed for calculating the value of n_1 by using equation (7) to

FIGURE 7: Loading-rate dependency of Young's modulus for different rocks: (a) Tage tuff, (b) Ogino tuff, (c) Emochi andesite, and (d) Jingkou sandstone.

simulate the loading-rate dependency of Young's modulus. Table 2 presents the parameter values calculated for the four rock types.

4.3. Calculated Results of Young's Modulus. Equation (7) was used to calculate $\Delta E/E$ for each tenfold increase in the

loading rate. The results for the four rock types are presented in Figure 11 and Tables 3–6. The calculated values agreed with the measured results. For Tage tuff (dry condition), Table 3 indicates that the measured values of $\Delta E/E$ at stress levels of 30%, 50%, and 70% were 0.0160, 0.0145, and 0.0196, respectively, uniaxial compression; the calculated values

FIGURE 8: Relationship between Young's modulus and loading rate for the four rocks.

FIGURE 9: Loading-rate dependency of Young's modulus for Tage tuff under direct and indirect tension.

FIGURE 10: Nonlinear Maxwell model.

TABLE 2: Calculated parameter values for the four rocks.

Parameters	Tage tuff		Ogino tuff	Emochi andesite		Jingkou sandstone
	Compression	Tension		Compression		
a_1	10^{-1}	10^{-4}	10^{-3}	10^{-1}		10^{-14}
a_3	1	1	1	1		1
m_1	45	20	35	65		40
m_3	25	40	23	39		33
n_1	60	46	30	60		40
n_3	45	46	43	42		35

FIGURE 11: Increase in Young's modulus at different stress levels.

were 0.0166, 0.0180, 0.0189, respectively. Under direct tension and indirect tension, the measured values were 0.0264 and 0.0200, respectively, and the calculated value for direct tension is 0.0211. In literature [15], the experimental value for $\Delta E/E$ was 0.0760 under wet conditions and 0.0470 under dry conditions. Thus, $\Delta E/E$ was clearly greater under wet conditions than under dry conditions for Tage tuff.

For Ogino tuff (dry condition), Table 4 indicates that the measured values of $\Delta E/E$ at stress levels of 30%, 50%, and 70% were 0.0295, 0.0371, and 0.0473, respectively, under uniaxial compression. The calculated results were 0.0375, 0.0135, and 0.0355, respectively. For Eموchi andesite, Table 5 indicates that the measured $\Delta E/E$ values were 0.0335, 0.0347, and 0.0321, respectively, at the same stress levels. The calculated values were 0.0207, 0.0204, and 0.0201, respectively. In literature [15], the experimental values of $\Delta E/E$ for Sanjome andesite were 0.0190, 0.0370, and 0.0210 under dry conditions and 0.0250 under wet conditions. Thus, Young's modulus always increased with a tenfold increase in the loading rate regardless of the dry or wet conditions. For Jingkou sandstone, the measured $\Delta E/E$ values were 0.0192, 0.0180, and

0.0182 at stress levels of 30%, 50%, and 70%, respectively, and the calculated values were 0.0206, 0.0087, and 0.0206, respectively. The experimental value of $\Delta E/E$ for Kiwai sandstone was 0.0170 under dry conditions, which indicates that Young's modulus increased for each tenfold increase in the loading rate. However, Shirahama sandstone had a value of -0.7 under dry conditions, which indicates that Young's modulus decreased with an increasing loading rate.

Based on the results in this study and literature [15], Young's modulus generally increases with the loading rate for both Class I and Class II rocks. However, Young's modulus of Shirahama sandstone slightly decreased with an increasing loading rate, and the loading-rate dependency of Young's modulus was more obvious under wet conditions than under dry conditions. Additional test data on Young's modulus will be obtained in future research.

5. Discussion

In this study, loading-unloading-reloading tests were performed at preset stress levels of 30%, 50%, and 70% of the peak strength to separate the irrecoverable strain under uniaxial compression. Figure 12 shows the average values of the unloading modulus K_u at stress levels of 30%, 50%, and 70%. The unloading modulus values for Tage tuff, Ogino tuff, Eموchi andesite, and Jingkou sandstone were 5.55, 7.8, 12, and 11.6 GPa, respectively. For direct tension, the initial modulus was used to separate the irrecoverable strain. Tables 2–5 present the measured values of the irrecoverable strain.

The correlation coefficient of Young's modulus with strength is given by $(\Delta E/E)/(\Delta \sigma/\sigma)$. Figure 13 shows the relation between $(\Delta E/E)/(\Delta \sigma/\sigma)$ and $\varepsilon_i/\varepsilon$ from Tables 3–6, which is approximated by equation (10) for rocks under uniaxial compression and tension:

$$\frac{\Delta E}{E} \approx \left(\frac{\varepsilon_i}{\varepsilon} \right) \cdot \left(\frac{\Delta \sigma}{\sigma} \right). \quad (10)$$

Equation (7) can be used to obtain

$$\frac{\Delta E}{E} = \left\{ 1 - \frac{1}{10^{1/(m_1+1)}} \right\} \cdot \frac{\varepsilon_i}{(\varepsilon_1 + \lambda \sigma_{psl})}. \quad (11)$$

The constitutive equations and test results showed that $\Delta E/E$ is related to the irrecoverable strain ε_i and preset stress level σ_{psl} . The rate of increase in Young's modulus increases with the irrecoverable strain, and it decreases with increasing stress. The irrecoverable strain showed a delay at high loading rates, which may explain the increase in Young's modulus with an increased loading rate.

TABLE 3: Statistical results for the measured and calculated Young's moduli of Tage tuff.

Rock Source Type	Tage tuff (dry), Tage tuff (wet)					Okubo et al.	
	In the study						
	E_{30}	E_{50}	E_{70}	E_{DT}	E_{TT}	E_{50}	E_{50}
$\Delta\sigma/\sigma$		0.0526		0.0607	—	0.0560	0.1010
$\varepsilon_1/\varepsilon$	0.2804	0.2383	0.2241	0.3605	—	—	—
$(\Delta\sigma/\sigma \times \varepsilon_1/\varepsilon)$	0.0142	0.0125	0.0118	0.0219	—	—	—
$\Delta E/E$	0.0160	0.0145	0.0196	0.0264	0.0200	0.0470	0.076
$(\Delta E/E)^*$	0.0166	0.0180	0.0189	0.0211	—	—	—

TABLE 4: Statistical results for the measured and calculated Young's moduli of Ogino tuff.

Rock Source Type	Ogino tuff (dry)		
	In the study		
	E_{30}	E_{50}	E_{70}
$\Delta\sigma/\sigma$		0.0497	
$\varepsilon_1/\varepsilon$	0.5693	0.5339	0.5144
$(\Delta\sigma/\sigma \times \varepsilon_1/\varepsilon)$	0.0283	0.0265	0.0256
$\Delta E/E$	0.0295	0.0371	0.0473
$(\Delta E/E)^*$	0.0375	0.0135	0.0355

TABLE 5: Statistical results for the measured and calculated Young's moduli of andesite.

Rock Source Type	Emochi andesite (dry)			Sanjome andesite			
	In the study			Okubo et al.		Okubo et al.	
	E_{30}	E_{50}	E_{70}	Dry E_{50}	Wet E_{50}	Dry E_{50}	Dry E_{50}
$\Delta\sigma/\sigma$		0.0521		0.0620	0.0800	0.0620	0.0620
$\varepsilon_1/\varepsilon$	0.4903	0.4299	0.3915	—	—	—	0.2725
$(\Delta\sigma/\sigma \times \varepsilon_1/\varepsilon)$	0.0255	0.0224	0.0204	—	—	—	0.0169
$\Delta E/E$	0.0335	0.0347	0.0321	0.0190	0.0250	0.0370	0.0210
$(\Delta E/E)^*$	0.0207	0.0204	0.0201	—	—	—	—

TABLE 6: Statistical results for the measured and calculated Young's moduli of sandstone.

Rock Source Type	Jingkou sandstone (dry)			Kiwai sandstone (dry)		Shirahama sandstone (dry)	
	In the study			Okubo et al.		Okubo et al.	
	E_{30}	E_{50}	E_{70}	E_{50}		E_{50}	
$\Delta\sigma/\sigma$		0.0613		—		—	
$\varepsilon_1/\varepsilon$	0.3508	0.2456	0.1792	0.1221		−1.000	
$(\Delta\sigma/\sigma \times \varepsilon_1/\varepsilon)$	0.0215	0.0153	0.0110	—		—	
$\Delta E/E$	0.0192	0.0180	0.0182	0.0170		−0.7	
$(\Delta E/E)^*$	0.0206	0.0087	0.0206	—		—	

* $(\Delta\sigma/\sigma)$: measured stress for a tenfold increase in the loading rate; $(\Delta E/E)$: measured Young's modulus for a tenfold increase in the loading rate; $(\Delta E/E)^*$: calculated Young's modulus for a tenfold increase in the loading rate; ε and ε_1 : measured failure strain and irrecoverable strain at the preset point, respectively. Source: experimental data in the study and from [15].

FIGURE 12: Separation of irrecoverable and recoverable strains. K_u is the slope of the unloading secant curves, σ_{psl} is the preset stress level, and ϵ_1 and ϵ_3 are the irrecoverable and recoverable strains, respectively.

FIGURE 13: Relation between $(\Delta E/E)/(\Delta \sigma/\sigma)$ and ϵ_1/ϵ for different rocks in compression (Com) and tension (Ten).

6. Conclusions

The loading-rate dependency of Young's modulus for Class I and Class II rocks was systematically studied through laboratory tests, and a variable-compliance constitutive equation was developed. The main conclusions are as follows:

- (1) Precise test data for Young's moduli of four rock types were obtained under uniaxial compression by improving the precision of the specimen dimensions. The initial modulus and split modulus of Tage tuff under direct and indirect tension were also obtained. Regardless of the test method, Young's modulus showed the same law of loading-rate dependency.
- (2) Irrecoverable and recoverable strains were separated by loading-unloading-reloading tests at preset stress levels, where the slope of the unloading secant curve was used as the elastic constant.
- (3) The variable-compliance constitutive equation was used to simulate the loading-rate dependency of Young's modulus. The calculated results showed good agreement with the measured values.
- (4) The constitutive equations and test data showed that the rate of increase in Young's modulus increased with the irrecoverable strain and decreased with increasing stress. The irrecoverable strain showed a delayed increase at high loading rates, which may be the main reason for the increase in Young's modulus with an increasing loading rate.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

The authors would like to thank the Natural Science Foundation of Chongqing, China (cstc2019jcyj-msxmX0488 and cstc2021jcyj-msxm3199), the Science and Technology Research Program of Chongqing Municipal Education Commission (KJQN201901338), and the China Postdoctoral Science Foundation (2021M693751) for their financial support.

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