

Research Article

Dynamic Topology Optimization of Long-Span Continuum Structures

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Herein, to improve the dynamic performance of continuum structures, their fundamental frequency is optimized using the topology optimization method. This helps to obtain the best material distribution in the design space and increases the fundamental frequency of the structure higher than the disturbance frequency. Using the variable density method, the dynamic topology optimization model of a long-span continuum structure is built based on the density interpolation model of a solid isotropic material with penalization (SIMP). The goal of this optimization is to maximize the first-order eigenvalue, and the optimization constraint is that the total volume of the structure is smaller than the given value. To improve the efficiency and accuracy of the model, sensitivity filtering is adopted to avoid numerical instability during calculation. Moreover, the optimization criterion method is used to iteratively solve the optimization results. Finally, the structural topology optimization method is implemented on the long-span single beam of a bridge crane at a construction site. The results show that the natural frequency of the structure is increased and the modal characteristics are improved, which lays the foundation for further optimization and dynamic-response analysis.

1. Introduction

The main concept underlying topology optimization is to maximize the material utilization rate and optimize the structural performance while satisfying certain constraints. Structural topology optimization can be traced back to the truss structure layout optimization theory proposed by Michell [1] at the beginning of the last century. In the 1980s, Bendsoe and Kikuchi [2] proposed the homogenization method, which marked the advent of research on the topology optimization of continuum structures. In recent years, topology optimization theory has developed rapidly, becoming one of the most popular and challenging fields of research in structural optimization. In actual engineering structures, particularly, long-span continuous structures—which include cranes, hydraulic press towers, and beam structures—strict requirements must be met in terms of dynamic performance. Therefore, research on topology optimization must be extended from static design to

dynamic design [3–5], and it is of great practical significance to implement dynamic topology optimization of long-span continuum structures.

When analyzing the dynamic characteristics of engineering structures, to prevent resonance and damage, the natural frequency of the structure should differ from the external excitation frequency. Moreover, it is important to study the response of a structure to various dynamic loads. Therefore, a modal analysis of the structure must be carried out first. The dynamic topology optimization of frequency focuses on improving the modal characteristics of a structure while satisfying the constraints regarding structural stiffness, such that the overall stiffness of the structure can be improved and the material can be optimized. Current topology optimization studies in the field of dynamics mainly focus on two facets of research. One is using the characteristic frequency as the objective function or constraint condition, and the other is using the structural flexibility as the objective function [6]. Li et al. [7] set up an optimization model with

the characteristic frequency as a dynamic constraint. In this model, the dynamic constraints change according to the principal structural vibrations. Diaz and Kikuchi [8] used the homogenization method to optimize the structural topology by maximizing the natural frequency as the objective function. Pedersen [9] used the variable density method to optimize the structural topology by maximizing the first eigenfrequency as the objective function. Based on the SIMP interpolation model, Qin et al. [10] established a minimum weight model using the displacement and fundamental frequency as a joint constraint, improving the dynamic performance of the structure. Xu and Ma [6] established a topology optimization model under frequency-excitation loads with the objective function of minimizing the dynamic flexibility. With the aim of minimizing the dynamic compliance of the structure, Jiang et al. [11] combined the equivalent static loads' method with the bidirectional evolutionary structural optimization method to optimize the continuum structural topology. Liu et al. [12] took the displacement response amplitude at a specified position of the structure in the steady state as the objective function and the structural volume as the constraint to investigate the topology optimization of a structure under harmonic-force excitation.

In comparison to static topology optimization, research on the dynamic topology optimization of engineering structures is still limited owing to the difficulty in setting up a model for continuum topology optimization, along with the large computational cost of using numerical algorithms in engineering applications. Yang et al. [13] proposed a structural topology optimization method with regular geometric constraints in combination with a method based on the changes in material properties and the bidirectional evolutionary structural optimization method, aimed at solving the structural optimization problem in the design of fuselage flutter models, which takes the modal values as the goal. Using the body's modal frequency and the vibration intensity of the engine block as the optimization objectives and the modal frequency as the constraint, Du et al. [14] established a structural vibration intensity optimization model. Taking the virtual prototype of a four-cylinder engine block as the design object, the variable density method was adopted to realize low-vibration optimization design. Jiao et al. [15] divided the optimization domain of a bridge crane girder into several subdomains. Then, the relationship between the subdomains and the optimization domain was constructed to establish a mathematical model for the periodic topology optimization of the girder. In this optimization problem, the relative density of the elements in the optimization domain was taken as the design variable and the minimum compliance under the volume constraint as the objective function. Jang et al. [16] designed a lightweight structure for a mobile harbor crane by considering the deadweight, inertial load, and wind force. Through the integrated design process of topology and shape optimization, the conceptual and basic designs of the MH crane were successfully obtained. Using the cross-section shape as the design variable, Kim et al. [17] conducted the topology optimization and shape optimization of a crane boom, with

the minimum mass as the design objective and the static strength and dynamic stiffness of the system as the constraints.

In this paper, a topology optimization model is established by combining the topology optimization method and finite element theory. In this model, maximizing the first-order eigenvalue is the goal, the total volume of the structure is the constraint, and the relative density of the elements is the design variable. To avoid a checkerboard lattice and the mesh-dependence phenomenon, the solid isotropic material with the penalization (SIMP) model and the optimization criterion (OC) method are used. Finally, the topology optimization calculation of a large-span continuum structure of construction machinery is implemented to improve the natural frequency and modal characteristics, which lays the foundation for further optimization design and dynamic-response analysis. The innovation of this paper is the presentation of an optimization criterion method by constructing the Lagrange function by introducing Lagrange multiplier based on Kuhn-Tucker condition to overcome the large computational cost and cope with the implicit nonlinear objective function in topology optimization of large-span engineering structures.

2. SIMP Method in Variable Density Theory

The material interpolation model of variable density theory converts the discrete optimization problem into a continuous optimization problem by introducing intermediate density elements [18]. Then, a penalty factor is introduced to punish the intermediate density so that the material density tends to discrete 0 and 1, which denote the material elimination and material retention, respectively.

The most widely used interpolation models in continuum topology optimization are SIMP [19] model and RAMP [20] (rational approximation of material properties) model. The algorithm of the SIMP model has fast convergence and simple sensitivity, which can be expressed as

$$E(x_i) = E_{\min} + x_i^p (E_0 - E_{\min}), \quad i = 1, 2, \dots, n, \quad (1)$$

where x_i is the relative density of the i th element, n is the total number of discrete elements, p is the penalty factor, $E(x_i)$ is the elastic modulus after interpolation, and E_0 and E_{\min} are the elastic modulus of the solid and eliminated material, respectively.

Equation (1) cannot be directly applied to the frequency optimization for dynamic characteristics of the structure. The reason is that the penalty factor only penalizes the stiffness in low-density region, which results in the local modal with large mass and small stiffness. To avoid this phenomenon, the modified SIMP interpolation model was introduced to weigh the ratio of mass to stiffness [21], which can be written as

$$E(x_i) = E_0 \left[\frac{x_{\min} - x_{\min}^p}{1 - x_{\min}^p} (1 - x_i^p) + x_i^p \right], \quad 0 < x_{\min} \leq x_i \leq 1, \quad (2)$$

where x_{\min} is the minimum density to avoid the singularity introduction of the stiffness matrix.

3. Dynamics Topology Optimization Model Subheadings

3.1. Optimization Model. Tens of thousands of points will be produced in long-span continuum structure when the finite element meshing is carried out. Therefore, a great amount of computational cost is involved to calculate the natural frequency and mode shape of each order of the structure. Moreover, the low-order mode has a stronger influence on the dynamic characteristics of the structure than the high-order mode [22]. Therefore, a dynamic optimization model with maximization of first-order eigenvalue as the goal, total volume of the structure as the constraint, and relative density of elements as the design variable is established in this paper, which is expressed as follows [23]:

$$\begin{aligned} \text{Find : } \mathbf{x} &= \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}^T, \\ \text{Maximum : } \lambda_{\min} &= \min(\lambda_1, \lambda_2, \dots, \lambda_{N_{\text{dof}}}), \\ \sum_{i=1}^n v_i x_i - f v_0 &\leq 0, \quad i = 1, 2, \dots, n, \\ \text{Subject to : } (\mathbf{K} - \lambda_j \mathbf{M})\phi &= 0, \quad j = 1, 2, \dots, N_{\text{dof}}, \\ &0 < x_{\min} \leq x_i \leq 1, \end{aligned} \quad (3)$$

where λ_j is the j th eigenvalue, N_{dof} is the number of freedom degrees of the structure, $\phi = \{\phi_1, \phi_2, \dots, \phi_j\}$, ϕ_j is the corresponding characteristic mode, \mathbf{K} and \mathbf{M} are the global stiffness matrix and mass matrix of the structure, respectively, v_i is the i th volume, f is the percentage of the retained volume, which is taken as 0.5 in this paper, and v_0 is the original volume of the structure, ρ_0 is the density of solid material, and $j = 1, 2, \dots, N_{\text{dof}}$ is the number of all eigenvalues corresponding to the degree of freedom of the structure.

3.2. Sensitivity Analysis. Sensitivity analysis model characteristics are sensitive to design variables. Through sensitivity analysis, design variables that have a greater impact on structural characteristics can be identified to find the optimal direction for each iteration.

Sensitivity of objective function to design variables is

$$\phi^T \frac{\partial \mathbf{K}}{\partial x_i} \phi - \frac{\partial \lambda_j}{\partial x_i} \phi^T \mathbf{M} \phi - \lambda_j \phi^T \frac{\partial \mathbf{M}}{\partial x_i} \phi = 0. \quad (4)$$

After derivation, the following equations can be obtained:

$$\frac{\partial \lambda_j}{\partial x_i} = \phi^T \left(\frac{\partial \mathbf{K}}{\partial x_i} - \lambda_j \frac{\partial \mathbf{M}}{\partial x_i} \right) \phi. \quad (5)$$

Substituting (2) into equations (4) and (5), we can obtain

$$\frac{\partial \lambda_j}{\partial x_i} = \phi^T \left(\sum_{i=1}^n \frac{1 - x_{\min}}{1 - x_{\min}^p} p x_i^{p-1} \mathbf{k}_i^0 - \lambda_j \sum_{i=1}^n \mathbf{m}_i^0 \right) \phi, \quad (6)$$

where \mathbf{k}_i^0 is the initial stiffness of the element and \mathbf{m}_i^0 is the initial mass of the element.

Sensitivity of the volume constraint relative to the design variable is as follows:

$$\frac{\partial (\sum_{i=1}^n v_i x_i - f v_0)}{\partial x_i} = v_i. \quad (7)$$

3.3. Optimization Criterion Method. With large-scale discrete elements and design variables, topology optimization of long-span continuum structures is very complex. To overcome the large computational cost and cope with the implicit nonlinear objective function, it is a good choice to adopt the optimization criterion method which has the obvious advantages of less iteration times and fast convergence. Based on Kuhn–Tucker condition, the optimization criterion method is to construct Lagrange function by introducing Lagrange multiplier. It requires that the number of reanalysis will not change with the complexity of structure and the number of design variables. The constrained extreme value problem is more suitable for solving the continuum topology optimization problem.

According to the Kuhn–Tucker condition, if the constraint is a nonactive constraint, then there exists a nonzero and nonnegative Lagrange multiplier l if $(\partial c(\mathbf{X})/\partial x_i) + l(\partial v(\mathbf{X})/\partial x_i) = 0$, and the solution reaches the extreme value.

The Lagrange function of the dynamic topology optimization problem is as follows:

$$\begin{aligned} L &= \lambda_{\min} + l_1 \left(\sum_{i=1}^n v_i x_i - f v_0 \right) + l_2 [(\mathbf{K} - \lambda_j \mathbf{M})\phi] \\ &+ l_3 \sum_{i=1}^n (x_{\min} - x_i) + l_4 \sum_{i=1}^n (x_i - x_{\max}), \end{aligned} \quad (8)$$

where l_1 , l_2 , l_3 , and l_4 are Lagrange multipliers.

When the design variables are the extreme value, the Kuhn–Tucker condition can be written as

$$\begin{aligned} \{\phi_{\min}\}^T \left(\frac{\partial \mathbf{K}}{\partial x_i} - \lambda_{\min} \frac{\partial \mathbf{M}}{\partial x_i} \right) \{\phi_{\min}\} \\ + l_2 \left(\frac{\partial \mathbf{K}}{\partial x_i} - \frac{\partial \lambda_j}{\partial x_i} \mathbf{M} - \lambda_j \frac{\partial \mathbf{M}}{\partial x_i} \right) \phi + l_1 v_i = 0. \end{aligned} \quad (9)$$

The following formula is used for iterative update of design variables:

$$x_i^* = \begin{cases} \max(x_{\min}, x_i - t), & \text{if } x_i B_i^\eta \leq \max(x_{\min}, x_i - t), \\ \min(1, x_i + t), & \text{if } x_i B_i^\eta \geq \min(1, x_i + t), \\ (x_i B_i^\eta)^q, & \text{if } \max(x_{\min}, x_i - t) < x_i B_i^\eta < \min(1, x_i + t), \end{cases} \quad (10)$$

where

$$B_i = \frac{\max(0, -(\partial\lambda_{\min}/\partial x_i))}{l_1 v_i}. \quad (11)$$

In equation (10), x_i^* is the iterative design variable, η is the damping coefficient, and t represents the limit constant of movement. In this paper, $\eta=0.5$ and $t=0.2$ are used to improve the convergence of the algorithm and ensure the stability of iteration. The Lagrange multiplier l_1 is obtained by dichotomy, and $q=1$ indicates that this is the standard optimization criterion method.

4. Filtering Function

The optimization calculation in this paper is conducted based on the finite element method, and numerical instability will occur during the optimization process, such as checkerboard pattern and mesh dependence [24]. These phenomena make the optimization results unavailable in actual engineering structures. In order to suppress this problem, it is necessary to introduce the filtering function in the postprocessing. The commonly used filtering methods include density filtering, sensitivity filtering, and gray filtering [25]. In this paper, the sensitivity filtering method is adopted; its basic idea is to use the weighted average sensitivity value to replace the old sensitivity to achieve numerically unstable filtering.

The sensitivity filtering method proposed by Sigmund [26] has a good effect in overcoming numerical instability. This method replaces the original sensitivity value of the central element with the weighted average of the sensitivity of the element within the filtering radius. The classical sensitivity filtering formula is

$$\frac{\partial \hat{c}}{\partial x_i} = \left(\frac{1}{x_i \sum_{k=1}^N H_k} \right) \sum_{k=1}^N H_k x_k \frac{\partial c}{\partial x_k}, \quad (12)$$

$$H_k = r - r_1(i, k),$$

where r is the filtering radius, c is the objective function, N is the total number of all elements in the range from the center element, H_k is the convolution factor, and $r_1(i, k)$ is the distance between the center of elements i and k , usually taking 1–3 times of the element size.

Gray filtering [27] is a nonlinear filtering method to obtain clearer black-and-white cells, also known as intermediate density filtering method. It can be realized by modifying the parameters in OC update criteria. For example, the coefficient q in equation (11) can be taken as 2.

5. Topology Optimization of a Large-Span Beam Structure

5.1. Mechanical Model. Taking the single main girder of a large-span normal-track double-beam bridge crane as an example, its mechanical model can be simplified to a simply supported solid girder structure. The force analysis is carried out when the trolley is fully loaded at the midpoint of the span, and the static wheel pressure of the trolley and the weight of the beam are equivalent to the contact point of the wheel and the beam, yielding the following parameters: $P=115$ kN, beam span $L=21.6$ m, height $B=1.3$ m, width $S=0.5$ m, small wheel pitch $C=3.6$ m, and $E_0=210$ GPa. The corresponding force diagram is shown in Figure 1. The beam is fixed at two ends of lower cover plate. An eight-node regular hexahedron element with a size of 0.1 m size was used for meshing. The optimal area is set as the solid portion between the upper and lower cover plates. In comparison to the main girder and trolley, the weight of the driver's cab, the walkway, and the electrical system is very small and is ignored in the design process.

5.2. Flowchart of Topology Optimization. The process of girder structural dynamic topology optimization is shown in Figure 2. The precision of convergence is prescribed to be 0.01.

5.3. Results and Analysis. It can be seen from equation (2) that intermediate densities incur a penalty. However, too large a penalty factor makes elements with relative densities approaching 1 tend to 0, and too small a penalty factor nullifies the effect of using penalties. Therefore, the selection of the penalty factor affects the optimized structure. Figure 3 shows the topology optimization results when $p=3, 4, 5,$ and 7 and $r=1.5$ m. Figure 4 shows the frontal view of the optimization results with different penalty factors. It can be seen that there is a hollow structure inside the beam, which is consistent with the general box-type structure of main beams of cranes in practice. Moreover, holes gradually appear in the web. With the increases in the penalty factor, the number, shape, and location of holes are different, and the retained material generally presents an inclined distribution. These results are consistent with [18].

The material on the web has an inclined distribution, the web material in the middle part is removed, and the trapezoidal structure is used to bear the pressure of the two wheels of the trolley. The structure on the web is generally inclined, indicating that the web material is best arranged in this way under these working conditions. Therefore, the

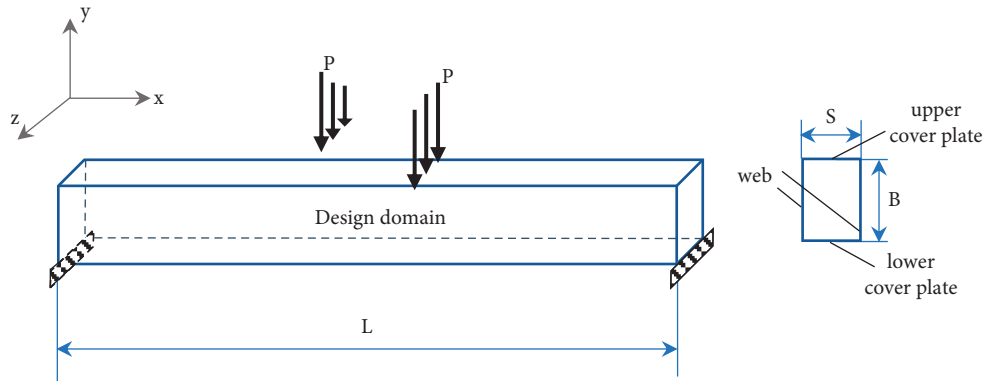


FIGURE 1: Force diagram of the single girder.

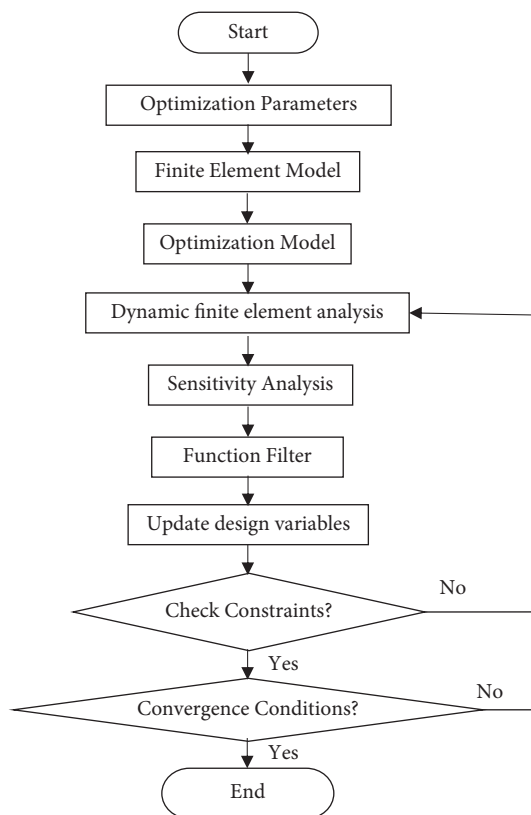


FIGURE 2: Flowchart of the proposed topology optimization method.

inclined layout of the internal ribs of the web can be considered in actual practice. When $p = 1$, there is no penalty effect and an unreasonable structure is obtained through optimization. When $p = 3$, a continuous inclined structure appeared on the web, indicating that the penalty factor began to take effect, and the thickness of the longest inclined structure was relatively thin. When $p = 4-5$, the penalty effect is more obvious. In comparison to $p = 3$, the inclined structure on the web is reduced, but the longest part is thickened. When $p = 7-9$, the penalty effect is too heavy; too much material is removed and the structure tends to become truss-like.

Figure 5 shows the fundamental frequency optimization process with different penalty factors when $r = 1.5$ m. The top

figure shows the complete iterative process, while the bottom figure shows a partially enlarged view of the first 40 iterations. With increases in the iteration number, the first-order natural frequency of the structure first decreases slightly and then increases rapidly, finally slowly increasing after 30 iterations until the end of the iteration process. Moreover, with increases in the penalty factor, the number of iterations increases, and the frequency optimization result gradually decreases.

The track direction of the bridge crane trolley reflects the first-order natural frequency of transverse horizontal vibrations. According to the corresponding design requirements in China, the frequency along the direction of the bridge crane on the trolley track should be greater than 1 Hz. As shown in Table 1—other than the fact that $p = 1$ does not reflect any effects of penalties—the fundamental frequency values before optimization are less than 1 Hz. When using other values for p , the optimal structure has different fundamental frequency values, all of which are greater than 1 Hz, which meet the requirements for dynamic design.

Figure 6 shows the optimization process of the fundamental frequency under different filtering radii at $p = 3$. The figure at the top shows the full iteration process, while the figure at the bottom shows a partially enlarged view of the first 40 iterations. As the number of iterations increases, the first-order natural frequency of the structure also increases. When $r = 1.0-1.2$ m, the frequency value increases slightly, and as the filter radius increases, the frequency optimization result gradually decreases. The fundamental frequency values of the structure after optimization from Table 1 are all greater than 1 Hz, which meets the requirements of dynamic characteristic design.

Figure 7 shows the front view of structural topology optimization results with different filter radii. When $r = 1$ m, it can be seen that a large amount of material is retained on the web, and there is an obvious checkerboard phenomenon. When $r = 1.2$ m, there are too many fine structures on the web, but the retained material has a clear inclined arrangement. When $r = 1.5$ m, there are no unnecessary details in the structure, and the material distribution is more reasonable. When $r = 1.8-2.5$ m, the web structure is removed excessively, and some structural features are deleted.

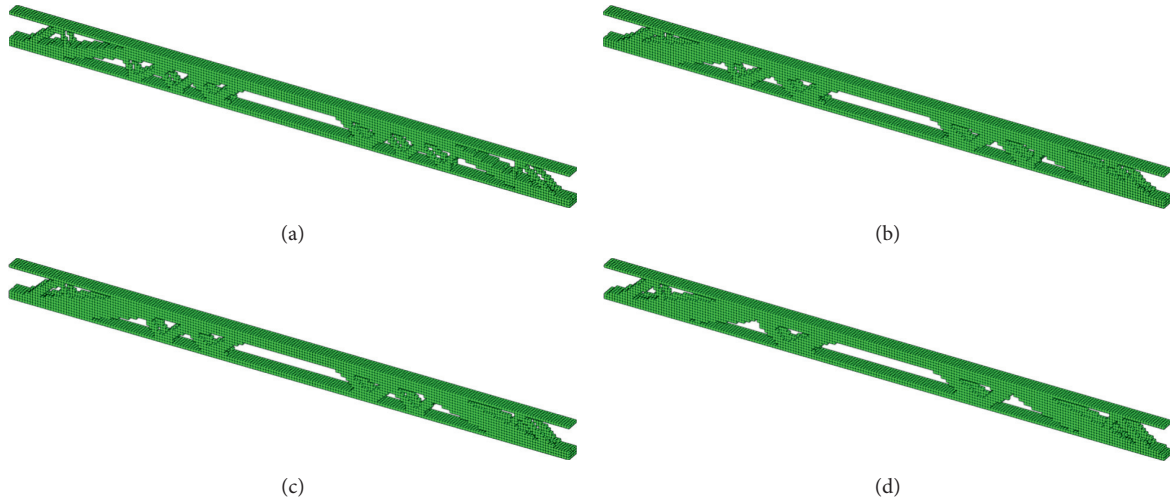


FIGURE 3: The result of structural topology optimization ($r = 1.5$ m). (a) $p = 3$. (b) $p = 4$. (c) $p = 5$. (d) $p = 7$.

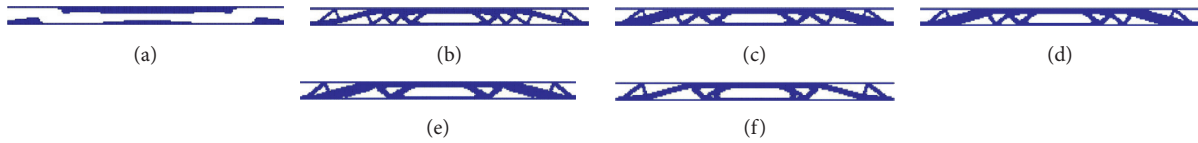


FIGURE 4: Front view of structural topology optimization results with different penalty factors ($r = 1.5$ m). (a) $p = 1$. (b) $p = 3$. (c) $p = 4$. (d) $p = 5$. (e) $p = 7$. (f) $p = 9$.

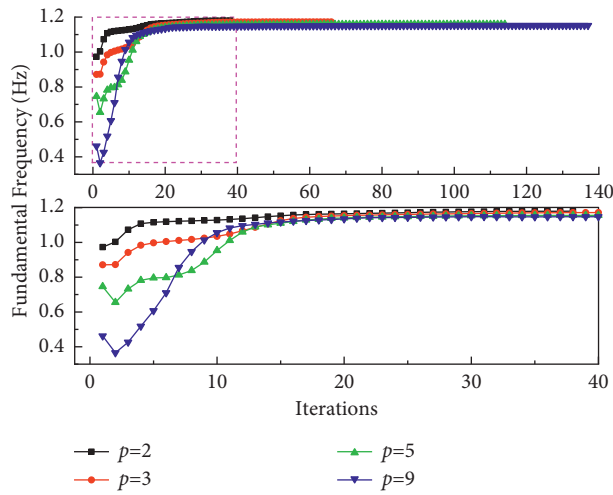


FIGURE 5: Optimization process of the fundamental frequency with different penalty factors ($r = 1.5$ m).

TABLE 1: Fundamental frequency values under different penalty factors and filter radii.

	$r = 1.5$ m					$p = 3$				
	$p = 2$	$p = 3$	$p = 5$	$p = 7$	$p = 9$	$r = 1$ m	$r = 1.2$ m	$r = 1.5$ m	$r = 1.8$ m	$r = 2$ m
Before optimization	0.973	0.871	0.746	0.615	0.462	0.871	0.871	0.871	0.871	0.871
The optimized	1.180	1.172	1.159	1.145	1.144	1.180	1.192	1.172	1.159	1.150

Figure 8 shows the volume ratio (v_i/v_0) curves in the iterative process with different penalty factors when $r = 1.5$ m. Figure 9 shows the volume ratio (v_i/v_0) curves in

the iterative process with a different filter radius when $p = 3$. The volume ratio increases slowly and then increases rapidly after about 30 iterations and finally converges to 0.5 after

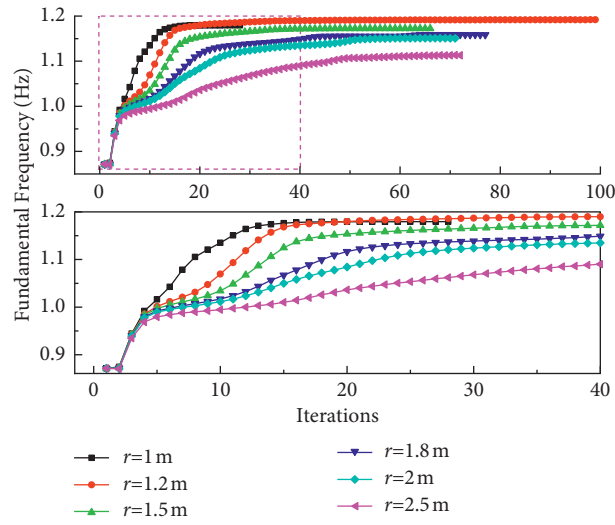


FIGURE 6: Optimization process of the fundamental frequency at different filter radii ($p = 3$).

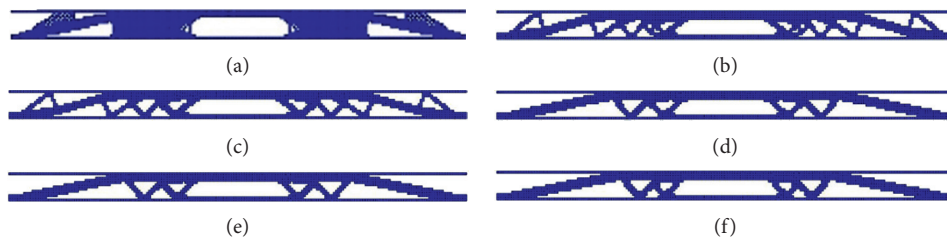


FIGURE 7: Front view of structural topology optimization results with different filter radius ($p = 3$). (a) $r = 1$ m. (b) $r = 1.2$ m. (c) $r = 1.5$ m. (d) $r = 1.8$ m. (e) $r = 2$ m. (f) $r = 2.5$ m.

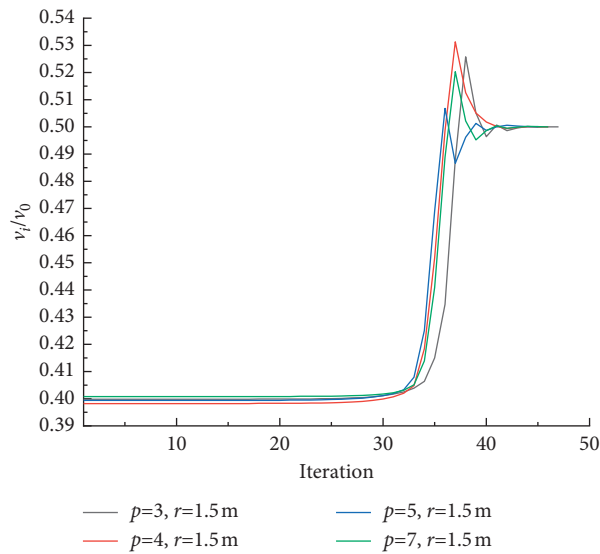


FIGURE 8: Volume ratio curves in the iterative process with different penalty factors ($r = 1.5$ m).

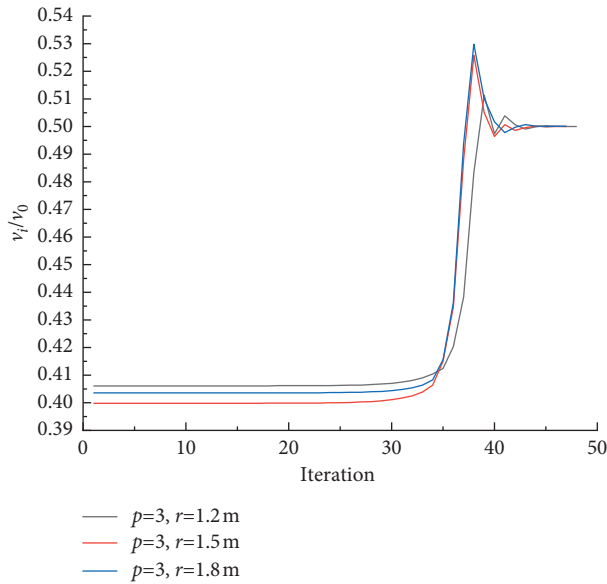


FIGURE 9: Volume curves in the iterative process with different filter radii ($p = 3$).

several oscillations. The curves and those in [18] have the similar variation tendency.

6. Conclusions

In construction machinery, large-span continuum structures have a large number of model elements and design variables. Therefore, a large computational cost is involved in their performance analysis and design optimization. In this article, the dynamic characteristics of the aforementioned structure are optimized based on the variable density method. The optimization model is established with the goal of maximizing first-order eigenvalues with a constraint regarding the total volume. Sensitivity filtering is used to suppress numerical instability while improving the efficiency and accuracy of calculations. The optimization criterion method is used to efficiently solve the structure and obtain the corresponding topological structure to increase its fundamental frequency. The proposed method can meet the requirements of dynamic characteristics design and improve the utilization rate of materials. Therefore, the obtained topological structure can be used as a reference to guide future studies on design.

Data Availability

The data used to support the findings of this study are included within the article. The topology optimization results in this paper have been obtained using the software Matlab. The codes have been uploaded as the supplementary material.

Conflicts of Interest

The authors declare no conflicts of interest.

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Supplementary Materials

MATLAB codes of the proposed topology optimization method. (*Supplementary Materials*)

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