Research Article

Dynamic Behavior Analysis of Intermediate Bearing-Squeeze Film Dampers-Rotor System under Constant Maneuvering Overload

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Under the flight maneuvering of an aircraft, the maneuvering load on the rotor is generated, which may induce the change of dynamic behavior of aeroengine rotor system. To study the influence on the rotor dynamic behavior of constant maneuvering overload, a nonlinear dynamic model of bearing-rotor system under arbitrary maneuver flight conditions is presented by finite element method. The numerical integral method is used to investigate the dynamic characteristics of the rotor model under constant maneuvering overload, and the simulation results are verified by experimental works. Based on this, the dynamic characteristics of a complex intermediate bearing-squeeze film dampers- (SFD-) rotor system during maneuvering flight are analyzed. The simulation results indicate that the subharmonic components are amplified under constant maneuvering overload. The amplitude of the combined frequency components induced by the coupling of the inner and outer rotors is weakened. The static displacements of the rotor caused by the additional excitation force are observed. Besides, the period stability of the movement of the rotor deteriorates during maneuver flight. The design of counterrotation of the inner and outer rotors can effectively reduce the amplitude of subharmonic under constant maneuvering overload.

1. Introduction

The maneuverability is an essential indicator in evaluating the performance of military aircraft. For military drones, it is unnecessary to consider the pilot's ability to withstand maneuvering loads, and the maneuvering overload will be significantly increased to improve the maneuverability in the future. Additional excitation load will be generated acting on the aeroengine rotor system under the condition of maneuver flight, which may affect the dynamic behavior of the rotor [1, 2]. As the maneuvering overload increases, such effect may be magnified.

In recent years, significant attention has been paid to the dynamic behavior of the rotor system during maneuvering flight. Xu and Liao [3] studied the dynamic behavior of Jeffcott rotor with squeeze film dampers (SFD) under the condition of maneuver flight. The numerical simulation results manifested that the additional maneuvering load induced by maneuvering flight changes the vibration

characteristics of the rotor system. The SFD can effectively suppress the transient vibration during the maneuvering process. Ananthan and Leishman [4] investigated the dynamic behavior of a turbo-shaft engine rotor system of a helicopter during maneuver flight. The numerical simulation results showed that the five types of maneuvering flight, that is, dive, jump, pulling up during descent, roll, and reverse roll, can affect the vibration amplitude of the rotor and generate unstable additional loads. Sakata et al. [5, 6] analyzed the steady-state vibration of a simply supported flexible rotor with rotary foundation through the methods of mathematical modeling, simulation calculation, and experimental methods.

Furthermore, Brentner et al. [7, 8] studied the noise problem of rotorcraft in maneuvering flight and developed a new system for maneuvering noise prediction. Yang et al. [9] studied the nonlinear dynamic response of a cracked rotor with rigid support during maneuvering flight, and three ways for the vibration responses leading to chaos were

observed. Lin and Meng [10, 11] studied the vibration behavior of Jeffcott rotor under the conditions of initial bending, asymmetric stiffness, and rub-impact fault in the environment of maneuvering flight. Zhu and Chen [12, 13] established a dynamic model of an aircraft rotor system in arbitrary maneuvering flight action using Lagrange equations, in which the added damping, stiffness, and maneuvering load were obtained, and the influence of maneuver flight on the movement of rotor was investigated. Hou and Chen [14–16] established a dynamic model of a bearing-rotor system under maneuver flight conditions with faults such as rub-impact and cracks. The nonlinear dynamics behavior of the system was analyzed through numerical integral and analytic methods. The subharmonic resonance and irregular motions caused by maneuver were observed. Gao et al. [17] established a flexible asymmetric rotor system with ball bearings and SFD during maneuver flight. The vibration characteristics in conditions of accelerative motion, angular motion, and pitching flight were studied.

Relative simple models, such as Jeffcott and lumped parameter rotor, are widely adopted in the above mentioned literature, which can efficiently disclose the nonlinear characteristics of rotor system. However, the analysis cannot precisely reflect the vibration behavior of the actual aero-engine complex rotor system. Using the finite element theory, the transient response of a rotor system with SFD during maneuver flight was investigated by Han and Ding [18], but a relative simple single rotor model was aimed in their research. Besides, there are few researches on the dynamic behavior of the rotor system during maneuver flight by experimental method.

This paper aims to establish a nonlinear dynamic model of bearing-rotor system under arbitrary maneuver flight conditions using FEM. The added damping, stiffness, and maneuvering load generated by maneuver flight are obtained. The numerical integral method is used to investigate the vibration behavior of the rotor under constant maneuvering overload, and the simulation results are verified by experiments. Based on this, the vibration characteristics of a complex intermediate bearing-SFD-rotor system under constant maneuvering overload are analyzed through the above methods.

2. Mathematical Model

2.1. Finite Element Formulation

2.1.1. Disk. To obtain the dynamic differential equations of a disk during maneuvering flight, a coordinate system is presented in Figure 1. OXYZ is the ground coordinate

system, which represents the center of gravity, flight velocity, and acceleration of the plane. OXYZ is the coordinate system used to describe the whirl of the rotor around its own axis. Assuming that the engine rotor is installed on the aircraft with one end, which coincides with the center of gravity of the aircraft, then OXYZ is also the relative coordinate system of the aircraft. The degree-of-freedom of the disk can be expressed as

$$q_d = [x \ y \ \theta_x \ \theta_y]. \quad (1)$$

And the kinetic energy of a disk of rotor system under arbitrary maneuvering flight actions can be written as

$$T = T_t + T_r, \quad (2)$$

where T_t and T_r are the translational and rotational kinetic energy of the disk, respectively. And the translational kinetic energy during the movement of the rotor can be expressed as [13]

$$T_t = \frac{1}{2} m (\dot{r}^T \dot{r} + v_B^T v_B - r^T \omega_B^2 r + 2\dot{r}^T \omega_B r + 2v_B^T \dot{r} + 2v_B^T \omega_B r), \quad (3)$$

where m denotes the quality of the disk, r represents the relative position of the disk in OXYZ coordinate system, and v_B and ω_B denote the velocity and angular velocity of the aircraft in OXYZ, respectively. r , v_B , and ω_B can be expressed as follows:

$$\begin{aligned} r &= \begin{bmatrix} x + e \cos(\Omega t + \varphi) \\ y + e \sin(\Omega t + \varphi) \\ z \end{bmatrix}, \\ v_B &= \begin{bmatrix} \dot{X}_B \\ \dot{Y}_B \\ \dot{Z}_B \end{bmatrix}, \\ \omega_B &= \begin{bmatrix} 0 & -\omega_{B,z} & \omega_{B,y} \\ \omega_{B,z} & 0 & -\omega_{B,x} \\ -\omega_{B,y} & \omega_{B,x} & 0 \end{bmatrix}, \end{aligned} \quad (4)$$

where Ω denotes the rotating speed of the rotor, φ is the initial phase angle, and $\dot{X}_B$, $\dot{Y}_B$, $\dot{Z}_B$ and $\omega_{B,x}$, $\omega_{B,y}$, $\omega_{B,z}$ are the velocities of the aircraft along each coordinate axis and the angular velocities around each coordinate axis in OXYZ coordinate system, respectively.

The rotational kinetic energy of the disk during the movement of the rotor is [13]

$$T_r = \frac{1}{2} I_d (\dot{\theta}_x + \omega_{B,x})^2 + \frac{1}{2} I_d (\dot{\theta}_y + \omega_{B,y})^2 + \frac{1}{2} I_p [\Omega^2 + 2\Omega(\theta_y + \theta_{B,y})(\dot{\theta}_x + \omega_{B,x})], \quad (5)$$

where $\theta_{B,y}$ denotes the angular displacement of the aircraft around Y axis in OXYZ coordinate system, and $\theta_y + \theta_{B,y} \approx \theta_y$, $(\theta_y + \theta_{B,y})' \approx \dot{\theta}_y + \omega_{B,y}$, I_d and I_p are the

diameter and polar moment of inertia of the disk, respectively. Considering the damping of the disk as a linear function of velocity, and assuming that the generalized

FIGURE 1: Coordinate system of disk.

damping matrix of the disk is C_d , the generalized stiffness matrix of the shaft at the position of the disk is K_d . Then, the dissipation energy of the disk can be expressed as equation (6) by the Rayleigh energy dissipation function. And the elastic potential energy of the flexible shaft at the position of the disk can be expressed as

$$\begin{aligned} D &= \frac{1}{2} \dot{q}_d^T C_d \dot{q}_d, \\ V &= \frac{1}{2} q_d^T K_d q_d. \end{aligned} \quad (6)$$

Substitute equations (1)–(6) into the Lagrange equations for nonconservative systems, and the dynamic differential equations of the disk during arbitrary maneuver flight actions can be deduced as

$$\begin{aligned} M_d \ddot{q}_d + (C_d + \Omega G_d + C_{db}) \dot{q}_d + (K_d + K_{db}) q_d &= F_u + F_{db} + Q_d, \\ C_{db} &= \begin{bmatrix} 0 & -2m\omega_{B,z} & 0 & 0 \\ 2m\omega_{B,z} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ K_{db} &= \begin{bmatrix} -m(\omega_{B,y}^2 + \omega_{B,z}^2) & m(\omega_{B,x}\omega_{B,y} - \dot{\omega}_{B,z}) & 0 & 0 \\ m(\omega_{B,x}\omega_{B,y} + \dot{\omega}_{B,z}) & -m(\omega_{B,x}^2 + \omega_{B,z}^2) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ F_{db} &= \begin{bmatrix} -m(\ddot{X}_B - \omega_{B,z}\dot{Y}_B + \omega_{B,y}\dot{Z}_B) - mz(\omega_{B,x}\omega_{B,z} + \dot{\omega}_{B,y}) \\ -m(\ddot{Y}_B + \omega_{B,z}\dot{X}_B - \omega_{B,x}\dot{Z}_B) - mz(\omega_{B,y}\omega_{B,z} - \dot{\omega}_{B,x}) \\ -I_p\Omega\omega_{B,y} - I_d\dot{\omega}_{B,x} \\ I_p\Omega\omega_{B,x} - I_d\dot{\omega}_{B,y} \end{bmatrix}, \end{aligned} \quad (7)$$

where M_d and G_d are the inertial matrix and gyro matrix of the disk with maneuver flight ignored, respectively. Their formulas are omitted in this paper, which can be found in Ref. [19]. C_{db} , K_{db} , and F_{db} denote the added damping

matrix, stiffness matrix, and maneuvering load caused by maneuver flight. F_u represents the unbalanced force of the disk, and Q_d denotes the generalized external forces acting on the disk.

FIGURE 2: Schematic diagram of beam element.

2.1.2. *Shaft.* The schematic diagram of a Timoshenko beam element is shown in Figure 2. Similar to the disk, the kinetic

energy of an arbitrary microelement segment of length $d\xi$ can be expressed as follows:

$$dT = dT_t + dT_r = \frac{1}{2}\rho_e A_e (\dot{r}_e^T \dot{r}_e + v_B^T v_B - r_e^T \omega_B^2 r_e + 2\dot{r}_e^T \omega_B r_e + 2v_B^T \dot{r}_e + 2v_B^T \omega_B r_e) d\xi$$

$$+ \frac{1}{2} dI_d (\dot{\theta}_e + \omega_{B,x})^2 + \frac{1}{2} dI_d (\dot{\varphi}_e + \omega_{B,y})^2 + \frac{1}{2} dI_p [\Omega^2 + 2\Omega(\dot{\theta}_e + \omega_{B,x})(\varphi_e + \theta_{B,y})],$$

$$r_e = \begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix}, \quad (8)$$

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \\ \varphi_e \end{bmatrix} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & N_3 & 0 & 0 & N_4 \\ 0 & N_1 & -N_2 & 0 & 0 & N_3 & -N_4 & 0 \\ 0 & -\tilde{N}_1 & \tilde{N}_2 & 0 & 0 & -\tilde{N}_3 & \tilde{N}_4 & 0 \\ \tilde{N}_1 & 0 & 0 & \tilde{N}_2 & \tilde{N}_3 & 0 & 0 & \tilde{N}_4 \end{bmatrix} q_e,$$

$$q_e = [x_1 \ y_1 \ \theta_{x1} \ \theta_{y1} \ x_2 \ y_2 \ \theta_{x2} \ \theta_{y2}]^T,$$

where q_e is the nodal displacement vector, and N_i ($i = 1 - 4$) and $\tilde{N}_i$ ($i = 1 - 4$) represent the shape functions and the expressions, which are referred to in Ref. [19]. r_e denotes the relative position of the centroid of the microelement segment in $oxyz$ coordinate system. Then, the kinetic energy expression of the beam element can be obtained by integrating equation (8) along the axis oz .

Similar to the derivation process of a disk, the dynamic differential equations of the beam element under arbitrary maneuver flight conditions can be expressed as

$$M_e \ddot{q}_e + (C_e + \Omega G_e + C_{eb}) \dot{q}_e + (K_e + K_{eb}) q_e = F_{eb} + Q_e, \quad (9)$$

where M_e , G_e , and K_e are the inertial matrix, gyro matrix, and stiffness matrix of the beam element, and the expressions are referred to in Ref. [19]. In this work, the internal damping of the material is ignored, and C_e is the external damping of the beam element. Q_e denotes the generalized external forces applied to the beam element. C_{eb} , K_{eb} , and F_{eb} denote the added damping, stiffness, and maneuvering load caused by maneuver flight, which can be expressed as follows:

$$C_{eb} = 2\rho_e A_e \omega_{B,z} H_1,$$

$$K_{eb} = \rho_e A_e (\omega_{B,y}^2 + \omega_{B,z}^2) H_2 + \rho_e A_e (\omega_{B,x}^2 + \omega_{B,z}^2) H_3 + \rho_e A_e (\omega_{B,x} \omega_{B,y} - \dot{\omega}_{B,z}) H_4$$

$$+ \rho_e A_e (\omega_{B,x} \omega_{B,y} + \dot{\omega}_{B,z}) H_5,$$

$$\begin{aligned}
F_{cb} &= \rho_e A_e \begin{bmatrix} (\dot{Y}_B \omega_{B,z} - \dot{Z}_B \omega_{B,y} - \dot{\omega}_{B,y} z_e - \ddot{X}_B - \omega_{B,x} \omega_{B,z} z_e) A_1 \\ (-\dot{X}_B \omega_{B,z} + \dot{Z}_B \omega_{B,x} + \dot{\omega}_{B,x} z_e - \ddot{Y}_B - \omega_{B,y} \omega_{B,z} z_e) A_1 \\ (\dot{X}_B \omega_{B,z} - \dot{Z}_B \omega_{B,x} - \dot{\omega}_{B,x} z_e + \ddot{Y}_B + \omega_{B,y} \omega_{B,z} z_e) A_2 \\ (\dot{Y}_B \omega_{B,z} - \dot{Z}_B \omega_{B,y} - \dot{\omega}_{B,y} z_e - \ddot{X}_B - \omega_{B,x} \omega_{B,z} z_e) A_2 \\ (\dot{Y}_B \omega_{B,z} - \dot{Z}_B \omega_{B,y} - \dot{\omega}_{B,y} z_e - \ddot{X}_B - \omega_{B,x} \omega_{B,z} z_e) A_3 \\ (-\dot{X}_B \omega_{B,z} + \dot{Z}_B \omega_{B,x} + \dot{\omega}_{B,x} z_e - \ddot{Y}_B - \omega_{B,y} \omega_{B,z} z_e) A_3 \\ (\dot{X}_B \omega_{B,z} - \dot{Z}_B \omega_{B,x} - \dot{\omega}_{B,x} z_e + \ddot{Y}_B + \omega_{B,y} \omega_{B,z} z_e) A_4 \\ (\dot{Y}_B \omega_{B,z} - \dot{Z}_B \omega_{B,y} - \dot{\omega}_{B,y} z_e - \ddot{X}_B - \omega_{B,x} \omega_{B,z} z_e) A_4 \end{bmatrix} + \rho_e I_e \begin{bmatrix} (2\Omega \omega_{B,x} - \dot{\omega}_{B,y}) B_1 \\ (2\Omega \omega_{B,y} + \dot{\omega}_{B,x}) B_1 \\ -(2\Omega \omega_{B,y} + \dot{\omega}_{B,x}) B_2 \\ (2\Omega \omega_{B,x} - \dot{\omega}_{B,y}) B_2 \\ (2\Omega \omega_{B,x} - \dot{\omega}_{B,y}) B_3 \\ (2\Omega \omega_{B,y} + \dot{\omega}_{B,x}) B_3 \\ (2\Omega \omega_{B,y} + \dot{\omega}_{B,x}) B_4 \\ -(2\Omega \omega_{B,x} - \dot{\omega}_{B,y}) B_4 \\ (2\Omega \omega_{B,x} - \dot{\omega}_{B,y}) B_4 \end{bmatrix}, \\
H_1 &= \begin{bmatrix} 0 & A_{11} & A_{12} & 0 & 0 & -A_{13} & A_{14} & 0 \\ A_{11} & 0 & 0 & A_{12} & A_{13} & 0 & 0 & A_{14} \\ -A_{12} & 0 & 0 & -A_{22} & -A_{23} & 0 & 0 & -A_{24} \\ 0 & -A_{12} & A_{22} & 0 & 0 & -A_{23} & A_{24} & 0 \\ 0 & -A_{13} & A_{23} & 0 & 0 & -A_{33} & A_{34} & 0 \\ A_{13} & 0 & 0 & A_{23} & A_{33} & 0 & 0 & A_{34} \\ -A_{14} & 0 & 0 & -A_{24} & -A_{34} & 0 & 0 & -A_{44} \\ 0 & -A_{14} & A_{24} & 0 & 0 & -A_{34} & A_{44} & 0 \end{bmatrix}, \\
H_2 &= \begin{bmatrix} -A_{11} & 0 & 0 & -A_{12} & -A_{13} & 0 & 0 & -A_{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -A_{12} & 0 & 0 & -A_{22} & -A_{23} & 0 & 0 & -A_{24} \\ -A_{13} & 0 & 0 & -A_{23} & -A_{33} & 0 & 0 & -A_{34} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -A_{14} & 0 & 0 & -A_{24} & -A_{34} & 0 & 0 & -A_{44} \end{bmatrix}, \\
H_3 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -A_{11} & A_{12} & 0 & 0 & -A_{13} & A_{14} & 0 \\ 0 & A_{12} & -A_{22} & 0 & 0 & A_{23} & -A_{24} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -A_{13} & A_{23} & 0 & 0 & -A_{33} & A_{34} & 0 \\ 0 & A_{14} & -A_{24} & 0 & 0 & A_{34} & -A_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
H_4 &= \begin{bmatrix} 0 & A_{11} & -A_{12} & 0 & 0 & A_{13} & -A_{14} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{12} & -A_{22} & 0 & 0 & A_{23} & -A_{24} & 0 \\ 0 & A_{13} & -A_{23} & 0 & 0 & A_{33} & -A_{34} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{14} & -A_{24} & 0 & 0 & A_{34} & -A_{44} & 0 \end{bmatrix},
\end{aligned}$$

$$H_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{11} & 0 & 0 & A_{12} & A_{13} & 0 & 0 & A_{14} \\ -A_{12} & 0 & 0 & -A_{22} & -A_{23} & 0 & 0 & -A_{24} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{13} & 0 & 0 & A_{23} & A_{33} & 0 & 0 & A_{34} \\ -A_{14} & 0 & 0 & -A_{24} & -A_{34} & 0 & 0 & -A_{44} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (10)$$

where

$$\begin{cases} A_i = \int_0^{l_e} N_i d\xi, \\ B_i = \int_0^{l_e} \tilde{N}_i d\xi, \\ A_{ij} = \int_0^{l_e} N_i N_j d\xi, \\ B_{ij} = \int_0^{l_e} \tilde{N}_i \tilde{N}_j d\xi, \end{cases} \quad (i = 1 \sim 4, j = 1 \sim 4), \quad (11)$$

in which ρ_e , A_e , and I_e denote the material density, the cross-sectional area, and the area moment of the shaft inertia, respectively.

Rearranging the differential equations of motion of the disks and beam elements, the dynamic differential equations of the rotor system under arbitrary maneuver flight actions can be expressed as

$$M\ddot{q} + (C + \Omega G + C_b)\dot{q} + (K + K_b)q = F_u + F_b + Q, \quad (12)$$

where M , C , G , and K represent the inertial matrix, damping matrix, gyro moment matrix, and stiffness matrix of the rotor system without maneuver flight conditions and bending-torsion coupling effects considered, respectively. C_b , K_b , and F_b are the added damping matrix, stiffness matrix, and maneuvering load vectors caused by maneuver flight. Q denotes the generalized external force, which is applied to the rotor system containing the bearing force at the support stage.

2.2. Nonlinear Forces of the Bearing. In the process of the whirling of the rotor, the nonlinear forces of rolling ball bearing can be written as follows [20]:

$$\begin{cases} f_x^b = -k_n \sum_{j=1}^{N_b} u_{\theta j}^\xi H(u_{\theta j}) \cos \theta_j, \\ f_y^b = -k_n \sum_{j=1}^{N_b} u_{\theta j}^\xi H(u_{\theta j}) \sin \theta_j, \end{cases} \quad (13a)$$

$$\theta_j = \frac{2\pi(j-1)}{N_b} + \omega_{bm}t, \quad (13b)$$

$$u_{\theta j} = x \cos \theta_j + y \sin \theta_j - \frac{\gamma}{2}, \quad (13c)$$

$$H(u_{\theta j}) = \begin{cases} 0, & u_{\theta j} \leq 0, \\ u_{\theta j}, & u_{\theta j} > 0, \end{cases} \quad (13d)$$

$$\omega_{bm} = \frac{\omega_{in}r_1 + \omega_{out}r_2}{r_1 + r_2}, \quad (13e)$$

where k_n denotes the contact stiffness between the roller and the ring. N_b is the number of rollers. θ_j denotes the circumferential position of the j -th roller at time t . And γ is the radial clearance. $u_{\theta j}$ is the elastic contact deformation in the radial direction of the j -th roller. Furthermore, ω_{bm} denotes the rotational speed of the revolution of rollers. r_1 and r_2 are the internal and external radius of the bearing. ω_{in} and ω_{out} represent the rotational speeds of the internal and external rings. Generally, $\xi = (3/2)$ is set for rolling ball bearing.

Substituting the nonlinear forces of bearing into the generalized external force Q in equation (12), the nonlinear dynamic model of the bearing-rotor system can be obtained.

3. Experimental Verification for the Bearing-Rotor System

3.1. Test Rig Description. To validate the correctness of the dynamic model presented in this paper, a test rig for simulating the maneuvering flight of a bearing-rotor system is proposed and designed, as shown in Figure 3. The rotor system is fixed at one end of the rotating arm structure using an installing seat. The other end of the rotating arm is mounted with balancing weights. During the experiments, the rotating arm structure is driven by the driving motor through the reduction gear box. It leads to the bearing-rotor system to rotate around the central spindle of the test rig. Through the different mounting holes reserved on the installing seat, the rotor can provide different pitch angles, as shown in Figure 4. When the pitch angle is set to 0° , the maneuver flight action of hovering around the central spindle is simulated. When the pitch angle is 90° , the maneuver flight action of rolling is performed. And if the pitch angle is between 0° and 90° , the synthetic maneuver of hovering and rolling can be studied. The test rig can apply the additional centrifugal force and gyro moment to the rotor system caused by maneuver flight. The maneuvering

FIGURE 3: Test rig for simulating maneuver flight of a bearing-rotor system.

FIGURE 4: Diagrams of pitch angle. (a) 0°. (b) 30°. (c) 90°.

FIGURE 5: Bearing-rotor system.

TABLE 1: Inertial properties and unbalance configuration.

Name	Disk 1	Disk 2	Disk 3
Mass (kg)	0.4	0.4	0.5
Polar moment of inertia ($\text{g}\cdot\text{m}^2$)	0.00038	0.00038	0.00041
Unbalance ($10^{-5}\text{kg}\cdot\text{m}$)	4	4	5

TABLE 2: Parameters of the bearings.

Name	Bearing 1	Bearing 2
Radius of outer ring (mm)	30	30
Radius of inner ring (mm)	10	10
No. of rollers	8	8
Radial clearance (μm)	6	6

TABLE 3: Maneuver flight parameters.

	Flight maneuvers	Overload (g)	$\dot{Z}_B$ (m/s)	$\omega_{B,y}$ (rad/s)	$\dot{Y}_B$ (m/s)	$\omega_{B,z}$ (rad/s)
1	Hovering	10	12.12	8.08	0	0
2	Hovering	20	17.15	11.43	0	0
3	Hovering	30	21	14	0	0
4	Rolling	10	0	0	12.12	8.08
5	Rolling	20	0	0	17.15	11.43
6	Rolling	30	0	0	21	14
7	Synthetic	10	10.5	7	6.06	4.03
8	Synthetic	20	14.85	9.90	8.57	5.72
9	Synthetic	30	18.19	12.12	10.5	7

FIGURE 6: The mesh diagram of the bearing-rotor system.

TABLE 4: Dimension and elements information of the bearing-rotor system.

Node number	Axial location (m)	Bearing/disk	Element number	Outer diameter (m)	Inner diameter (m)
1	0.001	Disk 3	1	0.010	0
2	0.015		2	0.010	0
3	0.030		3	0.010	0
4	0.051		4	0.010	0
5	0.066		5	0.010	0
6	0.081		6	0.010	0
7	0.096		7	0.010	0
8	0.101	Bearing 2	8	0.012	0
9	0.116		9	0.012	0
10	0.133		10	0.012	0
11	0.148		11	0.012	0
12	0.163		12	0.012	0
13	0.178		13	0.012	0
14	0.193		14	0.012	0

TABLE 4: Continued.

Node number	Axial location (m)	Bearing/disk	Element number	Outer diameter (m)	Inner diameter (m)
15	0.208	Disk 2	15	0.012	0
16	0.225		16	0.012	0
17	0.240		17	0.012	0
18	0.255		18	0.012	0
19	0.270		19	0.012	0
20	0.285		20	0.012	0
21	0.300		21	0.012	0
22	0.315		22	0.012	0
23	0.330	Disk 1	23	0.012	0
24	0.341		24	0.012	0
25	0.356		25	0.012	0
26	0.371		26	0.012	0
27	0.386		27	0.012	0
28	0.401		28	0.010	0
29	0.406		29	0.010	0
30	0.419		30	0.010	0
31	0.436	Bearing 1	31	0.010	0
32	0.451		32	0.010	0
33	0.466		33	0.010	0
34	0.480		34	0.010	0
35	0.490		35	0.010	0
36	0.500

FIGURE 7: Waterfall spectra plots for the vibration of disk 3 without maneuver flight. (a) Numerical simulation. (b) Experimental results.

FIGURE 8: Continued.

FIGURE 8: Waterfall spectra plots for the vibration of disk 3 during hovering. (a) Horizontal response, numerical simulation. (b) Horizontal response, experimental results. (c) Vertical response, numerical simulation. (d) Vertical response, experimental results.

FIGURE 9: Waterfall spectra plots for the response of disk 3 during rolling. (a) Horizontal response, numerical simulation. (b) Horizontal response, experimental results. (c) Vertical response, numerical simulation. (d) Vertical response, experimental results.

FIGURE 10: Continued.

FIGURE 10: Waterfall spectra plots for the vibration of disk 3 during the synthetic maneuver of hovering and rolling. (a) Horizontal response, numerical simulation. (b) Horizontal response, experimental results. (c) Vertical response, numerical simulation. (d) Vertical response, experimental results.

FIGURE 11: Variation result of the static displacements of disk 3 with overload under maneuver flight conditions. (a) Horizontal magnitude, numerical simulation. (b) Horizontal magnitude, experimental results. (c) Vertical magnitude, numerical simulation. (d) Vertical magnitude, experimental results.

FIGURE 12: Structural diagram of the dual rotor system.

TABLE 5: Inertial properties and unbalance configuration of disks.

Name	Inner rotor		Outer rotor	
	Disk 1	Disk 4	Disk 2	Disk 3
Mass (kg)	2.41	2.38	3.17	1.57
Polar moment of inertia ($\text{kg}\cdot\text{m}^2$)	0.0082	0.0082	0.0146	0.0059
Unbalance ($10^{-5} \text{ kg}\cdot\text{m}$)	2	2	1.5	3

TABLE 6: Radial stiffness of elastic supports.

Name	I	II	III
Stiffness (N/m)	2.9×10^7	4.4×10^7	2×10^7

TABLE 7: Parameters of SFDs.

Name	Inner rotor		Outer rotor
	SFD I	SFD II	SFD III
Radius (mm)	25	18	35
Length (mm)	13	13	14
Radial clearance (mm)	0.25	0.18	0.35
Dynamic viscosity ($10^{-2} \text{ Pa}\cdot\text{s}$)	1.0752	—	—

overload of the centrifugal force can be adjusted through controlling the rotating speed of the rotating arm structure.

Figure 5 shows the bearing-rotor system, which is composed of a motor, a flexible coupling, three disks, and two supports. Rolling bearings are adopted for the supports, and the three disks contain two simply supported disks and one cantilever disk. Inertia properties and unbalance configuration of the disks are summarized in Table 1. Related parameters of bearings are provided in Table 2.

To validate the correctness of the dynamic model, nine kinds of maneuver flight conditions are investigated by numerical simulation and experimental methods, respectively, as shown in Table 3, where g denotes the gravitational acceleration. And the pitch angle is set to 30° when the synthetic maneuver of hovering and rolling is studied.

3.2. Numerical and Experimental Results. The finite element model is established, which consists of 40 elements,

including 35 beam elements, 3 disk elements, and 2 supporting elements, as shown in Figure 6. The geometric dimensions and information of each element are listed in Table 4.

The Newmark- β method and Newton-Raphson method are used to solve the differential equations of motion. For the Newmark- β method, two parameters are set as $\alpha = 0.3$ and $\beta = 0.6$. Without maneuver flight condition, the waterfall spectra plots for the vibration of disk 3 with the rotor operating in 20–600 rad/s using numerical simulation and experimental methods are shown in Figure 7. The comparison results manifest that the characteristics of the response obtained by numerical simulation and experimental approach agree well with each other. Compared with the numerical simulation results, the amplitudes of the double frequency and triple frequency components in the experimental test are more extensive due to the misalignment of shafting and coupling or asymmetry of the stiffness of bearings, which is not considered in the simulation calculation.

TABLE 8: Parameters of the intermediate bearing.

Name	Radius of inner ring (mm)	Radius of outer ring (mm)	No. of rollers	Radial clearance (μm)
Value	9.35	14.15	9	6

TABLE 9: Dimension and elements information of the dual rotor system.

Node number	Axial location (m)	Bearing/disk	Element number	Outer diameter (m)	Inner diameter (m)
1	0.100		1	0.018	0
2	0.130		2	0.018	0
3	0.160		3	0.018	0
4	0.190		4	0.018	0
5	0.220		5	0.018	0
6	0.255	Disk 1	6	0.018	0
7	0.289		7	0.018	0
8	0.323		8	0.018	0
9	0.359		9	0.018	0
10	0.384		10	0.018	0
11	0.409		11	0.018	0
12	0.433		12	0.018	0
13	0.445		13	0.018	0
14	0.490		14	0.022	0
15	0.560		15	0.022	0
16	0.650	Bearing 1	16	0.022	0
17	0.753		17	0.022	0
18	0.820		18	0.022	0
19	0.958		19	0.022	0
20	1.030		20	0.022	0
21	1.065		21	0.022	0
22	1.087	Bearing 4	22	0.022	0
23	1.100		23	0.022	0
24	1.140		24	0.022	0
25	1.170	Disk 4	25	0.022	0
26	1.190		26	0.022	0
27	1.211		27	0.017	0
28	1.215		28	0.014	0
29	1.220		29	0.014	0
30	1.230	Bearing 2	End of the inner rotor		
31	0.642		30	0.035	0.03
32	0.655		31	0.035	0.03
33	0.679		32	0.035	0.03
34	0.687	Bearing 3	33	0.038	0.03
35	0.711		34	0.038	0.03
36	0.737		35	0.038	0.03
37	0.762		36	0.038	0.03
38	0.808		37	0.038	0.03
39	0.854	Disk 2	38	0.038	0.03
40	0.899		39	0.038	0.03
41	0.945		40	0.038	0.03
42	0.960		41	0.038	0.03
43	0.980		42	0.038	0.03
44	1.000	Disk 3	43	0.07	0.03
45	1.020		44	0.06	0.03
46	1.040		45	0.06	0.03
47	1.064	Bearing 4	46	0.06	0.03
48	1.074				

TABLE 10: Parameters of maneuver flight.

Maneuvers	$v_{B,z}$ (m/s)	$\omega_{B,y}$ (rad/s)	Overload (g)
Horizontal turn	98	1	10

The waterfall spectra plots for the vibration of disk 3 with the rotor operating in 20–600 rad/s under the maneuver flight conditions of hovering, rolling, and the synthetic maneuver of hovering and rolling with the overload of 20 g

FIGURE 13: Waterfall spectra plots for the vibration of disk 3. (a) Horizontal response, straight flight. (b) Horizontal response, hovering. (c) Vertical response, straight flight. (d) Vertical response, hovering.

FIGURE 14: Waterfall spectra plots for the vibration of disk 4. (a) Horizontal response, straight flight. (b) Horizontal response, hovering. (c) Vertical response, straight flight. (d) Vertical response, hovering.

FIGURE 15: Spectrograms for the vertical vibration of disk 4. (a) 300 rad/s, straight flight. (b) 300 rad/s, hovering. (c) 400 rad/s, straight flight. (d) 400 rad/s, hovering. (e) 500 rad/s, straight flight. (f) 500 rad/s, hovering. (g) 550 rad/s, straight flight. (h) 550 rad/s, hovering.

FIGURE 16: Bifurcation diagrams for the vibration of disk 4. (a) Horizontal response, straight flight. (b) Horizontal response, hovering. (c) Vertical response, straight flight. (d) Vertical response, hovering.

FIGURE 17: Continued.

FIGURE 17: Orbit of disk 4. (a) 300 rad/s, straight flight. (b) 300 rad/s, hovering. (c) 400 rad/s, straight flight. (d) 400 rad/s, hovering. (e) 500 rad/s, straight flight. (f) 500 rad/s, hovering. (g) 550 rad/s, straight flight. (h) 550 rad/s, hovering.

are shown in Figures 8–10, respectively. The frequency component ω_g in experimental results is induced by the internal gear transmission in the reduction gearbox, which is not discussed in this work. By comparing the vibration response under conditions of straight flight and maneuver flight, a zero-frequency vibration component is observed during maneuver flight; that is, the static displacements of the rotor are generated under constant maneuvering

overload. The static displacements in the horizontal direction caused by the additional centrifugal force do not vary with the rotating speed of the rotor. And the static displacements in the vertical direction induced by the additional gyro moment rise as the rotating speed increases. Especially, when the aircraft makes the flight maneuver of rolling, the rotation direction of the rotor is the same as its revolution. No maneuvering gyro torque is generated;

FIGURE 18: Poincaré maps of disk 4. (a) 300 rad/s, straight flight. (b) 300 rad/s, hovering. (c) 400 rad/s, straight flight. (d) 400 rad/s, hovering. (e) 500 rad/s, straight flight. (f) 500 rad/s, hovering. (g) 550 rad/s, straight flight. (h) 550 rad/s, hovering.

FIGURE 19: Vertical response of disk 3. (a) $\lambda = 1.6$. (b) $\lambda = -1.6$. FIGURE 20: Vertical response of disk 4. (a) $\lambda = 1.6$. (b) $\lambda = -1.6$.

therefore, there is no static displacement in the vertical direction, as shown in Figures 9(c) and 9(d).

With comparison of the influence of maneuver flight actions on the magnitude of the static displacements in numerical simulation and experimental works, Figure 11 shows the variation of the static displacements of disk 3 with the amount of overload under the maneuver flight conditions of hovering, rolling, and the synthetic maneuver when the rotor operates at 600 rad/s. It can be observed that the magnitude of the static displacements through numerical simulation and experimental methods is basically consistent.

According to the above result analysis, the dynamic model of the bearing-rotor system under arbitrary maneuver flight conditions presented in this paper has the superior ability to predict the dynamic behavior of the rotor system under constant maneuvering overload.

4. Complex Dual Rotor System

The dynamic model of the bearing-rotor system under arbitrary maneuver flight conditions has been established and verified by experimental works. Based on this, the dynamic behavior of a complex intermediate bearing-SFD-rotor

system under constant maneuvering overload is investigated using the finite element model proposed in this paper.

4.1. Nonlinear Forces of the Supports. The nonlinear oil film forces of SFD in the horizontal and vertical directions can be obtained by Reynolds equation [21, 22], which are presented as follows:

$$\begin{cases} f_x^{sfd} = -\frac{\mu RL^3}{c^2(x^2 + y^2)^{(1/2)}} [x(\dot{\epsilon}I_2 + \epsilon\dot{\psi}I_1) + y(\dot{\epsilon}I_1 + \epsilon\dot{\psi}I_3)], \\ f_y^{sfd} = -\frac{\mu RL^3}{c^2(x^2 + y^2)^{(1/2)}} [y(\dot{\epsilon}I_2 + \epsilon\dot{\psi}I_1) - x(\dot{\epsilon}I_1 + \epsilon\dot{\psi}I_3)], \end{cases} \quad (14a)$$

$$\epsilon = \frac{\sqrt{x^2 + y^2}}{c}, \quad (14b)$$

$$\dot{\epsilon} = \frac{x\dot{x} + y\dot{y}}{c\sqrt{x^2 + y^2}}, \quad (14c)$$

FIGURE 21: Spectrograms for the vertical vibration of disk 3. (a) 300 rad/s, $\lambda = 1.6$. (b) 300 rad/s, $\lambda = -1.6$. (c) 400 rad/s, $\lambda = 1.6$. (d) 400 rad/s, $\lambda = -1.6$. (e) 500 rad/s, $\lambda = 1.6$. (f) 500 rad/s, $\lambda = -1.6$. (g) 550 rad/s, $\lambda = 1.6$. (h) 550 rad/s, $\lambda = -1.6$.

FIGURE 22: Spectrograms for the vertical vibration of disk 4. (a) 300 rad/s, $\lambda = 1.6$. (b) 300 rad/s, $\lambda = -1.6$. (c) 400 rad/s, $\lambda = 1.6$. (d) 400 rad/s, $\lambda = -1.6$. (e) 500 rad/s, $\lambda = 1.6$. (f) 500 rad/s, $\lambda = -1.6$. (g) 550 rad/s, $\lambda = 1.6$. (h) 550 rad/s, $\lambda = -1.6$.

$$\dot{\psi} = \frac{y\dot{x} - x\dot{y}}{x^2 + y^2}, \quad (14d)$$

$$\tan \psi = \frac{x}{y}, \quad (14e)$$

where x and y denote the eccentric position of the journal in the Cartesian coordinate system. R , L , and c are the radius, length, and radial clearance of SFD, respectively, and μ denotes the dynamic viscosity. I_j ($j = 1 - 3$) are Sommerfeld integrals.

Similar to the nonlinear forces of rolling ball bearing as equations (13a)–(13e), the nonlinear forces of intermediate bearing can be represented as equations (15a) and (15b).

$$\begin{cases} f_x^b = -k_n \sum_{j=1}^{N_b} u_{\theta_j}^\xi H(u_{\theta_j}) \cos \theta_j, \\ f_y^b = -k_n \sum_{j=1}^{N_b} u_{\theta_j}^\xi H(u_{\theta_j}) \sin \theta_j, \end{cases} \quad (15a)$$

$$u_{\theta_j} = (x^{ir} - x^{or}) \cos \theta_j + (y^{ir} - y^{or}) \sin \theta_j - \frac{y}{2}, \quad (15b)$$

where the superscripts “*ir*” and “*or*” represent the internal and external rings of the bearing, respectively.

4.2. Calculation Model. The intermediate bearing-SFD-rotor system is presented in Figure 12. The rotor system is composed of two shafts, rotating around the same axis oz , which are linked by an intermediate bearing. There are two disks in each rotor, disks 1 and 4 in the inner rotor, and disks 2 and 3 in the outer rotor. The rotors are supported by four supports, and the supporting form of supports I, II, and III is elastic support + rolling ball bearing + SFD. Support IV is the intermediate bearing with internal ring connected with the inner rotor and external ring linked with the outer rotor. Inertia properties and unbalance configuration of the disks are shown in Table 5. Table 6 shows the radial stiffness of elastic supports. And the parameters of SFDs and intermediate bearing are shown in Tables 7 and 8. The material density of the shafts is 7810 kg/m^3 , Young's modulus is 196 GPa , and the shear modulus is 75.5 GPa . The speed ratio of the rotor system is defined as $\lambda = \omega_2/\omega_1$, where ω_1 and ω_2 are the rotation speeds of the inner and outer rotors, respectively, and $\lambda = 1.6$ in this work.

4.3. Response Analysis. The finite element model of the dual rotor system is established, and the geometric dimensions and information of each element are listed in Table 9.

Taking hovering as the research object, the dynamic behavior of the complex intermediate bearing-SFD-rotor system under constant maneuvering overload is analyzed. The maneuvering parameters of the aircraft are shown in Table 10. Except for the maneuvering parameters mentioned here, the values of other parameters are set to 0.

Solving the dynamic differential equations under the conditions of straight flight and maneuver flight, respectively, Figures 13 and 14 show the waterfall spectra plots for the vibration of disks 3 and 4. And the spectrograms for the vertical vibration of disk 4 are shown in Figure 15. For the condition of straight flight, the frequency components mainly include the fundamental frequencies ω_1 and ω_2 and some combined frequency components such as $2\omega_2 - \omega_1$, $3\omega_2 - 2\omega_1$, caused by the coupling effect of the inner and outer rotors through the intermediate bearing, and the subharmonic vibration induced by the nonlinear characteristics of SFDs, which is defined as ω_s . Compared with the straight flight condition, the combined frequency components are reduced, and the subharmonic vibration component is amplified under the maneuvering flight of hovering; see Figures 13(d), 14(d), and 15. It can be observed that the amplitude of the subharmonic component is relatively small when the rotating speed of the inner rotor is below 400 rad/s , the 3rd critical speed of the rotor system. However, after the rotating speed exceeds 400 rad/s , the subharmonic vibration is rapidly amplified with the continuous increase of the rotational speed, and it will far exceed the amplitude of fundamental frequency vibration when the rotating speed of the inner rotor reaches 550 rad/s , as shown in Figure 15(h). Besides, similar to the bearing-rotor system, static displacements of the rotor can be observed under constant maneuvering overload. For $10g$ overload in this work, the magnitude of the static displacement of the rotor in the horizontal direction caused by the additional centrifugal force has exceeded the resonance amplitude at the 3rd critical speed of the rotor system. Therefore, for the future aircraft with high maneuverability and overload, the clearance of rotors and stators needs to be specially designed to prevent rub-impact fault.

The bifurcation diagrams of the vibration response varying with the rotational speed of disk 4 are shown in Figure 16. The orbits and Poincare maps for the vibration response of disk 4 are illustrated in Figures 17 and 18, respectively. By comparison, there is little difference in periodicity between the two conditions of straight flight and hovering at low speeds. However, as the rotating speed of the rotor increases, the vibration response periodicity will deteriorate under the condition of maneuver flight. Significantly, after the rotating speed of the inner rotor exceeds the 3rd critical speed of the rotor system, the movement of the rotor system gradually develops from pseudo-period to chaos under constant maneuvering overload.

The coaxial co- or counterrotating rotors exhibit different vibration characteristics [21, 23]. Under the condition of maneuver flight of hovering, the vibration characteristics of the dual rotor system in the case of counterrotation of the inner and outer rotors ($\lambda = -1.6$) are analyzed in this work. In comparison with the two points, the waterfall spectra plots for the vibration of disks 3 and 4 are shown in Figures 19 and 20, respectively. The comparison results manifest that there are some differences in the static displacements between the two points under constant maneuvering overload. It is because the additional gyro torque acting on the rotor caused by maneuver flight is different in

the two situations. And the magnitude of the static displacements relates to the specific structure of the rotor system, such as the axial locations of disks and bearings, the inertia properties of each disk, and the stiffness of each bearing.

The spectrograms for the vertical vibration of disks 3 and 4 are shown in Figures 21 and 22, respectively. Compared with the corotating dual rotor systems, the amplitude of subharmonic can be reduced in varying degrees under constant maneuvering overload in the case of counter-rotation, as highlighted in Figures 21(e)–21(h) and 22(e)–22(h). Therefore, the design of counterrotation of the inner and outer rotors can effectively reduce the amplitude of subharmonic induced by large maneuvering overload.

5. Conclusions

A nonlinear dynamic model of the bearing-rotor system under arbitrary maneuver flight conditions is modeled in this paper, and the added damping, stiffness, and maneuvering load generated by maneuver flight are obtained. The numerical integral method is used to investigate the vibration behavior of the rotor under constant maneuvering overload. Moreover, the simulation results are compared and verified by the experimental results. Based on this, the vibration characteristics of a complex intermediate bearing-SFD-rotor system under constant maneuvering overload are analyzed. Conclusions are as follows:

- (1) The maneuver flight action can lead to some static displacements of the rotor. It is found that the static displacements in the horizontal direction caused by the additional centrifugal force do not change with the rotating speed of the rotor. And the static displacements in the vertical direction induced by the additional gyro moment increase with the rise in the rotating speed. For the future aircraft with high maneuverability and overload, the clearance of rotors and stators needs to be specially designed to prevent rub-impact fault induced by the static displacements.
- (2) Compared with the condition of straight flight, the combined frequency components induced by the coupling of the inner and outer rotors through the intermediate bearing are weakened. In contrast, the subharmonic vibration component caused by the nonlinear characteristics of SFDs is amplified under constant maneuvering overload. The amplitude of subharmonic vibration may far exceed the fundamental frequency vibration under certain maneuver flight conditions, which may do great harm to the bearings of rotor system, and it needs to be paid great attention in the design of aeroengine.
- (3) The periodicity of motion of the rotor deteriorates during maneuver flight. For the dual rotor system under the maneuver action of hovering studied in this paper, when the rotating speed exceeds the 3rd critical speed of the rotor system, the movement of the rotor system develops from pseudo-period to chaos gradually as the rotational speed continues to increase.
- (4) The co- and counterrotating dual rotor systems have different vibration characteristics under constant maneuvering overload. The design of counter-rotation of the inner and outer rotors can effectively reduce the amplitude of subharmonic induced by large maneuvering overload.

Data Availability

The data that support the findings of this study are available upon request from the corresponding author.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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