Research Article

Isogeometric Analysis for Active Control of Piezoelectric Functionally Graded Plates in Thermal Environment

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An isogeometric analysis (IGA) method is proposed for investigating the active shape and vibration control of functionally graded plates (FGPs) with surface-bonded piezoelectric materials in a thermal environment. A simple first-order shear deformation theory (S-FSDT) with four variables is used to describe the displacement field of the plates. To ensure the investigation of smart piezoelectric structure in the thermal environment closer to the actual situation, a modified piezoelectric constitutive equation with consideration of the temperature effect of dielectric and piezoelectric strain coefficients is implemented to replace the traditional linear piezoelectric constitutive equation. Meanwhile, the neutral surface is adopted to avoid the stretching-bending coupling. The accuracy and effectiveness of the proposed S-FSDT-based IGA method are verified by comparing with several existing numerical examples. Then, the static bending and open-loop control of the plates under mechanical and thermal loads are further studied. Finally, the active control including static bending control and vibration control of piezoelectric functionally graded plates (PFGPs) is also investigated by using a displacement-velocity feedback control law.

1. Introduction

Smart piezoelectric structures are commonly used for active control in the fields of aerospace engineering [1]. As a type of smart structures, piezoelectric functionally graded plates (PFGPs) refer to using the functionally graded plates (FGPs) [2] as the substrate and using the piezoelectric materials as sensors or actuators. In the last few decades, many scholars have studied the active control of FGPs with surface-bonded piezoelectric layers. For instance, He et al. [3] used the finite element method (FEM) to study the active vibration control of PFGPs and the voltage response of the actuators and sensors. Nguyen-Quang et al. [4, 5] analyzed the vibration control of FGPs and functionally graded carbon nanotube reinforced composite (FG-CNTRC) plates. To avoid the reduction in calculation accuracy caused by element distortion [6], in FEM, Selim et al. [7] used the element-free method to research the active vibration control of

functionally graded multilayer graphene nanoplatelets reinforced composite plates.

Indeed, the functional graded materials (FGMs) [8, 9] are often used in the thermal environment. Hence, the investigation of mechanical behavior of PFGPs in the thermal environment is conducive to improving the accuracy of structural analysis and design. Yang et al. [10, 11] estimated the nonlinear free vibration and static bending analysis of PFGPs in the thermal environment. Liew et al. [12] analyzed the static shape and vibration control of PFGPs. Dai et al. [13] researched the vibration control of PFGPs with the element-free method. However, they did not consider the temperature field of piezoelectric layers. An exact approach for nonlinear vibration and dynamic responses of PFGPs in the thermal environment was developed by Huang and Shen [14] and Xia and Shen [15]. Similarly, Fakhari et al. [16, 17] developed a nonlinear investigation for the transient response and vibration control of PFGPs. Phung-Van et al.

[18] employed isogeometric analysis (IGA) [19] to investigate the nonlinear dynamic response of PFGPs. Although the temperature field of piezoelectric materials was considered in [14–18], the effect of temperature on the properties of piezoelectric materials was not considered.

Different from these references, Shen [20] assumed that the material properties of PZT-5A vary linearly with temperature to research the thermal bending of PFGPs, but the effect of temperature on the dielectric parameter and piezoelectric strain parameter was not considered. The authors in [21] prove that the dielectric and piezoelectric strain parameters are sensitive to temperature. Undoubtedly, it is helpful to make the analysis closer to the actual situation if the influence of temperature on dielectric and piezoelectric strain parameters can be considered in the analysis of smart piezoelectric structure in the thermal environment.

For the numerical method of smart piezoelectric structures and nonhomogeneous composites, although FEM [22, 23] and meshless methods [24] have been proved to be successful and effective, some scholars are still trying to find superior numerical methods for research. Owing to the outstanding features of the exact geometrical modeling, high-order continuity, and simple meshing [25], IGA has been used in a variety of engineering applications [26–31]. Besides, the mathematical model of smart piezoelectric structures is complicated due to the existence of the electric field. Thus, an accurate method with low calculation cost is distinctly important. Obviously, the calculation efficiency and accuracy are directly affected by the number of variables in plate theories. In plate theories, classical plate theory (CPT) [3] has only three variables, whereas it is only appropriate for thin plates. There are five variables in first-order shear deformation theory (FSDT) [32], but the shear-locking effect will occur in the analysis of thin plates. Higher-order shear deformation theories (HSDTs) [33] and some other theories [34] have five or more variables, which make the discretization process more complicated. It is worth noting that, by adopting a simple first-order shear deformation theory (S-FSDT) with only four variables, Yu et al. [35] used the IGA method to investigate the free vibration and nonlinear static bending responses of FGP. The results indicated that the S-FSDT-based IGA method is both applicable for thin and thick plates and naturally free from shear-locking.

This paper thus aims to develop an investigation for the active shape and vibration control of PFGPs in the thermal environment based on the S-FSDT and isogeometric analysis method. The neutral surface is introduced into the S-FSDT for avoiding the stretching-bending coupling. Additionally, to make the investigation of smart piezoelectric structure in the thermal environment close to the actual situation, a modified piezoelectric constitutive equation considering the temperature effect of dielectric and piezoelectric stress parameters is applied to substitute the traditional linear piezoelectric constitutive equation in the derivation of isogeometric finite element equation of PFGPs.

2. PFGPs in Thermal Environment

The sizes of the plate depicted in Figure 1 are $a \times b \times h_t$ (length $\times$ width $\times$ thickness). $h_t = h_f + 2h_p$, where h_f and h_p are the thickness of FGP and piezoelectric layers. z_0 is the distance between the midsurface and neutral surface. T_u and T_c are the temperature of the top surface of PFGP and FGP. T_l and T_m are the temperature of the bottom surface of PFGP and FGP.

2.1. Material Properties. The material properties of the FGP are defined as

$$P(z, T) = (P_c - P_m) \left(\frac{1}{2} + \frac{z}{h_f} \right)^n + P_m, \quad (1)$$

where z is the thickness coordinate and $z \in [h_f/2 - h_f/2]$. n is the gradient index. T is the temperature. P_c and P_m represent the material properties of ceramic and metal.

In the thermal environment, P_c and P_m can be obtained as follows [16]:

$$P_{c/m} = P_0 (P_{-1} T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3), \quad (2)$$

where P_{-1} , P_0 , P_1 , P_2 , and P_3 are the material coefficients related to the temperature and are unique to the constituent materials.

2.2. Temperature Distribution. The temperature along the thickness can be acquired by calculating the steady-state heat transfer equation [36]:

$$-\frac{d}{dz} \left(k(z) \frac{dT(z)}{dz} \right) = 0, \quad (3)$$

where k is the thermal conductivity.

Combined with the thermal boundary conditions and continuity conditions [37], the temperature distribution can be given by

$$T(z) = T_l + \frac{T_u - T_l}{\int_{-h_t/2}^{h_t/2} (1/k(z)) dz} \int_{-h_t/2}^z \frac{1}{k(z)} dz. \quad (4)$$

2.3. Constitutive Equation. By introducing the influence of temperature on dielectric and piezoelectric stress parameters into the traditional linear piezoelectric constitutive equation [38], the modified piezoelectric constitutive equation is defined as

$$\begin{aligned} \sigma &= Q\tilde{\epsilon} - \hat{e}^T E - Q\alpha\Delta T, \\ D &= \tilde{e}\tilde{\epsilon} + \hat{g}E + P\Delta T, \\ S &= \alpha^T Q^T \tilde{\epsilon} + P^T E + \bar{C}\Delta T, \end{aligned} \quad (5)$$

where σ and $\tilde{\epsilon}$ are stress and strain. D is the electric displacement vector. S and $\bar{C}$ are entropy and specific heat. E is

FIGURE 1: Piezoelectric functionally graded plate.

the electric field vector, and $\mathbf{Q}$ is the elastic constant matrix. $\Delta T = T(z) - T_0$, in which T_0 is the reference temperature. α and $\mathbf{P}$ are thermal expansion coefficient and pyroelectric vectors. $\hat{\mathbf{e}}$ and $\hat{\mathbf{g}}$ are temperature-dependent piezoelectric stress parameter and dielectric parameter matrices. The specific forms of α , $\mathbf{P}$, $\hat{\mathbf{e}}$, and $\hat{\mathbf{g}}$ are

$$\alpha = \{\alpha_x \quad \alpha_y \quad 0 \quad 0 \quad 0\}^T, \quad (6)$$

$$\mathbf{P} = \{P_1 \quad P_2 \quad P_3\}^T, \quad (7)$$

$$\hat{\mathbf{e}} = \mathbf{e} + \tilde{\mathbf{e}}\Delta T, \quad (8)$$

$$\hat{\mathbf{g}} = \mathbf{g} + \tilde{\mathbf{g}}\Delta T, \quad (9)$$

with

$$\begin{aligned} \hat{\mathbf{e}} &= [\hat{\mathbf{e}}_m \quad \hat{\mathbf{e}}_s], \\ \hat{\mathbf{e}}_m &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hat{e}_{31} & \hat{e}_{32} & 0 \end{bmatrix}, \\ \hat{\mathbf{e}}_s &= \begin{bmatrix} 0 & \hat{e}_{24} \\ \hat{e}_{15} & 0 \\ 0 & 0 \end{bmatrix}, \\ \hat{\mathbf{g}} &= \begin{bmatrix} \hat{g}_{11} & 0 & 0 \\ 0 & \hat{g}_{22} & 0 \\ 0 & 0 & \hat{g}_{33} \end{bmatrix}, \end{aligned} \quad (10)$$

where $\mathbf{e}$ and $\mathbf{g}$ are piezoelectric stress and dielectric constant matrices at reference temperature. $\tilde{\mathbf{e}}$ and $\tilde{\mathbf{g}}$ are the coefficients that need to be confirmed through experiments.

2.4. Displacement Field and Strain. By introducing the neutral surface, the displacement field in S-FSDT [35] is given as

$$\begin{aligned} u(x, y, z) &= u_0(x, y) - (z - z_0) \frac{\partial w_b}{\partial x}, \\ v(x, y, z) &= v_0(x, y) - (z - z_0) \frac{\partial w_b}{\partial y}, \end{aligned} \quad (11)$$

$$w(x, y, z) = w_b(x, y) + w_s(x, y),$$

where u_0 and v_0 denote the displacements of neutral surface. z_0 is given by

$$z_0 = \frac{\int_{-h_t/2}^{h_t/2} z E(z) dz}{\int_{-h_t/2}^{h_t/2} E(z) dz}. \quad (12)$$

It is seen that the transverse displacement $w_0(x, y)$ in FSDT [39, 40] is expressed by bending (w_b) and shear (w_s) terms; and the rotation variables are denoted by using the partial derivatives of the transverse bending components ($\beta_x = -\partial w_b / \partial x$, $\beta_y = -\partial w_b / \partial y$) in S-FSDT. Through the above expressions, we can see that there are only four variables in S-FSDT.

The strain-displacement relation is

$$\tilde{\boldsymbol{\epsilon}} = \begin{Bmatrix} \boldsymbol{\epsilon}_m \\ 0 \end{Bmatrix} + \begin{Bmatrix} -(z - z_0) \boldsymbol{\epsilon}_b \\ \boldsymbol{\gamma} \end{Bmatrix}, \quad (13)$$

where

$$\boldsymbol{\epsilon}_m = \{u_{0,x} \quad v_{0,y} \quad u_{0,y} + v_{0,x}\}^T, \quad (14a)$$

$$\boldsymbol{\epsilon}_b = \left\{ \frac{\partial^2 w_b}{\partial x^2} \quad \frac{\partial^2 w_b}{\partial y^2} \quad \frac{2\partial^2 w_b}{\partial x \partial y} \right\}^T, \quad (14b)$$

$$\boldsymbol{\gamma} = \{\gamma_{xz} \quad \gamma_{yz}\}^T = \left\{ \frac{\partial w_s}{\partial x} \quad \frac{\partial w_s}{\partial y} \right\}^T. \quad (14c)$$

The stress resultants of PFGPs can be calculated by

$$\mathbf{N} = \{N_x^T \quad N_y^T \quad N_{xy}^T\}^T = \int_{-h_t/2}^{h_t/2} \{\sigma_x^T \quad \sigma_y^T \quad \sigma_{xy}^T\}^T dz, \quad (15a)$$

$$\begin{aligned} \mathbf{M} &= \{M_x^T \quad M_y^T \quad M_{xy}^T\}^T \\ &= \int_{-h_t/2}^{h_t/2} \{\sigma_x^T \quad \sigma_y^T \quad \sigma_{xy}^T\}^T (z - z_0) dz, \end{aligned} \quad (15b)$$

$$\begin{aligned} \overline{\mathbf{Q}} &= \{\overline{\mathbf{Q}}_x^T \quad \overline{\mathbf{Q}}_y^T\}^T \\ &= \int_{-h_t/2}^{h_t/2} \{\sigma_{xz}^T \quad \sigma_{yz}^T\}^T dz, \end{aligned} \quad (15c)$$

where $\mathbf{N}$, $\mathbf{M}$, and $\overline{\mathbf{Q}}$ denote in-plane forces, moments, and shear forces.

Substituting equation (5) into equations (15a)–(15c) yields

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \\ \overline{\mathbf{Q}} \end{Bmatrix} = \widehat{\mathbf{c}} \begin{Bmatrix} \boldsymbol{\varepsilon}_m \\ \boldsymbol{\varepsilon}_b \\ \boldsymbol{\gamma} \end{Bmatrix} - \begin{Bmatrix} \mathbf{N}^p \\ \mathbf{M}^p \\ 0 \end{Bmatrix} - \begin{Bmatrix} \mathbf{N}^{\text{th}} \\ \mathbf{M}^{\text{th}} \\ 0 \end{Bmatrix}, \quad (16)$$

where

$$\widehat{\mathbf{c}} = \begin{bmatrix} \mathbf{D}_{mb} & 0 \\ 0 & \mathbf{C}_s \end{bmatrix}, \quad (17)$$

with

$$\mathbf{D}_{mb} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix}, \quad (18a)$$

$$\mathbf{C}_s = k_s \int_{-ht/2}^{ht/2} \mathbf{Q}_{ij} dz, \quad i, j = 4, 5,$$

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-ht/2}^{ht/2} Q_{ij} (1, (z - z_0), (z - z_0)^2) dz, \quad i, j = 1, 2, 6, \quad (18b)$$

in which the shear correction factor $k_s = 5/6$ and $B_{ij} = 0$ [41].

The thermal and electric stress resultants can be calculated by

$$(\mathbf{N}^p, \mathbf{M}^p) = \int_{(h_f/2)}^{(h_f/2)+h_p} \widehat{\mathbf{e}}_m^T \mathbf{E} (1, z - z_0) dz + \int_{-(h_f/2)-h_p}^{-(h_f/2)} \widehat{\mathbf{e}}_m^T \mathbf{E} (1, z - z_0) dz, \quad (19a)$$

$$(\mathbf{N}^{\text{th}}, \mathbf{M}^{\text{th}}) = \int_{(h_f/2)}^{(h_f/2)+h_p} \mathbf{Q}_m \boldsymbol{\alpha}_m \Delta T (1, z - z_0) dz + \int_{-(h_f/2)}^{-(h_f/2)-h_p} \mathbf{Q}_m \boldsymbol{\alpha}_m \Delta T (1, z - z_0) dz + \int_{-(h_f/2)-h_p}^{-(h_f/2)} \mathbf{Q}_m \boldsymbol{\alpha}_m \Delta T (1, z - z_0) dz, \quad (19b)$$

with

$$\mathbf{Q}_m = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}, \quad (20)$$

$$\boldsymbol{\alpha}_m = \{\alpha_x \quad \alpha_y \quad 0\}^T.$$

2.5. Weak Form of Governing Equation. The equations of equilibrium and electrostatics can be denoted as follows [12]:

$$\begin{aligned} \sigma_{ij,j} + f_{bi} &= \rho \ddot{u}_i, \\ D_{i,j} &= 0, \end{aligned} \quad (21)$$

where σ_{ij} and f_{bi} are the components of the stress tensor and the body force. D represents the component of the electric displacement vector. ρ is the density.

By introducing the mechanical and electric boundary conditions [12, 30], the weak form of PFGPs can be obtained through Hamilton's principle which can be expressed as

$$\delta \int_{t_1}^{t_2} L dt = 0, \quad (22)$$

in which

$$\begin{aligned} L &= \frac{1}{2} \int_{\Omega} (\rho \dot{u}^T \dot{u} - \boldsymbol{\sigma}^T \boldsymbol{\varepsilon} + \mathbf{D}^T \mathbf{E}) d\Omega + \int_{\Gamma_s} \mathbf{u}^T f_s d\Gamma - \int_{\Gamma_\phi} \boldsymbol{\varphi} \mathbf{q}_s d\Gamma_\phi \\ &\quad + \sum \mathbf{u}^T \mathbf{f}_p, \end{aligned} \quad (23)$$

where $\mathbf{u}$ and $\dot{u}$ are the displacement and velocity. $\mathbf{f}_p$ and $\mathbf{f}_s$ represent mechanical point and surface loads, respectively. $\mathbf{q}_s$ is the surface charge. Γ_s and Γ_ϕ denote boundaries of Ω corresponding to the external mechanical and the electrical loading surface, respectively.

2.6. Isogeometric Analysis. The NURBS basis function for the two-dimensional problems is expressed as follows [42]:

$$N_{i,j}^{p,q}(\xi, \eta) = \frac{B_{i,p}(\xi) B_{j,q}(\eta) w_{i,j}}{\sum_{i=1}^n \sum_{j=1}^m B_{i,p}(\xi) B_{j,q}(\eta) w_{i,j}}, \quad (24)$$

where $B_{i,p}(\xi)$ and $B_{j,q}(\eta)$ denote the B-spline basis functions with orders p and q in ξ and η directions, respectively. $w_{i,j}$ is the weight.

The displacements of the plate can be approximated as

$$\mathbf{u}(\xi, \eta) = [u_0 \quad v_0 \quad w_b \quad w_s]^T = \sum_{I=1}^{m \times n} N_I(\xi, \eta) \mathbf{d}_I, \quad (25)$$

where $\mathbf{d}_I = [u_{0I} \quad v_{0I} \quad w_{bI} \quad w_{sI}]^T$ represents the displacement vector of control point I . $N_I(\xi, \eta)$ denotes the NURBS basis function.

Substituting equation (25) into equations (14a)–(14c) yields

$$\boldsymbol{\varepsilon}_m = \sum_{I=1}^{m \times n} \mathbf{B}_I^m \mathbf{d}_I, \quad \boldsymbol{\varepsilon}_b = \sum_{I=1}^{m \times n} \mathbf{B}_I^b \mathbf{d}_I, \quad \boldsymbol{\gamma} = \sum_{I=1}^{m \times n} \mathbf{B}_I^s \mathbf{d}_I, \quad (26)$$

where

$$\mathbf{B}_I^m = \begin{bmatrix} N_{I,x} & 0 & 0 & 0 \\ 0 & N_{I,y} & 0 & 0 \\ N_{I,y} & N_{I,x} & 0 & 0 \end{bmatrix}, \quad (27a)$$

$$\mathbf{B}_I^b = \begin{bmatrix} 0 & 0 & N_{I,xx} & 0 \\ 0 & 0 & N_{I,yy} & 0 \\ 0 & 0 & 2N_{I,xy} & 0 \end{bmatrix}, \quad (27b)$$

$$\mathbf{B}_I^s = \begin{bmatrix} 0 & 0 & 0 & N_{I,x} \\ 0 & 0 & 0 & N_{I,y} \end{bmatrix}. \quad (27c)$$

Similarly, the electric potential of piezoelectric layers can be approximated as

$$\varphi = \sum_{I=1}^{m \times n} \mathbf{N}_I \boldsymbol{\Phi}_I. \quad (28)$$

The electric field is then given by

$$\mathbf{E} = -\nabla \varphi = - \sum_{I=1}^{m \times n} \mathbf{B}_{\varphi I} \boldsymbol{\Phi}_I. \quad (29)$$

The electric field in x and y directions is ignored; hence,

$$\mathbf{B}_{\varphi I} = \left\{ 0 \ 0 \ \frac{N_I}{h_p} \right\}^T. \quad (30)$$

Then, by substituting equations (5) and (26) and (25)–(29) into equation (25), the governing equations of PFGPs are expressed as

$$\mathbf{M}_{uu} \ddot{\mathbf{d}} + \mathbf{K}_{uu} \mathbf{d} + \mathbf{K}_{u\varphi} \boldsymbol{\Phi} = \mathbf{f} + \mathbf{f}_{th}, \quad (31a)$$

$$\mathbf{K}_{\varphi u} \mathbf{d} - \mathbf{K}_{\varphi\varphi} \boldsymbol{\Phi} = \mathbf{f}_\varphi - \mathbf{f}_{\varphi\theta}. \quad (31b)$$

After simplification, equations (31a) and (31b) are rewritten as

$$\mathbf{M}_{uu} \ddot{\mathbf{d}} + \overline{\mathbf{K}} \mathbf{d} = \mathbf{F}, \quad (32)$$

$$\overline{\mathbf{K}} = \mathbf{K}_{uu} + \mathbf{K}_{u\varphi} \mathbf{K}_{\varphi\varphi}^{-1} \mathbf{K}_{\varphi u}, \quad (33)$$

$$\mathbf{F} = \mathbf{f} + \mathbf{f}_{th} + \mathbf{K}_{u\varphi} \mathbf{K}_{\varphi\varphi}^{-1} \mathbf{f}_\varphi - \mathbf{K}_{u\varphi} \mathbf{K}_{\varphi\varphi}^{-1} \mathbf{f}_{\varphi\theta}. \quad (34)$$

The specific form of items in equation (32) is given as

$$\begin{aligned} \mathbf{M}_{uu} &= \int_V \overline{\boldsymbol{\Lambda}}^T \mathbf{m} \overline{\boldsymbol{\Lambda}} dV, \\ \mathbf{K}_{uu} &= \int_V \mathbf{B}_u^T \hat{\mathbf{c}}_{\mathbf{B}_u} dV, \\ \mathbf{K}_{u\varphi} &= \int_V \mathbf{B}_u^T \hat{\mathbf{e}}^T \mathbf{B}_\varphi dV, \\ \mathbf{K}_{\varphi\varphi} &= \int_V \mathbf{B}_\varphi^T \hat{\mathbf{g}} \mathbf{B}_\varphi dV, \\ \mathbf{K}_{\varphi u} &= \mathbf{K}_{u\varphi}^T, \\ \mathbf{f} &= \mathbf{R}^T \mathbf{f}_p + \int_\Omega \mathbf{R}^T \mathbf{f}_s d\Omega + \int_V \mathbf{R}^T \mathbf{f}_b dV, \\ \mathbf{f}_{th} &= \int_V ((\mathbf{B}_m)^T \mathbf{N}^{th} + (\mathbf{B}_b)^T \mathbf{M}^{th}) dV, \\ \mathbf{f}_\varphi &= \int_\Omega \mathbf{R}^T q_E d\Omega, \\ q_E &= \hat{\mathbf{g}}_{33} \frac{\overline{U}}{h_p}, \\ \mathbf{f}_{\varphi\theta} &= \int_V \mathbf{B}_\varphi^T \mathbf{P} \Delta T dV, \end{aligned} \quad (35)$$

with

$$\begin{aligned} \mathbf{m} &= \begin{bmatrix} I_0 & I_1 \\ I_1 & I_2 \end{bmatrix}, \\ \overline{\boldsymbol{\Lambda}} &= [\boldsymbol{\Lambda}_1 \ \boldsymbol{\Lambda}_2]^T, \\ (I_0, I_1, I_2) &= \int_{-h_t/2}^{h_t/2} \rho(z) (1, (z - z_0), (z - z_0)^2) dz, \\ \boldsymbol{\Lambda}_1 &= \begin{bmatrix} N_I & 0 & 0 & 0 \\ 0 & N_I & 0 & 0 \\ 0 & 0 & N_I & N_I \end{bmatrix}, \\ \boldsymbol{\Lambda}_2 &= \begin{bmatrix} 0 & 0 & N_{I,x} & 0 \\ 0 & 0 & N_{I,y} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \quad (36)$$

3. Closed-Loop Control

The active control of smart piezoelectric structures mainly includes two types: one is the open-loop control, which uses the inverse piezoelectric effect of piezoelectric materials to

change the shape of the structures by applying voltage; the other is the closed-loop control, that is, using piezoelectric materials as sensors to sense the state of the structures; then, the output voltage of sensors is amplified as the input voltage of the actuators to control the deformation or suppress the vibration of structures.

In this paper, the upper piezoelectric layer is applied as an actuator, whereas the lower piezoelectric layer is used as a sensor. In general, the external charge of the sensor is zero. Therefore, using the subscripts a and s to express the actuator and sensor, equation (31b) can be rewritten as

$$\mathbf{K}_{\varphi ua} \mathbf{d} - \mathbf{K}_{\varphi \varphi a} \boldsymbol{\varphi} = \mathbf{f}_{\varphi a} - \mathbf{f}_{\varphi \theta a}, \quad (37)$$

$$\mathbf{K}_{\varphi us} \mathbf{d} - \mathbf{K}_{\varphi \varphi s} \boldsymbol{\varphi} = -\mathbf{f}_{\varphi \theta s}. \quad (38)$$

The electric potential generated in the sensor is given by

$$\boldsymbol{\varphi}_s = \mathbf{K}_{\varphi \varphi s}^{-1} (\mathbf{K}_{\varphi us} \mathbf{d} + \mathbf{f}_{\varphi \theta s}). \quad (39)$$

According to the displacement-velocity feedback control law [43], φ_a can be calculated by

$$\boldsymbol{\varphi}_a = \mathbf{G}_d \boldsymbol{\varphi}_s + \mathbf{G}_v \dot{\boldsymbol{\varphi}}_s, \quad (40)$$

where $\mathbf{G}_d$ and $\mathbf{G}_v$ represent the displacement and velocity feedback control gains.

Substituting equations (39) and (40) into equation (37) yields

$$\begin{aligned} \mathbf{f}_{\varphi a} = & \mathbf{K}_{u\varphi a} \mathbf{d} + \mathbf{f}_{\varphi \theta a} - \mathbf{G}_d \mathbf{K}_{\varphi \varphi a} \mathbf{K}_{\varphi \varphi s}^{-1} (\mathbf{K}_{\varphi us} \mathbf{d} + \mathbf{f}_{\varphi \theta s}) \\ & - \mathbf{G}_v \mathbf{K}_{\varphi \varphi a} \mathbf{K}_{\varphi \varphi s}^{-1} (\mathbf{K}_{\varphi us} \dot{\mathbf{d}} + \dot{\mathbf{f}}_{\varphi \theta s}). \end{aligned} \quad (41)$$

Substituting equation (41) into equation (32) and introducing the Rayleigh damping $\mathbf{C}_s$, the final form of governing equation is expressed as

$$\mathbf{M}_{uu} \ddot{\mathbf{d}} + (\mathbf{C} + \mathbf{C}_s) \dot{\mathbf{d}} + \bar{\mathbf{K}}^* \mathbf{d} = \mathbf{F}^*, \quad (42)$$

$$\mathbf{C} = \mathbf{G}_v \mathbf{K}_{u\varphi a} \mathbf{K}_{\varphi \varphi s}^{-1} \mathbf{K}_{\varphi us}, \quad (43)$$

$$\bar{\mathbf{K}}^* = \mathbf{K}_{uu} + \mathbf{K}_{u\varphi s} \mathbf{K}_{\varphi \varphi s}^{-1} \mathbf{K}_{\varphi us} + \mathbf{G}_d \mathbf{K}_{u\varphi a} \mathbf{K}_{\varphi \varphi s}^{-1} \mathbf{K}_{\varphi us}, \quad (44)$$

$$\begin{aligned} \mathbf{F}^* = & \mathbf{f} + \mathbf{f}_{\text{th}} - \mathbf{G}_d \mathbf{K}_{u\varphi a} \mathbf{K}_{\varphi \varphi s}^{-1} \mathbf{f}_{\varphi \theta s} - \mathbf{K}_{u\varphi s} \mathbf{K}_{\varphi \varphi s}^{-1} \mathbf{f}_{\varphi \theta s} \\ & - \mathbf{G}_v \mathbf{K}_{u\varphi a} \mathbf{K}_{\varphi \varphi s}^{-1} \dot{\mathbf{f}}_{\varphi \theta s}. \end{aligned} \quad (45)$$

The Rayleigh damping $\mathbf{C}_s$ in equation (42) can be calculated by

$$\mathbf{C}_s = \alpha_R \mathbf{M}_{uu} + \beta_R \mathbf{K}_{uu}. \quad (46)$$

The Rayleigh damping coefficients α_R and β_R can be confirmed through [4].

4. Numerical Results

In this section, the convergence and accuracy of the S-FSDT-based IGA method is verified through some numerical examples. Then, the open-loop control and closed-loop control of piezoelectric functionally graded plates are

investigated by using the modified piezoelectric constitutive equation. In the vibration control, the Newmark- β method [44] is adopted for researching the transient response of PFGPs, and the parameters γ and β are set to be 0.5 and 0.25, respectively.

4.1. Convergence Verification. Let us consider a simply supported PFGP with sizes of $a = b = 400$ mm, $h_f = 5$ mm, and $h_p = 0.1$ mm. The functionally graded plate is composed of ZrO_2 and Al. The material parameters of metal and ceramic are reported in Table 1. The piezoelectric layers are PZT-G1195N, and the material properties are $E_{11} = E_{22} = E_{33} = 63$ GPa, $\nu_{12} = \nu_{13} = \nu_{23} = 0.3$, $\rho = 7600$ kg/m³, $d_{31} = d_{32} = 254 \times 10^{-12}$ m/V, and $g_{33} = 15 \times 10^{-9}$ F/m.

Using the cubic NURBS basis functions, Table 2 lists the first six natural frequencies of the plate with different mesh levels. One can see that the results of this work are in good agreement with those in [13]. Besides, the difference between the levels of 16×16 and 18×18 can be negligible. Therefore, the mesh of 16×16 is employed in the following analysis.

4.2. Comparison Studies

4.2.1. Piezoelectric Bimorph Beam. As shown in Figure 2, the beam consists of two PVDFs with opposite polarization directions. The sizes of the beam are 100 mm $\times$ 5 mm $\times$ 1 mm. The material properties are $E_{11} = E_{22} = 2.0$ GPa, $G_{12} = 1.0$ GPa, $\nu_{12} = 0$, $e_{31} = e_{32} = 0.046$ C/m², and $g_{11} = g_{22} = g_{33} = 0.1062 \times 10^{-9}$ F/m. Figure 3 shows the centerline deflection of the beam under 1V voltage. One can see that the results in this work match well with the theory solution presented by Tzou [45].

4.2.2. Static Bending of FGP in Thermal Environment. The length and thickness of a square SSSS ZrO_2/Al FGP are 200 mm and 10 mm. Assumed that the plate is subjected to the temperature gradient of $T_c = 300^\circ\text{C}$ and $T_m = 20^\circ\text{C}$. q_0 is an increasing transverse distributed load from 0 to -1×10^{-7} N/m². Figure 4 plots the trend of the dimensionless central deflection ($\bar{w} = w/h_f$) with the parameterized load ($\bar{P} = q_0 a^4/E_m h_f^4$). Also based on the IGA, although the S-FSDT has only four variables, it is still as accurate and effective as HSDT [46].

4.3. Parametric Studies

4.3.1. Piezoelectric Stress and Dielectric Parameters. Similar to equations (8) and (9), we can also define the temperature-dependent piezoelectric stress parameter $\hat{e}_{31}$ and dielectric parameter $\hat{g}_{33}$ as

$$\begin{aligned} \hat{e}_{31} &= e_{31} + \tilde{e}_{31} \Delta T, \\ \hat{g}_{33} &= g_{33} + \tilde{g}_{33} \Delta T, \end{aligned} \quad (47)$$

where e_{31} and g_{33} are piezoelectric stress and dielectric constants at temperature $T_0 = 0^\circ\text{C}$.

Wang et al. [21] measured the piezoelectric strain constant $\hat{d}_{31}$ and dielectric constant $\hat{g}_{33}/\epsilon_0$ of PZT-5A at

different temperature, where $\epsilon_0 = 8.85 \times 10^{-12}$ farad/m. Therefore, PZT-5A is selected as the actuators and sensors in subsequent investigations. Figure 5 plots the experimental data of $\hat{d}_{31}$ and $\hat{g}_{33}/\epsilon_0$ and their fitted curves. Through the equations of the fitted curves, $\hat{e}_{31}$ and $\hat{g}_{33}$ can be expressed as

$$\begin{aligned}\hat{e}_{31} &= -34.1599 - 0.1053\Delta T, \\ \hat{g}_{33} &= 2.717835 \times 10^{-8} + 1.3629 \times 10^{-10}\Delta T \\ &\quad + 2.43375 \times 10^{-13}\Delta T^2 - 7.08 \times 10^{-15}\Delta T^3 \\ &\quad - 4.2834 \times 10^{-17}\Delta T^4 + 7.2039 \times 10^{-19}\Delta T^5.\end{aligned}\quad (48)$$

4.3.2. Temperature Distribution of PFGP. We now study the temperature distribution of a PFGP with sizes of $a = b = 400$ mm, $h_f = 5$ mm, and $h_p = 0.1$ mm. The FGP is composed of ZrO_2 and Al. The material properties of PZT-5H are $E = 63$ Gpa, $\nu = 0.3$, $k = 2.1$ W/mK, $\alpha = 5 \times 10^{-9}/^\circ\text{C}$, $\rho = 7500$ kg/m³, and $P_3 = 4 \times 10^{-4}$ (C/m²°C).

Many studies [12, 13] of PFGPs ignored the heat conduction of the piezoelectric layers and directly applied thermal loads to the FGPs. Figure 6 shows the temperature distribution along the thickness of FGP under the temperature gradient of $T_c = 100^\circ\text{C}$ and $T_m = 0^\circ\text{C}$. It is seen that the temperature through the thickness shows a linear distribution in the case that the plate is homogeneous. However, the temperature presents a nonlinear distribution and is lower than that in the homogeneous plate when the plate is non-homogeneous. Figure 7 shows the temperature distribution of PFGP under the temperature gradient of $T_u = 100^\circ\text{C}$ and $T_l = 0^\circ\text{C}$. One can find that the temperature distribution of considering the heat conduction of the piezoelectric layers is completely different from that without considering the heat conduction. Besides, the temperature also presents a linear distribution when n is equal to zero. This is because the thermal conductivity of ZrO_2 and PZT-5A are almost identical.

4.3.3. Static Bending of PFGPs. Figure 8 plots the centerline deflection of the plate subjected to a surface load of -100 N/m². It is seen that the deflection increases with the increase in n . This is because the increase in n decreases the stiffness of the plate.

Figure 9 visualizes the centerline deflection under the thermal load $T_c = 100^\circ\text{C}$ and $T_m = 0^\circ\text{C}$. It can be seen that all the deflections are downward, and the deflection of the plate with $n = \infty$ is larger than the deflection with $n = 0$. This is because the thermal expansion coefficient of Al is larger than ZrO_2 .

Figure 10(a) plots the centerline deflection under thermal load of $T_u = 100^\circ\text{C}$ and $T_l = 0^\circ\text{C}$. The deflections are no longer all downward, and the deflection with $n = 0$ is greater than that with $n = \infty$. This is on account of the thickness of the piezoelectric layer is very small, and the deflection is mainly generated by the temperature gradient of FGP. From Figure 7, we can see that the temperature distribution of the FGP with $n = 0$ is obviously larger than that of $n = \infty$. Additionally, there is a difference between the deflection that

considers the temperature effect of the piezoelectric parameters (in solid line) and the deflection that does not consider the temperature effect (in dash line). Figure 10(b) plots the centerline deflection with $h_p = 0.5$ mm. One can observe that, with the increase in the thickness of the piezoelectric layers, the deformation of the plate is changed and the difference is amplified. Figure 11 shows the centerline deflection under the temperature gradient of $T_u = 160^\circ\text{C}$ and $T_l = 0^\circ\text{C}$. The results indicate that, with the increase in temperature rise, the difference also increases because the modified piezoelectric constitutive method used in this paper is established according to the experimental data of the dielectric and stress parameters in the thermal environment. Certainly, it can help to calculate and simulate the mechanical behavior of smart piezoelectric structures closer to the actual situation.

Next, the open-loop control of PFGPs is studied, in which both two piezoelectric layers act as actuators. Figure 12 shows the centerline deflection of the plate with $n = 0.5$ under a mechanical load of -100 N/m² and different voltages. Figure 13 plots the centerline deflection under the temperature gradient of $T_u = 100^\circ\text{C}$ and $T_l = 0^\circ\text{C}$ and different voltages. As expected, the deflection under mechanical and thermal loads can be reduced by increasing the input voltage.

4.3.4. Static Bending Closed-Loop Control. In the analysis of static bending closed-loop control, G_d is used to control the static deformation of the plate, and G_v is set to be zero. Figures 14 and 15 show the control effect of centerline deflection under the mechanical load of -100 N/m² and temperature gradient of $T_u = 100^\circ\text{C}$ and $T_l = 0^\circ\text{C}$, respectively. We can see that the deflection can be effectively controlled by using the displacement feedback control gain G_d .

4.3.5. Active Vibration Control of PFGPs. The former CFFF PFGP is employed for investigating the active vibration control. Assumed that the plate suffers an initial transverse surface load of -100 N/m², and then, the load is subsequently removed. Figures 16(a) and 16(b) are the transient responses at the tip of the plate under the temperature rise of $T_u = 0^\circ\text{C}$, $T_l = 0^\circ\text{C}$ and $T_u = 100^\circ\text{C}$, $T_l = 0^\circ\text{C}$, respectively. It is seen that, as G_v increases, the vibration disappears faster. Besides, because of the existence of thermal load, the vibration amplitude in Figure 16(b) is larger than that in Figure 16(a), and there is still a static offset after the vibration disappears in the thermal environment.

Finally, the active forced vibration control of a SSSS PFGP subjected to four types of distributed transverse loads is investigated. The index $n = 1$ and the sizes of plate are $a = b = 200$ mm, $h_f = 10$ mm, and $h_p = 0.5$ mm. Four types of loads are defined as follows:

$$q = q_0 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) F(t), \quad (49)$$

where $q_0 = 10^5$ N/m² and

TABLE 1: Material parameters of FGMs.

Materials	E (Gpa)	ν	k (W/mK)	α ($10^{-6}/K$)	ρ (kg/m^3)
ZrO ₂	151	0.3	2.09	10	3000
Al	70	0.3	204	23	2707

TABLE 2: The first six natural frequencies (Hz) of the plate with different mesh levels.

Mode	Method	n			
		0	1	5	∞
1	IGA (12×12)	205.88	177.95	167.74	150.99
	IGA (16×16)	205.88	177.95	167.74	150.99
	IGA (18×18)	205.88	177.95	167.74	150.99
	[13]	207.02	178.93	168.45	151.51
2/3	IGA (12×12)	514.26	444.50	418.95	377.12
	IGA (16×16)	514.24	444.48	418.94	377.11
	IGA (18×18)	514.24	444.48	418.94	377.11
	[13]	518.72	448.52	422.18	379.88
4	IGA (12×12)	822.06	710.55	669.66	602.82
	IGA (16×16)	822.04	710.54	669.64	602.81
	IGA (18×18)	822.04	710.54	669.64	602.81
	[13]	826.56	714.49	672.57	604.99
5/6	IGA (12×12)	1027.20	887.86	836.70	753.22
	IGA (16×16)	1026.99	887.71	836.56	753.09
	IGA (18×18)	1026.97	887.68	836.54	753.07
	[13]	1038.50	898.23	845.34	760.93

FIGURE 2: Piezoelectric bimorph beam.

FIGURE 3: Centerline deflection of the beam under 1V voltage.

FIGURE 4: Trend of the dimensionless central deflection with the parameterized load.

$$F(t) = \begin{cases} 1, & \text{Step load,} \\ 1 - t/t_1 & \text{Triangular load,} \\ \sin(\pi t/t_1) \text{ Sinusoidal load, } e^{-\bar{\gamma}t} \text{ Explosive load, if } 0 \leq t \leq t_1, 0, \text{ if } t \geq t_1, & \end{cases} \quad (50)$$

in which $\bar{\gamma} = 330 \text{ s}^{-1}$ and $t_1 = 0.003 \text{ s}$.

Figures 17–20 show the central deflection of the plate subjected to step, triangular, sinusoidal, and explosive loads. As expected, both in the forced and free vibration stages, the

velocity feedback gain G_v can control the vibration of the plate effectively. Because the deflection generated by temperature rise is opposite to that by mechanical loads, the deflection is negative after the vibration disappears in the thermal environment.

FIGURE 5: The experimental data of $\hat{d}_{31}$ and $\hat{g}_{33}/\epsilon_0$ of PZT-5A and their fitted curves.

FIGURE 6: Temperature distribution along the thickness of FGP under a temperature gradient of $T_c = 100^\circ\text{C}$ and $T_m = 0^\circ\text{C}$.

FIGURE 7: Temperature distribution along the thickness of PFGP under a temperature gradient of $T_u = 100^\circ\text{C}$ and $T_l = 0^\circ\text{C}$.

FIGURE 8: Centerline deflection under mechanical load.

FIGURE 9: Centerline deflection under the thermal load $T_c = 100^\circ\text{C}$ and $T_m = 0^\circ\text{C}$.

FIGURE 10: Centerline deflection with $h_p = 0.1$ and 0.5 mm under the thermal load $T_u = 100^\circ\text{C}$ and $T_l = 0^\circ\text{C}$: (a) $h_p = 0.1$ mm; (b) $h_p = 0.5$ mm.

FIGURE 11: Centerline deflection with $h_p = 0.1$ and 0.5 mm under the thermal load $T_u = 160^\circ\text{C}$ and $T_l = 0^\circ\text{C}$: (a) $h_p = 0.1$ mm; (b) $h_p = 0.5$ mm.

FIGURE 12: Centerline deflection under mechanical load and different voltages.

FIGURE 13: Centerline deflection under thermoelectric load.

FIGURE 14: Control effect of centerline deflection under mechanical load: (a) CFFF plate; (b) SSSS plate.

FIGURE 15: Control effect of centerline deflection under thermal load: (a) CFFF plate; (b) SSSS plate.

FIGURE 16: Transient responses at the tip of the plate: (a) $T_u = 0^\circ\text{C}$, $T_l = 0$; (b) $T_u = 100^\circ\text{C}$, $T_l = 0$.

FIGURE 17: The central deflection under the step load: (a) $T_u = 0^\circ\text{C}$, $T_l = 0$; (b) $T_u = 100^\circ\text{C}$, $T_l = 0$.

FIGURE 18: The central deflection under the triangular load: (a) $T_u = 0^\circ\text{C}$, $T_l = 0$; (b) $T_u = 100^\circ\text{C}$, $T_l = 0$.

FIGURE 19: The central deflection under the sinusoidal load: (a) $T_u = 0^\circ\text{C}$, $T_l = 0$; (b) $T_u = 100^\circ\text{C}$, $T_l = 0$.

FIGURE 20: The central deflection under the explosive load: (a) $T_u = 0^\circ\text{C}$, $T_l = 0$; (b) $T_u = 100^\circ\text{C}$, $T_l = 0$.

5. Conclusion

In this work, we employed a modified piezoelectric constitutive equation with consideration of the temperature effect of dielectric and piezoelectric strain parameters to replace the traditional linear piezoelectric constitutive equation for ensuring the investigation of smart piezoelectric structure in the thermal environment closer to the actual situation. In the selection of numerical methods, an isogeometric analysis (IGA) method combined with a simple first-order shear deformation theory (S-FSDT) was used.

From some numerical examples, some meaningful conclusions can be obtained:

- (1) The temperature distribution of considering the heat conduction of piezoelectric layers is completely different from that without considering.
- (2) In thermal bending analysis of PFGPs, there is a difference between the results using the modified piezoelectric constitutive equation and those using the traditional linear piezoelectric constitutive equation. And this difference increases with the

increase in temperature and thickness of piezoelectric layers.

- (3) The static deformation of the plates can be controlled according to the requirements by adjusting the control gain G_d .
- (4) The free and forced vibration of the plates can be controlled effectively by using the velocity feedback control gain G_v . When the plates are subjected to the thermal load, there is a static offset after the vibration disappears.
- (5) On the premise of ensuring the working performance of smart piezoelectric structures, to maintain the stability of piezoelectric structure in the thermal environment, the material whose piezoelectric parameters are not sensitive to temperature should be selected as far as possible.

Although the study in this paper has certain reference significance for the engineering application of smart piezoelectric structures, the NURBS-based IGA method still has some shortcomings [47], for instance, the waste of control points caused by refinement. Fortunately, these drawbacks can be avoided by using the rational splines over hierarchical T-meshes (RHT-splines) [31] and polynomial splines over hierarchical T-meshes (PHT-splines) [48]. Thus, introducing these splines techniques into the present formulation would be an interesting work for our future studies.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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