Research Article

Elastodynamic Solutions of a Finite Soil Layer under Interior Distributed Actions

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Through a method of displacement potentials, Fourier series, and Hankel integral transformation, the generalized solutions of an elastic layer resting on a rigid base under arbitrary, distributed, buried, and time-harmonic loads are developed in this study. With the proposed solution, the specific results for two kinds of uniform distributions as a kind of fundamental solutions in the interaction analysis of media and inclusions by the method of boundary integral equations are included as illustrations. Finally, numerical examples involving surface and buried patch loads are presented to validate the solutions and examine the effects of the thickness of the elastic layer. The results show that the proposed solution can cover the classical half-space solution by taking enough large thickness of the elastic layer (e.g., the ratio of the layer thickness beneath the load to the load radius ≥ 50) and the surface load solution by setting the load depth to zero; the underlying rigid base has significant and complex influence on the dynamic response of the thin layer due to wave reflections, which needs to be considered in the design and practice of related engineering.

1. Introduction

Mechanical behaviors of soils under dynamic loads, such as traffic loads, construction operations, machinery vibrations, and earthquakes, are of fundamental importance in pavement engineering, geotechnical, and seismic engineering [1–3]. By treating soil materials as single-phase elastic or viscoelastic media, their dynamic responses to external loads have been studied extensively in classical treatments [4–6]. Using the theories and methods developed for single-phase solids, research on the propagation of elastic waves in poroelastic materials also has continued to increase since the pioneering work of Biot [7].

The research began with the surface load cases involving either point, line, or patch loads, which characterize certain kinds of spatial symmetry and simplification. For instance, Lee [8], Yan et al. [9], Jones [10], Liu and Pan [11], Andersen

et al. [12], and You et al. [13] studied the dynamic response problems of single-phase elastic layered half-spaces subjected to vertical surface loads. Paul [14, 15], Halpern and Christiano [16], Puswewala and Rajapakse [17], Senjuntichai and Rajapakse [18], Jin and Liu [19], and Feng et al. [20] investigated the axisymmetric responses of poroelastic half-spaces under vertical or radial ring surface loads.

More general and mathematically demanding cases are those when asymmetric sources are buried inside media, which are generally corresponding to the interaction problems of media and embedded structures (e.g., geogrid reinforcement, pile foundation, or buried pipes), blast loads, or seismic loads. For example, Khojasteh et al. [21], Liu et al. [22], Lin et al. [23], Park and Kaynia [24], Noori et al. [25], Ai and Li [26], and Ai et al. [27] analyzed the dynamic responses of single-phase multi-layered half-spaces to buried vertical or horizontal loads.

Philippacopoulos [28], Jin and Liu [29], Zhou et al. [30], Chen et al. [31], Zheng et al. [32, 33], and Pooladi et al. [34] studied the dynamic responses of poroelastic half-spaces due to buried point or distributed loads.

The media in all the foregoing studies are modeled as half-spaces, which are available when their thicknesses are large enough. However, a more common case of many soil systems found in practice is that of finite soil stratum underlain by stiff bedrock. The existence of the bedrock boundary in this physical system leads to dynamic behavior with both resonant and cutoff frequency phenomena in wave mechanics. The significant differences in geometry and boundary conditions of the finite-layered media and the well-known infinite half-space clearly warrant careful attention in design and practice. For the finite-layered medium problems, Zheng [35] studied the dynamic response of an elastic layer on stiff foundation under time-harmonic surface vertical concentrated load. Chen et al. [36] presented a semianalytical solution for the axisymmetric consolidation of a transversely isotropic soil layer resting on a rough impervious base and subjected to a uniform circular pressure at the ground surface. Kim [37] proposed the general viscoelastic solutions for multilayered systems subjected to static and moving loads. Dan et al. [38] performed the coupled hydromechanical response analysis of saturated asphalt pavement under moving traffic load. Recently, Yuan and Wang [39] investigated the effects of surface tension on the two-dimensional contact problems of an elastic layer bonded to the rigid substrate.

These studies, however, were only suitable for surface load cases, while unable to reflect the effect of interior actions. Together with the fundamental importance of buried load solutions in the soil-structure interaction analysis, the expected generality significance that buried load solutions can be reduced to surface load solutions, and finite-layered solutions to half-space solutions. To this end, an analytical investigation for time-harmonic response analysis of an elastic layer resting on rigid base under arbitrary, distributed, and interior excitations is initiated to enhance the current framework for this class of problems. The remainder of this study is structured as follows: In Section 2, displacement potentials are introduced to fully decouple the field equations into three independent wave equations, followed by a finite Fourier transformation and a Hankel transformation, which reduce the dimension of wave equations to one. Then, general solutions to arbitrary, distributed, and buried loads are obtained by imposing the boundary condition and the conditions at the load level. In Section 3, specific responses to buried circular patch loads with uniform distribution are presented as application illustrations of the generalized solution. By degenerating the model to a half-space and comparing it with classical results and comparing the present solution with FEM results, the present solution is validated in Section 4. Also, the effect of the thickness of the elastic layer on its response of displacements is investigated with numerical examples.

2. Statement of Problem and Its Solutions

As mentioned earlier, the problem under consideration is that of an arbitrary internal body force field distributed on the plane $z = s$ in a finite stratum of homogeneous, isotropic, and linearly elastic medium resting on rigid base, as shown in Figure 1. In terms of elastodynamics, the equation of motion for the elastic layer can be expressed as

$$(\lambda^s + \mu^s)\nabla(\nabla \cdot \mathbf{u}_s) + \mu^s \nabla \cdot (\nabla \mathbf{u}_s) - \rho^s \ddot{\mathbf{u}}_s = 0, \quad (1)$$

where λ^s and μ^s are the two Lamé constants of the layer and ρ^s represents the density of the layer. $\mathbf{u}_s$ is the displacement vector of the layer, and $\ddot{\mathbf{u}}_s$ is its acceleration vector. ∇ denotes the gradient operator.

From Figure 1, the buried load source at $z = s$ can be written as

$$\sigma_{zr}^s(r, \theta, s^-, t) - \sigma_{zr}^s(r, \theta, s^+, t) = \begin{cases} P(r, \theta, t) & (r, \theta, s) \in \pi_s \\ 0 & (r, \theta, s) \notin \pi_s \end{cases}, \quad (2a)$$

$$\sigma_{z\theta}^s(r, \theta, s^-, t) - \sigma_{z\theta}^s(r, \theta, s^+, t) = \begin{cases} Q(r, \theta, t) & (r, \theta, s) \in \pi_s \\ 0 & (r, \theta, s) \notin \pi_s \end{cases}, \quad (2b)$$

$$\sigma_{zz}^s(r, \theta, s^-, t) - \sigma_{zz}^s(r, \theta, s^+, t) = \begin{cases} R(r, \theta, t) & (r, \theta, s) \in \pi_s \\ 0 & (r, \theta, s) \notin \pi_s \end{cases}, \quad (2c)$$

where P , Q , and R represent the stress distributions in directions r , θ , and z , respectively, σ_{zr}^s , $\sigma_{z\theta}^s$, and σ_{zz}^s are stress components, and π_s is the distribution area of applied loads.

Here it is assumed that the surface of the layer is free-traction and its bottom is bonded on the base, i.e.,

$$\sigma_{zr}^s(r, \theta, 0, t) = \sigma_{z\theta}^s(r, \theta, 0, t) = \sigma_{zz}^s(r, \theta, 0, t) = 0, \quad (3a)$$

$$u_s(r, \theta, L, t) = v_s(r, \theta, L, t) = w_s(r, \theta, L, t) = 0, \quad (3b)$$

and all displacements at $z = s$ are continuous, where u_s , v_s , and w_s are displacement components.

Since time-harmonic loads with the time term $e^{i\omega t}$ are considered in this paper, field variables can be written as

$$\begin{aligned} \sigma_{zr}^s(r, \theta, z, t) &= \sigma_{zr}^s(r, \theta, z)e^{i\omega t}, \\ \sigma_{z\theta}^s(r, \theta, z, t) &= \sigma_{z\theta}^s(r, \theta, z)e^{i\omega t}, \\ \sigma_{zz}^s(r, \theta, z, t) &= \sigma_{zz}^s(r, \theta, z)e^{i\omega t}, \\ u_s(r, \theta, z, t) &= u_s(r, \theta, z)e^{i\omega t}, \\ v_s(r, \theta, z, t) &= v_s(r, \theta, z)e^{i\omega t}, \\ w_s(r, \theta, z, t) &= w_s(r, \theta, z)e^{i\omega t}. \end{aligned} \quad (4)$$

For brevity, the time term $e^{i\omega t}$ is suppressed in the following analyses. Therefore, equation (1) can be represented as

FIGURE 1: An elastic layer under buried, arbitrary, distributed, and time-harmonic loads.

$$(\lambda^s + \mu^s) \nabla (\nabla \cdot \mathbf{u}_s) + \mu^s \nabla \cdot (\nabla \mathbf{u}_s) + \rho^s \omega^2 \mathbf{u}_s = 0. \quad (5)$$

Important to our further development is the resolution of displacement field by three scalar potentials inspired by Pak [6], i.e.,

$$\mathbf{u}_s(r, \theta, z) = \nabla \phi_s(r, \theta, z) + \nabla \times (\chi_s(r, \theta, z) \mathbf{e}_z + \nabla \times (\eta_s(r, \theta, z) \mathbf{e}_z)), \quad (6)$$

where $\mathbf{e}_z$ is the unit vector along the z -direction of the cylindrical coordinate system.

Then, the component form of equation (6) is further written as

$$\begin{aligned} u_s &= \frac{\partial \phi_s}{\partial r} + \frac{\partial \chi_s}{r \partial \theta} + \frac{\partial^2 \eta_s}{\partial r \partial z}, \\ v_s &= \frac{\partial \phi_s}{r \partial \theta} + \frac{\partial^2 \eta_s}{r \partial \theta \partial z} - \frac{\partial \chi_s}{\partial r}, \\ w_s &= \frac{\partial \phi_s}{\partial z} - \left(\frac{\partial^2 \eta_s}{\partial r^2} + \frac{\partial^2 \eta_s}{r^2 \partial \theta^2} + \frac{\partial \eta_s}{r \partial r} \right). \end{aligned} \quad (7)$$

By the imposition of equation (6), the equations of motion in terms of potential functions can be obtained from equation (5) as

$$\begin{aligned} \nabla^2 \phi_s(r, \theta, z) + k_1^2 \phi_s(r, \theta, z) &= 0, \\ \nabla^2 \chi_s(r, \theta, z) + k_2^2 \chi_s(r, \theta, z) &= 0, \\ \nabla^2 \eta_s(r, \theta, z) + k_2^2 \eta_s(r, \theta, z) &= 0, \end{aligned} \quad (8)$$

where $k_1^2 = \omega^2/V_d^2$ and $k_2^2 = \omega^2/V_s^2$. $V_d = \sqrt{\lambda^s + 2\mu^s/\rho^s}$ denotes the speed of the dilatational wave, and $V_s = \sqrt{\mu^s/\rho^s}$ is the speed of the shearing wave. $\nabla^2 = (\partial^2/\partial r^2) + (\partial/\partial r \partial r) + (\partial^2/r^2 \partial \theta^2) + (\partial^2/\partial z^2)$ is the Laplacian operator.

Next integral transformations are introduced to solve equation (8). First, by a Fourier expansion with respect to θ , these potential functions can be written as

$$\begin{aligned} \phi_s(r, \theta, z) &= \sum_{n=-\infty}^{+\infty} \phi_{sn}(r, z) e^{in\theta}, \\ \chi_s(r, \theta, z) &= \sum_{n=-\infty}^{+\infty} \chi_{sn}(r, z) e^{in\theta}, \\ \eta_s(r, \theta, z) &= \sum_{n=-\infty}^{+\infty} \eta_{sn}(r, z) e^{in\theta}. \end{aligned} \quad (9)$$

The displacement components can be written as

$$\begin{aligned} u_s(r, \theta, z) &= \sum_{n=-\infty}^{+\infty} u_{sn}(r, z) e^{in\theta}, \\ v_s(r, \theta, z) &= \sum_{n=-\infty}^{+\infty} v_{sn}(r, z) e^{in\theta}, \\ w_s(r, \theta, z) &= \sum_{n=-\infty}^{+\infty} w_{sn}(r, z) e^{in\theta}, \end{aligned} \quad (10)$$

and the source distributions in equations (2a)–(2c) can be written as

$$\begin{aligned} P(r, \theta, z) &= \sum_{n=-\infty}^{+\infty} P_n(r, z) e^{in\theta}, \\ Q(r, \theta, z) &= \sum_{n=-\infty}^{+\infty} Q_n(r, z) e^{in\theta}, \\ R(r, \theta, z) &= \sum_{n=-\infty}^{+\infty} R_n(r, z) e^{in\theta}. \end{aligned} \quad (11)$$

The equations for the Fourier series coefficients of the potentials can then be derived from the three wave equations (8), i.e.,

$$\begin{aligned} \left(\frac{\partial^2}{\partial r^2} + \frac{\partial}{r \partial r} - \frac{n^2}{r^2} + \frac{\partial^2}{\partial z^2} \right) \phi_{sn}(r, z) + k_1^2 \phi_{sn}(r, z) &= 0, \\ \left(\frac{\partial^2}{\partial r^2} + \frac{\partial}{r \partial r} - \frac{n^2}{r^2} + \frac{\partial^2}{\partial z^2} \right) \chi_{sn}(r, z) + k_2^2 \chi_{sn}(r, z) &= 0, \\ \left(\frac{\partial^2}{\partial r^2} + \frac{\partial}{r \partial r} - \frac{n^2}{r^2} + \frac{\partial^2}{\partial z^2} \right) \eta_{sn}(r, z) + k_2^2 \eta_{sn}(r, z) &= 0. \end{aligned} \quad (12)$$

Then, applying the Hankel transformation,

$$\tilde{f}^n(\xi) = \int_0^\infty r f(r) J_n(\xi r) dr, \quad (13)$$

whose inverse transformation is

$$f(r) = \int_0^\infty \xi \tilde{f}^n(\xi) J_n(\xi r) d\xi, \quad (14)$$

to equation (12) yields

$$\begin{aligned}
\frac{\partial^2}{\partial z^2} \tilde{\phi}_{sn}^n(\xi, z) - (\xi^2 - k_1^2) \tilde{\phi}_{sn}^n(\xi, z) &= 0, \\
\frac{\partial^2}{\partial z^2} \tilde{\chi}_{sn}^n(\xi, z) - (\xi^2 - k_2^2) \tilde{\chi}_{sn}^n(\xi, z) &= 0, \\
\frac{\partial^2}{\partial z^2} \tilde{\eta}_{sn}^n(\xi, z) - (\xi^2 - k_2^2) \tilde{\eta}_{sn}^n(\xi, z) &= 0.
\end{aligned} \tag{15}$$

The general solutions of equation (15) can be easily found to be

$$\begin{aligned}
\tilde{\phi}_{sn}^n(\xi, z) &= A_n^I e^{\alpha z} + B_n^I e^{-\alpha z}, \\
\tilde{\chi}_{sn}^n(\xi, z) &= C_n^I e^{\beta z} + D_n^I e^{-\beta z}, \\
\tilde{\eta}_{sn}^n(\xi, z) &= E_n^I e^{\beta z} + F_n^I e^{-\beta z}, \quad \text{in region I,}
\end{aligned} \tag{16a}$$

and

$$\begin{aligned}
\tilde{\phi}_{sn}^n(\xi, z) &= A_n^{II} e^{\alpha z} + B_n^{II} e^{-\alpha z}, \\
\tilde{\chi}_{sn}^n(\xi, z) &= C_n^{II} e^{\beta z} + D_n^{II} e^{-\beta z}, \\
\tilde{\eta}_{sn}^n(\xi, z) &= E_n^{II} e^{\beta z} + F_n^{II} e^{-\beta z}, \quad \text{in region II,}
\end{aligned} \tag{16b}$$

where $\alpha = (\xi^2 - k_1^2)^{1/2}$, $\beta = (\xi^2 - k_2^2)^{1/2}$, and $A_n^I \sim F_n^I$, $A_n^{II} \sim F_n^{II}$ are unknown coefficients of integration which are determined by the interfacial and boundary conditions. The branch cut of the radicals α and β is specified such that $\text{Re}(\alpha) \geq 0$ and $\text{Re}(\beta) \geq 0$.

Now, it is noted that the free surface and bottom conditions provide 6 equations (equation (3)); the stress discontinuity across the region interface provides 3 equations (equations (2a)–(2c)); and the continuity of soil displacements provides the other 3 equations.

For further usage, the displacement-potential relationship in the cylindrical coordinate systems and integral transformation domains obtained by substituting equations (9), (10), and (13) into equation (7) are given below:

$$\begin{aligned}
\tilde{u}_{sn}^{n+1}(\xi, z) + i\tilde{v}_{sn}^{n+1}(\xi, z) &= -\xi \tilde{\phi}_{sn}^n(\xi, z) + i\xi \tilde{\chi}_{sn}^n(\xi, z) - \xi \frac{\partial}{\partial z} \tilde{\eta}_{sn}^n(\xi, z), \\
\tilde{u}_{sn}^{n-1}(\xi, z) + i\tilde{v}_{sn}^{n-1}(\xi, z) &= -\xi \tilde{\phi}_{sn}^n(\xi, z) + i\xi \tilde{\chi}_{sn}^n(\xi, z) + \xi \frac{\partial}{\partial z} \tilde{\eta}_{sn}^n(\xi, z), \\
\tilde{w}_{sn}^n(\xi, z) &= \frac{\partial}{\partial z} \tilde{\phi}_{sn}^n(\xi, z) + \xi^2 \tilde{\eta}_{sn}^n(\xi, z),
\end{aligned} \tag{17}$$

and the stress-potential relationship derived by using Hook's law and equations (7), (9), (10) and (13) can be expressed as

$$\begin{aligned}
\tilde{\sigma}_{zzn}^{sn}(\xi, z) &= \left[2\mu^s \frac{\partial^2}{\partial z^2} \tilde{\phi}_{sn}^n(\xi, z) - \lambda^s k_1^2 \tilde{\phi}_{sn}^n(\xi, z) \right] + 2\mu^s \xi^2 \frac{\partial}{\partial z} \tilde{\eta}_{sn}^n(\xi, z), \\
\tilde{\sigma}_{zrn}^{s(n+1)}(\xi, z) + i\tilde{\sigma}_{z\theta n}^{s(n+1)}(\xi, z) &= -2\mu^s \xi \frac{\partial}{\partial z} \tilde{\phi}_{sn}^n(\xi, z) + i\mu^s \xi \frac{\partial}{\partial z} \tilde{\chi}_{sn}^n(\xi, z) - \mu^s \xi \left[\frac{\partial^2}{\partial z^2} \tilde{\eta}_{sn}^n(\xi, z) + \xi^2 \tilde{\eta}_{sn}^n(\xi, z) \right], \\
\tilde{\sigma}_{zrn}^{s(n-1)}(\xi, z) - i\tilde{\sigma}_{z\theta n}^{s(n-1)}(\xi, z) &= 2\mu^s \xi \frac{\partial}{\partial z} \tilde{\phi}_{sn}^n(\xi, z) + i\mu^s \xi \frac{\partial}{\partial z} \tilde{\chi}_{sn}^n(\xi, z) + \mu^s \xi \left[\frac{\partial^2}{\partial z^2} \tilde{\eta}_{sn}^n(\xi, z) + \xi^2 \tilde{\eta}_{sn}^n(\xi, z) \right].
\end{aligned} \tag{18}$$

After the determination of the unknown coefficients by the set of boundary condition equations, one can obtain the solutions of field variables in forms

$$\begin{aligned}
\bar{u}_{sn}^{n+1}(\xi, z) + i\bar{v}_{sn}^{n+1}(\xi, z) &= \gamma_1 \frac{X_n - Y_n}{2\mu^s} + \gamma_2 \frac{Y_n + X_n}{2\mu^s} - \gamma_3 \frac{Z_n}{(\lambda^s + 2\mu^s)}, \\
\bar{u}_{sn}^{n-1}(\xi, z) - i\bar{v}_{sn}^{n-1}(\xi, z) &= -\gamma_1 \frac{X_n - Y_n}{2\mu^s} + \gamma_2 \frac{Y_n + X_n}{2\mu^s} + \gamma_3 \frac{Z_n}{(\lambda^s + 2\mu^s)}, \\
\bar{w}_{sn}^n(\xi, z) &= -\gamma_4 \frac{X_n - Y_n}{2\mu^s} + \gamma_5 \frac{Z_n}{(\lambda^s + 2\mu^s)},
\end{aligned} \tag{19}$$

and

$$\begin{aligned}
\bar{\sigma}_{zsn}^{sn}(\xi, z) &= -\gamma_6 \frac{X_n + Y_n}{2\mu^s} + \gamma_7 \frac{Z_n}{(\lambda^s + 2\mu^s)}, \\
\bar{\sigma}_{zrn}^{s(n+1)}(\xi, z) + i\bar{\sigma}_{z\theta n}^{s(n+1)}(\xi, z) &= \gamma_8 \frac{X_n - Y_n}{2} + \gamma_9 \frac{Y_n + X_n}{2} - \gamma_{10} \frac{Z_n}{(\lambda^s + 2\mu^s)}, \\
\bar{\sigma}_{zrn}^{s(n-1)}(\xi, z) - i\bar{\sigma}_{z\theta n}^{s(n-1)}(\xi, z) &= -\gamma_8 \frac{X_n - Y_n}{2} + \gamma_9 \frac{Y_n + X_n}{2} + \frac{Z_n}{(\lambda^s + 2\mu^s)},
\end{aligned} \tag{20}$$

where

$$\gamma_i = \begin{cases} \gamma_i^- & z < s \\ \gamma_i^+ & z > s \end{cases}, \quad i = 1 \sim 10, \tag{21}$$

and the coefficients γ_i^- , γ_i^+ , X_n , Y_n , and Z_n are provided in Appendix.

By applying the inverse Hankel transformation to the solutions in equations (19) and (20) and then inserting the inverted Fourier components into the Fourier series expansion in equations (9)–(11), the time-harmonic response of the layer to the general buried loads is formally developed below.

$$\begin{aligned}
u_s(r, \theta, z, t) &= \frac{1}{4\mu^s} e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_1 (X_n - Y_n) [J_{n+1}(\xi r) - J_{n-1}(\xi r)] d\xi \right\} \\
&+ \frac{1}{4\mu^s} e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_2 (Y_n + X_n) [J_{n+1}(\xi r) + J_{n-1}(\xi r)] d\xi \right\} \\
&- \frac{1}{2(\lambda^s + 2\mu^s)} e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_3 Z_n [J_{n+1}(\xi r) - J_{n-1}(\xi r)] d\xi \right\},
\end{aligned} \tag{22a}$$

$$\begin{aligned}
v_s(r, \theta, z, t) &= \frac{1}{4i\mu^s} e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_1 (X_n - Y_n) [J_{n+1}(\xi r) + J_{n-1}(\xi r)] d\xi \right\} \\
&+ \frac{1}{4i\mu^s} e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_2 (Y_n + X_n) [J_{n+1}(\xi r) - J_{n-1}(\xi r)] d\xi \right\} \\
&- \frac{1}{2i(\lambda^s + 2\mu^s)} e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_3 Z_n [J_{n+1}(\xi r) + J_{n-1}(\xi r)] d\xi \right\},
\end{aligned} \tag{22b}$$

$$\begin{aligned}
w_s(r, \theta, z, t) &= -\frac{1}{2\mu^s} e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_4 (X_n - Y_n) J_n(\xi r) d\xi \right\} \\
&+ \frac{1}{(\lambda^s + 2\mu^s)} e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_5 Z_n J_n(\xi r) d\xi \right\},
\end{aligned} \tag{22c}$$

$$\begin{aligned}\sigma_{zr}^s(r, \theta, z, t) = & -\frac{1}{4}e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_8 (X_n - Y_n) [J_{(n-1)}(\xi r) - J_{(n+1)}(\xi r)] d\xi \right\} \\ & + \frac{1}{4}e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_9 (Y_n + X_n) [J_{(n-1)}(\xi r) + J_{(n+1)}(\xi r)] d\xi \right\} \\ & + \frac{1}{2(\lambda^s + 2\mu^s)} e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_{10} Z_n [J_{(n-1)}(\xi r) - J_{(n+1)}(\xi r)] d\xi \right\},\end{aligned}\quad (22d)$$

$$\begin{aligned}\sigma_{z\theta}^s(r, \theta, z, t) = & \frac{1}{4i}e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_8 (X_n - Y_n) [J_{(n+1)}(\xi r) + J_{(n-1)}(\xi r)] d\xi \right\} \\ & + \frac{1}{4i}e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_9 (Y_n + X_n) [J_{(n+1)}(\xi r) - J_{(n-1)}(\xi r)] d\xi \right\} \\ & - \frac{1}{2i(\lambda^s + 2\mu^s)} e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_{10} Z_n [J_{(n+1)}(\xi r) + J_{(n-1)}(\xi r)] d\xi \right\},\end{aligned}\quad (22e)$$

$$\begin{aligned}\sigma_{zz}^s(r, \theta, z, t) = & -\frac{1}{2\mu^s} e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_6 (X_n - Y_n) J_n(\xi r) d\xi \right\} \\ & + \frac{1}{(\lambda^s + 2\mu^s)} e^{i\omega t} \sum_{n=-\infty}^{+\infty} \left\{ e^{in\theta} \int_0^\infty \xi \gamma_7 Z_n J_n(\xi r) d\xi \right\}.\end{aligned}\quad (22f)$$

3. Green's Functions of the Elastic Layer under Specific Loads

In this section, two kinds of special loads are considered to introduce the application of the obtained equations (22a)–(22f).

3.1. Buried Vertical Uniformly Distributed Circular Patch Load. Such a load can be expressed as

$$R(r, \theta) = R_0(r) = \begin{cases} \frac{1}{\pi r_0^2}, & r < r_0, \\ 0, & r \geq r_0, \end{cases} \quad (23)$$

and $R_n(r) = 0$ for $n \neq 0$, and $P_n(r) = 0$, $Q_n(r) = 0$ for all n .

Combining equations (23) and (22a)–(22f), one obtains

$$u_s(r, \theta, z) = -\frac{1}{(\lambda^s + 2\mu^s)\pi r_0} \int_0^\infty \gamma_3(\xi, z; s) J_1(\xi r_0) J_1(\xi r) d\xi, \quad (24a)$$

$$v_s(r, \theta, z) = 0, \quad (24b)$$

$$w_s(r, \theta, z) = \frac{1}{(\lambda^s + 2\mu^s)\pi r_0} \int_0^\infty \gamma_5(\xi, z, s) J_1(\xi r_0) J_0(\xi r) d\xi, \quad (24c)$$

$$\sigma_{zr}^s(r, \theta, z) = -\frac{1}{(\lambda^s + 2\mu^s)\pi r_0} \int_0^\infty \gamma_{10}(\xi, z; s) J_1(\xi r_0) J_1(\xi r) d\xi, \quad (24d)$$

$$\sigma_{z\theta}^s(r, \theta, z) = 0, \quad (24e)$$

$$\sigma_{zz}^s(r, \theta, z) = \frac{1}{(\lambda^s + 2\mu^s)\pi r_0} \int_0^\infty \gamma_7(\xi, z, s) J_1(\xi r_0) J_0(\xi r) d\xi. \quad (24f)$$

3.2. Buried Horizontal Uniformly Distributed Circular Patch Load. Such a load can be written as

$$P(r, \theta) = \begin{cases} \frac{1}{\pi r_0^2} \cos(\theta), & r < r_0, \\ 0, & r \geq r_0, \end{cases} \quad (25a)$$

$$Q(r, \theta) = \begin{cases} -\frac{1}{\pi r_0^2} \sin(\theta), & r < r_0, \\ 0, & r \geq r_0, \end{cases} \quad (25b)$$

such that

$$P_1(r) = P_{-1}(r) = \frac{1}{2\pi r_0^2}, \quad (26a)$$

and $P_n(r) = 0$ for $n \neq \pm 1$.

$$Q_1(r) = -Q_{-1}(r) = i \frac{1}{2\pi r_0^2}, \quad (26b)$$

and $Q_n(r) = 0$ for $n \neq \pm 1$, and $R_n(r) = 0$ for all n .

Combining equations (26a) and (26b) and (22a)–(22f), one gets

$$u_s(r, \theta, z) = \frac{\cos(\theta)}{2\mu^s \pi r_0} \left\{ - \int_0^\infty \gamma_1(\xi, z; s) J_1(\xi r_0) [J_0(\xi r) - J_2(\xi r)] d\xi \right. \\ \left. + \int_0^\infty \gamma_2(\xi, z; s) J_1(\xi r_0) [J_0(\xi r) + J_2(\xi r)] d\xi \right\}, \quad (27a)$$

$$v_s(r, \theta, z) = -\frac{\sin(\theta)}{2\mu^s \pi r_0} \left\{ - \int_0^\infty \gamma_1(\xi, z; s) J_1(\xi r_0) [J_0(\xi r) + J_2(\xi r)] d\xi \right. \\ \left. + \int_0^\infty \gamma_2(\xi, z; s) J_1(\xi r_0) [J_0(\xi r) - J_2(\xi r)] d\xi \right\}, \quad (27b)$$

$$w_s(r, \theta, z) = -\frac{\cos(\theta)}{\mu^s \pi r_0} \int_0^\infty \gamma_4(\xi, z, s) J_1(\xi r_0) J_1(\xi r) d\xi, \quad (27c)$$

$$\sigma_{zr}^s(r, \theta, z) = \frac{\cos(\theta)}{2\pi r_0} \left\{ - \int_0^\infty \gamma_8(\xi, z; s) J_1(\xi r_0) [J_0(\xi r) - J_2(\xi r)] d\xi \right. \\ \left. + \int_0^\infty \gamma_9(\xi, z; s) J_1(\xi r_0) [J_0(\xi r) + J_2(\xi r)] d\xi \right\}, \quad (27d)$$

$$\sigma_{z\theta}^s(r, \theta, z) = \frac{\sin(\theta)}{2\pi r_0} \left\{ - \int_0^\infty \gamma_8(\xi, z; s) J_1(\xi r_0) [J_0(\xi r) - J_2(\xi r)] d\xi \right. \\ \left. + \int_0^\infty \gamma_9(\xi, z; s) J_1(\xi r_0) [J_0(\xi r) + J_2(\xi r)] d\xi \right\}, \quad (27e)$$

$$\sigma_{zz}^s(r, \theta, z) = -\frac{\cos(\theta)}{\mu^s \pi r_0} \int_0^\infty \gamma_6(\xi, z, s) J_1(\xi r_0) J_1(\xi r) d\xi. \quad (27f)$$

In a similar way, Green's functions of the elastic layer to point loads or ring loads can also be obtained from equations (22a)–(22f).

4. Reduction of the Solutions and Numerical Examples

When the layer thickness tends to infinity, it is natural to expect that the present problem should be reduced to a half-space case. Thus, by setting $L \rightarrow \infty$, one can derive from equation (19)

$$\begin{aligned} \tilde{u}_{sn}^{n+1}(\xi, z) + i\tilde{v}_{sn}^{n+1}(\xi, z) &= -\bar{\gamma}_1 \frac{X_n - Y_n}{2\mu^s} + \bar{\gamma}_2 \frac{Y_n + X_n}{2\mu^s} - \bar{\gamma}_3 \frac{Z_n}{(\lambda^s + 2\mu^s)}, \\ \tilde{u}_{sn}^{n-1}(\xi, z) - i\tilde{v}_{sn}^{n-1}(\xi, z) &= \bar{\gamma}_1 \frac{X_n - Y_n}{2\mu^s} + \bar{\gamma}_2 \frac{Y_n + X_n}{2\mu^s} + \bar{\gamma}_3 \frac{Z_n}{(\lambda^s + 2\mu^s)}, \\ \tilde{w}_{sn}^n(\xi, z) &= \bar{\gamma}_4 \frac{X_n - Y_n}{2\mu^s} + \bar{\gamma}_5 \frac{Z_n}{(\lambda^s + 2\mu^s)}, \end{aligned} \quad (28)$$

where

FIGURE 2: Lateral displacement u_s induced by unit lateral load on π_s ($\bar{\omega} = \omega r_0 / \sqrt{\mu^s / \rho^s} = 0.5$): (a) $s = 0$; (b) $s = 20r_0$.

$$\begin{aligned}
 \bar{y}_1 &= \frac{\xi^2}{2\alpha k_2^2} e^{-\alpha|z-s|} - \frac{\beta}{2k_2^2} e^{-\beta|z-s|} - \frac{1}{2k_2^2} \frac{a_1}{a_2} \left[\frac{\xi^2}{\alpha} e^{-\alpha(s+z)} + \beta e^{-\beta(s+z)} \right] + \frac{2\mu^s (2\xi^2 - k_2^2) \beta \xi^2}{a_2 k_2^2} [e^{-\alpha s - \beta z} + e^{-\beta s - \alpha z}], \\
 \bar{y}_2 &= \frac{1}{2\beta} [e^{-\beta|z-s|} + e^{-\beta(s+z)}], \\
 \bar{y}_3 &= \text{sgn}(z, s) \frac{\xi}{2k_1^2} [e^{-\alpha|z-s|} - e^{-\beta|z-s|}] + \frac{\xi}{2k_1^2} \frac{a_1}{a_2} [e^{-\alpha(s+z)} + e^{-\beta(s+z)}] - \frac{2\mu^s (2\xi^2 - k_2^2) \xi}{a_2 k_1^2} [\alpha \beta e^{-\alpha s - \beta z} + \xi^2 e^{-\beta s - \alpha z}], \\
 \bar{y}_4 &= -\text{sgn}(z, s) \frac{\xi}{2k_2^2} [e^{-\alpha|z-s|} - e^{-\beta|z-s|}] + \frac{\xi}{2k_2^2} \frac{a_1}{a_2} [e^{-\alpha(s+z)} + e^{-\beta(s+z)}] - \frac{2\mu^s (2\xi^2 - k_2^2) \xi}{a_2 k_2^2} [\xi^2 e^{-\alpha s - \beta z} + \alpha \beta e^{-\beta s - \alpha z}], \\
 \bar{y}_5 &= -\frac{\alpha}{2k_1^2} e^{-\alpha|z-s|} + \frac{\xi^2}{2\beta k_1^2} e^{-\beta|z-s|} - \frac{1}{2k_1^2} \frac{a_1}{a_2} \left[\alpha e^{-\alpha(s+z)} + \frac{\xi^2}{\beta} e^{-\beta(s+z)} \right] + \frac{2\mu^s (2\xi^2 - k_2^2) \alpha \xi^2}{a_2 k_1^2} [e^{-\alpha s - \beta z} + e^{-\beta s - \alpha z}],
 \end{aligned} \tag{29}$$

and

$$\begin{aligned}
 a_1 &= \mu^s [(2\xi^2 - k_2^2)^2 + 4\xi^2 \alpha \beta], \\
 a_2 &= \mu^s [(2\xi^2 - k_2^2)^2 - 4\xi^2 \alpha \beta], \\
 k_1^2 &= \frac{\rho^s \omega^2}{\lambda^s + 2\mu^s}, \\
 k_2^2 &= \frac{\rho^s \omega^2}{\mu^s}, \\
 \text{sgn}(z, s) &= \begin{cases} -1, & z < s, \\ 1, & z > s. \end{cases}
 \end{aligned} \tag{30}$$

It is easy to show that the reduced results are identical to the classical half-space solution from Pooladi et al. [34], which can also be graphically confirmed by the excellent

agreement between the reduced results and the half-space results as shown in Figure 2. Note that in the following computation analyses, the numerical quadrature method suggested by Pooladi et al. [34] is used to evaluate the present solution. Unless otherwise stated, the material parameters $\lambda^s = \mu^s = 10 \text{ MPa}$ and $\rho^s = 2500 \text{ kg} \cdot \text{m}^{-3}$, the load radius $r_0 = 1 \text{ m}$, and the observation point coordinates $r = 0, \theta = 0$ are employed in the whole numerical examples.

Also, as depicted in Figure 3(a), an axisymmetric finite element model for buried vertical load case with ADINA is established to verify the present solution. In this model, the above material parameters are used; for dynamic case, the dimensionless excitation frequency $\bar{\omega} = \omega r_0 \sqrt{\rho^s / \mu^s} = 0.5$; 9-node rectangle element is employed; the model length of 50 m has been able to eliminate the boundary effect. It should be noted that due to the software display setting for two-dimensional problems, the coordinate system of the axisymmetric FEM model is shown as the Cartesian coordinate system. The coordinate systems in the present paper and the FEM model are equivalent. It is observed from

FIGURE 3: Comparisons of the present solution with the corresponding FEM solution: (a) axisymmetric finite element model by ADINA; (b) modulus of vertical displacement along with depth.

FIGURE 4: Vertical displacement w_s induced by unit vertical load on π_s : (a) real part of vertical displacement; (b) imaginary part of vertical displacement.

FIGURE 5: Vertical stress σ_{zz}^s induced by unit vertical load on π_s : (a) real part of vertical stress; (b) imaginary part of vertical stress.

FIGURE 6: Lateral displacement u_s induced by unit lateral load on π_s : (a) real part of lateral displacement; (b) imaginary part of lateral displacement.

Figure 3(b) that these solutions agree well, and the present solution is thus validated.

Figures 4 and 5 show the displacement and stress distributions of the layer along the z axis under vertical loads. From Figure 4, sharp corners are observed at the load plane $z = s$, and the displacements vanish at the soil bottom; these results are consistent with the boundary conditions (2c) and (3b). From Figure 5, the stress vanishes at the soil surface $z = 0$, and the stress discontinuity is $1/\pi$ at the load plane $z = s$; these results are consistent with the boundary conditions (3a) and (2c). With the increase of the excitation frequency, the fluctuation of the dynamic response of the layer increases. Similar behaviors can also be observed in

Figures 6 and 7 that represent lateral displacement and stress distributions of the layer along the z axis under lateral loads.

The effects of the layer thickness on the vertical displacements induced by vertical loads are shown in Figures 8 and 9. Obviously, the layer thickness has significant and complicated effects on the displacements. When its thickness tends to infinity, the displacements converge to the half-space results. This is because the waves are reflected by the rigid base and then disturb the behavior of the layer. When the layer is very thick, which means the base is located far away, fewer waves are reflected and have smaller amplitude due to attenuation, such that in this case the reflected components have negligible effects on the layer. This also

FIGURE 7: Lateral stress σ_{zr}^s induced by unit lateral load on π_s : (a) real part of lateral stress; (b) imaginary part of lateral stress.

FIGURE 8: Vertical displacement w_s induced by unit vertical load on π_s ($s = 2r_0$, $\bar{\omega} = 1.5$): (a) real part of vertical displacement; (b) imaginary part of vertical displacement.

confirms that when the layer is thick enough (e.g., the ratio of the layer thickness beneath the load to the load radius ≥ 50), it can be modeled as half-spaces for simplification.

Similar conclusions can also be drawn from Figures 10 and 11, which represent the lateral displacements induced by lateral loads.

FIGURE 9: Vertical displacement w_s at the surface $z = 0$ induced by unit vertical load on π_s ($s = 2r_0$, $\bar{w} = 0.5$): (a) real part of vertical displacement; (b) imaginary part of vertical displacement.

FIGURE 10: Lateral displacement u_s induced by unit lateral load on π_s ($s = 2r_0$, $\bar{w} = 1.5$): (a) real part of lateral displacement; (b) imaginary part of lateral displacement.

FIGURE 11: Lateral displacement u_s at the surface $z = 0$ induced by unit lateral load on π_s ($s = 2r_0$, $\bar{w} = 0.5$): (a) real part of lateral displacement; (b) imaginary part of lateral displacement.

5. Conclusions

In this paper, three-dimensional general solutions of an elastic layer resting on rigid base to arbitrary, distributed, and buried loads are presented by a method of displacement potentials and integral transforms. As illustrations, results for vertical and horizontal loads of uniform circular distributions are also included. The solution is demonstrated with numerical examples and the effects of the layer thickness are examined as well. Compared with the classical results of half-spaces or surface load problems, the present solution can be used to deal with a variety of axisymmetric and asymmetric wave propagation problems in finite-layered media.

Also, it is worth noting that from the previous analysis, the generalized solutions presented in this work cover many

classical results, such as half-space solutions and surface load solutions, and may have a wide range of application. By using the present solutions, other Green's functions corresponding to ring loads and point loads can be easily derived, which are of fundamental importance to analyze the problems associated with soil-structure interactions by the method of boundary integral equations.

Appendix

Coefficients in Equations (19) and (20)

The coefficients in equations (19) and (20) are listed below:

$$\gamma_1^- = \frac{\xi}{2k_2^2} [\xi(c_1^- + c_3^-) + \beta(c_5^- + c_7^-)], \quad (\text{A.1})$$

$$\gamma_2^- = \frac{1}{2\beta(1 + e^{-2\beta L})} [e^{\beta(z-s)} - e^{-2\beta L} e^{\beta(z+s)} + e^{-\beta(z+s)} - e^{-2\beta L} e^{-\beta(z-s)}], \quad (\text{A.2})$$

$$\gamma_3^- = \frac{\xi}{2k_1^2} [c_2^- + c_4^- + \beta(c_6^- + c_8^-)], \quad (\text{A.3})$$

$$\gamma_4^- = \frac{\xi}{2k_2^2} [\alpha(c_1^- + c_3^-) + \xi(c_5^- + c_7^-)], \quad (\text{A.4})$$

$$\gamma_5^- = \frac{1}{2k_1^2} [\alpha(c_2^- - c_4^-) + \xi^2(c_6^- - c_8^-)], \quad (\text{A.5})$$

$$\gamma_6^- = \frac{\xi}{2k_2^2} [(2\mu^s \alpha^2 - \lambda^s k_1^2)(c_1^- + c_3^-) + 2\mu^s \xi \beta(c_5^- + c_7^-)], \quad (\text{A.6})$$

$$\gamma_7^- = \frac{1}{2k_1^2} \left[(2\mu^s \alpha^2 - \lambda^s k_1^2) (c_2^- + c_4^-) + 2\mu^s \xi^2 \beta (c_6^- + c_8^-) \right], \quad (\text{A.7})$$

$$\gamma_8^- = \frac{\xi}{2k_2^2} \left[2\xi \alpha (c_1^- - c_3^-) + (\beta^2 + \xi^2) (c_5^- + c_7^-) \right], \quad (\text{A.8})$$

$$\gamma_9^- = \frac{1}{2(1 + e^{-2\beta L})} \left[e^{\beta(z-s)} - e^{-\beta(z+s)} - e^{-2\beta L} e^{\beta(z+s)} + e^{-2\beta L} e^{-\beta(z-s)} \right], \quad (\text{A.9})$$

$$\gamma_{10}^- = \frac{\mu^s \xi}{2k_1^2} \left[2\alpha (c_2^- - c_4^-) + (\beta^2 + \xi^2) (c_6^- - c_8^-) \right], \quad (\text{A.10})$$

$$\gamma_1^+ = \frac{\xi}{2k_2^2} \left[\xi c_1^+ \frac{1}{e^{\alpha L}} + \xi c_3^+ \frac{1}{R} (\alpha\beta + \xi^2) + \beta c_5^+ \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-\beta L} + \beta c_7^+ \right], \quad (\text{A.11})$$

$$\gamma_2^+ = \frac{1}{2\beta(1 + e^{-2\beta L})} \left[e^{-\beta(z+s)} + e^{-\beta(z-s)} - e^{-2\beta L} e^{\beta(z-s)} - e^{-2\beta L} e^{\beta(z+s)} \right], \quad (\text{A.12})$$

$$\gamma_3^+ = \frac{\xi}{2k_1^2} \left[c_2^+ \frac{1}{e^{\alpha L}} + c_4^+ \frac{1}{R} (\alpha\beta + \xi^2) + \beta c_6^+ \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-\beta L} + \beta c_8^+ \right], \quad (\text{A.13})$$

$$\gamma_4^+ = \frac{\xi}{2k_2^2} \left[\alpha c_1^+ \frac{1}{e^{\alpha L}} - \alpha c_3^+ \frac{1}{R} (\alpha\beta + \xi^2) + \xi c_5^+ \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-\beta L} - \xi c_7^+ \right], \quad (\text{A.14})$$

$$\gamma_5^+ = \frac{1}{2k_1^2} \left[\alpha c_2^+ \frac{1}{e^{\alpha L}} - \alpha c_4^+ \frac{1}{R} (\alpha\beta + \xi^2) + \xi^2 c_6^+ \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-\beta L} - \xi^2 c_8^+ \right], \quad (\text{A.15})$$

$$\gamma_6^+ = \frac{\xi}{2k_2^2} \left[(2\mu^s \alpha^2 - \lambda^s k_1^2) c_1^+ \frac{1}{e^{\alpha L}} + (2\mu^s \alpha^2 - \lambda^s k_1^2) c_3^+ \frac{1}{R} (\alpha\beta + \xi^2) + 2\mu^s \xi^2 \beta c_5^+ \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-\beta L} + 2\mu^s \xi^2 \beta c_7^+ \right], \quad (\text{A.16})$$

$$\gamma_7^+ = \frac{1}{2k_1^2} \left[(2\mu^s \alpha^2 - \lambda^s k_1^2) c_2^+ \frac{1}{e^{\alpha L}} + (2\mu^s \alpha^2 - \lambda^s k_1^2) c_4^+ \frac{1}{R} (\alpha\beta + \xi^2) + 2\mu^s \xi^2 \beta c_6^+ \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-\beta L} + 2\mu^s \xi^2 \beta c_8^+ \right], \quad (\text{A.17})$$

$$\gamma_8^+ = \frac{\xi}{2k_2^2} \left[2\xi \alpha c_1^+ \frac{1}{e^{\alpha L}} - 2\xi \alpha c_3^+ \frac{1}{R} (\alpha\beta + \xi^2) + (\beta^2 + \xi^2) c_5^+ \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-\beta L} - (\beta^2 + \xi^2) c_7^+ \right], \quad (\text{A.18})$$

$$\gamma_9^+ = -\frac{1}{2(1 + e^{-2\beta L})} \left[e^{-\beta(z+s)} + e^{-\beta(z-s)} + e^{-2\beta L} e^{\beta(z-s)} + e^{-2\beta L} e^{\beta(z+s)} \right], \quad (\text{A.19})$$

$$\gamma_{10}^+ = \frac{\mu^s \xi}{2k_1^2} \left[2\alpha c_2^+ \frac{1}{e^{\alpha L}} - 2\alpha c_4^+ \frac{1}{R} (\alpha\beta + \xi^2) + (\beta^2 + \xi^2) c_6^+ \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-\beta L} - (\beta^2 + \xi^2) c_8^+ \right], \quad (\text{A.20})$$

where

$$c_1^- = \left(b_1 \frac{1}{e^{\alpha L}} - 1 \right) \frac{1}{\alpha} e^{\alpha(z-s)} - b_2 \frac{1}{e^{\alpha L}} 2\beta (\beta^2 + \xi^2) e^{\alpha z - \beta s} - b_3 \frac{1}{e^{\alpha L}} \frac{1}{\alpha} e^{\alpha(z+s)} + b_4 \frac{1}{e^{\alpha L}} 2\beta e^{-\beta L} (\beta^2 + \xi^2) e^{\alpha z + \beta s}, \quad (\text{A.21})$$

$$c_2^- = \left(b_1 \frac{1}{e^{\alpha L}} - 1 \right) e^{\alpha(z-s)} - b_2 \frac{1}{e^{\alpha L}} 2\xi^2 (\beta^2 + \xi^2) e^{\alpha z - \beta s} + b_3 \frac{1}{e^{\alpha L}} e^{\alpha(z+s)} - b_4 \frac{1}{e^{\alpha L}} 2\xi^2 e^{-\beta L} (\beta^2 + \xi^2) e^{\alpha z + \beta s}, \quad (\text{A.22})$$

$$c_3^- = (a_4 a_1 - a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \frac{1}{R} (\alpha\beta + \xi^2) \frac{1}{\alpha} e^{-\alpha(z+s)} - (a_4 2\mu^s - a_5 e^{-\beta L}) \frac{1}{R} (\alpha\beta + \xi^2) 2\beta(\beta^2 + \xi^2) e^{-\alpha z - \beta s} \\ - \left[(a_4 a_2 + a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \frac{1}{R} (\alpha\beta + \xi^2) - 1 \right] \frac{1}{\alpha} e^{-\alpha(z-s)} + a_5 e^{-\beta L} \frac{1}{R} (\alpha\beta + \xi^2) 2\beta(\beta^2 + \xi^2) e^{-\alpha z + \beta s}, \quad (\text{A.23})$$

$$c_4^- = (a_4 a_1 - a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \frac{1}{R} (\alpha\beta + \xi^2) e^{-\alpha(z+s)} - (a_4 2\mu^s - a_5 e^{-\beta L}) \frac{1}{R} (\alpha\beta + \xi^2) 2\xi^2 (\beta^2 + \xi^2) e^{-\alpha z - \beta s} \\ + \left[(a_4 a_2 + a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \frac{1}{R} (\alpha\beta + \xi^2) - 1 \right] e^{-\alpha(z-s)} - a_5 e^{-\beta L} \frac{1}{R} (\alpha\beta + \xi^2) 2\xi^2 (\beta^2 + \xi^2) e^{-\alpha z + \beta s}, \quad (\text{A.24})$$

$$c_5^- = \left[e^{-\beta L} - a_3 \frac{1}{R} (a_4 a_1 - a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \right] \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-\beta L} 2\xi e^{\beta z - \alpha s} \\ - \left\{ \left[e^{-\beta L} - a_3 \frac{1}{R} 2\alpha 2\xi^2 \beta (a_4 2\mu^s - a_5 e^{-\beta L}) \right] \frac{1}{a_4} e^{-\beta L} (\alpha\beta + \xi^2) (\beta^2 + \xi^2) - 1 \right\} \frac{1}{\xi} e^{\beta(z-s)} \\ + \left[e^{-\beta L} + a_3 \frac{1}{R} (a_4 a_2 + a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \right] \frac{1}{a_4} e^{-\beta L} 2(\alpha\beta + \xi^2) \xi e^{\beta z + \alpha s} \\ - \left(1 + \frac{1}{R} a_3 a_5 2\alpha 2\xi^2 \beta \right) \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-2\beta L} (\beta^2 + \xi^2) \frac{1}{\xi} e^{\beta(z+s)}, \quad (\text{A.25})$$

$$c_6^- = \left[e^{-\beta L} - a_3 \frac{1}{R} (a_4 a_1 - a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \right] \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-\beta L} 2\alpha e^{\beta z - \alpha s} \\ - \left\{ \left[e^{-\beta L} - a_3 \frac{1}{R} 2\alpha 2\xi^2 \beta (a_4 2\mu^s - a_5 e^{-\beta L}) \right] \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-\beta L} (\beta^2 + \xi^2) - 1 \right\} \frac{1}{\beta} e^{\beta(z-s)} \\ - \left[e^{-\beta L} + a_3 \frac{1}{R} (a_4 a_2 + a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \right] \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-\beta L} 2\alpha e^{\beta z + \alpha s} \\ + \left(1 + \frac{1}{R} a_3 a_5 2\alpha 2\xi^2 \beta \right) \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-2\beta L} (\beta^2 + \xi^2) \frac{1}{\beta} e^{\beta(z+s)}, \quad (\text{A.26})$$

$$c_7^- = b_5 2\xi e^{-\alpha s - \beta z} - b_6 \frac{1}{\xi} e^{-(z+s)} - b_7 2\xi e^{-\beta z + \alpha s} + (b_8 - 1) \frac{1}{\xi} e^{-\beta(z-s)}, \quad (\text{A.27})$$

$$c_8^- = b_5 2\alpha e^{-\beta z - \alpha s} - b_6 \frac{1}{\beta} e^{-\beta(z+s)} + b_7 2\alpha e^{-\beta z + \alpha s} - (b_8 - 1) \frac{1}{\beta} e^{-\beta(z-s)}, \quad (\text{A.28})$$

$$c_1^+ = b_1 \frac{1}{\alpha} e^{\alpha(z-s)} - b_2 2\beta(\beta^2 + \xi^2) e^{\alpha z - \beta s} - b_3 \frac{1}{\alpha} e^{\alpha(z+s)} + b_4 2\beta e^{-\beta L} (\beta^2 + \xi^2) e^{\alpha z + \beta s}, \quad (\text{A.29})$$

$$c_2^+ = b_1 e^{\alpha(z-s)} - b_2 2\xi^2 (\beta^2 + \xi^2) e^{\alpha z - \beta s} + b_3 e^{\alpha(z+s)} - b_4 2\xi^2 e^{-\beta L} (\beta^2 + \xi^2) e^{\alpha z + \beta s}, \quad (\text{A.30})$$

$$c_3^+ = (a_4 a_1 - a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \frac{1}{\alpha} e^{-\alpha(z+s)} - (a_4 2\mu^s - a_5 e^{-\beta L}) (\beta^2 + \xi^2) 2\beta e^{-\alpha z - \beta s} \\ - (a_4 a_2 + a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \frac{1}{\alpha} e^{-\alpha(z-s)} + a_5 e^{-\beta L} 2\beta(\beta^2 + \xi^2) e^{-\alpha z + \beta s}, \quad (\text{A.31})$$

$$c_4^+ = (a_4 a_1 - a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) e^{-\alpha(z+s)} - (a_4 2\mu^s - a_5 e^{-\beta L}) 2\xi^2 (\beta^2 + \xi^2) e^{-\alpha z - \beta s} \\ + (a_4 a_2 + a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) e^{-\alpha(z-s)} - a_5 e^{-\beta L} 2\xi^2 (\beta^2 + \xi^2) e^{-\alpha z + \beta s}, \quad (\text{A.32})$$

$$c_5^+ = \left[e^{-\beta L} - a_3 \frac{1}{R} (a_4 a_1 - a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \right] 2\xi e^{\beta z - \alpha s} - \left[e^{-\beta L} - a_3 \frac{1}{R} 2\alpha 2\xi^2 \beta (a_4 2\mu^s - a_5 e^{-\beta L}) \right] (\beta^2 + \xi^2) \frac{1}{\xi} e^{\beta(z-s)} \\ + \left[e^{-\beta L} + a_3 \frac{1}{R} (a_4 a_2 + a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \right] 2\xi e^{\beta z + \alpha s} - \left(1 + \frac{1}{R} a_3 a_5 2\alpha 2\xi^2 \beta \right) e^{-\beta L} (\beta^2 + \xi^2) \frac{1}{\xi} e^{\beta(z+s)}, \quad (\text{A.33})$$

$$c_6^+ = \left[e^{-\beta L} - a_3 \frac{1}{R} (a_4 a_1 - a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \right] 2\alpha e^{\beta z - \alpha s} - \left[e^{-\beta L} - a_3 \frac{1}{R} 2\alpha 2\xi^2 \beta (a_4 2\mu^s - a_5 e^{-\beta L}) \right] (\beta^2 + \xi^2) \frac{1}{\beta} e^{\beta(z-s)} \\ - \left[e^{-\beta L} + a_3 \frac{1}{R} (a_4 a_2 + a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \right] 2\alpha e^{\beta z + \alpha s} + \left(1 + \frac{1}{R} a_3 a_5 2\alpha 2\xi^2 \beta \right) e^{-\beta L} (\beta^2 + \xi^2) \frac{1}{\beta} e^{\beta(z+s)}, \quad (\text{A.34})$$

$$c_7^+ = b_5 2\xi e^{-\beta z - \alpha s} - b_6 \frac{1}{\xi} e^{-\beta(z+s)} - b_7 2\xi e^{-\beta z + \alpha s} + b_8 \frac{1}{\xi} e^{-\beta(z-s)}, \quad (\text{A.35})$$

$$c_8^+ = b_5 2\alpha e^{-\beta z - \alpha s} - b_6 \frac{1}{\beta} e^{-\beta(z+s)} + b_7 2\alpha e^{-\beta z + \alpha s} - b_8 \frac{1}{\beta} e^{-\beta(z-s)}, \quad (\text{A.36})$$

$$b_1 = (\alpha\beta - \xi^2) (a_4 a_1 - a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \frac{1}{R} e^{-\alpha L} - 2\xi^2 \beta 2\alpha \frac{1}{a_4} \left[e^{-\beta L} - a_3 \frac{1}{R} (a_4 a_1 - a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \right], \quad (\text{A.37})$$

$$b_2 = (\alpha\beta - \xi^2) e^{-\alpha L} (a_4 2\mu^s - a_5 e^{-\beta L}) \frac{1}{R} - \left[e^{-\beta L} - a_3 \frac{1}{R} 2\alpha 2\xi^2 \beta (a_4 2\mu^s - a_5 e^{-\beta L}) \right] \frac{1}{a_4}, \quad (\text{A.38})$$

$$b_3 = (\alpha\beta - \xi^2) e^{-\alpha L} (a_4 a_2 + a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \frac{1}{R} + 2\xi^2 \beta \left[e^{-\beta L} + a_3 \frac{1}{R} (a_4 a_2 + a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \right] \frac{1}{a_4} 2\alpha, \quad (\text{A.39})$$

$$b_4 = (\alpha\beta - \xi^2) e^{-\alpha L} a_5 \frac{1}{R} + \left(1 + \frac{1}{R} a_3 a_5 2\alpha 2\xi^2 \beta \right) \frac{1}{a_4}, \quad (\text{A.40})$$

$$b_5 = \frac{1}{(\beta^2 + \xi^2)} b_1 \frac{1}{e^{\alpha L}} - \frac{1}{(\beta^2 + \xi^2)} (a_4 a_1 - a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \frac{1}{R} (\alpha\beta + \xi^2) \\ + \left[e^{-\beta L} - a_3 \frac{1}{R} (a_4 a_1 - a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \right] \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-\beta L} - \frac{1}{(\beta^2 + \xi^2)}, \quad (\text{A.41})$$

$$b_6 = 2\alpha b_2 \frac{1}{e^{\alpha L}} 2\xi^2 \beta - 2\alpha (a_4 2\mu^s - a_5 e^{-\beta L}) \frac{1}{R} (\alpha\beta + \xi^2) 2\xi^2 \beta \\ + \left[e^{-\beta L} - a_3 \frac{1}{R} 2\alpha 2\xi^2 \beta (a_4 2\mu^s - a_5 e^{-\beta L}) \right] \frac{1}{a_4} e^{-\beta L} (\alpha\beta + \xi^2) (\beta^2 + \xi^2) - 1, \quad (\text{A.42})$$

$$b_7 = \frac{1}{(\beta^2 + \xi^2)} b_3 \frac{1}{e^{\alpha L}} - \frac{1}{(\beta^2 + \xi^2)} (a_4 a_2 + a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \frac{1}{R} (\alpha\beta + \xi^2) \\ - \left[e^{-\beta L} + a_3 \frac{1}{R} (a_4 a_2 + a_5 2\xi^2 \beta e^{-\beta L} 2\alpha) \right] \frac{1}{a_4} e^{-\beta L} (\alpha\beta + \xi^2) + \frac{1}{(\beta^2 + \xi^2)}, \quad (\text{A.43})$$

$$b_8 = 2\alpha b_4 \frac{1}{e^{\alpha L}} 2\xi^2 \beta e^{-\beta L} - 2\alpha a_5 e^{-\beta L} \frac{1}{R} (\alpha\beta + \xi^2) 2\xi^2 \beta - \left(1 + \frac{1}{R} a_3 a_5 2\alpha 2\xi^2 \beta \right) \frac{1}{a_4} (\alpha\beta + \xi^2) e^{-2\beta L} (\beta^2 + \xi^2) + 1, \quad (\text{A.44})$$

$$R = [a_2 (\alpha\beta + \xi^2) + a_1 (\alpha\beta - \xi^2) e^{-2\alpha L}] a_4 - 4\xi^2 \alpha \beta a_3 a_5, \quad (\text{A.45})$$

$$a_1 = (\beta^2 + \xi^2) (2\mu^s \alpha^2 - \lambda^s k_1^2) + 4\mu^s \xi^2 \alpha \beta, \quad (\text{A.46})$$

$$a_2 = (\beta^2 + \xi^2) (2\mu^s \alpha^2 - \lambda^s k_1^2) - 4\mu^s \xi^2 \alpha \beta, \quad (\text{A.47})$$

$$a_3 = (\beta^2 + \xi^2) e^{-\alpha L} - (\alpha\beta + \xi^2) e^{-\beta L} + (\alpha\beta - \xi^2) e^{-(2\alpha+\beta)L}, \quad (\text{A.48})$$

$$a_4 = (\beta^2 + \xi^2) (\alpha\beta - \xi^2) + (\beta^2 + \xi^2) (\alpha\beta + \xi^2) e^{-2\beta L} - 4\alpha \beta \xi^2 e^{-(\alpha+\beta)L}, \quad (\text{A.49})$$

$$a_5 = 2\mu^s (\beta^2 + \xi^2) (\alpha\beta + \xi^2) e^{-\beta L} - a_1 e^{-\alpha L}, \quad (\text{A.50})$$

and

$$\begin{aligned} X_n &= \tilde{P}_n^{n-1}(\xi) - i\tilde{Q}_n^{n-1}(\xi), \\ Y_n &= \tilde{P}_n^{n+1}(\xi) - i\tilde{Q}_n^{n+1}(\xi), \\ Z_n &= \tilde{R}_n^n(\xi). \end{aligned} \quad (\text{A.51})$$

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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