Research Article

Study on Instability Mechanism of Extraction Structure under Undercut Space Based on Thin Plate Theory in Block Caving Method

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The instability of extraction structure under the undercut space in the block caving stope presents specific characteristics: rib spalling and floor heave in ore-loading roadway and collapse of major apexes. In order to study the stress and displacement evolution law of extraction structure under undercut space and reveal the instability mechanism of extraction structure, the numerical simulation model of block caving stope was established using the finite difference software FLAC 3D. According to the postundercutting strategy in Tongkuangyu Mine in China, extraction structure was formed first in the simulation process, and then the undercut level was divided into eight units for excavation step by step. The stress and displacement of extraction structure after each step of undercutting were monitored and analyzed. Based on the thin plate theory, the mechanism of stress change and deflection deformation of extraction structure was revealed. The research results show that, under the action of high horizontal tectonic stress and vertical stress, the extraction structure under undercut space produces vertical upward bending deformation after undercutting during the block caving. The tension stress concentration gradually appears in the side wall of the ore-loading roadway and the tip of the major apexes; with the increase of the undercutting area, the degree of tensile stress concentration gradually becomes strong; when the tensile strength of the rock mass in extraction structure is exceeded, extraction structure presents instability. It is necessary to make the overlying ore collapse on extraction structure as soon as possible after undercutting, which is beneficial to release the tension stress in the extraction structure under undercutting space.

1. Introduction

As a large-scale, low-cost, and high-efficiency underground mining method, block caving mining method is the preferred method for deep ore resources when conditions permit [1–3]. The extraction structure of block caving mining method refers to a series of roadways and excavations between the undercut level and the production level [4, 5]. The extraction structure undertakes the task of ore drawing, and all ores need to be transported out of the stope through the extraction structure. Therefore, ensuring the safety and stability of the extraction structure is one of the key factors for the successful application of the block caving method. Due to the particularity of block caving mining, the service life of extraction structure is long, and thus extraction structure is in a complex and changing high stress environment, which makes it difficult to maintain its stability, so that it has high risk of instability.

Many scholars have studied the theories and methods related to the stability of the extraction structure and ground pressure behavior in block caving method. Trueman et al. [6–8] used numerical simulation, theoretical analysis, on-
site monitoring, and other means to study the influence of undercut strategy, tectonic stress, extraction structure height, advancing undercut front retention time, and other factors on the extraction structure of block caving method, and control measures were put forward. Rojas et al. [9, 10] studied the causes of the failure of the extraction structure in the ore caving stage by monitoring the stress, deformation, crack propagation, and microseismic events of the ore-loading roadway. Xia et al. [11, 12] studied the instability mechanism of the extraction structure under the conditions of multiple production levels in one panel and poor undercutting in block caving method by using three-dimensional numerical simulation method. The research on the force of ore bulk on the extraction structure under the undercut space has gradually attracted the attention of mining scholars. Pierce [13–15] used limit equilibrium method, empirical formula, and numerical simulation to study the force of ore bulk on the extraction structure in the ore drawing stage, so as to guide the maintenance of the extraction structure in the ore drawing process. Castro et al. [16, 17] studied the variation of vertical stress of extraction structure in stope under three kinds of ore drawing conditions by using laboratory physical model; the research shows that the vertical stress was mainly affected by the area of ore drawing area and the distance from the ore drawing advancing front.

In the actual production of block caving mine, under the condition of large high horizontal tectonic stress, with the advance of undercutting, the extraction structure under the undercut space presented instability in some areas. However, previous studies have not really revealed the root cause of stress and deformation of the extraction structure under undercut space in the block caving. Based on the engineering background of Tongkuangyu Mine, a typical block caving mine in China, the evolution characteristics of the stress and displacement of the extraction structure under the undercut space in the process undercutting of block caving were simulated by using the three-dimensional finite difference software FLAC3D, the stress of the extraction structure under the undercut space was analyzed based on the thin plate theory, and the instability mechanism of the extraction structure under the undercut pulling space was revealed. It is of great significance to guide the stability maintenance of the extraction structure in block caving mines.

2. Instability Characteristics of Extraction Structure under Undercut Space in Tongkuangyu Mine

Postundercutting strategy is adopted in Tongkuangyu Mine; the excavation of the extraction structure precedes the blasting of undercut level. The disadvantage of the post-undercutting strategy is that stress concentration is formed easily in the extraction structure in front of an advancing undercut front, which has been confirmed by many scholars [6]. The in-situ stress of Tongkuangyu Mine was measured by stress relief method; the maximum principal stress was in the horizontal direction, which was almost consistent with the strike of ore body. According to the investigation on-site, it was found that under the influence of high horizontal tectonic stress, with the increase of undercut area, the extraction structure under undercut space presented partial instability; the instability characteristics presented rib spalling and floor heave in ore-loading roadway and collapse of major apexes. As shown in Figure 1, the most easily observed features at the production level were the deformation and failure of the ore-loading roadway.

3. Stress Distribution and Evolution Law of Extraction Structure under Undercut Space

3.1. Model Construction and Simulation Process. In this study, a three-dimensional finite difference software FLAC3D was used to establish the numerical model, and the model range was taken from the actual mining range of Tongkuangyu Mine. The model was 385 m long, 250 m wide, and 260 m high, with 989824 units in total. The overall structure of the model is shown in Figure 2(a). There are 8 ore-crossing veins and 31 ore-loading roadways and draw-bells in production level, and the burial depth of production level is 550 m. The draw-bell is 13 m long, 10 m high, and 6.4 m wide at the bottom and 11 m wide at the top. The size of ore-loading roadways and the ore drawing roadways is 3.8 m × 3.2 m (width × height). The internal structure diagram and roadway name of the model are shown in Figure 2(b).

In the simulation process, the extraction structure was formed firstly, and then undercut level was divided into 7 units for excavation step by step. The width of the undercutting unit advanced in each step was 30 m, which was the distance between the two adjacent ore-loading roadways. The stress and displacement of extraction structure under undercut space were monitored after the excavation of each unit. The excavation sequence of undercut unit is shown in Figure 2(c).

The rock mass of Tongkuangyu Mine was sampled on-site and then made into standard rock samples for laboratory test. The measured rock parameters were reduced and weakened to obtain the parameters of engineering rock mass which could be used for numerical simulation. The Mohr–Coulomb failure criterion was used in the model calculation, but it could not effectively describe the influence of joints, fracture, and structural planes on the strength of rock mass; however, Hoek–Brown criterion could accurately characterize the failure of rock mass materials with a large number of joints and fracture. The cohesion and internal friction angle in the equivalent Mohr–Coulomb failure criterion were converted by Hoek–Brown criterion, so that the Mohr–Coulomb failure criterion could be better applied to the engineering rock mass in the numerical model. The physical and mechanical parameters required for simulation are shown in Table 1. The vertical stress was applied on the top of the model and the in-situ stress was applied inside the model. The horizontal movement is limited on the side of the model, and the vertical movement was limited on the bottom of the model.

The characteristic equations of in-situ stress are shown as follows:
Figure 1: Deformation and failure of ore-loading roadway in Tongkuangyu Mine. (a) Rib spalling in ore-loading roadway. (b) Floor heave in ore-loading roadway.

Figure 2: Numerical model. (a) Overall structure of the model. (b) Internal structure of the model. (c) Excavation sequence of undercut unit.

Table 1: Physical and mechanical parameters of rock mass in the model.

<table>
<thead>
<tr>
<th>Item</th>
<th>Elasticity modulus, $E$ (GPa)</th>
<th>Bulk density, $\rho$ (kg/m$^3$)</th>
<th>Cohesive strength, $C$ (MPa)</th>
<th>Shear modulus, $G$ (GPa)</th>
<th>Internal friction angle, $\Phi$ (°)</th>
<th>Tensile strength, $\sigma_t$ (MPa)</th>
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\[
\begin{align*}
\sigma_x &= 22.8954 - 0.0399Z, \\
\sigma_y &= 11.6484 - 0.0204Z, \\
\sigma_z &= 14.6600 - 0.0269Z.
\end{align*}
\]

3.2. Distribution Characteristics of Stress and Displacement of Extraction Structure under Undercut Space. After three-dimensional numerical simulation, the characteristic cutting planes of stress and displacement contours after each excavation step were selected for comparative analysis, and the stress concentration area and large deformation area after each excavation step could be found, thus the location of the extraction structure prone to ground pressure failure could be obtained.

Evolution characteristics contours of minimum principal stress in XOZ cutting plane of extraction structure are shown in Figure 3. In the contours, the negative value represents the compressive stress, and the positive value represents the tensile stress. The red area is the high stress concentration area of the minimum principal stress, which is positive, so it represents tension stress. Figure 3(a) shows the stress state of the extraction structure when the undercut strike length is 90 m. It can be seen that the side wall of ore-loading roadways and the tip of major apexes under the undercut space are in the tension stress concentration area, and the maximum tension stress value reaches 4.82 MPa. With the continuous advancement of undercutting, the strike length of undercut gradually reaches 210 m, and the stress state of the extraction structure is shown in Figure 3(b) at this time. It can be seen that the concentration of tension stress in the side wall of ore-loading roadways and the tip of major apexes under the undercut space increases, and the maximum tensile stress value reaches 5.07 MPa. Therefore, with the increase of undercut area, it is easy to reach the tension failure condition at the side wall of ore-loading roadways and the tip of major apexes under the undercut space, which leads to rib spalling of ore-loading roadways and the collapse of the tip of major apexes.

Evolution characteristics contours of vertical displacement in XOZ cutting plane of extraction structure are shown in Figure 4. In the contours, the negative value represents the vertical downward displacement, and the positive value represents the vertical upward displacement. Figure 4(a) shows the vertical displacement state of the extraction structure when the undercut strike length is 90 m. It can be seen that the whole extraction structure under the undercut space has a vertical upward displacement, and the vertical upward displacement of major apexes tip in the middle of the stope is the largest, reaching 98 mm. With the continuous advancement of undercutting, the strike length of undercut gradually reaches 210 m, and the vertical displacement state of the extraction structure is shown in Figure 4(b) at this time. It can be seen that the vertical upward displacement of the side wall of the ore-loading roadways and the tip of major apexes under the undercut space increases, and the maximum vertical displacement reaches 116 mm. Therefore, with the increase of the undercut area, the vertical displacement of the extraction structure under the undercut space increases.

3.3. Evolution Law of Stress and Displacement of Extraction Structure under Undercut Space. The stress and displacement monitoring points were arranged at the side wall of the ore-loading roadways and the tip of major apexes under the undercut space. The variation law of the maximum tensile stress and maximum vertical displacement with the undercut strike length is shown in Figure 5. It can be seen from Figure 5(a) that with the increase of the advancing strike length of the undercut, the tension stress concentration of the major apexes under the undercut space becomes intense, the tension stress of the side wall of the ore-loading roadways increases in an oscillatory manner, and they are all in a state of high tension stress concentration. It can be seen from Figure 5(b) that the maximum vertical displacement of the major apexes and the side wall of the ore-loading roadways under the undercut space increases with the increasing of the advancing strike length of the undercut.

4. Analysis of Instability Mechanism of Extraction Structure under Undercut Space Based on Thin Plate Theory

4.1. Establishment of Equivalent Model of Extraction Structure Thin Plate. In order to reveal the mechanism of the bending deformation and tension stress increase of the extraction structure under the undercut space, based on the thin plate theory of elasticity, an equivalent model of the extraction structure under the lateral and longitudinal compression of Winkler elastic support was proposed in this section. The extraction structure rock mass under the undercut space was equivalent to a rectangular plate with reduced elastic modulus, which was simply supported on four sides, as shown in Figure 6. Due to the bottom of the thin plate equivalent model is connected with the rock mass floor, it can be assumed that the displacement \( \omega \) at any point on the floor surface is proportional to the stress \( q \) at that point, that is, the rock mass floor is composed of a series of spring elements. The lateral compressive stress is \( q_x \) perpendicular to the plate surface, and the longitudinal pressure is the in-plane pressure of the thin plate; there are the two horizontal principal stresses \( q_x \) and \( q_y \). The \( x \)-axis direction of the model is the inclination of extraction structure in the stope, and the length is set as \( a \). The direction of \( y \)-axis is the strike of extraction structure in the stope, and the length is set as \( b \). The direction perpendicular to the extraction structure of stope is \( z \)-axis, and the thickness of thin plate is \( h \).

Because the height of the extraction structure is about 16 m, the minimum width of the undercut range of the stope in Tongkuangyu Mine is 80 m. According to the definition of the thin plate, the ratio of the thickness \( h \) of the thin plate to the minimum size of the middle surface is in the range of \( 1/80 \sim 1/5 \), which is the thin plate. It is obvious that the model is in the range of the thin plate definition. When the thin plate is bent, the middle surface becomes a curved surface, and the
Figure 3: Evolution characteristics contour of minimum principal stress in XOZ cutting plane of extraction structure. (a) The undercut strike length was 90 m. (b) The undercut strike length was 210 m.

Figure 4: Evolution characteristics contour of vertical displacement in XOZ cutting plane of extraction structure. (a) The undercut strike length was 90 m. (b) The undercut strike length was 210 m.

Figure 5: Stress and displacement evolution law of extraction structure. (a) Evolution law of maximum tension stress. (b) Evolution law of maximum vertical displacement.
$z$-direction displacement of each point on the middle surface is called deflection ($\omega$). When $(a/h) \leq (1/5)$, it is called small deflection bending. According to the field investigation results of extraction structure instability, the extraction structure deformation meets the condition of small deflection bending.

The extraction structure rock mass under the undercut space is approximately equivalent to the elastic thin plate after the reduction of elastic modulus. Due to the extraction structure thin plate is an incomplete thin plate, it would cause incompatible deformation and stress inside. However, the causes and influencing factors of the extraction structure bending deformation and tension stress are analyzed in this section only, so the extraction structure elastic thin plate is equivalent to a homogeneous thin plate. The elastic modulus of the equivalent elastic thin plate should be reduced according to the excavation rate of the roadway. According to statistics, the excavation rate of the roadway in the extraction structure rock space of Tongkuangyu Mine is about 40%, thus the elastic modulus $E_D$ of the equivalent elastic thin plate can be calculated according to the following formula:

$$E_D = (1 - r)E = 0.6E.$$  \hspace{1cm} (2)

In formula (2), $E_D$ is the equivalent elastic modulus of the extraction structural thin plate, $E$ is the elastic modulus of complete rock mass, and $r$ is the excavation rate of extraction structure rock mass.

### 4.2. Solution of Deflection and Tension Stress of Extraction Structure under Undercut Space

According to the static equilibrium condition of the equivalent thin plate differential body, the small deflection bending differential equation can be derived as follows:

$$DV^4\omega = \left( N_x \frac{\partial^2 \omega}{\partial x^2} + 2N_y \frac{\partial^2 \omega}{\partial x \partial y} + N_y \frac{\partial^2 \omega}{\partial y^2} \right)$$  \hspace{1cm} (3)

$$= q_0 - k\omega,$$

$$D = \frac{E_D h^3}{12(1 - \mu^2)}.$$  \hspace{1cm} (4)

where $D$ is the bending stiffness of the extraction structure thin plate, $\omega$ is the deflection of the extraction structural plate, $V$ is the Laplacian operator, $N_x$, $N_y$, and $N_{xy}$ are the internal forces of thin plate, $q_0$ is the initial lateral stress of thin plate model of extraction structure, $k$ is the foundation modulus of floor under extraction structure, $h$ is the thickness of extraction structure thin plate, and $\mu$ is Poisson’s ratio of thin rock mass of extraction structure.

Due to the thin plate of the extraction structure is approximately regarded as satisfying the boundary condition of simply supported four edges, the deflection of the extraction structure can be calculated by the Navier method, and the deflection expression is assumed to be a double trigonometric series:

$$\omega = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}.$$  \hspace{1cm} (5)

According to the longitudinal load of the thin plate, the internal force of the middle plane is synthesized as follows:

$$N_x = q_x h, \quad N_y = q_y h,$$

$$N_{xy} = 0.$$  \hspace{1cm} (6)

By combining formula (6) with formula (3), the following formula can be given:

$$DV^4\omega = q_0 - k\omega - q_x h \frac{\partial^2 \omega}{\partial x^2} - q_y h \frac{\partial^2 \omega}{\partial y^2}.$$  \hspace{1cm} (7)

In order to obtain $A_{mn}$ in the deflection expression, we need to expand $q_0$ in the form of double trigonometric series:

$$q_0 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}.$$  \hspace{1cm} (8)

In order to find a suitable coefficient $a_{m'n'}$ of this trigonometric series, multiply $\sin(n'\pi y/b)dy$ on both sides of equation (8) to find the integral from 0 to b, when $n \neq n'$,

$$\int_0^b \sin(n'\pi y/b)\sin(n\pi y/b)dy = 0,$$

and when $n = n'$,

$$\int_0^b \sin(n'\pi y/b)\sin(n'\pi y/b)dy = (b/2).$$

Therefore, the following formula can be given:

$$\int_0^b q_0 \sin \frac{n'\pi y}{b}dy = \frac{b}{2} \sum_{m=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a}.$$  \hspace{1cm} (9)

Multiply $\sin(n'\pi x/a)dx$ on both sides of formula (8) and find the integral from 0 to a,

$$\int_0^a \int_0^b q_0 \sin \frac{m'\pi x}{a} - \sin \frac{n'\pi x}{b}dx dy = \frac{ab}{4} a_{m'n'}.$$  \hspace{1cm} (10)

When $m$ and $n$ are odd numbers, a suitable coefficient $a_{m'n'}$ is obtained:
\[
\begin{align*}
q_0 &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{16q_0}{\pi mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}, \quad m = 1, 3, 5, \ldots, n = 1, 3, 5, \ldots \tag{12}
\end{align*}
\]

By combining formula (12), deflection expression (5), and formula (7), the following formula can be given:

\[
\begin{align*}
\pi^4 D \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{16q_0}{\pi mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \\
&- k \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} + q_x h \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2\pi^2}{a^2} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \\
&+ q_y h \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{n^2\pi^2}{b^2} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \\
&= 0. \tag{13}
\end{align*}
\]

In order to simplify the calculation, according to the actual deflection convergence degree of the extraction structure under the undercut space, when \(m\) and \(n\) are taken as 1, the accuracy requirement of deflection can be basically met. Therefore, formula (13) can be written as

\[
\begin{align*}
\pi^4 A_{11} \left( \frac{1}{a^2} + \frac{1}{b^2} \right)^2 - \frac{16q_0}{\pi} + kA_{11} - \frac{\pi^2 q_x h}{a^2} A_{11} - \frac{\pi^2 q_y h}{b^2} A_{11} = 0. \tag{14}
\end{align*}
\]

Due to \(\sin(\pi x/a)\sin(\pi y/b)\) cannot be 0, the following formula can be given:

\[
\begin{align*}
\pi^4 A_{11} \left( \frac{1}{a^2} + \frac{1}{b^2} \right)^2 - \frac{16q_0}{\pi} + kA_{11} - \frac{\pi^2 q_x h}{a^2} A_{11} - \frac{\pi^2 q_y h}{b^2} A_{11} = 0. \tag{15}
\end{align*}
\]

From formula (15), it can be obtained that

\[
A_{11} = \frac{16q_0}{\pi^2 \left[ \pi^4 D \left( \frac{1}{a^2} + \frac{1}{b^2} \right)^2 + k - \left( \frac{\pi^2 q_x h}{a^2} \right)^2 - \left( \frac{\pi^2 q_y h}{b^2} \right)^2 \right]}.
\]

Therefore, the double trigonometric series expansion of \(q_0\) can be obtained:
The deflection formula of the equivalent thin plate model of the extraction structure can be obtained:

$$\omega = \frac{16q_0}{\pi^2 \left[ \pi^4 D\left(\frac{1}{a^2}\right) + \left(\frac{1}{b^2}\right)^2 + k - \left(\pi^2 q_x h/a^3\right) - \left(\pi^2 q_y h/b^3\right) \right]} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}$$ (17)

According to the physical equation of thin plate, three main stress components are expressed as deflection:

$$\sigma_x = \frac{E_D z}{1 - \mu^2} \frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial y^2}$$,

$$\sigma_y = \frac{E_D z}{1 - \mu^2} \frac{\partial^2 \omega}{\partial y^2} + \mu \frac{\partial^2 \omega}{\partial x^2}$$,

$$\tau_{xy} = \frac{E_D z}{1 + \mu} \frac{\partial^2 \omega}{\partial x \partial y}$$ (18)

By combining formula (17) and deflection expression (18), the stress of each point of the extraction structure thin plate model can be obtained:

$$\sigma_x = \frac{8E_D hq_0}{\pi^2 \left[ \pi^4 D\left(\frac{1}{a^2}\right) + \left(\frac{1}{b^2}\right)^2 + k - \left(\pi^2 q_x h/a^3\right) - \left(\pi^2 q_y h/b^3\right) \right]} \frac{a^2 \mu + b^2}{\pi^4 D\left(\frac{1}{a(b/a)^2}\right)^2 + \left(\frac{1}{b^2}\right)^2} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b}$$

$$\sigma_y = \frac{8E_D hq_0}{\pi^2 \left[ \pi^4 D\left(\frac{1}{a^2}\right) + \left(\frac{1}{b^2}\right)^2 + k - \left(\pi^2 q_x h/a^3\right) - \left(\pi^2 q_y h/b^3\right) \right]} \frac{a^2 + b^2 \mu}{\pi^4 D\left(\frac{1}{a(b/a)^2}\right)^2 + \left(\frac{1}{b^2}\right)^2} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b}$$

$$\tau_{xy} = \frac{8E_D hq_0}{\pi^2 \left[ \pi^4 D\left(\frac{1}{a^2}\right) + \left(\frac{1}{b^2}\right)^2 + k - \left(\pi^2 q_x h/a^3\right) - \left(\pi^2 q_y h/b^3\right) \right]} \frac{ab}{\pi^4 D\left(\frac{1}{a(b/a)^2}\right)^2 + \left(\frac{1}{b^2}\right)^2} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b}$$ (19)

When \(x = (a/2)\) and \(y = (b/2)\), the deflection of the extraction structure thin plate model is the largest:

$$\omega_{\text{max}} = \frac{16q_0}{\pi^2 \left[ \pi^4 D\left(\frac{1}{a^2}\right) + \left(\frac{1}{b^2}\right)^2 + k - \left(\pi^2 q_x h/a^3\right) - \left(\pi^2 q_y h/b^3\right) \right]}$$ (20)

When \(x = (a/2), y = (b/2), \) and \(z = (h/2)\), at this point, the bending moment of the thin plate is the largest, and the tension stress is the largest at the upper part of the thin plate:

$$\sigma_{x_{\text{max}}} = \frac{8E_D hq_0}{\pi^2 \left[ \pi^4 D\left(\frac{1}{a^2}\right) + \left(\frac{1}{b^2}\right)^2 + k - \left(\pi^2 q_x h/a^3\right) - \left(\pi^2 q_y h/b^3\right) \right]} \frac{a^2 \mu + b^2}{\pi^4 D\left(\frac{1}{a(b/a)^2}\right)^2 + \left(\frac{1}{b^2}\right)^2}$$

$$\sigma_{y_{\text{max}}} = \frac{8E_D hq_0}{\pi^2 \left[ \pi^4 D\left(\frac{1}{a^2}\right) + \left(\frac{1}{b^2}\right)^2 + k - \left(\pi^2 q_x h/a^3\right) - \left(\pi^2 q_y h/b^3\right) \right]} \frac{a^2 + b^2 \mu}{\pi^4 D\left(\frac{1}{a(b/a)^2}\right)^2 + \left(\frac{1}{b^2}\right)^2}$$

$$\tau_{xy_{\text{max}}} = 0.$$ (21)
As the strike length of stope b is greater than the dip length of stope a, according to equation (21), it can be concluded that $\sigma_{x_{\text{max}}} > \sigma_{y_{\text{max}}}$, thus $\sigma_{x_{\text{max}}}$ is the maximum tensile stress of thin plate.

4.3. Variation Law of Deflection and Tension Stress of Extraction Structure under Undercut Space. In order to study the variation law of the maximum deflection and maximum tension stress of the equivalent model of the extraction structure thin plate with the advancing of the undercutting and the initial lateral stress, according to the geological data of Tongkuangyu Mine, the calculation parameters in the formula $\omega_{\text{max}}$ and $\sigma_{\text{max}}$ are assigned; the calculation parameters are shown in Table 2.

According to the actual process of extraction drawing in Tongkuangyu Mine, when the horizontal inclined length of undercut reached a certain value, it remained unchanged, and the horizontal strike length of undercut increased continuously until the end of ore drawing in the middle section. Therefore, the variation of the maximum deflection and the maximum tension stress of the thin plate equivalent model with the increase of the undercut area can be simplified as the variation with the increase of the horizontal strike length of the undercut. After assignment, the variation rules of the maximum tension stress and the maximum deflection with the increase of the horizontal strike length under different initial stresses are shown in Figure 7.

It can be seen from Figure 7(a) that the maximum tension stress of thin plate increases with the increase of strike length, but the growth rate slows down and finally tends to be stable. The decrease of initial lateral stress $q_0$ weakens the maximum tension stress concentration but does not affect the variation trend of maximum tension stress with strike length. Therefore, with the increase of the undercut area, the maximum tension stress of the extraction structure rock mass under the undercut space would increase. When the tensile strength of the rock mass is exceeded, extraction structure presents instability. To make overlying ore cave on the extraction structure as soon as possible, the initial lateral stress in the vertical direction can be reduced, thus the tension stress of the extraction structure below the undercut space can be released to a certain extent, which can be conducive to the stability of the extraction structure. It can be seen from Figure 7(b) that the maximum vertical displacement of thin plate increases with the increase

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<th>Serial number</th>
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<tr>
<td>2</td>
<td>Foundation modulus, $k$ (MPa/m)</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>The inclination of extraction structure, $a$ (m)</td>
<td>80</td>
</tr>
<tr>
<td>4</td>
<td>Equivalent thickness of thin plate, $h$ (m)</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>Poisson’s ratio, $\mu$</td>
<td>0.22</td>
</tr>
<tr>
<td>6</td>
<td>Horizontal initial stress in x direction, $q_x$ (MPa)</td>
<td>11.6</td>
</tr>
<tr>
<td>7</td>
<td>Horizontal initial stress in y direction, $q_y$ (MPa)</td>
<td>22.9</td>
</tr>
</tbody>
</table>

Figure 7: Stress and displacement evolution law of equivalent thin plate. (a) Evolution law of maximum tension stress in equivalent thin plate. (b) Evolution law of maximum vertical displacement in equivalent thin plate.
of strike length, but the growth rate slows down and finally tends to be stable. The decrease of initial lateral stress $q_0$ weakens the maximum vertical displacement, but does not affect the variation trend of maximum vertical displacement with strike length. Therefore, with the increase of the undercut area, the maximum vertical displacement of the extraction structure rock mass under the undercut space would increase. To make overlying ore cave on the extraction structure as soon as possible, the vertical displacement of the extraction structure below the undercut space can be released to a certain extent.

5. Analysis of Instability Mechanism of Extraction Structure under Undercut Space

The extraction structure under the undercut space is similar to a weak “thin rock stratum” at the bottom of the stope. Under the influence of high horizontal tectonic stress and vertical stress, the excavation space is squeezed, and the extraction structure would produce upward bending deformation. In order to resist the deflection deformation, the extraction structure gradually presents tension state. With the increasing of the undercut space, the concentration degree of tension stress becomes higher and higher. When the tension strength of the rock mass is exceeded, extraction structure presents instability. The tension stress is easy to concentrate on the side wall of the ore-loading roadways and the tip of major apexes under the undercut space. Therefore, with the increase of the undercut area and the insufficient support strength of the extraction structure, the ore-loading roadways would show spalling, and the major apexes would gradually collapse. Due to the whole extraction structure would have vertical upward bending deformation, if the mine only strengthens the support of side wall and roof and does not pay attention to the maintenance of the floor of the production roadways, the floor of the production roadway would heave. At the same time, inducing the overlying strata to collapse on the extraction structure and carrying out orderly and balanced ore drawing can reduce the vertical deflection and tension stress concentration of the extraction structure under the undercut space, so as to ensure the stability of the extraction structure. The combined support form of bolt mesh cable shotcreting and floor concrete reverse arch could effectively control the loose deformation of the surrounding rock of the ore-loading roadway and improve the overall strength of the extraction structure and increase the stability of the extraction structure.

6. Conclusions

The evolution characteristics of the stress and displacement of the extraction structure under the undercut space in the process undercutting of block caving were simulated using FLAC³D based on the engineering background of Tongkuangyu Mine, the stress of the extraction structure under the undercut space was analyzed based on the thin plate theory, and the instability mechanism of the extraction structure under the undercut pulling space was revealed. The conclusions are as follows:

1. Under the action of high horizontal tectonic stress and vertical stress, the extraction structure under undercut space produces vertical upward bending deformation after undercutting during the block caving.
2. The tension stress concentration gradually appears in the side wall of the ore-loading roadway and the tip of the major apexes; with the increase of the undercutting area, the degree of tensile stress concentration gradually becomes strong; when the tensile strength of the rock mass in extraction structure is exceeded, extraction structure presents instability.
3. It is necessary to make the overlying ore collapse on extraction structure as soon as possible after undercutting, which is beneficial to release the tension stress in the extraction structure under undercutting space.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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