

Research Article

Insights into Underrail Rubber Pad's Effect on Vehicle-Track-Viaduct System Dynamics

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Received 7 February 2021; Accepted 22 May 2021; Published 7 June 2021

Academic Editor: Paola Forte

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To scientifically characterize the dynamic mechanical characteristics of the rubber pad under the rail of fasteners and its influence on the dynamic response of the vehicle-rail-viaduct system, taking the rubber pad under the rail of WJ-7B high-speed railway (HSR) with constant resistance as an example, a TFDV model was applied to characterize the viscoelasticity of the rubber pad and the theoretical model in the dynamic coupling of vehicle-rail-bridge was also studied. The results show that the energy storage modulus and loss factor of rubber pad under rail show a curved surface relation versus the change of frequency-temperature. In a certain frequency/temperature range, the energy storage modulus and loss factor of rubber pad under rail increase with the decrease of temperature and the increase of frequency, and the influence of low temperature on dynamic parameters is more significant. With the decrease of temperature, the minimum value of total dynamic flexibility decreases, and the corresponding extreme frequency shifts to high frequency. Viscoelastic dynamic features of rubber pad under rail mainly affect the dynamic response of vehicle subsystem and rail-bridge subsystem. With the decrease of ambient temperature of rubber pad, the dominant frequency band of power spectrum curve of each structure shifts to high frequency.

1. Introduction

High speed railway (HSR) serves as the effective solution to reduce the private transport and enhance the public transport, as well to reorganize the urban zone due to its high load, fast speed, high safety, and comfort. HSR has developed rapidly in China [1–3]. Because the elevated bridge track transportation can occupy less land and can more reliably control the long-term settlement of the foundation, it has good line smoothness and stability [4, 5]. When a high-speed train is running on a bridge, the wheelrail system will vibrate severely due to the irregularity and be transmitted to the foundation under the rail, which will cause the vibration of the vehicle-track-bridge system, leading to the change of the operating quality and service life [6]. To ensure the high quality of high-speed rail operation, high-speed rail tracks often use rail fasteners

with higher flexibility. The fastener not only is a necessary connecting device in the railway structure, but also plays a role of track vibration reduction. The main component that provides vibration reduction performance is the under-rail rubber pad in the fastener. In previous studies on vibration of vehicle-track-bridge systems at home and abroad, the dynamic mechanical parameters (stiffness, damping) of under rail rubber pads often adopt fixed values [7-10]. Because the underrail rubber pad component contains rubber polymer materials, its mechanical performance parameters under dynamic force are not constant. According to the literature [11-15], rubber polymer materials have viscoelasticity (referring to the mechanical properties of rubber materials under the influence of factors such as ambient temperature and excitation frequency which are also called temperature/ frequency dynamic characteristics). This dynamic

mechanical characteristic will change the dynamic mechanical parameters of the underrail rubber pad [16], which will affect the vibration response of the vehicletrack-viaduct system.

Relevant research studies were carried out on the dynamic parameters of rail fasteners. Yin et al. [17] analyzed the effect of the frequency variation characteristics of fastener's stiffness on wheel-rail vibration and noise based on the vehicle-track coupling dynamics theory and acoustic theory; Wang et al. [18, 19] used mathematical method and virtual excitation method to analyze the vehicle-rail vertical coupled random vibration considering the frequency change of the fastener pad's stiffness. The effect of the viscoelastic dynamic characteristics of the rubber pad on the vertical coupled vibration of the vehicle-rail-viaduct system is still rarely studied.

To explore the influence of viscoelasticity of the rubber pad under the track on the vehicle-track-viaduct vertical coupling dynamic response and accurately predict the dynamic response of the vehicle-track-viaduct system, this study takes the underrail rubber pad of the HSR WJ-7B constant resistance fastener as example and establishes a constitutive model that comprehensively considering the viscoelasticity of the under-rail rubber pad, and uses genetic algorithm to identify the structural model parameters by combining the established constitutive model with the separated parameter model and the vehicle-rail-viaduct coupled vibration frequency domain model to form a frequency domain viaduct vibration model that takes into account the viscoelastic dynamic characteristics of the underrail rubber pad. The viscoelasticity effect of underrail rubber pads on the dynamic response of the vehicle-railviaduct system is studied.

2. Vehicle-Rail-Viaduct Coupling Model

Dynamic flexibility method is used to construct the vehiclerail-bridge coupling model shown in Figure 1. Since the parameters involved in the calculation of vehicle-track coupled vibration in the under-rail pad are stiffness and loss factor, it is necessary to convert the modulus value into a stiffness value according to the structural size of the material. The calculation is as follows:

$$k_f = \frac{AE_s}{h},\tag{1}$$

where k_f indicates the stiffness of the rubber pad under the rail and *A* and *h* mean the bottom effective area (according to the actual bearing area and the measured size of the rubber pad under the rail, the effective area is reduced by 30% of the actual size) and thickness of the rubber pad under the buckle rail, respectively. E_s is the storage modulus of the rubber pad under the rail.

The vehicle-rail-viaduct coupling dynamics model used in this study is shown in Figure 2. The vehicle system uses the CRH380A high-speed train, using a 10-DOF longitudinal half-vehicle model of a single carriage [20]. The rail is simplified to be an infinite length Timoshenko beam; the track plate and the base version are simplified to Shock and Vibration

Euler–Bernoulli beams with free ends, and the bridge is simplified to a simple supported Euler beam at both ends. Fasteners, CA mortar layer, sliding layer, and bridge support are all considered as discrete viscoelastic support unit [21]. The model calculation process is shown in Figure 3.

2.1. Vehicle Vibration Model. The vertical vibration equation of a single carriage is as follows:

$$[M_{\nu}]\{\ddot{Z}_{\nu}\} + [C_{\nu}]\{\dot{Z}_{\nu}\} + [K_{\nu}]\{Z_{\nu}\} = \{P_{\nu}\}, \qquad (2)$$

where $[M_{\nu}]$ means the quality matrix of the carriage; $[C_{\nu}]$ represents the damping matrix of the carriage; $[K_{\nu}]$ indicates the stiffness matrix of the carriage; $\{Z_{\nu}\}$, $\{\dot{Z}_{\nu}\}$, and $\{\ddot{Z}_{\nu}\}$ indicate the displacement, velocity, and acceleration of the carriage, respectively, and $\{P_{\nu}\}$ denotes the wheel-rail force.

Separating the variables of (2) shall obtain

$$[\beta_w] = \frac{[Z_w]}{\{P_v\}},\tag{3}$$

where $[\beta_{\omega}]$ means the dynamic flexibility matrix of the wheel at the wheel-rail contacting point and $[Z_{\omega}]$ represents the vertical displacement matrix of the wheel pair.

2.2. Track-Viaduct Vertical Coupled Vibration Model. The rail viaduct coupling system mainly includes steel rails, track plate, base plate, and box beams. Fasteners connect the rail and track plate, and CA mortar connects the track plate and base plate, and sliding layers connect the base plates and box beams. The box type beams are supported by bearings installed on the piers.

The rail is simplified as Timoshenko beam, and its dynamic flexibility is as follows [22]:

$$\beta(x_1, x_2) = u_1 e^{-ik_1 |x_1 - x_2|} + u_2 e^{-ik_2 |x_1 - x_2|}.$$
 (4)

The vibration displacement of the rail is as follows:

$$Z_{r}(x) = \sum_{w=1}^{N_{w}} \beta_{r}(x, x_{w}) P_{w} - \sum_{n=1}^{N} \beta_{r}(x, x_{n}) F_{fn}, \qquad (5)$$

where P_w means the wheel-rail force of w^{th} wheel pair; N_w means the amount of the wheel pair; F_{fn} means the n^{th} fastener's force; and N means the amount of the fasteners for certain rail with 32 m.

The base plate and bearing plate are simplified as freefree Euler–Bernoulli beam, and the dynamic flexibility is as follows:

$$\beta_b(x_1, x_2) = \sum_{n=1}^{N} \frac{\varphi_n(x_1)\varphi_n(x_2)}{\left[\omega_n^2 (1+i\eta) - \omega^2\right]},$$
(6)

where $\varphi_n(x)$ means the n^{th} order mode shape at the x location, ω_n represents the n^{th} resonant circular frequency of free-free Euler beam, ω denotes the excitation circular frequency, η indicates the loss factor, and N describes the cutting-off order.

The displacement of the track plate follows:



FIGURE 1: Vertical coupling vibration analysis of vehicle-rail-bridge.



FIGURE 2: Subsystems of the vehicle-rail-viaduct system.



FIGURE 3: Calculation flow chart.

$$Z_{s}(x) = \sum_{n=1}^{N} \beta_{s}(x, x_{n}) F_{fn} - \sum_{m=1}^{M} \beta_{s}(x, x_{m}) F_{jm},$$
(7)

where F_{jn} means the *n*th fastener's force, *N* means the amount of the fasteners, F_{jm} means the *n*th discrete spring force at CA mortar layer, and *M* means the amount of the discrete springs.

When calculating the dynamic flexibility of the base plate, the base plate is simplified as a free-free Euler–Bernoulli beam, and its dynamic flexibility is as follows:

$$\beta_d(x_1, x_2) = \sum_{n=1}^{N} \frac{\varphi_n(x_1)\varphi_n(x_2)}{\left[\omega_n^2 (1+i\eta) - \omega^2\right]},$$
(8)

where $\varphi_n(x)$ means the n^{th} mode shape at the x^{th} location, ω_n indicates the n^{th} order circular frequency, ω means the circular frequency, η means the loss factor, and *N* means the cutting-off order.

The displacement of the base plate is

$$Z_{d}(x) = \sum_{m=1}^{M} \beta_{d}(x, x_{m}) F_{jm} - \sum_{h=1}^{H} \beta_{d}(x, x_{h}) F_{zh},$$
(9)

where F_{jm} is the discrete spring force of the m^{th} CA mortar layer, F_{zh} is the discrete spring force of the h^{th} sliding layer, M is the amount of discrete springs in the CA mortar layer, and H is the amount of discrete springs in the sliding layer.

The viaduct is simplified as simply supported Euler beam, and the dynamic flexibility is expressed as

$$\beta_d(x_1, x_2) = \sum_{n=1}^N \frac{\varphi_{bn}(x_1)\varphi_{bn}(x_2)}{\left[\omega_{bn}^2(1+i\eta_b) - \omega^2\right]},$$
(10)

where $\varphi_{bn}(x)$ means the n^{th} order mode shape of the simply supported beam, ω_{bn} means the n^{th} order resonant frequency, *N* means the amount of the modes, and ω means the excitation circular frequency. The displacement of the viaduct is as follows:

$$Z_{b}(x) = \sum_{h=1}^{H} \beta_{b}(x, x_{h}) F_{zh} - \sum_{i=1}^{2} \beta_{b}(x, x_{qi}) F_{qi}, \qquad (11)$$

where F_{qi} means the force applied by the *i*th bearing to X_{qi} on the viaduct.

The fastener force F_{fn} , mortar layer discrete support force F_{jm} , sliding layer discrete support force F_{zh} , and bearing force F_{qi} are as follows:

$$\begin{cases}
F_{fn} = K_f (Z_r(x_n) - Z_s(x_n)), \\
F_{jm} = K_j (Z_s(x_m) - Z_d(x_m)), \\
F_{zh} = K_z (Z_d(x_h) - Z_b(x_h)), \\
F_{qi} = K_q Z_b(x_i),
\end{cases}$$
(12)

where K_f means complex stiffness for fasteners; K_j means discrete spring complex stiffness for CA mortar layer; K_z means sliding layer discrete spring complex stiffness; and K_q means discrete spring complex stiffness for bridge support. And certain complex stiffness factors are as follows:

$$\begin{cases}
K_f = k_f (1 + i\eta_f), \\
K_j = k_j (1 + i\eta_j), \\
K_z = k_z (1 + i\eta_z), \\
K_q = k_q (1 + i\eta_q),
\end{cases}$$
(13)

where $k_f \eta_f$ mean the stiffness and the loss factor of the fastener, respectively; k_j , η_j represent the stiffness and loss factor of the CA mortar layer, respectively; k_z , η_z indicate the stiffness and loss factor of the sliding layer, respectively; and k_q , η_q denote the stiffness and loss factor of the viaduct bearing, respectively.

Substituting (14) into (5), (7), (9) and (11) can obtain

$$\begin{cases} Z_{r}(x) + \sum_{n=1}^{N} \beta_{r}(x,x_{n})K_{f}Z_{r}(x_{n}) - \sum_{n=1}^{N} \beta_{r}(x,x_{n})K_{f}Z_{s}(x_{n}) = \sum_{w=1}^{N_{w}} \beta_{r}(x,x_{w})P_{w} \\ - \sum_{n=1}^{N} \beta_{s}(x,x_{n})K_{f}Z_{r}(x_{n}) + Z_{s}(x) + \sum_{n=1}^{N} \beta_{s}(x,x_{n})K_{f}Z_{s}(x_{n}) + \sum_{m=1}^{M} \beta_{s}(x,x_{m})K_{j}Z_{s}(x_{m}) - \sum_{m=1}^{M} \beta_{s}(x,x_{m})K_{j}Z_{d}(x_{m}) = 0 \\ - \sum_{m=1}^{M} \beta_{d}(x,x_{m})K_{j}Z_{s}(x_{m}) + Z_{d}(x) + \sum_{m=1}^{M} \beta_{d}(x,x_{n})K_{j}Z_{d}(x_{m}) + \sum_{h=1}^{H} \beta_{d}(x,x_{h})K_{z}Z_{d}(x_{h}) - \sum_{h=1}^{H} \beta_{d}(x,x_{h})K_{z}Z_{b}(x_{h}) = 0 \\ - \sum_{h=1}^{H} \beta_{b}(x,x_{h})K_{z}Z_{s}(x_{h}) + Z_{b}(x) + \sum_{h=1}^{H} \beta_{b}(x,x_{h})K_{z}Z_{b}(x_{h}) + \sum_{i=1}^{I} \beta_{B}(x,x_{i})K_{q}Z_{b}(x_{i}) = 0. \end{cases}$$

$$(14)$$

Equation (14) can be written in matrix form:

$$[\beta K]\{Z\} = \{P\},\tag{15}$$

where $[\beta K]$ means result of the dynamic flexibility multiplied with complex stiffness of each structural layer of the rail viaduct system; $\{Z\}$ means the displacement matrix of

each layer of rail viaduct system; and $\{P\}$ means the load matrix.

From (15), the dynamic flexibility of the rail viaduct coupling system follows

$$\beta^{TB} = \frac{Z^{TB}}{P},\tag{16}$$

where Z^{TB} means displacement of rail viaduct coupling system under wheel-rail force and β^{TB} denotes dynamic flexibility representing the displacement of the rail viaduct coupling system under unit harmonic force.

2.3. Wheel-Rail Contact Model. Assuming the wheel-rail contact as Hertz contact, k_c means the contact spring stiffness coefficient; the dynamic flexibility follows

$$\beta^c = \frac{1}{k_c}.$$
 (17)

2.4. Virtual Excitation Method. This section computes the vehicle-rail-viaduct random dynamic responses via the virtual excitation method. Assuming the first wheel-rail contact point to be uneven, the power spectrum density (PSD) is $S_{rr}(\omega)$, and the virtual excitation vector of the two wheel pairs of the entire carriage follows

$$\mathbf{R}(\omega) = \left\{ 1e^{-2i\omega l_t/V} e^{-2i\omega l_c/V} e^{-2i\omega \left(l_t + l_c\right)/V} \right\}^T \sqrt{S_{rr}(\omega)}.$$
 (18)

The relative position of the carriage and the track remains unchanged, and the track irregularity spectrum moves along the rail at the speed of the vehicle, generating displacement excitation between the wheel and the rail. The wheel-rail force can be computed as follows:

$$\mathbf{P} = -\left(\beta_{\omega} + \beta^{TB} + I_{4\times 4}\beta^{c}\right)^{-1}\mathbf{R}(\omega).$$
(19)

Combining (19) with (2) and (15), the displacements of vehicle, track, and viaduct can be obtained. Through the displacement response $Z(\omega)$, the acceleration is further derived, and the acceleration response follows

$$a(\omega) = -\omega^2 \times Z(\omega), \qquad (20)$$

where $a(\omega)$ and $Z(\omega)$ mean the acceleration and displacement, respectively.

3. Viscoelasticity Characterization of Underrail Rubber Pad

Mathematical model is usually used to characterize the dynamic viscoelasticity of the viscoelastic damping material. This study applies the constant frequency but variable temperature test method to obtain the measured data in order to expand to high frequency domain via temperature-frequency equivalence. The Temperature-Frequency Dependent Viscoelastic (TFDV) model is applied to characterize the dynamic mechanical properties of the underrail rubber pads.

3.1. Temperature-Frequency Equivalence. For certain temperature range, most viscoelastic damping materials share the same dynamic mechanical characteristics between the low temperature condition and high frequency condition. Such temperature-frequency conversion relationship is socalled the Time-Temperature Equivalence (TTE) [23]. This gives the possibility of obtaining the high frequency dynamic mechanical characteristics of the viscoelastic damping materials at low temperature rather than at high frequency, which is arduous for most instruments. The TTE follows

$$E_e(\omega,T) = \frac{\rho T}{\rho_s T_s} E_e(\alpha_T \omega, T_s), \qquad (21)$$

$$E_l(\omega, T) = \frac{\rho T}{\rho_s T_s} E_l(\alpha_T \omega, T_s), \qquad (22)$$

where T_s means the reference temperature value, T means the measured temperature, $E_e(\omega, T)$ means the energy storage modulus, $E_l(\omega, T)$ represents the energy consumption modulus, ω denotes the excitation circular frequency, and α_T indicates the transferring factor [23].

3.2. FVMP Model. The fraction Voigt and Maxwell mode in parallel (FVMP) model is a high-order fractional derivative model, which can accurately describe the dynamic mechanical characteristics of various mechanical parameters of the underrail cushion in a certain frequency band. Its time domain constitutive equation [24] follows

$$\frac{c_2}{\eta_1}\sigma(t) + \frac{1}{\eta_1}D^{\alpha}\sigma(t) = D^{\alpha+\gamma}\varepsilon(t) + c_1D^{\alpha}\varepsilon(t) + \frac{\mu_2}{\eta_1}D^{\beta}\varepsilon(t) + c_2D^{\gamma}\varepsilon(t) + c_1c_2\varepsilon(t),$$
(23)

where $c_1 = \mu_1/\eta_1$ and $c_2 = \mu_2/\eta_2$. μ_1 and μ_2 mean the elastic parameters of the FVMP model, η_1 and η_2 represent the viscosity parameters of the FVMP model, and α , β , and γ indicate the fractional derivative order.

After transforming and separating (23), the storage modulus, dissipation modulus, and loss factor are obtained as

$$E_{s}(\omega) = \mu_{1} + \eta_{1}\omega^{\gamma}\cos\frac{\gamma\pi}{2} + \mu_{2}\frac{\omega^{\alpha+\beta}\cos(((\beta-\alpha)/2)\pi) + c_{2}\omega^{\beta}\cos((\beta/2)\pi)}{(c_{2})^{2} + \omega^{2\alpha} + 2c_{2}\omega^{\alpha}\cos((\alpha/2)\pi)},$$
(24)

$$E_{l}(\omega) = \eta_{1}\omega^{\gamma}\sin\frac{\gamma\pi}{2} + \mu_{2}\frac{\omega^{\alpha+\beta}\sin\left(((\beta-\alpha)/2)\pi\right) + c_{2}\omega^{\beta}\sin\left((\beta/2)\pi\right)}{(c_{2})^{2} + \omega^{2\alpha} + 2c_{2}\omega^{\alpha}\cos\left((\alpha/2)\pi\right)},$$
(25)

$$\tan \delta = \frac{E_l}{E_s}.$$
(26)

3.3. TFDV (Temperature Frequency Dynamic Viscoelastic) Model. This study tries to establish a comprehensive dynamic model with considering both temperature variation and frequency variation simultaneously to analyze the concurrent effect of vibration frequency and ambient temperature. The TTE is introduced to the FVMP model firstly, and substituting (21) and (22) into (24) and (25), respectively, leads to the temperature-frequency variation model as

$$E_{s}(\omega,T) = \frac{\rho T}{\rho_{s}T_{s}} \left(\mu_{1} + \eta_{1}\omega_{T}^{\gamma}\cos\frac{\gamma\pi}{2} + \mu_{2}\frac{\omega_{T}^{\alpha+\beta}\cos\left(\left((\beta-\alpha)/2\right)\pi\right) + c_{2}\omega_{T}^{\beta}\cos\left((\beta/2)\pi\right)}{(c_{2})^{2} + \omega_{T}^{2\alpha} + 2c_{2}\omega_{T}^{\alpha}\cos\left((\alpha/2)\pi\right)} \right),$$

$$E_{l}(\omega,T) = \frac{\rho T}{\rho_{s}T_{s}} \left(\eta_{1}\omega_{T}^{\gamma}\sin\frac{\gamma\pi}{2} + \mu_{2}\frac{\omega_{T}^{\alpha+\beta}\sin\left(\left((\beta-\alpha)/2\right)\pi\right) + c_{2}\omega_{T}^{\beta}\sin\left((\beta/2)\pi\right)}{(c_{2})^{2} + \omega_{T}^{2\alpha} + 2c_{2}\omega_{T}^{\alpha}\cos\left((\alpha/2)\pi\right)} \right),$$

$$(27)$$

$$\tan \delta(\omega,T) = \frac{E_{l}(\omega,T)}{E_{s}(\omega,T)},$$

where $\omega_T = \alpha_T \omega$.

t

The schematic diagram of the constitutive model (TFDV model) for the underrail rubber pad established in this study is shown in Figure 4.

3.4. Temperature Variant Mechanical Test of Underrail Rubber Pad. The stiffness and damping of the under-rail rubber pad account for about 95% of the fastener system, so the stiffness and damping of the underrail rubber pad can be considered as the mechanical parameters of the fastener system [25]. Underrail rubber pad mainly provides the dynamic stiffness and loss factor of fasteners. The test object in this paper is the underrail rubber pad in the HSR WJ-7B constant resistance fastener system, as shown in Figure 5.

The test uses the EPLEXOR 500N Dynamic Thermomechanical Analyzer (DMA, as shown in Figure 6) produced by the German GABO Company for compression test.

The sample is a cylinder $\emptyset 15 \text{ mm} \times \emptyset 10 \text{ mm}$, the excitation frequency is 2 Hz, the static strain is controlled at 1.0%, the dynamic strain is controlled at 0.1%, and the temperature control system is used to adjust the test environment temperature from -60° C to 40° C, extract the test

values of modulus and loss factor at this time, and finally get the temperature spectrum of the underrail rubber pad, as shown in Figure 7.

3.5. Dynamic Characteristics of Underrail Rubber Pads

3.5.1. Temperature Variant Dynamic Characteristics of Underrail Rubber Pad. There are three forms of viscoelastic damping materials at different temperatures: glassy, rubbery, and viscous flow. As the temperature changes, the corresponding mechanical parameters of viscoelastic damping materials will also change in different forms accordingly. Figure 7 shows the DMA test results of the underrail rubber pad in the range [-60, 40]°C.

In Figure 7, at low temperatures, the underrail rubber pad behaves in a glassy state, its storage modulus value is large and slowly decreases with the increase of temperature, and its energy dissipation modulus increases with the increase of temperature; when the temperature rises, the material is in the glass transition zone; in this temperature range, the storage modulus decreases sharply, and the energy dissipation modulus decreases sharply as the temperature



FIGURE 4: Schematic diagram of the TFDV model of rubber pad under rail.



FIGURE 5: WJ-7B fastener system: (a) rail fastener system; (b) underrail rubber pad.

increases after reaching the peak temperature of the energy dissipation modulus $(-50.7^{\circ}C)$. When the temperature rises to a certain range, the modulus value of the rubber pad under the rail tends to be stable, and this state is in a rubber state.

The loss factor of the rubber pad under the rail has a peak at the glass transition temperature T_g (-45.7°C). When the temperature is less than T_g , the loss factor increases sharply with the increase in temperature. When the temperature is greater than T_g , the loss factor decreases sharply and the rate of decrease gradually slows down. When the temperature rises to a certain range, the value of the loss factor tends to stabilize. The glass transition temperature T_g (-45.7°C) of the underrail rubber pad obtained from the test data and the corresponding loss factor are 0.55.

In a certain temperature range, the temperature change test value of the underrail rubber pad shows obvious lowtemperature sensitivity and high-temperature stability. Under low-temperature conditions, the mechanical properties of the underrail rubber pad have direct correlation with change to the mechanical response of the vehicle, track, and bridge subsystems.

3.5.2. Frequency-Varying Dynamic Characteristics of the Underrail Rubber Pad

(1) Temperature-Frequency Equivalence. To obtain the frequency-varying viscoelastic dynamic characteristics, combining the DMA temperature sweep test and the principle of temperature-frequency equivalence, the dynamic performance of the underrail rubber pad can be obtained. The frequency-varying mechanical characteristics of the under rail rubber pad when the reference temperature is 20°C is taken as an example. Figure 8 shows the frequency-dependent modulus value and loss factor after temperature-frequency equivalent treatment when the reference temperature of the underrail rubber pad is 20°C.

(2) Frequency Domain Data Model Fitting. To ensure the fitting accuracy of the multiobjective function to the data, the frequency domain discrete data obtained in (1) is fitted with the high-order fractional derivative FVMP model, and the FVMP model parameters are identified in combination with the genetic algorithm. Model parameters are shown in Table 1. To verify the fitting effect of the model, this paper



FIGURE 6: Dynamic thermomechanical analyzer.



FIGURE 7: DMA test of rubber pad: (a) modulus; (b) loss factor.

compares the frequency-domain discrete dynamic parameter data of the undertrack rubber pad with the fitting curve of the FVMP model, shown in Figure 8.

Figure 8 shows the comparison between the test values of the modulus value and loss factor of the underrail rubber

pad and the fitting curve of the FVMP parameter model. In Figure 8(a), both the modulus value and loss factor of the underrail rubber pad increase with the increase in frequency and increase of amplitude gradually slows down. In Figure 8(b), the frequency domain discrete dynamic



FIGURE 8: Comparison of fitted data and the tested data for the rubber pad: (a) modulus; (b) loss factor.

TABLE 1: Model parameters of underrail rubber pads.

Parameter	Energy storage modulus model	Energy consumption modulus
μ_1	0.008	_
η_1	0.004	2.757
μ_2	59.672	0.001
η_2	17.537	0.001
α	0.041	1.754
β	0.08	2.813
γ	0.232	0.216

parameter values of the underrail rubber pad are compared with the FVMP parameter model simulation value. The FVMP parameter model can better describe the frequency change trend of its modulus value and loss factor, indicating that the FVMP model can accurately reflect frequency variation characteristics of rubber pads under derailment.

3.5.3. Characterization of Underrail Rubber Pad via the TFDV Model. Based on the TFDV model, the temperature variation $(-30, -20)^{\circ}$ C test data of the underrail rubber pad is processed; combined with the FVMP model parameters identified by the genetic algorithm, the three-dimensional dynamics of the temperature-frequency variation of the underrail rubber pad's storage modulus and loss factor can be obtained; see Figure 9.

In Figure 9, the storage modulus value and loss factor of the underrail rubber pad characterized by the TFDV model show a curved relationship with the frequency-temperature change; within a certain frequency/temperature range, the storage modulus value of the underrail rubber pad and the loss factor increase with decreasing temperature and increasing frequency. The dynamic parameters of underrail rubber pads have obvious temperature and frequency dependence, and the influence of low temperature on the dynamic parameters is more significant—the energy storage of underrail rubber pads. The dynamic properties of modulus viscoelasticity illustrate the characteristics of underrail rubber pads being rigid at low temperature/high frequency and soft at high temperature/low frequency.

4. Viscoelastic Effect of the Undertrack Rubber Pad on the Vehicle-Track-Bridge System

4.1. Computation Condition. The computation condition is illustrated in Tables 2 and 3.

The track irregularity adopts the German spectrum (wavelength [1, 100] m), shown in Figure 10, the low-frequency dynamic response of the vehicle-rail-bridge system can be calculated, and when the Sato spectrum (wavelength < 1 m) is used as short-wave irregularity spectrum, the high-frequency dynamic response of the vehicle-rail-bridge system can be computed; this study uses the German low interference spectrum and the Sato spectrum as the track irregularity excitation [26].

In terms of German spectral density function, it is defined as

$$S_{\nu}(\omega) = \frac{A_{\nu}\omega_c^2}{\left(\omega^2 + \omega_r^2\right)\left(\omega^2 + \omega_s^2\right)} \left(\mathrm{m}^2/\mathrm{rad}/\mathrm{m}\right),\tag{28}$$



FIGURE 9: Three-dimensional diagram of viscoelastic dynamic characteristics of rubber pad under rail: (a) energy storage modulus; (b) loss factor.

TABLE 2: Parameters of CRH380A high-speed train.

Parameters	Values
Rated load car body mass (kg)	42934
Bogie mass (kg)	3300
Wheelset mass (kg)	1780
Moment of inertia of car body nodding (kg·m ²)	1.712×106
Bogie nodding moment of inertia (kg·m ²)	1807
Vertical stiffness of primary suspension (N·m ⁻¹)	1.176×106
Primary suspension damping (N·s/m)	1.0×104
Secondary suspension stiffness (N·m ⁻¹)	2.4×105
Secondary suspension damping (N·s/m)	2.0×104
Car length (m)	25
Vehicle distance (m)	17.5
Fixed wheelbase (m)	2.5

where the low interference spectrum is used, and the parameters are $A_v = 4.032e - 7 \text{ m} \cdot \text{rad}$, $\omega_c = 0.8246 \text{ rad/m}$, $\omega_r = 0.0206 \text{ rad/m}$, and $\omega_s = 0.4380 \text{ rad/m}$.

Sato spectral density is defined as

$$S(\omega) = \frac{A}{\omega^{-n}} \left(\frac{m^2}{rad} \right),$$
(29)

where the surface of the track is in good condition, A = 0.065, and N = 3.06.

4.2. Results Analysis. The influence of the frequency variation parameters of the underrail rubber pads on the dynamic response of the vehicle-rail-bridge coupling system at ambient temperatures of 20° C, -20° C, and -30° C is used, shown in Table 4. According to the storage modulus of the underrail rubber pads measured by the DMA test and the external dimensions of the test material, the energy storage stiffness and loss factor temperature change values of the underrail rubber pads at different temperatures can be converted, and the corresponding temperature-frequency equivalent treatment can be used to obtain the frequency change data.

4.2.1. Viscoelastic Effect of the Underrail Rubber Pad on the Dynamic Flexibility. To analyze the influence of the viscoelastic dynamic characteristics of the underrail rubber pad on the dynamic flexibility of the vehicle-track-bridge system, it is necessary to analyze the dynamic flexibility of the vehicle-rail-bridge coupling system and the rail-bridge under the temperature change characteristics of the underrail rubber pad—the longitudinal attenuation rate of the system. Load the unit simple harmonic force on the wheel, and the obtained wheel displacement is the wheel dynamic flexibility; when the unit simple harmonic force is loaded on the rail; the obtained rail-bridge system displacement is the rail-bridge system dynamic flexibility.

(1) Vehicle-Rail-Bridge Dynamic Flexibility Amplitude and Phase. Based on the obvious vibration response of the wheelrail contact position, this paper analyzes the system dynamic flexibility of the first wheel-rail contact position. Under the frequency change conditions of the rubber pads under different ambient temperatures, the dynamic flexibility amplitude and phase of the vehicle-rail-bridge system are shown in Figures 11–14.

From Figures 11–13, the dynamic flexibility of the wheel is the largest at 1 Hz, which corresponds to the natural frequency of the vehicle's secondary suspension. In the frequency range [1, 200] Hz, the dynamic flexibility of the wheel gradually decreases with the increase of frequency; the dynamic flexibility of the contact spring is considered as constant; the peak value of rail-bridge dynamic flexibility at 5 Hz corresponds to the first-order natural frequency of the bridge-bearing system.

Components	Item/symbol/unit	Values
	Elastic modulus (N/m ²)	2.1 e11
	Moment of inertia of section (m ⁴)	3.217 <i>e</i> – 5
	Density (kg/m ³)	7850
D a il	Sectional area (m ²)	7.745e - 3
Ruit	Shear modulus (N/m ²)	7.7 <i>e</i> 10
	Section factor (κ)	0.4
	Loss factor (η_r)	0.01
	Stiffness (MN/m)	Temperature/frequency variant
	Loss factor	Temperature/frequency variant
Eastanan	Fastener space (m)	0.625
Fastener	Elastic modulus (N/m ²)	3.6 <i>e</i> 10
	Moment of inertia of section (m ⁴)	1.7e - 3
	Density (kg/m ³)	2500
C1 -1	Sectional area (m ²)	0.51
Siab	Loss factor	0.1
	Stiffness (N/m)	3.79 <i>e</i> 11
	Loss factor	0.2
CA mortar	Elastic modulus (N/m ²)	3.0 <i>e</i> 10
	Moment of inertia of section (m ⁴)	7.3125 <i>e</i> – 3
	Density (kg/m ³)	2500
Dave blate	Sectional area (m ²)	0.975
base plate	Loss factor	0.1
	Stiffness (N/m)	2.1 <i>e</i> 11
	Loss factor	0.2
Clipping Lawar	Length (m)	32
Slipping layer	Elastic modulus (N/m ²)	3.6 <i>e</i> 10
	Moment of inertia of section (m ⁴)	11.056
	Density (kg/m ³)	2650
Viaduct	Sectional area (m ²)	9.089
v iuuuci	Loss factor	0.1
	Stiffness (N/m)	6 <i>e</i> 9
Viaduct hearing	Bearing spatial interval (m)	32
v iadaci bearing	Loss factor	0.25

TABLE 3: Dynamic parameters of track-bridge structure.



FIGURE 10: Irregularity spectrum.

In Figure 11, the total dynamic flexibility in the frequency band below 25 Hz is mainly affected by the wheels. The total dynamic flexibility in the frequency band [25, 120] Hz is mainly determined by the wheel and track-bridge dynamic flexibility. The total dynamic flexibility appears at a minimum value at 59 Hz. The total dynamic flexibility in the frequency band [120, 200] Hz is mainly affected by the track-bridge dynamic flexibility.

In Figure 12, the total dynamic flexibility in the frequency band below 30 Hz is mainly affected by the dynamic flexibility of the wheels. The total dynamic flexibility in the frequency band [30, 150] Hz is mainly determined by the dynamic flexibility of the wheels and the rail-bridge. The total dynamic flexibility appears extremely small at 75 Hz. The total dynamic flexibility in the frequency band [150, 200] Hz is mainly affected by the track-bridge dynamic flexibility.

In Figure 13, the total dynamic compliance in the frequency range below 40 Hz is mainly affected by the wheels. The total dynamic compliance in the frequency range [40, 170] Hz is mainly determined by the wheel and track-bridge dynamic compliance. The total dynamic compliance appears at a minimum at 93 Hz. The total dynamic compliance of the frequency band [170, 200] Hz is mainly affected by the trackbridge dynamic compliance; The amplitudes of the dynamic compliances of the wheel and the track-bridge at the



TABLE 4: Temperature/frequency characteristic calculation conditions.

FIGURE 11: Dynamic compliance amplitude and phase of vehicle-track-bridge system with rubber pad at 20°C: (a) amplitude; (b) phase.



FIGURE 12: Dynamic compliance amplitude and phase of vehicle-track-bridge system with constant resistance elastic pad at -20° C: (a) amplitude; (b) phase.



FIGURE 13: Dynamic compliance amplitude and phase of vehicle-track-bridge system with constant resistance elastic pad at -30° C: (a) amplitude; (b) phase.

frequency corresponding to the minimum point of the total dynamic compliance are equal, and phases are opposite; this frequency point is the natural frequency of the rail-bridge system.

In Figure 14, the total dynamic compliance amplitude curve and phase curve at different temperatures basically coincide in the frequency band below 25 Hz. This is because the total dynamic compliance in this frequency band is mainly determined by the wheel dynamic flexibility, and the viscoelasticity of the underrail rubber pad affects the wheel movement. The main frequency of the minimum point and the phase main frequency of the three total dynamic flexibility curves shift to high frequency with the decrease of temperature. This is because the lower the temperature, the greater the stiffness of the rubber pad under the rail, which leads the overall rigidity of the rail-bridge system to be larger.

(2) Track Decay Rate (TDR) of Track-Bridge System. The unit simple harmonic force is applied at wheel-rail contact point 1, and the response at wheel-rail contact point 3 is called the transmission dynamic flexibility between wheel-rail contact points 1 and 3. This section uses the transmission dynamic compliance decay rate to describe the track-viaduct relation, i.e., the relation between the dynamic flexibility of the bridge origin and the dynamic flexibility of the span; the TDR of the structure [27] is expressed as

$$\Lambda = \frac{20 \lg \left(\left| z_0 \right| / \left| z_i \right| \right)}{L},\tag{30}$$

where z_0 is the displacement of contact point 1 when unit simple harmonic force is applied at wheel-rail contact point 1 and z_i is the displacement of contact point 3 when unit simple harmonic force is applied at wheel-rail contact point 1. Figure 15 shows the TDR of the track-bridge system under the temperature/frequency change conditions of the undertrack rubber pad.

In Figure 15, in the frequency range [0, 122] Hz, the TDR of the track-bridge system decreases with the decrease of temperature; that is, the decrease in temperature in this frequency band increases the longitudinal transmission of vibration along the track-bridge system, which leads to vibration due to energy conservation. The vertical transmission along the track-bridge system is reduced; in the frequency range [181, 200] Hz, the TDR of the track-bridge system increases with the decrease of temperature; that is, the decrease in temperature in this frequency band causes the vibration to travel along the track-bridge system, while longitudinal transmission decreases. Due to the conservation of energy, the vertical transmission of vibration along the track-bridge system increases; the change trend of the TDR spectrogram of the track-bridge system at different temperatures is basically the same.

To summarize, the viscoelasticity of the underrail rubber pad has a certain effect on the total dynamic flexibility and the TDR of the rail-bridge system. The lower the temperature, the smaller the minimum value of the total dynamic flexibility, and the frequency of the minimum value of the total dynamic flexibility will shift to high frequency. This is due to the increase in the stiffness of the rubber pad under the rail due to the decrease in temperature, which causes the overall track-bridge system stiffness increases; in the low frequency band, the decrease in temperature causes the TDR of the track-bridge system to decrease; that is, the longitudinal transmission of vibration along the track-bridge system increases and the vertical transmission decreases; in the higher frequency band, the temperature decrease causes the decay rate of vibration transmission of the track-bridge system to increase; that is,



FIGURE 14: Amplitude and phase of total dynamic flexibility of rubber cushion car-rail-bridge system under different temperatures: (a) amplitude; (b) phase.



FIGURE 15: TDR of the track-bridge system.

the longitudinal transmission of vibration along the trackbridge system decreases and the vertical transmission increases.

4.2.2. Viscoelastic Effect of the Rubber Pad on the Random Vibration of the Vehicle-Track-Bridge System. This section uses virtual excitation method to calculate the random dynamic response of the vehicle-rail-bridge system at a speed of 350 km/h.

(1) Vertical Wheel-Rail Force. Figure 16 shows the wheel-rail force spectrum of the underrail rubber pad at different temperatures. To accurately analyze the influence of the viscoelasticity of the underrail rubber pad on the wheel-rail force, the main peak value of the wheel-rail force and the corresponding peak frequency in Figure 16 are extracted; see Table 5.

From Figure 16 and Table 5, the changing trend of the wheel-rail force amplitude calculated by the underrail rubber pad at different temperatures is consistent. Generally speaking, there is basically no difference in the wheel-rail force calculated in the frequency band below 25 Hz. The difference in the frequency range [25, 200] Hz is more obvious; the main peak frequency of the wheel-rail force amplitude curve is in the frequency range [55, 85] Hz, which is related to the coupled vibration of the wheel and the track-bridge; the stiffness of the rubber pad under the track decreases with the decrease of temperature. Therefore, the peak frequency corresponding to the main peak of the wheel-rail force shifts to high frequency, which shows that the amplitude of the high-frequency wheel-rail force increases with the decrease of temperature.

(2) Vehicle Body Vibration Acceleration Power Spectral Density. Figure 17 shows the acceleration PSD of car body nodding and ups and downs of the underrail rubber pad at three temperatures.

Extract the peak value of the PSD of the vehicle body nodding and the ups and downs vibration acceleration in Figure 17 and the corresponding peak frequency; see Table 6.

In Figures 17(a) and 17(b), the vehicle body ups and downs/nodding acceleration power spectrum curves calculated under different ambient temperatures for



FIGURE 16: Wheel-rail force amplitude.

TABLE 5. Wheel-Tall peak loree and peak negueiley	TABLE 5:	Wheel-rail	peak force	and peak	frequency
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Condition	Ambient temperature (°C)	Resonant frequency		
Condition		Peak (N)	Frequency at peak (Hz)	
	20	3.646e4	55	
Under rubber pad	-20	1.470 <i>e</i> 4	75	
	-30	1.203 <i>e</i> 4	85	



FIGURE 17: Power spectrum of vehicle body vibration acceleration when (a) carriage up and down vibration and (b) carriage nodding.

TABLE 6: Car body acceleration power spectrum peak and peak frequency.

A surface terms	Ambient temperature (°C)	First resonant frequency		Second resonant frequency	
Acceleration type		Peak $((m/s^2)^2/Hz)$	Frequency (Hz)	Peak $((m/s^2)^2/Hz)$	Frequency (Hz)
	20	0.40840	1	0.06800	5
Vehicle body ups and downs vibration acceleration	-20	0.40860	1	0.06732	5
	-30	0.40870	1	0.06700	5
	20	0.01041	1	0.00081	7
Vehicle body nodding vibration acceleration	-20	0.01040	1	0.00080	7
	-30	0.01039	1	0.00079	7

underrail rubber pads basically overlap, mainly in the frequency band below 10 Hz; from Table 6, the vehicle body ups and downs vibration acceleration are in different working conditions. In this case, the calculated first main frequency is 1 Hz, the corresponding maximum peak value is $0.4087 (m/s^2)^2/Hz$, the minimum peak value is $0.4084 \,(\text{m/s}^2)^2/\text{Hz}$, and the difference between the two is only 0.07%. The second main frequencies are both 5 Hz, the corresponding maximum peak value is $0.06800 \text{ (m/s}^2)$ 2 /Hz, the minimum peak value is 0.06700 (m/s²)²/Hz, and the difference between the two is only 1.5%, which can be ignored; the car body nods. The first dominant frequency of vibration acceleration calculated under different working conditions is 1 Hz, the corresponding maximum peak value is $0.01041 \text{ (m/s}^2)^2/\text{Hz}$, the minimum peak value is $0.01039 \,(\text{m/s}^2)^2/\text{Hz}$, and the difference between the two is only 0.2%; the second main frequency is 7 Hz, the corresponding maximum peak value is $0.0081 \text{ (m/s}^2)^2/\text{Hz}$, the minimum peak value is $0.0079 \,(m/s^2)^2/Hz$, and the difference between the two is only 2.5%, which can be ignored and not counted; therefore, the viscoelastic dynamic characteristics of the underrail rubber pads have basically no effect on the dynamic response of the car body.

(3) Wheelset and Rail Vibration Acceleration Power Spectral Density. The dynamic response law of wheel-rail direct contact is similar. This article puts it together for analysis. Figure 18 shows the power spectral density of wheelset and rail vibration acceleration calculated under different temperatures for the rubber pad under the rail.

Extract the more obvious peaks and corresponding peak frequencies in the power spectrum of wheel sets and rails in Figure 18 and Tables 7 and 8.

From Figure 18(a) and Table 7, the power spectrum of wheelset acceleration of the underrail rubber pad at different ambient temperatures has no obvious difference within the frequency range of 25 Hz; the peak frequency is between 55 and 85 Hz; as the temperature decreases, the peak frequency of the wheelset acceleration power spectrum gradually

increases; in the low frequency band within 61 Hz, the lower the temperature, the smaller the amplitude of the wheelset vibration acceleration power spectrum; in the higher frequency band [81, 200] Hz, the higher the temperature, the lower the wheelset vibration acceleration power spectrum amplitude; this is similar to the wheel-rail force amplitude curve law.

From Figure 18(b) and Table 8, the power spectrum of rail vibration acceleration under different ambient temperatures has no obvious difference within the frequency range of 20 Hz; the peak frequency is between 55 and 98 Hz. As the temperature decreases, the peak frequency of the rail acceleration power spectrum curve gradually increases; in the low frequency stage within 66 Hz, the lower the temperature, the smaller the amplitude of the rail vibration acceleration power spectrum; in the [85, 200] Hz high frequency band, the lower the temperature, the higher the overall power spectral density of the vibration acceleration of the upper rail.

The viscoelastic dynamic characteristics of the underrail rubber pad have an impact on the vehicle system and the random dynamic response of the rail. The viscoelasticity of the underrail rubber pad has almost no effect on the low-frequency acceleration PSD curve of the carriage body, wheel set, and rail because the dynamic response of the vehicle system and the rail and the total flexibility of the vehicle-rail-bridge system are closely related. In the low frequency band, the total dynamic flexibility is mainly determined by the wheel dynamic flexibility, while the viscoelasticity of the underrail rubber pad has no effect on the wheel dynamic flexibility; the viscoelasticity of the underrail rubber pad affects the wheel-rail force and the wheelset. The influence of certain rubber pad in the medium and high frequency range to the PSD of the rail is obvious. As the temperature decreases, the peak frequency of the PSD increases; in the lower frequency range, the lower the temperature is, the smaller the power spectrum amplitude will be; in the higher frequency band, the higher the temperature is, the greater the power spectrum amplitude will be.



FIGURE 18: Power spectrum of temperature/frequency variable wheelset and rail acceleration of elastic cushion pad with constant resistance: (a) wheelset; (b) rail.

TABLE 7: Wheelset power spectrum peak and peak frequency.

Condition	Ambient temperature (°C)	Peak $((m/s^2)^2/Hz)$	Frequency (Hz)
	20	1703	55
Under the constant resistance rail pad	-20	275.6	75
	-30	184	85

TABLE 8: Power spectrum peak and peak frequency of the rail.

Condition	Ambient temperature (°C)	Peak value ((m/s ²) ² /Hz)	Frequency (Hz)
	20	3074	55
Under the constant resistance rail pad	-20	625.7	75
	-30	473.8	98

5. Conclusions

This paper combines the TFDV model of the underrail rubber pad and the vehicle-track-bridge vertical coupling frequency domain analysis model, which uses the German spectrum and the Sato spectrum as the track irregularity. The conclusions are summarized as follows:

- (1) The temperature change test value of the underrail rubber pad shows obvious low-temperature sensitivity and high-temperature stability within a certain temperature range; as to the storage of the underrail rubber pad characterized by the TFDV model, both the energy modulus value and the loss factor show a curved relationship with the frequency-temperature change; within a certain frequency/temperature range, the storage modulus value and loss factor of the underrail rubber pad both decrease with temperature and increase frequency.
- (2) The lower the temperature is, the smaller the minimum value of the total dynamic flexibility will be; and the frequency of the minimum value of the total dynamic flexibility shifts to high frequency. In the low frequency band, the temperature decrease causes the TDR of the track-bridge system to decrease; that is, the longitudinal transmission of vibration along the track-bridge system increases, and the vertical transmission decreases; in the higher frequency band, the temperature decreases while the dynamic flexibility TDR of the track-bridge system increases; that is, the dynamic flexibility decreases along the track-bridge system's longitudinal transmission and increases the vertical transmission.
- (3) The viscoelastic dynamic characteristics of the rubber pad have little effect on the amplitude of the power spectrum of the vehicle body vibration acceleration. Since the total flexibility of the vehiclerail-bridge system is closely related to its dynamic

response, the wheels in the low frequency range mainly determine the total flexibility. The viscoelasticity of the rubber pad under the rail has no effect on the dynamic flexibility of the wheel; the primary and secondary suspension of the vehicle dissipate a large amount of low-frequency vibration energy transmitted to the car body.

(4) The viscoelastic dynamic characteristics of the underrail rubber pad have the same effect on the wheel-rail force, wheelset, and rail vibration acceleration power spectrum. As the temperature decreases, the peak frequency of the power spectrum curve gradually increases; in the lower frequency band, the lower the temperature is, the smaller the power spectrum amplitude will be; in the higher frequency band, the lower the temperature is, the greater the power spectrum amplitude will be. The viscoelasticity of the underrail rubber pad has a similar effect on the vibration of the track-bridge system. As the ambient temperature of the underrail rubber pad decreases, the dominant frequency band of the power spectrum curve shifts to high frequencies.

Data Availability

The data are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Natural Science Foundation (Grant nos. 51968025 and 51578238), the Key Projects of Jiangxi Natural Science Foundation (Grant no. 20192ACBL20009), and the Key Research and Development Program of Jiangxi (Grant no. 20181BBE50013) of China.

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