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Research Article

A Proposed Bearing Load Identification Method to Uncertain Rotor Systems

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Bearings are considered as important mechanical components in rotating machines. Bearing load is used as an indication of monitoring rotor system health, but there are interval and probability uncertain parameters in the process of obtaining bearing load from the rotor system. A bearing load strip enclosed by two bounding distributions is then formed, rather than a single distribution that we usually obtain through the load identification method for a deterministic rotor system. In this paper, a computational inverse approach that combines the interval and perturbation analysis method with regularization is presented to stably identify bearing load strip. Using an interval analysis method, a calculated transient response of the rotor structure only subjecting to the bearing load can be approximated as a linear function of the interval parameters in the rotor system. The perturbation analysis method based on Taylor expansion is used to transform the problem of the bearing load identification involving in probability parameters into two kinds of certain inverse problem, namely, the bearing load identification combining the mean value of uncertain parameters with calculated transient response function and the sensitivity identification of bearing load to each probability parameter. Regularization is used to overcome ill-posedness of bearing load identification arising from the noise-contaminated observed response. A rotor system with two bearings is investigated to demonstrate the effectiveness and accuracy of the presented method.

1. Introduction

Safety and reliability of the high-speed rotating mechanical equipment has attracted much attention with the increase in rotational speed and power [1–3]. In order to reduce the loss caused by mechanical equipment faults and achieve efficient fault diagnosis, it is necessary to perceive and manage the running status and health level of its key systems and components in real time [4]. Bearing is the main basic supporting structure of rotating machinery, and the running state of the rotor system can be assessed by bearing load [5]. At present, the theoretical system mostly uses the sensor to collect the fault characteristic signal of the bearing under specific working conditions and uses it as the basic data of the fault diagnosis model [6, 7]. Any deviations between the referenced bearing load and the observed value can be inferred as an indication of a change or damage in the rotor system. The approach for reliably monitoring the rotor system health is to create a real bearing load from the rotor system in its undamaged state [8]. However, in the actual work site, the acquisition system to determine the real bearing load may be affected by the noise in the environment or the uncertain factors caused by the manufacturing and service environment of the rotating machinery structure [9]. As a result, the bearing load identified by the collected signals do not fully reflect the real operation of the rotor system. Because of the existence of uncertain factors, the bearing load distribution will form a strip enclosed by two bounding distributions, rather than a single distribution that we usually obtain through a deterministic rotor system. If the observed bearing load is within the boundary of the referenced bearing load, the rotor system runs without fault.

In general, the problem to determine the bearing load strip from the collected output signals considering the uncertain factors is called an uncertain inverse problem. This type of inverse problem has recently attracted more and more attention. As the most popular uncertainty modeling strategy, probability [10, 11], interval [12, 13], and combined method [14, 15] have been proposed to applied in engineering for uncertain parameter identification and load identifications. However, the probability method is a quantitative description, which describes the uncertainty parameters accurately but requires a lot of information coming from many expensive experimental tests to construct the probability density function; the interval method is a qualitative description, which handles the uncertainty with limited information, but it will lose the actual value of the project due to the expansion of the interval estimation. Thus, while proposing a new uncertainty inverse analysis framework for gaining the bearing load strip to evaluate uncertain rotor system fault, a more efficient computing method needs to be developed together.

In this paper, a hybrid method based on interval theory [16], perturbation theory [17], and regularization method [18] is proposed to construct the inverse analysis framework to deal with the uncertainty with imprecise information; utilizing Taylor expansion, the problem of bearing load identification of the uncertain rotor system is transformed into two kinds of deterministic bearing load identification problems. One kind is the middle bearing load identification taking the uncertain parameters as the mean value, and other is the reverse calculation of the sensitivity of the bearing load with respect to the uncertain parameters; the regularization method is used to treat the unstable solution coming from the noise in the measurement response.

2. Bearing Load Identification Problem

The transient response [19] coming from vibration analysis of the bearing-rotor system is shown in the following:

$$\begin{bmatrix} \mathbf{M}_{II} & \mathbf{O} \\ \mathbf{O} & \mathbf{M}_{BB} \end{bmatrix} \{\ddot{q}_{\mathrm{I}}\,\ddot{q}_B\} + \begin{bmatrix} \mathbf{C}_{II} & \mathbf{O} \\ \mathbf{O} & \mathbf{C}_{BB} + \mathbf{C}_B \end{bmatrix} \begin{Bmatrix} \dot{q}_I \\ \dot{q}_B \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{II} & \mathbf{K}_{IB} \\ \mathbf{K}_{IB} & \mathbf{K}_{BB} + \mathbf{K}_B \end{bmatrix} \begin{Bmatrix} \mathbf{q}_I \\ \mathbf{q}_B \end{Bmatrix} = \omega^2 \begin{Bmatrix} \mathbf{F}_{eI}^T \\ \mathbf{F}_{eB}^T \end{Bmatrix}, \tag{1}$$

where $\mathbf{O}$ is the zero matrix; $\mathbf{M}$, $\mathbf{K}$, and $\mathbf{C}$, respectively, represent the mass, stiffness, and damping matrices of the separate rotor structure; the index B denotes the node of the rotor with the bearing support and the index I denotes the node of the rotor without the bearing support; $\mathbf{K}_B$ and $\mathbf{C}_B$ represent the bearing stiffness and damping matrices, respectively; the transient displacement $\mathbf{q}$, velocity $\dot{q}$, and acceleration $\ddot{q}$ represent the structural response of the bearing-rotor system under unbalanced force $\mathbf{F}_e$ with the rotational speed ω.

Denoting $\mathbf{F}^T = \mathbf{C}_B\dot{q}_B + \mathbf{K}_B\mathbf{q}_B$ and moving $\mathbf{F}^T$ from the left side of equation (1) to the right side, the equivalent kinetic equation of the rotor structure subjecting to the unbalanced force $\mathbf{F}_e$ and the bearing load $\mathbf{F}^T$ is shown in the following:

$$\begin{aligned} &\begin{bmatrix} \mathbf{M}_{II} & \mathbf{O} \\ \mathbf{O} & \mathbf{M}_{BB} \end{bmatrix} \{\ddot{q}_{\mathrm{I}}\,\ddot{q}_B\} + \begin{bmatrix} \mathbf{C}_{II} & \mathbf{O} \\ \mathbf{O} & \mathbf{C}_{BB} \end{bmatrix} \begin{Bmatrix} \dot{q}_I \\ \dot{q}_B \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{II} & \mathbf{K}_{IB} \\ \mathbf{K}_{IB} & \mathbf{K}_{BB} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_I \\ \mathbf{q}_B \end{Bmatrix} \\ &= \omega^2 \begin{Bmatrix} \mathbf{F}_{eI}^T \\ \mathbf{F}_{eB}^T \end{Bmatrix} - \begin{bmatrix} \mathbf{O} & \mathbf{O} \\ \mathbf{O} & C_B \end{bmatrix} - \begin{bmatrix} \mathbf{O} & \mathbf{O} \\ \mathbf{O} & K_B \end{bmatrix} \begin{Bmatrix} q_I \\ q_B \end{Bmatrix} = \omega^2 \begin{Bmatrix} \mathbf{F}_{eI}^T \\ \mathbf{F}_{eB}^T \end{Bmatrix} \begin{Bmatrix} \mathbf{O} \\ \mathbf{F}^T \end{Bmatrix}, \end{aligned} \tag{2}$$

or simplified to

$$[\mathbf{M}]\{\ddot{q}\} + [\mathbf{C}_2]\{\dot{q}\} + [\mathbf{K}_2]\{\mathbf{q}\} = \omega^2\{\mathbf{F}_e\} - \{\mathbf{F}^T\}. \tag{3}$$

The measurable structural response $\mathbf{q}$ can be thought to be the superposition of responses of the rotor separately subjecting to F_e and $-\mathbf{F}^T$. The damping matrix $\mathbf{C}_2$ and stiffness matrix $\mathbf{K}_2$ of the separate rotor in equation (3) do not combine the effects of the bearing.

As the rotor structure is linear time invariant, the following relationship is obtained according to the superposition principle

$$[\mathbf{M}]\{\ddot{q}_T\} + [\mathbf{C}_2]\{\dot{q}_T\} + [\mathbf{K}_2]\{\mathbf{q}_T\} = \{\mathbf{F}^T\}, \tag{4}$$

where $\mathbf{q}_T = \mathbf{q}^e - \mathbf{q}$, $\mathbf{q}_T$ represent the transient response of the rotor structure only subjecting to the bearing load $\mathbf{F}^T$. The unbalance response $\mathbf{q}^e$ only subjecting to the unbalance load F_e can be numerically and accurately calculated by combining the information of the unbalance load F_e together with the matrices $\mathbf{M}$, $\mathbf{C}_2$, and $\mathbf{K}_2$ into the forward solver [20]. The calculated unbalance response $\mathbf{q}^e$ together with the measurable structural response q is combined into Green's function method and regularization operation [21] to reconstruct the bearing load $\mathbf{F}^T$.

3. Bearing Load Identification Method for Uncertain Rotor Systems

When the parameters in the rotor system, such as material properties and geometric structure, cannot be determined completely, the Green kernel function in the load identification used to characterize the dynamic characteristics of the rotor system will be uncertain. When this convolution integral in the time domain is discretized, the whole concerned time period is separated into equally spaced intervals, and the response $\mathbf{q}_T$ only subjecting to the bearing load $\mathbf{F}^T$ can be expressed by a matrix form:

$$\mathbf{q}_T = \mathbf{H}(\lambda, \eta) \cdot \mathbf{F}^T(\lambda, \eta), \tag{5}$$

where $\mathbf{G}$ is the Green function matrix coming from the bearing load $\mathbf{F}^T$ to the response $\mathbf{q}_T$.

In general, the Green function matrix $\mathbf{H}$ of equation (5) is ill-conditioned, and the response $\mathbf{q}_T$ calculated by the measurable structural response $\mathbf{q}$ according to $\mathbf{q}_T = \mathbf{q}^e - \mathbf{q}$ inevitably carries noise. If the inverse of the matrix $\mathbf{H}$ exists, the bearing load $\mathbf{F}^T$ can be obtained by using the following equation based on regularization:

$$\begin{aligned}\mathbf{F}^T &= \mathbf{H}^\alpha \cdot \mathbf{q}_{T\delta} = \mathbf{V}\mathrm{Diag}\left(f(\alpha, \sigma_i)\sigma_i^{-1}\right)\mathbf{U}\mathbf{q}_{T\delta} \\ &= \sum_{i=1}^{m} f(\alpha, \sigma_i)\sigma_i^{-1}\left(\mathbf{U}_i^T \mathbf{q}_{T\delta}\right)\mathbf{V}_i,\end{aligned} \tag{6}$$

where $\mathbf{q}_{T\delta}$ represents the response with noise, $\mathbf{U} = [u_1, u_2, \ldots, u_m]$ is the left singular vector of $\mathbf{H}$ and $\mathbf{V} = [v_1, v_2, \ldots, v_m]$ is the right singular vector of $\mathbf{H}$, which are two normalized orthogonal matrices, and $f(\alpha, \sigma_i)$ denotes a filter function to attenuate the amplification effect of small singular value on noise.

The Green function matrix $\mathbf{H}(\lambda, \eta)$ is expressed as a set of uncertain parameters containing interval and probability parameters, so the bearing load is no longer a solution, but a solution set. In rotor system health monitoring, the upper and lower boundaries of the solution set are often concerned, and it is not necessary to solve all the possible values. According to the interval mathematics theory, when the uncertain parameter λ is in the interval form, the corresponding response $\mathbf{q}_T$ is also in the interval form. In order to obtain the maximum and minimum values of the bearing load, all the possible values of the interval uncertain parameter λ should be selected in turn, the bearing load should be identified based on the response $\mathbf{q}_T$ distribution strip, and the maximum and minimum values of the bearing load should be searched in the identification results. The recognition process with a traditional Monte Carlo simulation (MCS) [22] will involve complex multilayer nesting solution, and the forward problem calculation model needs to be called repeatedly, which will inevitably lead to the inefficiency of the solution. In this paper, the bearing load identification problem involving interval and probability uncertain parameters in equation (5) is transformed into a series of deterministic problems utilizing the method of interval analysis and matrix perturbation.

3.1. Boundary of Transient Response. According to the monotonicity analysis theory [23], the maximum and minimum values of bearing load must correspond to the boundary of interval uncertain parameters. The lower boundary λ^L and upper boundary λ^R of n-dimensional interval uncertain parameters can be described by the midpoint λ^c and radius λ^w of the interval. When the uncertainty level is small, the first-order Taylor expansion is carried out at the midpoint λc of the interval. The minimum and maximum values of the response $\mathbf{q}_T$ can be obtained directly and explicitly.

$$\begin{gathered}\begin{cases} q_{T\min}(t) = \min\limits_{\boldsymbol{\lambda}\in\Gamma} q_T(t, \boldsymbol{\lambda}) = q_T(t, \boldsymbol{\lambda}^c) - \sum\limits_{j=1}^{n}\left|\dfrac{\partial q_T(t, \boldsymbol{\lambda}^c)}{\partial \boldsymbol{\lambda}_j}\right|, \\ q_{T\max}(t) = \max\limits_{\boldsymbol{\lambda}\in\Gamma} q_T(t, \boldsymbol{\lambda}) = q_T(t, \boldsymbol{\lambda}^c) - \sum\limits_{j=1}^{n}\left|\dfrac{\partial q_T(t, \boldsymbol{\lambda}^c)}{\partial \boldsymbol{\lambda}_j}\right|, \end{cases} \\ \boldsymbol{\lambda} \in \boldsymbol{\lambda}^I = \left[\boldsymbol{\lambda}^L, \boldsymbol{\lambda}^R\right] = \left[\boldsymbol{\lambda}^c - \boldsymbol{\lambda}^w, \boldsymbol{\lambda}^c + \boldsymbol{\lambda}^w\right], \\ \lambda_j \in \lambda_j^I = \left[\lambda_j^L, \lambda_j^R\right] = \left[\lambda_j^c - \lambda_j^w, \lambda_j^c + \lambda_j^w\right], \\ \lambda = \lambda^c + \delta\lambda, \quad \delta\lambda \in [-1, 1]\lambda^w, \\ \lambda_j \in [-1, 1]\lambda_j^w, \quad j = 1, 2, \ldots, n.\end{gathered} \tag{7}$$

3.2. Boundary of Bearing Load. The k-dimensional probabilistic uncertainty parameter η can be expressed as the mean value η_d and disturbance part $\Delta\eta_r$. According to the perturbation theory [24], the Green kernel function matrix of the corresponding rotor system and the bearing load to be identified contain disturbance parts.

$$\begin{aligned}\eta &= \eta_d + \Delta\eta_r, \\ \eta_i &= \eta_{di} + \Delta\eta_{ri}, \quad i = 1, 2, \ldots, q, \\ \mathbf{q}_T &= \left(\mathbf{H}_d + \Delta\mathbf{H}_r\right) \cdot \left(\mathbf{F}_d^T + \Delta\mathbf{F}_r^T\right),\end{aligned} \tag{8}$$

where the subscript d and r represent the mean value and disturbance part of the probabilistic uncertain parameters, respectively.

$$\begin{aligned}\mathbf{q}_T &= \mathbf{H}_d\mathbf{F}_d^T, \\ -\Delta\mathbf{H}_r\mathbf{F}_d^T &= \mathbf{H}_d\Delta\mathbf{F}_r^T.\end{aligned} \tag{9}$$

The Green kernel function matrix $\mathbf{H}_d$ in equation (8) can be obtained when the probabilistic uncertain parameters are taken as the mean value. Based on the response $\mathbf{q}_T$, the mean value $\mathbf{F}_d^T$ of the bearing load can be inversely calculated by the deterministic bearing load identification method. The disturbance part $\Delta\mathbf{F}_r$ of the bearing load can be identified through the reverse calculation of the sensitivity of the bearing load with respect to the probabilistic uncertain parameters. Among the solving the mean value $\mathbf{F}_d^T$ and the disturbance part $\Delta\mathbf{F}_r$, the Green function matrix $\mathbf{H}_d$ in equation (9) is the same, and only one matrix singular value decomposition operation is needed in regularization. Through the above matrix perturbation analysis method, the bearing load identification problem with k-dimensional probabilistic uncertain parameters can be transformed into the following $k + 1$ deterministic problem. The bearing load with interval and probability uncertainty can be described using the Taylor expansion of first order regarding near their means (λ^c, η_d).

$$\begin{cases} \mathbf{F}_L^T = \min\limits_{\boldsymbol{\lambda}\in\Gamma,\boldsymbol{\eta}\in\Omega} F_d^T(t,\boldsymbol{\lambda},\boldsymbol{\eta})|\mathbf{q}_{Tm} - \sum\limits_{i=1}^{n}\left|\dfrac{\partial\mathbf{F}^T(t,\boldsymbol{\eta}_d)}{\partial\boldsymbol{\eta}_i}\right|\Delta\boldsymbol{\eta}_{ri}|\mathbf{q}_{T\min}q_{T\max}, \\ \mathbf{F}_R^T = \max\limits_{\boldsymbol{\lambda}\in\Gamma,\boldsymbol{\eta}\in\Omega} F_d^T(t,\boldsymbol{\lambda},\boldsymbol{\eta})|\mathbf{q}_{Tm} - \sum\limits_{i=1}^{n}\left|\dfrac{\partial F^T(t,\boldsymbol{\eta}_d)}{\partial\boldsymbol{\eta}_i}\right|\Delta\boldsymbol{\eta}_{ri}|\mathbf{q}_{T\min}q_{T\max}, \end{cases} \tag{10}$$

where the transient response $\mathbf{q}_{Tm} = \mathbf{q}^e - \mathbf{q}^m$ for identifying the mean value $\mathbf{F}_d^T$ is obtained from the measurable structural response $\mathbf{q}^m$ to means (λ^c, η_d) together with the calculated unbalance response $\mathbf{q}^e$, and he disturbance part $\Delta\mathbf{F}_r$ centering on the $\mathbf{q}_{T\min}$ and $q_{T\max}$ from equation (7) using the deterministic bearing load identification method is computed. The first-order partial derivative $(\partial F^T(t,\eta_d)/\partial\eta_i)$ to the probabilistic parameters can be dealt by the difference method [25], and the difference scheme is used to transform the partial differential equation into an algebraic equation to solve, for example, $(\partial\mathbf{F}^T(t,\eta_d)/\partial\eta_i) = (\Delta\mathbf{F}^T(t,\eta_d)/\Delta\eta_i)$, and the increment $\Delta\mathbf{F}^T(t,\eta_d)$ of the bearing load is obtained by small disturbance $\Delta\eta_{ri}$ at the midpoint. With the mean value $\mathbf{F}_d^T$ and the disturbance part $\Delta\mathbf{F}_r$, the minimum $\mathbf{F}_L^T$ and maximum $\mathbf{F}_R^T$ of the bearing load can easily calculated based on equation (10).

4. Bearing Load Identification Process of Uncertain Rotor Systems

Based on the above discussion, the solution procedure towards a bearing load identification of the rotor system involving interval and probability uncertain parameters can be described as follows (see Figure 1):

(1) Construct a forward solver to calculate the unbalance response $\mathbf{q}^e$ or structural response q utilizing the mass, stiffness, and damping matrices and the corresponding load

(2) Construct an inverse problem solver for the deterministic load identification algorithm, combining the calculated or measured structural response $\mathbf{q}^m$ and the unbalance response $\mathbf{q}^e$ with the known mass, stiffness, and damping matrices into the inverse problem solver to calculate the bearing load

(3) Assume the uncertain parameters (λ, η) from engineering experience, combining the mean of uncertain parameters (λ^c, η_d) together with the measured structural response $\mathbf{q}^m$ into the inverse problem solver to calculate the mean value $\mathbf{F}_d^T$

(4) Calculate $\mathbf{q}_{T\min}$ and $\mathbf{q}_{T\max}$ from the forward solver using the obtained $\mathbf{F}_d^T$ and the known interval parameters λ based on the interval analysis method

(5) Combine the probabilistic parameters η together with the obtained $q_{T\min}$ and $q_{T\max}$ into the inverse problem solver to calculate the disturbance part $\Delta\mathbf{F}_r$ based on the difference method

(6) Utilize the mean value $\mathbf{F}_d^T$ and the disturbance part $\Delta\mathbf{F}_r$ to calculate $\mathbf{F}_L^T$ and $\mathbf{F}_R^T$, as shown in equation (10)

5. Examples

The numerical model in reference [26, 27] is provided to verify the feasibility of the proposed uncertain bearing load identification algorithm. Figure 2 depicts a rotor system with three discs supported by two fault predicted journal bearings, the geometrical parameters of the rotor structure are partly given in Figure 2(a), the transfer matrix model of the rotor structure is established as shown in Figure 2(b), and the rotor system is divided into 33 elements consisting of 34 nodes with three discs at element 9, 17, and 28. The bearing acts on the nodes 7 and 24. The angular speed is 4000 rpm. The elastic modulus E of the rotor material is 210 GPa, the shear elastic modulus G is 80 GPa, and the density is 7650 kg/m^3. The disc weight is 14 kg, and the unbalance mass to construct unbalanced force F_e of 14 kg at 0° is placed at 1 mm radii from the center of the middle disc. The mass, stiffness, and damping matrices of the rotor system are obtained by the transfer matrix method.

5.1. Identification of Bearing Load under Determined Structure. The structural response q can be easily obtained by using a forward solver (such as TMM) reported by Mao et al. [27] with the information of the mass, stiffness, damping matrices, and the corresponding unbalance load and bearing load. For testing the inverse problem solver for the deterministic load identification algorithm, the structural response at the nodes 3, 8, 20, and 25 used to identify the bearing load is gained by noise-contaminated response (a 3% Gaussian noise is directly added to the computer-generated response coming from the assumed value of bearing stiffness and damping coefficient in Table 1 into the TMM forward solver), as shown in Figure 3. With the help of regularization, the bearing loads are stably obtained and shown in Figure 4. It is known that the load time histories of

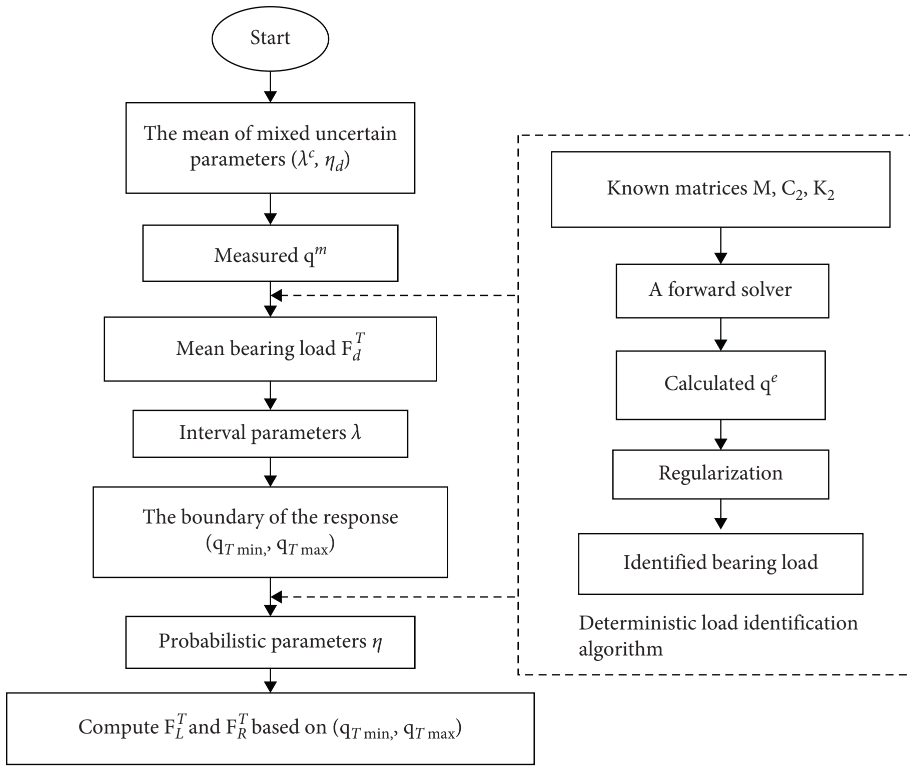


Figure 1: Solution procedure towards bearing load identification with uncertainty.

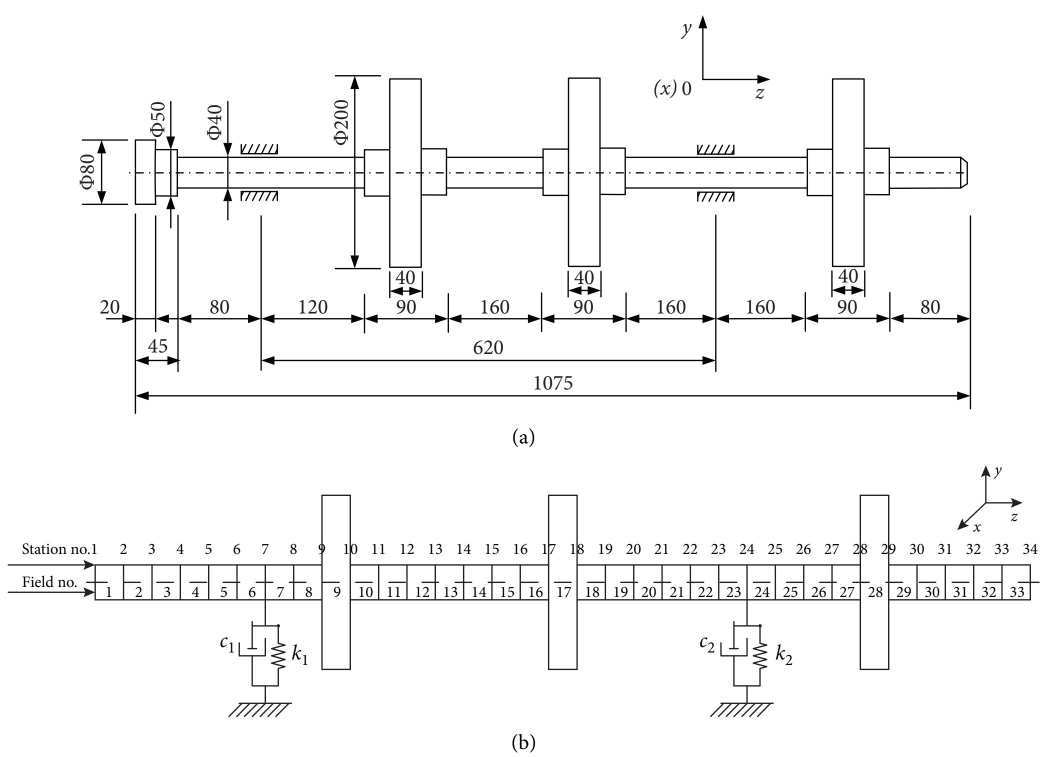


Figure 2: The model of a rotor system: (a) a bearing-rotor system with three discs; (b) the TMM model.

Table 1: Bearing dynamic parameters.

Bearing	Stiffness coefficients ($\mathrm{MN \cdot m^{-1}}$)				Damping coefficients ($\mathrm{KN \cdot s \cdot m^{-1}}$)			
	K_{xx}	K_{xy}	K_{yx}	K_{yy}	C_{xx}	C_{xy}	C_{yx}	C_{yy}
Left	46.36	83.4	−64.33	41.27	70.28	71.63	71.63	88.57
Right	13.38	29.16	−21.78	8.36	69.67	19.98	19.98	79.93

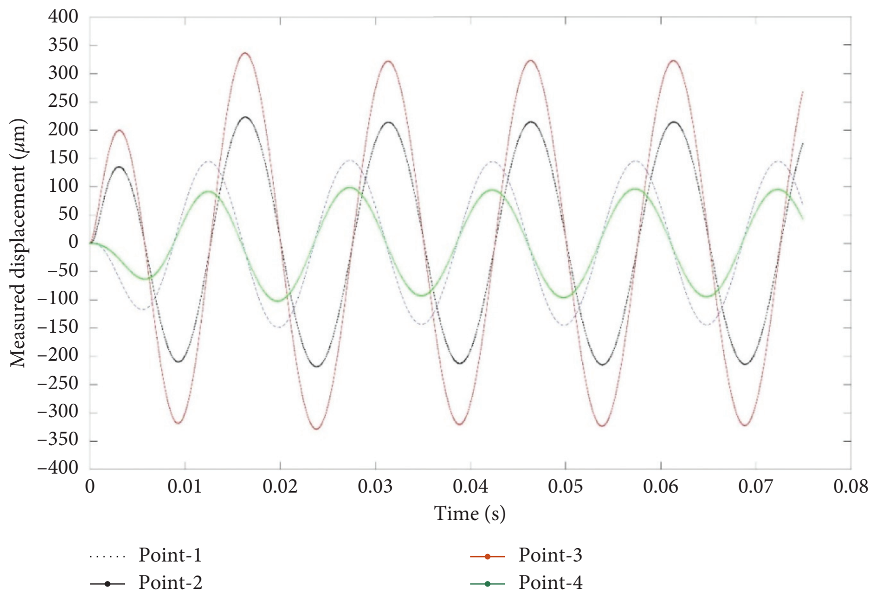


Figure 3: The structural response at four observed point.

the three cases are in good agreement, and in the acceptable range, it shows that the accuracy of the deterministic bearing load identification algorithm is feasible.

5.2. Bearing Load Identification under 3% Uncertainty. For testing the proposed uncertain bearing load identification algorithms, the unbalance parameters $\{m, e, \varphi\}$ are interval uncertain and the rotor material parameters $\{E, G\}$ are probability uncertain. Assumed the probabilistic uncertainty parameters are taken as the mean (that is, $\{E = 210\,\mathrm{GPa}, G = 80\,\mathrm{GPa}\}$), adjusting the interval uncertainty parameters under 3% uncertainty (that is, 3% off from the mean $\{m = 14\,\mathrm{kg}, e = 1\,\mathrm{mm}, \phi = {}^{0^\circ}\}$), and combining them together with the obtained mean bearing load as shown in Figure 4 into the forward solver (TMM) to calculate the response $\mathbf{q}_T$, the sensitivity curves of the response $\mathbf{q}_T$ with respect to the interval uncertainty parameters $\{m, e, \phi\}$ are obtained, as shown in Figure 5(a). It can be seen from the diagram that the response $\mathbf{q}_T$ is more sensitive to the unbalance parameters $\{m, e, \phi\}$ and the unbalance parameters have great influence on the identification of bearing load. Combining with the first-order derivatives to the unbalance parameters together with the response $\mathbf{q}_T(t, \lambda)$ coming from the obtained mean bearing load $\mathbf{F}_d^T$ into equation (7), the bounds of the response, $\mathbf{q}_{T\min}$ and $q_{T\max}$, can be obtained, shown in Figure 5(b).

Utilizing the obtained response, $\mathbf{q}_{T\min}$ and $q_{T\max}$, and the probability uncertainty parameters under 3% uncertainty into the inverse problem solver for the deterministic load identification algorithm, the bearing load with respect to each probability uncertain parameters is calculated and the results are plotted in Figure 6(a). It shows that the sensitivity curves vary greatly in the whole-time history, and the probability parameters will have a relatively huge single impact on the accuracy of the identified boundaries of the bearing load. Combining with the disturbance part $\Delta\mathbf{F}_r$ coming from the probabilistic parameters η together with the obtained $\mathbf{q}_{T\min}$ and $q_{T\max}$ together with the obtained mean bearing load $\mathbf{F}_d^T$ into equation (10), the bounds of bearing load, $\mathbf{F}_L^T$ and $\mathbf{F}_R^T$, can be obtained, shown in Figure 6(b). The identified bearing load boundary can effectively describe the time history of the bearing load change, the upper and lower boundaries of the bearing load can better contain the middle load, and the middle load is within the upper and lower boundaries. In addition, at the peak of the bearing load, the boundary of the identified bearing load is wide, so the interval and probability uncertain parameters have a great influence on the identification results of the bearing load currently.

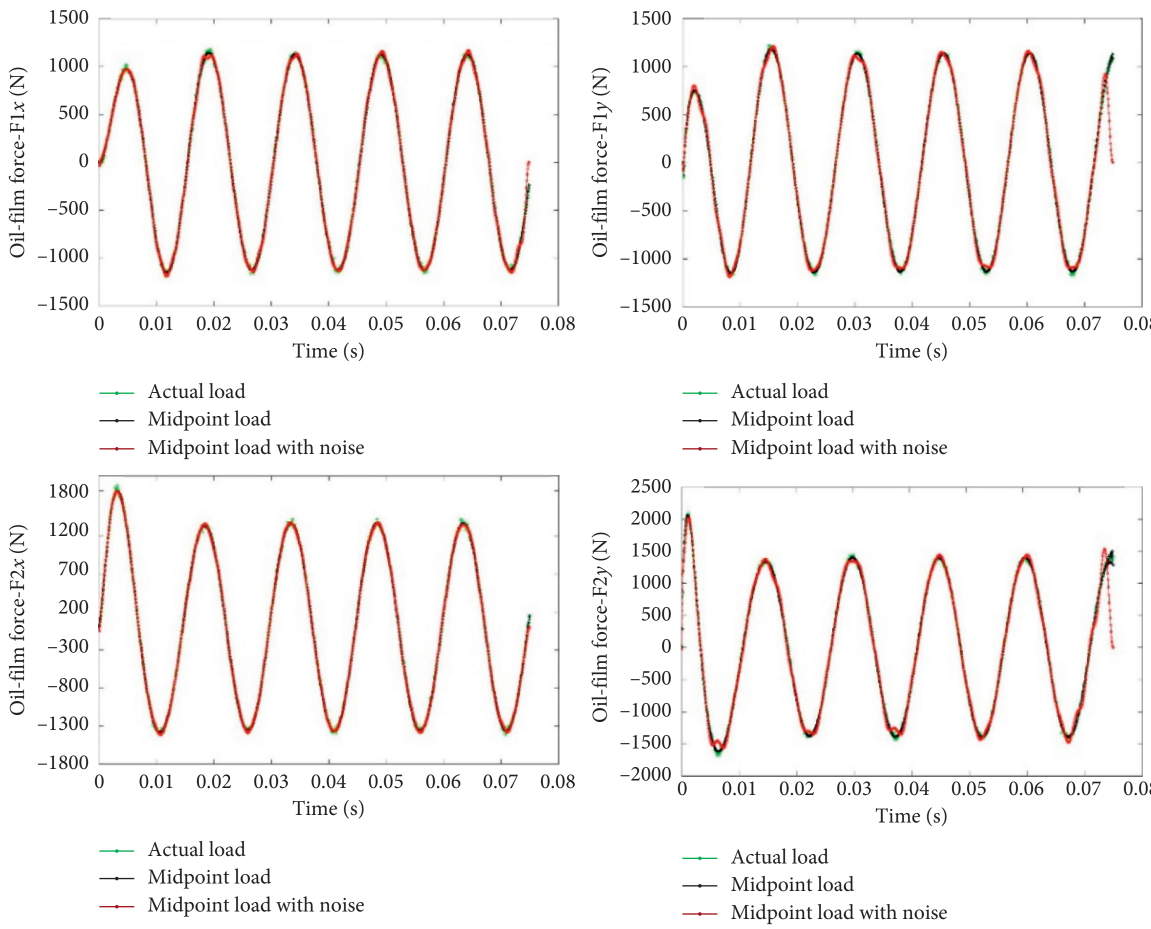
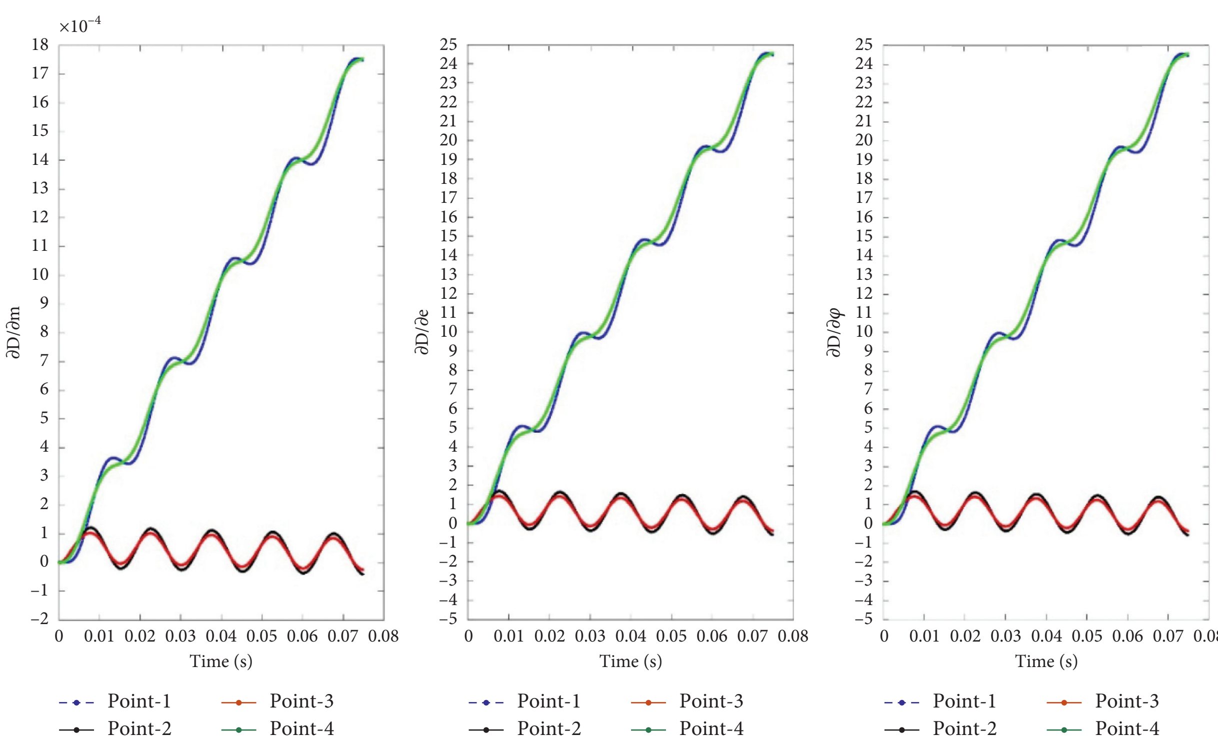
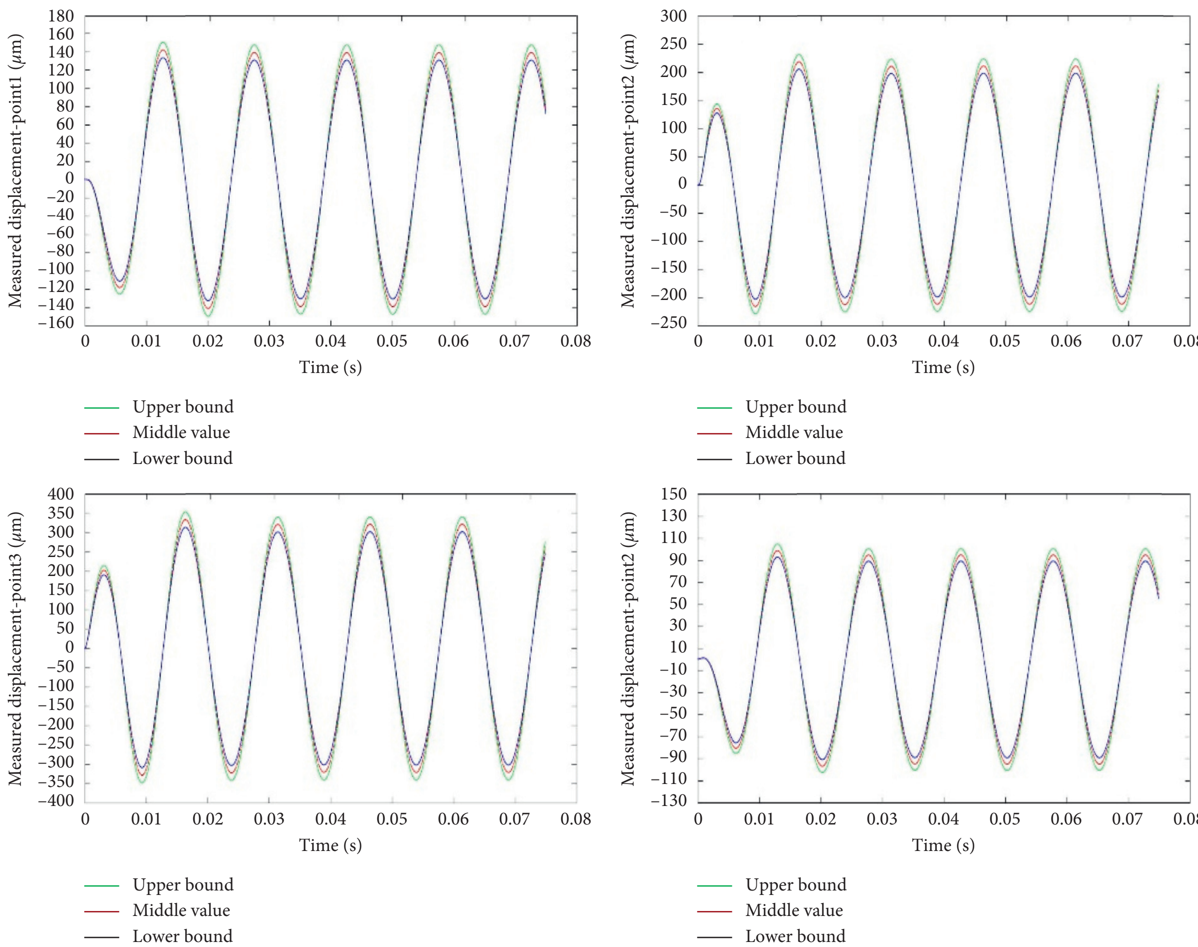


Figure 4: The identified bearing loads.



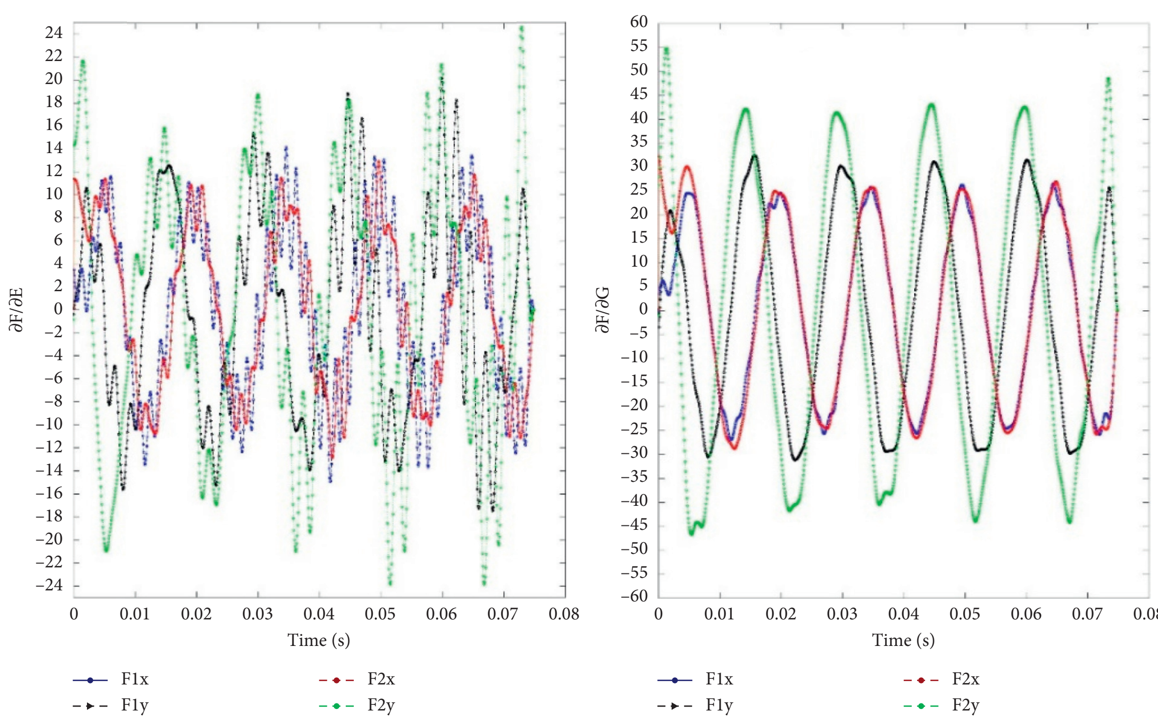
(a)

Figure 5: Continued.



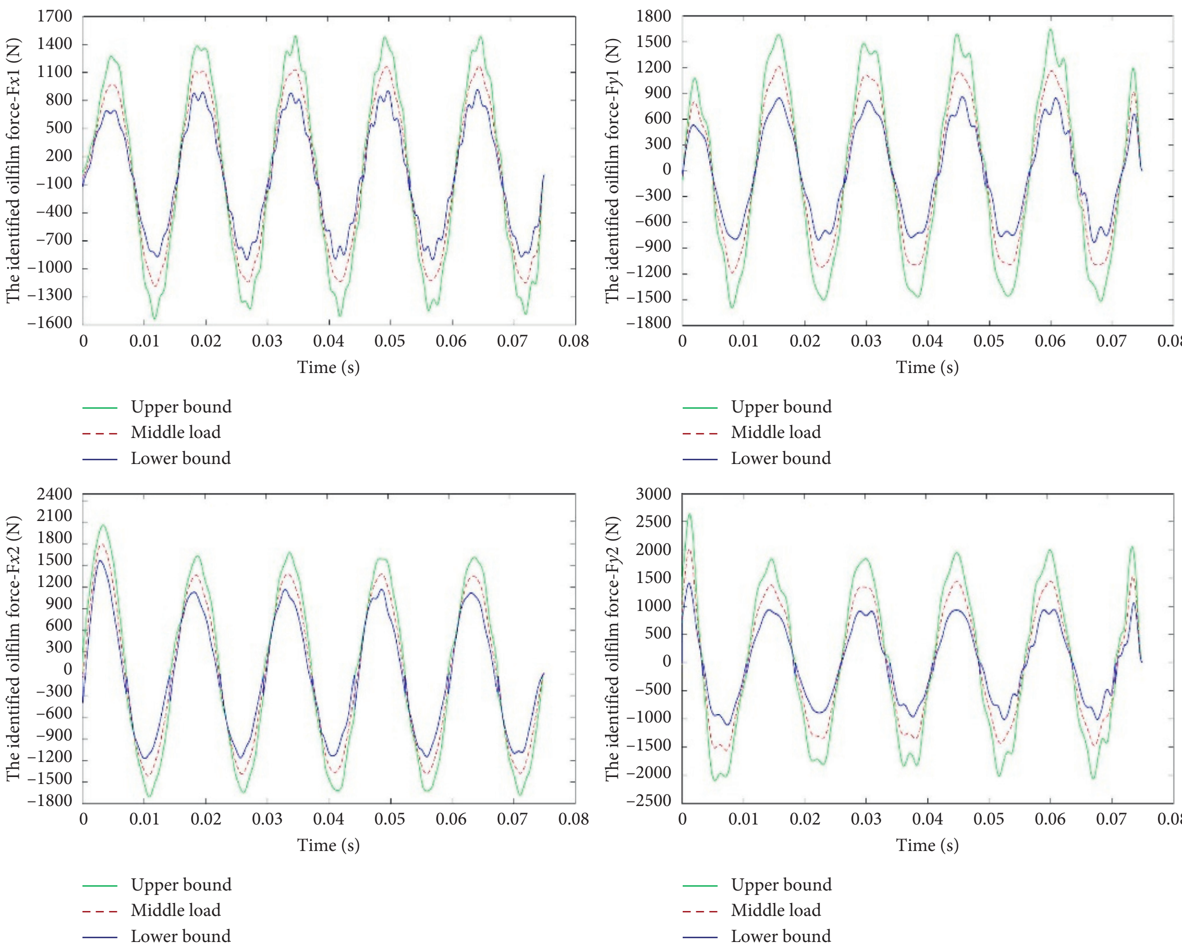
(b)

Figure 5: The rotor structure transient response: (a) the sensitivity curve of the transient response to the interval parameters; (b) the bounds of the rotor structure transient response.



(a)

Figure 6: Continued.



(b)

Figure 6: The identified uncertain bearing load: (a) the sensitivity curve of the bearing load to probabilistic parameters; (b) identified bearing load boundary at 3% uncertainty level.

6. Conclusions

Bearing load is the key factor that determines the lifetime and reliability of the rotor system. Bearing load plays a great role in health monitoring and fault diagnosis. Due to the existence of uncertain parameters, bearing load is a distribution strip. Based on the interval and perturbation theory and the improved regularization method, this paper proposes a new bearing load identification framework for the bearing-rotor system with uncertainty and measurement noise. The problem of bearing load identification is transformed into two kinds of certain inverse problems, namely, the bearing load identification on the mean value of uncertain parameters and the sensitivity identification of bearing load with respect to each uncertain parameter. The improved regularization method can overcome the ill-posedness of bearing load reconstruction for a deterministic rotor system. In the numerical example, the present method can stably identify the bounds of the bearing load only by knowing the bounds of the interval parameters and the statistical characteristics of the probability parameters, which has a certain practical value in engineering.

Data Availability

The data supporting the conclusion of the article are included in the relevant figures and tables in the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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