Research Article

Effect of Lubricant Starvation on the Tribo-Dynamic Behavior of Linear Roller Guideway

Parivash Soleimanian ¹, Mahdi Mohammadpour ² and Hamid Ahmadian ¹

¹Center of Excellence in Experimental Solid Mechanics and Dynamics, School of Mechanical Engineering, Iran University of Science and Technology, Narmak, Tehran 1684613114, Iran

²Wolfson School of Mechanical, Electrical and Manufacturing Engineering, Loughborough University, Loughborough, UK

Correspondence should be addressed to Mahdi Mohammadpour; m.mohammad-pour@lboro.ac.uk

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This paper presents an experimentally validated numerical approach linear roller guideways considering coupled vertical and horizontal (feed) motions and taking into account lubricant starvation. The inlet starvation is considered by incorporating potential flow method. Results show that starvation has pronounced effect on the lubricant film thickness, friction, and applied load on contact by up to 32%. Localised pressure values may vary by up to 100%. The severity of starvation effect is frequency dependent. It is also revealed that the starvation effect can be controlled by the amount of preload on linear guideway.

1. Introduction

Linear guideways are replacing sliding components due to higher demand for ultraprecision machining. Important features of linear guides are reduced friction, stiffer contacts, and improved precision. Studies on the gantry machines show that 60% of their total stiffness as well as 90% of damping in the system are contributed by their joints. Therefore, linear guides are the key components of these machines and determine the dynamics of the system and its accuracy [1, 2].

These guideways experience horizontal and vertical motions simultaneously. Consequently, rolling elements undergo coupled motions such as slipping and squeeze conditions. These motions are excited by machining loads as well as the contact entertainment conditions [3], leading to instability [4–7], noise [8, 9], fatigue [10–12], and wear [13, 14]. This dynamic behavior also governs the frictional behavior and losses in the system.

Friction in the linear guideway acts as a source of damping, affecting the precision of machine. Also, frictional heat generation results in the variation of stiffness and thermal distortion [15]. Consequently, considering coupled

vertical and feed motions and the effect of system's tribology, any model requires to consider the multiphysics effects of dynamics and tribological.

Some studies on the dynamics of linear guideways with the purpose of improving linear guideway stability and to increase the accuracy of machining [4, 6, 14, 16–19] have assumed Hertzian contacts in conjunctions neglecting the effect of lubrication. Considering the effects of tribology on the linear guideway dynamics, these effects cannot be neglected. Numerous studies are reported investigating friction in linear guideways [20–24]. In all the above investigations, the tribological effects are neglected. The proposed models also do not cover the frictional phenomena such as sliding, presliding, and lag in these modes. Soleimanian et al. [25] pioneered a tribo-dynamic system. The model showed the coupling between tribology and dynamics but did not take into account detailed realistic boundary conditions.

For accurate prediction of lubricant film thickness and, as a result, the dynamic and frictional behavior in linear guideways, application of realistic boundary conditions is essential. A key aspect in applying realistic boundary conditions is the consideration of inlet starvation. It is

demonstrated numerically [25–27] and experimentally [28–30] that at contact reversal, mixed or boundary regimes of lubrication are dominant. Therefore, determination of appropriate boundary conditions considering contact starvation is essential to accurately ascertain the tribological conditions. Fundamentally, there are two sources of starvation in lubricated contacts. The first type of starvation is originated from lack of lubricant supply, leaving the lubricated conjunction starved. This type of starvation may particularly be important due to reflow of oil in the groove of rolling elements. At high speed, high viscosity oil, or inefficient oil feeding conditions, the amount of lubricant available will be insufficient to produce a full oil film. The second type of starvation is due to the complex flow at the inlet, leading to reversal flow and hence inappropriate supply of lubricant and starvation. In this paper, the latter is being investigated, assuming that there is enough physical supply of lubricant, and the main source of starvation is due to reversal flow at the inlet.

Numerically, the starvation effect can be applied by a starved inlet condition proposed by Mohammadpour et al. [31], which should be under the starvation threshold defined by Hamrock and Dowson [32]. The realistic position for the starved boundary condition (and hence the amount of starvation) can be predicted using potential flow method, which accounts for reversing flow at the inlet, leading to starvation. The inlet stagnation point, which no reverse flow occurs beyond, is calculated by solving the potential flow equations. It is assumed that beyond the stagnation point, the remaining lubricant contributes to the fluid film formation. This approach was first pioneered by Tipei [33] for sliding contacts and recently employed for numerical predictions by Mohammadpour et al. [34] and revealed good agreement with the measurements conducted by Johns-Rahnejat and Gohar [35].

Some references have studied the effect of starvation on oil film and its resulting fatigue in elastohydrodynamic contact [36, 37]. Liu et al. [38] studied the friction and wear behavior of laser textured surfaces in nonconformal contact under starved lubrication. Zhang et al. [39] worked on influence of lubrication starvation and surface waviness on the oil film stiffness of elastohydrodynamic lubrication line contact. In this work, normal contact stiffness and oil film stiffness of line contact are studied numerically and analytically. Also, they investigated the effects of the normal load, rolling speed, regular surface waviness, and starved lubrication level on the oil film stiffness. Liu et al. [40] proposed a fatigue life prediction model for finite line contact under starved thermal elastohydrodynamic lubrication condition. In this paper, the results show that by introducing the starved lubrication condition, the lubrication characteristics and fatigue life of finite line contact are noticeably different from those of fully flooded condition. For example, the severe starvation conditions result in a significant increase in the coefficient of friction and a decrease in fatigue life. Highly loaded EHL contacts often operate under starved lubrication and mixed lubrication regime. Gupta et al. [41] studied the tribological and vibrational behaviors of conventional and textured spur gear

pairs experimentally and under fully flooded and starved lubricating conditions at different operating parameters. Yahiaoui et al. [42] analyzed the film formation mechanisms in a smooth EHL contact moved by oscillating velocities considering both squeeze and starvation. Based on the work of recent theory [43], the authors proposed an analytical solution to the Reynolds equation and validated it experimentally in order to better understand the mechanisms of lubrication under time-varying conditions.

This paper proposes an experimentally validated tribo-dynamic approach for linear roller guideways considering the coupled vertical and horizontal (feed) motions and taking into account the lubricant starvation. The developed dynamic model employs a tribological approach implicitly for the starved lubricated contacts which employs extrapolated film thickness equation for line contact predictions. The friction in this implicitly utilised tribo-dynamic approach is calculated considering thermal and non-Newtonian effects based on the provided model by Evans and Johnson [44], which was developed based on an experimental-analytical approach under elastohydrodynamic conditions. The effect of inlet starvation is considered by incorporating Tipei potential flow method [33]. After completing the implicit model with starvation effects, contact conditions for each rolling element are provided to a numerical mixed-elastohydrodynamic model of the rolling element to obtain tribological performance such as pressure and film thickness distributions. This explicit model also considers the effect of starvation by adjusting the inlet of computational domain to the calculated starvation boundary condition. Such multiphysics approach including starvation effects for linear roller guideways, coupling horizontal (feed), and vertical dynamics has not been reported hitherto. Results show that starvation has pronounced effect on the lubricant film, friction, and applied load on contacts by up to 32%. The severity of starvation effect is frequency dependent. The pressure distributions obtained from numerical EHL model show considerable differences between starved and fully flooded conditions, leading up to nearly 100% variation in the maximum pressure values. It is also revealed that the starvation effect can be controlled by the amount of preload on linear guideway.

2. Linear Roller Guideway Implicit Tribo-Dynamics

Total load applied to the linear roller guideway is shared between rollers. Under dynamic conditions, rollers' contribution is time-variable. So, using a quasi-static analysis to determine the contact load of each roller is unrealistic. The contact in the linear guideway has an oscillating nature that comes from the machining (vertical) and feed (horizontal) dynamics. Hence, rollers undergo fluctuating horizontal and vertical motions. From tribological point of view, the vertical motion contributes to the squeeze effect and the horizontal motion contributes to the lubricant entrainment. Schematic of recirculating linear roller bearing guideway is shown in Figure 1 comprising a rail, a carriage, and rolling elements. For the dynamic analysis of this system, the rail and carriage

FIGURE 1: Recirculating linear roller bearing guideway.

are assumed to be rigid, the effect of side leakage is neglected, edge effects are considered negligible and manufacturing imperfection, and thermal expansions are not considered.

Figure 2 represents a two-dimensional model of linear guideway, which is subject to normal force F and feed velocity. The force F represents cutting force, and horizontal oscillation velocity expresses feed velocity. As shown in Figure 2, the guideway has four grooves marked as 1, 2, 3, and 4. The applied force on roller set of groove i is Q_i ($i = 1, 2, 3, 4$). Rollers of each groove are under preload F_0 , with the associated deformation of δ_0 . θ is the angle between normal direction to the contact and the x -direction. Since the structure is symmetric, it is assumed that $Q_1 = Q_2 = F_L$ and $Q_3 = Q_4 = F_U$.

The feed degree of freedom (DOF) is the constraint with the feed velocity. This motion is in equation (1a). Therefore, under oscillating horizontal velocity in z -direction, the equations of motion for linear guideway are reduced to a single vertical DOF of equation (1b):

$$u = A_1 \sin(2\pi f_v t + \varphi_1) + A_2 \sin(2\pi f_v t + \varphi_2), \quad (1a)$$

$$M\ddot{y} + 2N \sin(\theta)(F_U - F_L) = Mg - F. \quad (1b)$$

The constant coefficients in the velocity equation are obtained from an instrumented linear guideway explained in the experimental section of this paper. Rolling elements have been assumed to take equally shared load. This assumption is in line with similar methodologies in the literature [6, 7, 45].

In equation (1b), N is the rolling element number and M is the rail mass. F_L and F_U represent applied load to each rolling element on the lower and upper grooves, respectively. The velocity in horizontal direction (feed velocity) is the source of coupling in the considered DOFs, affecting the lubricant film thickness and ultimately forces. F is the excitation force.

The force in the contact can be achieved from contact compliance relationship [5]. Elastic deformation is assumed in the contact of a rigid half space and deformable cylinder:

$$F_U = k\delta_U^{10/9}, \quad (2a)$$

$$F_L = k\delta_L^{10/9}, \quad (2b)$$

where k represents the coefficient for stiffness in the contact. The deformation at the contact comprises the dynamic load in the normal direction and an initial preload, as well as the space occupied by lubricant film [3]:

$$\delta_U = \frac{(y + 2 \sin(\theta)h_{c_U})}{2 \sin(\theta)} + \delta_0, \quad (3a)$$

$$\delta_L = \delta_0 - \frac{(y - 2 \sin(\theta)h_{c_L})}{2 \sin(\theta)}. \quad (3b)$$

The central thickness of lubricant film at the center in the lower and upper grooves is expressed as [3]

$$h_{c_U} = R \left[13.924 \bar{F}_U^{-0.045} U^{0.647} G^{0.46} \left(1 - 0.75 e^{132 \left(\frac{\partial h_{c_U}}{\partial t} \right) / u} \right) \right], \quad (4a)$$

$$h_{c_L} = R \left[13.924 \bar{F}_L^{-0.045} U^{0.647} G^{0.46} \left(1 - 0.75 e^{132 h_{c_L} / \partial t / u} \right) \right]. \quad (4b)$$

$\bar{F}_U$, $\bar{F}_L$, U , and G are dimensionless parameters as

$$\bar{F}_U = \frac{F_U}{E' R l'}, \quad (5a)$$

$$\bar{F}_L = \frac{F_L}{E' R l'}, \quad (5b)$$

$$U = \frac{\mu_0 u}{R E'}, \quad (5c)$$

$$G = \alpha E', \quad (5d)$$

where u is the velocity of feed motion, applied as a non-holonomic constraint. Equations (4a)–(4b) are regressed equations that are obtained from regression and fitting to data obtained from tests or numerical solutions performed for different conditions.

The above film thickness equations establish the coupling between the tribology and dynamics in the implicit model. ODE45 function in Matlab environment is used for integration of equations to obtain $\ddot{y}$, $\dot{y}$, and other dependent parameters.

Greenwood and Tripp [46] method is used to predict the boundary friction. The method, originally pioneered by Greenwood and Tripp, calculates the boundary friction based on information from surface level interactions rather than macroscale measurements. This allows the model to capture the friction value based on surface topography and material interactions and applicable for wide range of working conditions. It is based on Gaussian distribution of surface asperities, which is applicable to the engineering surfaces use in this study. In the mixed regime of lubrication, a proportion of applied load is carried by the asperities. This share is obtained as

FIGURE 2: Load on each roller of groove in the linear guideway system.

$$W_a = \frac{8\sqrt{2}}{15} \pi (n\sigma\beta)^2 \sqrt{\frac{\sigma}{\beta}} E' A F_{5/2}(\lambda), \quad \lambda = \frac{h}{\sigma} \quad (6)$$

The area occupied by asperities is [40]

$$A_a = \pi^2 (n\sigma\beta)^2 A F_2(\lambda). \quad (7)$$

The statistical functions $F_2(\lambda)$ and $F_{5/2}(\lambda)$ can be expressed through polynomial function as [47, 48]

$$F_2(\lambda) = \begin{cases} -0.002\lambda^5 + 0.028\lambda^4 - 0.173\lambda^3 + 0.526\lambda^2 - 0.804\lambda + 0.5, & \text{for } \lambda \leq 3, \\ 0, & \text{for } \lambda > 3, \end{cases} \quad (8a)$$

$$F_{5/2}(\lambda) = \begin{cases} -0.004\lambda^5 + 0.057\lambda^4 - 0.296\lambda^3 + 0.784\lambda^2 - 1.078\lambda + 0.617, & \text{for } \lambda \leq 3, \\ 0, & \text{for } \lambda > 3. \end{cases} \quad (8b)$$

Boundary friction can then be presented as [49]

$$F_b = \tau_l A_a, \quad (9)$$

where τ_l is the limiting shear stress of lubricant expressed as [47, 49]

$$\tau_l = \tau_{l_0} + \beta_m P_m, \quad (10)$$

where $P_m = W_a/A_a$. This limit is the highest shear stress that a lubricant can exhibit under extreme pressure at asperity contacts [44].

Evans and Johnson [44] expression is used for viscous friction F_v , under EHL conditions, which considers thermal effects:

$$F_v = w \left(0.87\alpha\tau_0 + 1.74 \frac{\tau_0}{\bar{p}} \ln \left(\frac{1.2}{\tau_0 h_c} \left(\frac{2\dot{K}\mu_0}{1 + 9.6\zeta} \right)^{1/2} \right) \right), \quad (11)$$

where

$$\zeta = \frac{4}{\pi} \frac{\dot{K}}{h_c/R} \left(\frac{\bar{p}}{E' R K' \rho' c' u} \right)^{1/2},$$

$$w = F_U \text{ or } F_L, \quad (12)$$

$$h_c = h_{c_U} \text{ or } h_{c_L}.$$

The total friction can then be calculated as

$$F = F_b + F_v. \quad (13)$$

3. Starvation Model

As mentioned in the introduction, in this study, the starvation effect of reverse flow at the inlet is investigated. The realistic inlet boundary condition is applied in both implicit tribo-dynamic analysis and explicit numerical model outlined in the next section. Hamrock and Dowson defined a parameter as starvation threshold using numerical methods [32], which was followed experimentally

using optical interferometric by Wedeven et al. [50]. If the boundary conditions in the contact, which is analogous to the inlet boundary position in the computation domain of a numerical model, is inside this threshold, starvation occurs, and the film thickness of lubricant is smaller than the one for fully flooded condition. These thresholds are defined as [32]

$$\begin{aligned} m_U^* &= 1 + 3.06 \left[\frac{R}{a_H^2} h_{c_U} \right]^{0.58}, \\ m_L^* &= 1 + 3.06 \left[\frac{R}{a_H^2} h_{c_L} \right]^{0.58}. \end{aligned} \quad (14)$$

The actual inlet distances, m_U and m_L , can be calculated based on an inlet boundary condition (see later). If $m_U < m_U^*$ and $m_L < m_L^*$, then in the implicit model, the film thickness values h_{c_U} and h_{c_L} are adjusted to consider the starvation effect as [32]

$$\begin{aligned} \left(\frac{h_{c_U,S}}{h_{c_U,F}} \right) &= \left(\frac{m_U - 1}{m_U^* - 1} \right)^{0.29}, \\ \left(\frac{h_{c_L,S}}{h_{c_L,F}} \right) &= \left(\frac{m_L - 1}{m_L^* - 1} \right)^{0.29}. \end{aligned} \quad (15)$$

Tipei [33] initially found that in the inlet zone, there are swirl flows, where some counter flow occurs. This is also noted in experimental observations by Birkhoff and Hays [51]. It means that regardless of available oil at the inlet, only a fraction of the inlet lubricant flow is admitted into the contact domain. This is a reason for starvation, which is investigated in this study.

The comparison of inlet distance and the starvation thresholds estimates the level of starvation. The proposed model by Tipei is based on a well-established method of potential flows (Q_1, Q_2, Q_3), as shown in Figure 3. This method yields [33] the following compatibility equation:

$$\begin{aligned} &\cot^2 \pi \left[\frac{1}{2} - \frac{1 - k_u}{f(k_u)} \right] - \cot^2 \pi \sqrt{\left[\frac{1}{2} - \frac{1 - k_u}{f(k_u)} \right]^2 - \frac{2k_u}{f(k_u)}} = \cot \pi \left\{ \frac{1}{2} - \frac{1 - k_u}{f(k_u)} - \sqrt{\left[\frac{1}{2} - \frac{1 - k_u}{f(k_u)} \right]^2 - \frac{2k_u}{f(k_u)}} \right\} \\ &\times \cot \pi \left\{ \frac{1}{2} - \frac{1 - k_u}{f(k_u)} + \sqrt{\left[\frac{1}{2} - \frac{1 - k_u}{f(k_u)} \right]^2 - \frac{2k}{f(k)}} \right\}. \end{aligned} \quad (16)$$

where $k_u = u_1/u_2$ and u_1 and u_2 are velocities of surfaces. $f(k_u)$ depends on the gradient of inlet pressure. $f(k_u)$ values for common conditions of k_u are presented in Table 1. It is shown that the Prandtl-Hopkins boundary conditions

cannot explain the inlet swirl flow [34]. Thus, the Swift-Stieber boundary condition is used in this study. This assumption results in

$$\cos \phi_{i_U} = - \left[1 - \frac{h_{c_U}}{R} (\cosh \vartheta_i - 1) \right] \cdot \cos \phi_{i_L} = - \left[1 - \frac{h_{c_L}}{R} (\cosh \vartheta_i - 1) \right], \quad (17)$$

where the dimensionless ϑ_i is the film thickness ratio between the inlet to the central film thickness. These are obtained by solving the following two equations:

$$\begin{aligned} \frac{\cosh \vartheta_i}{\cosh \vartheta_e} &= \frac{1 - (1/3)(1 + (2\sqrt{k}/1 + k))}{1 - (f(k)/6(1 + k))}, \\ \left[1 - \frac{1}{3} \left(1 + \frac{2\sqrt{k_u}}{1 + k_u} \right) \right] \tanh \vartheta_e - \left[1 + \frac{f(k_u)}{6(1 + k_u)} \right] \tanh \vartheta_i \\ &- \left[1 - \frac{f(k_u)}{6(1 + k_u)} \right] \cosh \vartheta_i [\arcsin(\tanh \vartheta_e) - \arcsin(\tanh \vartheta_i)] = 0. \end{aligned} \quad (18)$$

TABLE 1: Commonly encounter values of $f(k_u)$.

k_u	$f(k_u)$
0	4
0.5	7.8
1	16

FIGURE 3: Flow through contact carried by the solid surfaces.

The physical inlet distances m_U and m_L are then found from the solution of equations (14)–(18) as

$$\begin{aligned} m_U &= R \sin \phi_{i_U}, \\ m_L &= R \sin \phi_{i_L}. \end{aligned} \quad (19)$$

The fully flooded regime is considered by a film thickness, which increases with increasing velocity. On the other hand, in the starved regime, with increasing velocity, the film decreases. This is mainly due to the reduced inlet distance represented in equation (19). The overall behavior of the starvation with respect to the rolling velocity is a result of these two competing factors.

4. EHL Contact Model

Film thickness, pressure distributions, and friction are calculated using a numerically solved EHL model in the mixed

regime of lubrication. Presented results are obtained for one limited cycle of feed velocity based on obtained load and kinematics in the previous section.

The model comprises simultaneous solution of Reynolds equation, load balance, and surface deformation. Under the mixed regime, contact load is shared between asperities and the fluid film. Hence, the load equilibrium should also be maintained. At any instant, the total pressure is obtained as

$$p = p_h + p_a, \quad (20)$$

where p is the total pressure, p_h is the hydrodynamic part of pressure, and p_a is the pressure at asperity contacts. A schematic representation of the film is provided in Figure 4.

Method provided by Greenwood et al. [38] is used to achieve asperity pressures. The method is based on the assumption of Gaussian distribution for asperities. The asperity pressure is calculated as

$$p_a = \frac{8\sqrt{2}}{15} \pi (n\sigma\beta)^2 \sqrt{\frac{\sigma}{\beta}} E' F_{5/2}(\lambda), \quad \lambda = \frac{h}{\sigma}, \quad (21)$$

where the function $F_{5/2}(\lambda)$ is statistical representation of Gaussian distribution, which is calculated by equation (8b).

Solution of Reynolds equation yields the hydrodynamic pressure distributions. The Reynolds equation in the following form is used in this study [52]:

$$\frac{\partial}{\partial x} \left(\frac{\rho h^3}{\mu} \frac{\partial p_h}{\partial x} \right) - 12u \frac{\partial(\rho h)}{\partial x} - 12 \frac{\partial(\rho h)}{\partial t} = 0. \quad (22)$$

In equation (22), viscosity and density are pressure dependent. Dowson and Higginson equation for the pressure-density relationship is used [53]:

$$\frac{\rho}{\rho_0} = 1 + \frac{0.6 p_h}{1 + 1.7 p_h}. \quad (23)$$

For pressure-viscosity dependency, Roelands' model [54] is employed:

$$\begin{aligned} \frac{\mu}{\mu_0} &= \exp \left((\ln \mu_0 + 9.67) \left(-1 + \left(1 + 5.1 \times 10^{-9} \times p_h \right)^Z \right) \right), \\ Z &= \frac{\alpha}{5.1 \times 10^{-9} (\ln(\mu_0) + 9.67)}, \end{aligned} \quad (24)$$

where ρ_0 and μ_0 are the atmospheric values of density and viscosity. Iso-thermal conditions are assumed.

The Swift–Stieber boundary condition is applied at the inlet and outlet, at the position calculated based on the starvation approach outlined in Section 3. These boundary conditions can be expressed as

$$\begin{aligned} p &= 0|_{x_{in}}, \\ p &= 0|_{x_{out}}, \\ \frac{dp}{dx} &= 0|_{x_{out}}. \end{aligned} \quad (25)$$

The hydrodynamic force is required to maintain the force equilibrium between the applied force and the reaction force. This is calculated as

FIGURE 4: Representation of mixed-EHL contact.

$$\frac{w}{l} = \int_{x_{in}}^{x_{out}} p_h(x) dx + \int_{x_{in}}^{x_{out}} p_a(x) dx, \quad (26)$$

where w is the total load at the contact. The lubricant film thickness is presented as a function of the geometry and elastic compliance:

$$h = h_0 + \frac{x^2}{2R} - \frac{2}{\pi E'} \int_{x_{in}}^{x_{out}} p \ln(x - x') dx'^2. \quad (27)$$

5. Method of Solution

The following solution procedures are undertaken:

- (i) First, we have the implicit tribo-dynamic model:
 - (1) At each time step of implicit model, Tipei's inlet distance (equations (16)–(19)) and inlet starvation thresholds (equation (14)) are calculated to find the extent of starvation in the contact.
 - (2) Equations (1a)–(13) (implicit dynamic model) are solved using the ODE45 solver in Matlab environment, taking into account the starvation level calculated in the previous step. At each iteration of each time step, the film thickness is updated using equation (15).
 - (3) The film thickness affects the results of step 1, by altering the inlet distance. Hence, at each iteration of each time step, the inlet distance is updated (back to step 1, forming a loop).
 - (4) The results of converged implicit model provide the force values, which are used in the numerical analysis, including starvation. Also, the viscous and boundary frictions of rollers are calculated using Evans and Johnson [38] and Greenwood and Tripp methods.
- (ii) Second, we have explicit EHL numerical model:
 - (5) An initial estimation for the central lubricant film thickness, h_0 , is made for full numerical explicit elastohydrodynamic solution. Results of converged implicit model are used.
 - (6) The inlet and outlet distances are calculated using this estimated lubricant film thickness and

with the simultaneous solution of equations (16)–(19). This inlet distance informs the dimension of computational domain.

- (7) Equations (20)–(24) are solved using finite difference approach. Newton–Raphson method is used for numerical solution of system of equations, including EHL contact and asperity forces. The detailed solution procedure for this part is provided in [26].
- (8) As the inlet distance varies with film thickness, the computational domain is adopted according to the calculated central film thickness (back to step 6, forming a loop).

6. Experimental Setup

An experimental rig for lubricated roller guideway is developed to validate the presented model, as well as obtaining the input velocity for the dynamic model (equation (1a)). The experimental setup comprises an INA RUE45-E [55] guideway system, shown in Figure 5. The specifications of linear guideway are presented in Table 2. Table 3 presents solids, lubricant, and surface characteristics. In this experiment, the rail is excited by a shaker, whilst carriage is fixed. To measure the driving force, a B&K 8200 transducer is installed between mechanism and the structure. Accelerometer of type DJB A/120/V is placed at the tip of rail to measure the horizontal motion. The rail excitation is single harmonic force at low frequency. The response and force signals of system are measured and transmitted to the data acquisition system. The sampling rate is 1600 Hz, which is used to measure the force and response signals. During the test, a hydraulic pump is used to provide the bulk lubricant to the space between the carriage and the rail to ensure that there is enough lubricant.

Tables 2 and 3 show the specification of guideway system used in the friction model, respectively. The contact friction force, F_f , is obtained as follows:

$$F_f = F_d - Ma, \quad (28)$$

where F_d , M , and a are the force, inertia, and rail acceleration, respectively. The measured friction of lubricated

FIGURE 5: The experimental test setup.

TABLE 2: Roller and guideway information.

Roller radius	2.5 mm
Roller length	8 mm
Roller radius	V3 (high preload)
Preload	9200 N
Number of grooves	4
Number of rollers in each groove	20
M	6.58 kg

TABLE 3: Materials, lubricant, and surface properties.

Parameter	Value
n	$0.54 \times 10^9 \text{ m}^{-2}$
β	$1.5 \times 10^{-4} \text{ m}$
σ	$0.45 \times 10^{-6} \text{ m}$
E'	$2 \times 10^{11} \text{ Pa}$
μ_0	0.195 N.s/m^2
α	$2.2 \times 10^{-8} \text{ Pa}^{-1}$
β_m	0.047
τ_{l0}	$2 \times 10^6 \text{ Pa}$
τ_0	$1 \times 10^5 \text{ Pa}$
K	0.14 J/kgK
ρ'	8750 kg/m^3
K'	46 W/mK
c'	470 J/kgK
f_v	5 Hz

guideway with the horizontal frequency of 5 Hz is shown in Figure 6.

The measured acceleration is integrated in time domain to obtain horizontal velocity of the rail. A Fourier function with 1st and 3rd multiples of feed frequency is fitted to the measured curve to be utilised in equation (1a). For the investigated case study, $A_1 = 0.2919 \text{ (m/s)}$, $A_2 = 0.0178 \text{ (m/s)}$, $\varphi_1 = 2.0588$, and $\varphi_2 = 1.2679$. Figure 7 shows the comparison of the measured, as well as fitted values of velocities for the feed frequency of 5 Hz. The fitted function is used as input to the model. As can be seen in Figure 7, the system response to the harmonic excitation contains odd multiples (1st and 3rd).

7. Results and Discussions

In this section, first, the presented model is validated at conjunction and system levels against experiment and other works in the open literature. Then, the significance of starvation effect on the tribo-dynamics of linear guideway is

investigated under externally loaded and unloaded conditions. Under unloaded condition, the effect of preload is also studied. Finally, the effect on the pressure and film thickness distributions are shown using the full numerical explicit EHL model.

7.1. Validation. The presented method in this paper is validated at conjunction and system levels. The validation at conjunction level is performed by comparing the dimensionless EHL pressure and film distributions of the presented model with those presented by Masjedi and Khonsari [48]. The comparison at $W = 1 \times 10^{-4}$, $U = 1 \times 10^{-11}$, and $G = 4500$ is presented in Figures 8 and 9. As can be seen, the present model is in close agreement with the reported results of Masjedi and Khonsari [56].

For the system level validation, the computed sliding friction is compared against the measured values. The friction force is calculated using equation (13). Figures 10 and 11 show the comparison. It should be noted that in this calculation, the numerical EHL outputs were used. As is revealed from Figures 10 and 11, there is a good agreement between the results of the presented model and the experimental results.

7.2. Externally Unloaded Case. The implicit tribo-dynamic model is solved in the absence of external load, representing idle rotation of a machine. The normal displacement, y , under both starved and fully flooded conditions are presented in Figure 12. As is revealed from this figure, no significant effect from starvation is observed on the dynamic response of linear guideway. This can be associated with larger film and reduced intensity of starvation due to the reverse flow. The frequency of fluctuations in vertical direction is 6590 Hz, which follows the base natural frequency of the linear guideway in the absence of external excitation.

Figures 13 reveals the values of central film thickness calculated from the implicit solution of analytical tribo-dynamics under fully flooded and starved conditions. This figure demonstrates that there is a pronounced reduction of 11% for the film thickness when applying the starvation boundary condition. This is despite the minimal effect of starvation on the dynamic response as seen in Figure 12, which reveals insignificant effect of film thickness variation on the contact stiffness under unloaded conditions. The EHL film thickness is less affected by the load, and hence regardless of the change in film thickness, dynamic response does not change equally. It should be noted that the frequency of fluctuations in Figure 13 follows the feed motion frequency (5 Hz), which governs the speed of entertaining motion.

Figure 14 shows the contact force calculated from the tribo-dynamic method under starved and fully flooded conditions. Figure 14 demonstrates that the effect of starvation on the contact force is also pronounced (10.3%). This is due to different lubricant film thickness, which leads to different deformation in the contact and hence contact load. This effect may change with different velocities as the film is significantly affected by the velocity in comparison with

FIGURE 6: Friction versus time and @ feed frequency of 5 Hz.

FIGURE 7: Feed velocity in time domain @ feed frequency of 5 Hz, measured (solid line) and fitted response (dashed line).

load. Figure 15 shows the FFT spectra of contact load, revealing the presence of both base natural frequency of the linear guideway and the feed frequency. It should be noted that the base natural frequency, also present in the force response of Figure 14, has no considerable effect on the film thickness of Figure 13. This is due to much smaller contribution of this frequency in the force response, which cannot reveal significant effect on the film thickness since film is mainly governed by the speed of entertaining motion rather than the force (see (4a) and (4b)). Figure 15 also shows the shift in the base natural frequency due to the altered contact stiffness.

Figure 16 presents the zero reverse inlet distance (the actual inlet distance) m_U using the outlined method. The figure also presents the threshold of starvation under the same condition. It can be seen that the actual calculated inlet is well within the starvation zone, confirming the presence of severe starvation as the root cause of observed effects on the thickness of lubricant film and the contact load.

FIGURE 8: Comparison of pressure distribution, current model (solid) and Masjedi and Khonsari (dash-dot) [56].

FIGURE 9: Comparison of film thickness distribution, current model (solid) and Masjedi and Khonsari (dash-dot) [56].

One of the important factors in the design of linear guideways is friction. Figure 17 shows the friction force for a roller from upper row under both starved and fully flooded conditions. These results are based on the lubricant film thickness of Figure 13, taking in to account the non-Newtonian and thermal effects as embedded in equation (24). It can be observed that the predicted friction is affected by starvation, showing an increase of 4.8%.

7.3. Preload Effects under Externally Unloaded Case. In this section, the effect of preload of linear guideway on the starvation of roller contacts is investigated. The preload is decreased from 9200 N in previous section to 4000 N. The normal displacement, y , under both starved and fully flooded conditions is presented in Figure 18. As is revealed from Figure 18, the frequency of fluctuations has reduced under lower preload from 6590 Hz to 6320 Hz. This is due to

FIGURE 10: Friction in time domain, experimental values (solid line) and current model (dashed line).

FIGURE 11: Friction versus sliding velocity, experimental values (solid line) and current model (dashed line).

the reduced contact stiffness under lower preload. Similar to higher preload of 9200 N in the previous section, there is no significant difference between the amplitude of dynamic response under starved and fully flooded conditions.

Figure 19 illustrates the variation of central film thickness under starved and fully flooded conditions and reduced preload of 4000 N. The figure shows 5.3% reduction in the film thickness in comparison with 11% under 9200 N of preload. This reveals that the effect of starvation of film thickness is more severe under higher preload values.

Figure 20 presents the contact load obtained under starved and fully flooded conditions. Similar to film thickness, the trend of contact load results is similar between both preload values. The reduced base natural frequency is also observed here. The contact load shows 4.5% difference between starved and fully flooded conditions in comparison with 10.3% of higher preload value, which is in line with the observed behavior for the film thickness.

FIGURE 12: Effect of starvation on the normal displacement (y) under externally unloaded condition, fully flooded (solid line), and starved (dashed line).

FIGURE 13: Starvation effect on the central film thickness under externally unloaded condition, fully flooded (solid line), and starved (dashed line).

Figure 21 shows the friction force for a roller of upper row under both starved and fully flooded conditions and lower preload of 4000 N. It is clear that the effect of starvation on friction values under lower preload value is less pronounced.

7.4. External Harmonic Excitation Case. In this section, the implicit model is solved taking into account external excitation force in normal direction. This represents the excitation force under machining conditions. Analysis is performed at different force amplitudes of 6000 N, 9000 N, and 12000 N. Quasi-dynamic analyses are performed at different frequencies between 2500 Hz, 4500 Hz, 5000 Hz, and 9000 Hz. These frequencies are selected to cover before resonance, during resonance and after resonance of the investigated linear guideway. The aim is to investigate the

FIGURE 14: Starvation effect on the roller contact load under externally unloaded condition, fully flooded (solid line, black color), and starved (dashed line, pink color).

FIGURE 15: FFT spectrum of the roller load under externally unloaded condition, fully flooded (solid line), and starved (dashed line): (a) lower frequencies and (b) higher frequencies.

effect of starvation on tribo-dynamic behavior of the system. The frequency of horizontal velocity is 5 Hz, governed by the machining process. Figure 22 shows the percentage difference of the peak-peak value of roller contact load between starved and fully flooded conditions. It is shown that the effect of starvation heavily depends on the excitation frequency and amplitude. The highest effect of starvation is observed at 9000 N and before resonance, reaching up to 20%.

Figure 23 shows the percentage difference of the friction force value between starved and fully flooded conditions under different excitation amplitude and frequencies. It is shown that the starvation effect on friction force is also

frequency and amplitude dependent, revealing up to 32% difference after resonance and under 12000 N of excitation amplitude.

7.5. Explicit Numerical Tribology Model. In the current section, explicit EHL model is solved, and numerical results are obtained for loaded (9000 N amplitude) and loaded scenarios. The aim is to investigate the effect of starvation on the pressure and film thickness distributions. Before conducting full numerical EHL analysis, the regime of lubrication should be ascertained to understand the nature of fluid film formation. This can be verified by mapping the

FIGURE 16: Threshold of starvation at inlet (m_U^*) and the predicted inlet distance m_U , calculated inlet distance, Tipei method (solid line), calculated inlet starvation boundary, and Hamrock and Dowson method (dashed line).

FIGURE 17: Effect of starvation on the friction force of a roller under externally unloaded condition, fully flooded (solid line), and starved (dashed line).

results on the Greenwood chart [57]. The charts display the significance of elastic and viscous effects by quantifying two dimensionless parameters, G_e and G_v , as defined in the following:

$$\begin{aligned} G_e &= \frac{W^*}{U^{*1/2}}, \\ G_v &= \frac{W^{*3/2} G^*}{U^{*1/2}}. \end{aligned} \quad (29)$$

Results are presented in Figures 24(a) and 24(b) for unloaded as well as loaded condition under 9000 N. There are four distinct regions on the chart: (1) viscous-elastic (VE) representing EHL regime of lubrication, (2) iso-viscous-rigid (IR) representing hydrodynamic regime of lubrication, (3) iso-viscous-elastic (IE), and (4) viscous-rigid. Presented results reveal that under both loading conditions and for both starved and fully flooded cases, EHL regime of

lubrication is dominant. This justifies the use of provided numerical method here.

7.5.1. Unloaded Condition. After conducting an implicit tribo-dynamic calculation for starved and fully flooded conditions, the predicted forces obtained from the previous implicit method are utilised as input for the numerical explicit EHL model under starved and fully flooded conditions. Figure 25 presents the time domain results of the upper rollers forces under feed frequency of 5 Hz. Three points along a limit cycle of base natural frequency are marked as A1, B1, and C1 to perform numerical calculations.

Pressure and film thickness distributions for points A1, B1, and C1 are shown in Figure 26. Results are presented for both starved and fully flooded conditions. It is revealed that under starved conditions, the film thickness has consistently reduced. This has consequences in terms of friction and abrasive wear characteristics. The fully flooded pressure

FIGURE 18: Effect of starvation on the normal displacement y under externally unloaded condition and lower preload of 4000 N, fully flooded (solid line), and starved (dashed line).

FIGURE 19: Effect of starvation on the central film thickness under externally unloaded condition and lower preload of 4000 N, fully flooded (solid line), and starved (dashed line).

FIGURE 20: Starvation effect on the roller contact force under externally unloaded condition and lower preload of 4000 N, fully flooded (solid line, black color), and starved (dashed line, pink color).

FIGURE 21: Effect of starvation on the friction force under externally unloaded condition and lower preload of 4000 N, fully flooded (solid line, black color), and starved (dashed line, pink color).

FIGURE 22: Percentage difference of the peak-peak value of roller contact load between starved and fully flooded conditions, 6000 N (solid line), 9000 N (dashed line), and 12000 N (dotted line).

distributions reveal higher secondary spike of up to two times of the maximum Hertzian pressure, whilst starved pressure distribution adheres more to the dry Hertzian pressure distribution. The secondary spike is due to continuity of flow condition when lubricant progresses toward the exit, where the pressure is at the ambient value. Thus, its viscosity decreases dramatically. Hence, thinning of film takes place and the pressure rises due to diminution of gap (as a hydrodynamic effect), giving rise to the appearance of a pressure spike, which falls to ambient value very quickly. Due to smaller film in the starved case and higher pressure in contact, the change from this higher value to the ambient pressure and the consequent spike is more pronounced. This

higher secondary spike can lead to incorrect estimation of the probability of subsurface originated wear.

7.5.2. Condition under 9000 N. The load in the contact for upper rollers under external force of 9000 N and 9000 Hz of frequency is shown in Figure 27 for starved and fully flooded conditions. EHL analysis is performed at A2-J2.

The pressure and film thickness distributions for points A2-J2 are presented in Figure 28 for fully flooded and starved conditions. The film thickness consistently drops. It is clear that the fully flooded and starved pressure distributions are much closer under externally loaded conditions.

FIGURE 23: Percentage difference of the friction force value between starved and fully flooded conditions, 6000 N (solid line), 9000 N (dashed line), and 12000 N (dotted line).

FIGURE 24: Starved (pink) and nonstarved (blue).

FIGURE 25: Contact load time history as input for numerical EHL model, fully flooded (solid line), and starved (dashed line).

FIGURE 26: Pressure and film thickness distributions at A1, B1, and C1, fully flooded (solid line), and starved (dashed line).

FIGURE 27: Contact load time history as input for numerical EHL model, fully flooded (solid line), and starved (dashed line).

FIGURE 28: Continued.

FIGURE 28: Pressure and film thickness distributions at A2-J2 under 9000 N, fully flooded (solid line), and starved (dashed line).

8. Conclusion

Experimentally validated tribo-dynamic models are presented for linear guideways subject to contact starvation. The starvation investigated in this study is assumed to be purely due to flow reversal at the inlet. An implicit model, comprising system dynamics and tribology, is initially solved to obtain contact load and kinematics in a timely manner. This model also allows for comprehensive studies. Then, the input data obtained from the implicit model are provided to an explicit numerical EHL model, taking into account the contact starvation. The following conclusions can be made from the presented results:

- (1) Under externally unloaded conditions, the starvation has negligible effect on the vertical dynamics of the linear guideway but shows pronounced effect on the film thickness (up to ~11%), friction (up to ~5%), and contact load (up to ~10%). The frequency content of the response comprises both base natural frequency of the linear guideway and the feed velocity frequency. The base natural frequency is affected by the level of starvation via different film thickness values and hence the contact stiffness. The above effects are less pronounced under lower values of the contact preload.
- (2) Under externally loaded conditions, the severity and effect of starvation depend on the frequency and amplitude of external excitation. Results show that the contact load can be affected by starvation by up to 32% (before resonance and under 9000 N). The friction force can also be affected by starvation by up to 20% (after resonance and under 12000 N).
- (3) Results of full numerical EHL model are provided for both externally loaded and unloaded conditions. It is shown that the film thickness distributions show consistent reduction under starved conditions, leading to higher probability of surface originated wear. The pressure distributions under externally loaded conditions are largely similar between starved and fully

flooded conditions. Under externally unloaded conditions, the starved pressure distributions are much closer to the Hertzian pressure distribution, whilst fully flooded conditions exhibit up to two times higher pressure peaks (secondary spike) than the maximum Hertzian value. The latter can lead to incorrect estimation of the subsurface originated wear.

- (4) It should be noted that there are other practical aspects which are not investigated in this study. These include the reflow of lubricant on the race as well as feed hole position of the lubricant. The latter can affect not only the oil availability, but also the inlet region temperature, which can affect the viscosity and reverse flow starvation.
- (5) It is revealed that unlike the fully flooded assumption, the prediction of tribology in the contact for real world starved conditions is far more complex. Unlike fully flooded film thickness, which increases with the rolling velocity, in a starved contact and due to increased starvation at higher velocities, which leads to lower film thickness, this is not necessarily the case. Similar effect can be observed for viscosity as is also experimentally shown by Cann et al. [58]. Higher viscosities, which generate higher film thickness values, cause higher degree of starvation occurring at much lower velocities. Hence, presented comprehensive method is required to realistically predict the contact behavior.

Nomenclature

- a_H : Hertzian contact half width (m)
 A : Apparent contact area (m²)
 A_a : Contact area of asperities (m²)
 c' : Thermal coefficient of bounding solid surfaces (J/kgK)
 E' : Equivalent modulus of elasticity,
 $1/E' = 0.5 [(1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2]$ (Pa)
 F : External force (N)
 F_b : Boundary friction (N)

F_L :	Lower rollers contact force (N)
F_U :	Upper rollers contact force (N)
f_v :	Sliding velocity frequency in the contact (Hz)
f_n :	Linear guideway natural frequency (Hz)
g :	Gravitational acceleration (ms^{-2})
h :	Thickness of the lubricant film (m)
h_c :	Central value of lubricant film thickness (m)
h_0 :	Constant in film thickness equation (m)
h_{cU} :	Upper roller film thickness (m)
h_{cL} :	Lower roller film thickness (m)
k_u :	Speed ratio
K :	Lubricant thermal conductivity (J/kgK)
K' :	Solid bodies thermal conductivity (W/mK)
m_L :	Boundary parameter at the inlet for lower rollers
m_U :	Boundary parameter at the inlet for upper rollers
m_L^* :	Starvation demarcation boundary parameter in lower groove rollers
m_U^* :	Starvation demarcation boundary parameter in upper groove rollers
n :	Density of number of asperities (m^{-2})
N :	Rollers per row (groove) (m)
p :	Total pressure (Pa)
p_a :	Asperity contact pressure (Pa)
p_h :	Hydrodynamic film pressure (Pa)
P_H :	Maximum value of Hertzian pressure (Pa)
P_m :	Average value of contact pressure (Pa)
R :	Equivalent radius of curvature (m)
t :	Time (s)
u :	Conjunction sliding Velocity (m/s)
W :	Dimensionless normal force, $W = w/IER$
W_a :	Contact load carried by asperities (N)
x :	Displacement (m)
Z :	Piezo-viscous index
α :	Piezo-viscosity coefficient (1/Pa)
β :	Asperity average radius (m)
β_m :	Limiting shear stress slope versus pressure
ϑ_i :	Inlet to the central film thickness ratio
ϑ_e :	Outlet to the central film thickness ratio
ρ :	Density of lubricant (kgm^{-3})
ρ_0 :	Density at atmospheric pressure (kgm^{-3})
ρ' :	Density of contacting solids (kgm^{-3})
σ :	Standard deviation of roughness heights (m)
τ_0 :	Eyring stress (Pa)
τ_L :	Limiting value of lubricant shear stress (Pa)
τ_{l0} :	Limiting stress of lubricant at atmospheric pressure (Pa)
ν :	Poisson ratio
ϕ_{iL} :	Inlet position angle of lower groove rollers
ϕ_{iU} :	Inlet position angle of upper groove rollers
μ :	Dynamic viscosity of lubricant (Nsm^{-2})
μ_0 :	Dynamic viscosity of lubricant at atmospheric pressure (Nsm^{-2}).

Data Availability

The data used to support the findings of this study are restricted by the institution in order to honour confidentiality of this research.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this study.

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