Research Article

Brake Noise Reduction Method Based on Monte Carlo Sampling and Particle Swarm Optimization

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Brake noise is one of the principal components of vehicle noise and is also one of the most critical measures of vehicle quality. During the braking process, the occurrence of brake noise has a significant relationship with the working conditions of the brake system. In the present study, dynamometer test data and the finite element method (FEM) were used to analyze the direct and indirect effects of variations in the working parameters on the brake noise, and a brake noise reduction method was developed. With this method, Monte Carlo sampling was used to consider variations in the parameters of the brake lining during the braking procedure, and the particle swarm optimization method was used to calculate the optimal parameter combination for the brake lining. A dynamometer test was carried out to validate the effect of optimization on brake noise mitigation.

1. Introduction

Brake noise, especially high-frequency brake squeal, is one of the principal components of vehicle noise. It may cause noise pollution and directly affect the ride comfort of a vehicle [1]. As the number of automobiles increases, the environmental impact caused by brake noise becomes more serious. Disc brakes are the most common type of brake system and are widely used in passenger cars and light commercial vehicles. Because reducing the brake noise is an obvious way to improve the riding comfort of a vehicle, the noise control of disc brakes has become a focus of vehicle noise, vibration, and harshness (NVH) engineering. It is also an important part of the development process for the brake system or entire vehicle [2].

The dynamometer test and finite element method (FEM) are important means of studying brake noise. The former is used to evaluate the brake noise under real braking conditions, and the latter is widely used to consider the geometry characteristics and material properties of parts for the development of a brake system. The frequency domain complex-eigenvalue method is a common analytical

approach using the FEM [3]. It was introduced in the 1980s and is currently one of the most important means of predicting brake noise and guiding brake design in engineering [4]. The FEM has been used in many studies to examine the effects of geometric characteristics or material properties of a brake lining on the brake noise. Studies on geometric characteristics have been focused on the geometry, structure, wear, or slot of the brake lining [5–7], and studies on material properties have concentrated on the elastic modulus, damping, frictional coefficient, or surface characteristics of the brake friction material [8–11].

Existing research has shown that, in addition to the shape and material parameters, the working conditions also have a significant impact on the occurrence of brake noise [12]. Because of the strong randomness during the braking process, research on the influence of working conditions on noise has been very limited. Fazio et al. [13] and Tison et al. [14] investigated the influence of working parameters on the contact interfaces of the brake system. Soobbarayen et al. [15] studied the influence of loading conditions on the vibrational and acoustic responses of the disc brake system. Lee et al. [16] studied the effect of humidity on the friction coefficient of a

brake material. Some studies introduced uncertainty of the working parameters into a numerical model of the brake system to study the effect of the brake noise [17–19].

In the present study, dynamometer test data and the FEM were used to analyze the direct and indirect effects of variations in the working parameters on the brake noise and examine the main cause for the occurrence of brake noise with changes in the working parameters. A brake noise reduction method based on Monte Carlo sampling and particle swarm optimization (PSO) was developed that is based on the randomness of the working parameters during the braking process. An optimization scheme for the brake lining structure and material parameters was developed, and a dynamometer test was carried out to test the effectiveness of the optimization method.

2. Analysis of Brake Noise with the FEM

For a general vibrating system, the vibration can be expressed as

$$M\ddot{x} + C\dot{x} + Kx = F, \quad (1)$$

where M , C , and K are the mass, damping, and stiffness matrices, respectively, of the system; x is the displacement vector; and F is the excitation force of the system. For a brake system, the friction force is introduced as the excitation force during the braking process. The force vector of F can be obtained after simplification:

$$F = \mu K_f x, \quad (2)$$

where μ is the friction coefficient between the brake lining and disc and K_f is the friction stiffness matrix. By combining equations (1) and (2), a homogeneous equation can be obtained [20]:

$$M\ddot{x} + C\dot{x} + (K - \mu K_f)x = 0, \quad (3)$$

Equation (3) shows that, because of the existence of the friction force, the system stiffness is coupled and the stiffness matrix is asymmetric, which in turn results in asymmetry of the characteristic matrix. Therefore, the eigenvalues of the characteristic matrix may be complex under certain conditions. For an underdamped system, the eigenvalues always occur in complex conjugate pairs. The characteristic equation is given by

$$\det(\lambda^2 M + \lambda C + K - \mu K_f) = 0, \quad (4)$$

$$\lambda_{i,1,2} = \sigma_i \pm j\omega_i,$$

where λ_i is the eigenvalue of the i^{th} mode of the system, σ_i is the real part of the eigenvalue and represents the attenuation coefficient of the corresponding mode, and ω_i is the imaginary part of the eigenvalue and represents the natural angular frequency of the corresponding mode. The damping ratio of the corresponding mode is

$$\zeta_i = \frac{\sigma_i}{\omega_i}. \quad (5)$$

Therefore, the stability of a system can be determined according to whether σ is positive or negative. If $\sigma_i < 0$, the

damping ratio is positive, the vibration of the system will decay over time, and the system is stable under this condition. If $\sigma_i > 0$, the damping ratio is negative, and the vibration of the system will not decay. This means that the corresponding mode is unstable. Under this condition, the absolute value of the damping ratio reflects the instability of system. A higher value increases the possibility of brake noise occurring [21].

When the brake pedal is stepped down, the piston and the caliper in the brake assembly were driven by the rising brake fluid pressure and pushed the brake pads to move towards the rotating brake disk. In the present study, the brake system of a passenger car's front axle was chosen as the subject of research. Hypermesh 13.0 was used to mesh the model after simplification, while different parts were set to element sizes of 3–5 mm [22]. The model was meshed into 151942 elements of the C3D10M (10-node modified tetrahedron element with hourglass control). Figure 1 shows the original model and FEM. Table 1 presents the material properties of each part of the brake system.

Table 2 lists all interactions defined in the finite element model of brake system. In this study, the tie and surf-surf contact interactions were used to simulate the contact relationship between the various surfaces. Brake noise is affected by the friction coefficient between the disk and lining; the nominal friction coefficient provided by the friction-material manufacturer (and corrected through dynamometer tests) was used in the present analysis.

The bracket of the brake system was fixed on the knuckle by its own threaded hole, so the inner surfaces of both threaded holes were constrained in the FEM. On the other hand, the disc was clamped by the wheel hub and knuckle through screws, so the inner and outer surfaces of the convex plate on the disc were constrained. A prestressed modal was built to simulate the brake process. At first, a load of 3 MPa was applied to the end face of the piston to simulate the brake pressure. And in the next step, a rotational speed of the disc was defined with an equivalent linear velocity of 3 km/h by editing the keywords of MOTION ROTATION in the inp file for Abaqus analysis.

The brake system model after preprocessing was submitted to Abaqus 6.13-4 for complex modal analysis by using standard solver, and the Lanczos method was used to extract the eigenvalues of the system. Considering that the model involves contact and friction, NLGEOM was turned on for geometric nonlinearity, and unsymmetric matrix storage and solution scheme was used for the complex-eigenvalue extraction procedure [23].

Based on engineering experience, modes in the analysis results with a damping ratio of less than -0.6×10^{-2} were treated as noise and are given in Table 2. In addition, a dynamometer test based on SAE J2521 was carried out to verify the accuracy of the FEM. Figure 2 shows the frequency distribution and sound pressure level data. In the dynamometer test, brake squeal occurred at frequencies of 7800, 8800, and 11 000 Hz. In the frequency band of 3000–4400 Hz, brake noise also occurred, but the frequency of the noise was fragmented. This was related to property changes of the brake system caused by low temperatures in

FIGURE 1: Original model and FEM of the brake system.

TABLE 1: Material properties of parts of the brake system.

Part	Material	Density (10^3 kg/m^3)	Elastic modulus (MPa)	Poisson's ratio	Element size (mm)
Disc	HT250	7.15	120000	0.26	5.0
Caliper	QT450	6.86	173500	0.3	3.5
Bracket	QT450	6.86	173500	0.3	3.5
Piston	35#	7.85	212000	0.31	3.0
Pin	Q345	7.85	206000	0.28	3.0
Backplane	Q345	7.85	206000	0.28	3.0
Lining	NAO	1.61	709	0.36	3.0

TABLE 2: Interactions in the brake system finite element model.

Contact pair	Interaction type	Number of contact pairs in the model	Friction coefficient
Backplate bracket	Surf-surf contact	4	0.16
Backplate piston	Surf-surf contact	1	0.10
Backplate caliper	Surf-surf contact	1	0.16
Bracket pin	Surf-surf contact	2	0.05
Piston caliper	Surf-surf contact	1	0.16
Lining disc	Surf-surf contact	2	0.48
Lining backplate	Tie	2	—
Pin caliper	Tie	2	—

FIGURE 2: Noise distribution in the dynamometer test.

the cold section of the test. When compared with the data in Table 3, most of the noisy modes were in agreement with the test results regarding the frequency distribution, and the 104th mode had a frequency error of less than 4%. Thus, the FEM was concluded to be sufficiently accurate to satisfy the demands for analysis.

3. Influence of Working Parameters on Brake Noise

3.1. Analysis of Dynamometer Test Results. To investigate the brake noise for different driving states of a vehicle, various combinations of working parameters including the brake pressure, brake speed, and initial rotor temperature were considered using the dynamometer test procedure of SAE J2521. The SAE J2521 disc and drum brake dynamometer squeal noise test procedure is intended to establish a common method to perform a series of screening test sequences that identify the propensity of a brake assembly to generate squeal noise under a variety of test conditions. It uses two main modes: (a) drag mode at low speed with a pressure application profile meant to induce instability and squeal noise and (b) deceleration braking at moderate speeds at constant brake pressure, to satisfy the strong need for global standardization of squeal noise dynamometer tests. This procedure is applicable to high-frequency squeal noise occurrences for on-road passenger cars and light trucks [24].

Based on the SAE J2521 test procedure, the dynamometer test of the brake assembly was performed by LINK Model 3900 NVH test dynamometer (Figure 3), and the noise signal during the test was recorded by a microphone. For the purposes of this test procedure, squeal noise is defined as peak noise levels equal to or above 70 dB(A) between 2 kHz and 16 kHz for brakes not using a full suspension corner [24]. The relationships between the absolute noise occurrence and the average brake pressure, initial brake speed, and initial rotor temperature were determined, as shown in Figures 4–6, respectively. The results of the dynamometer test showed significant differences in the absolute noise occurrence under different working conditions. This proved that the working parameters can have significant effects on the generation of brake noise.

The brake noise may be affected by the working parameters directly or indirectly. Direct effects are caused by changes in the working parameters themselves. Indirect effects are caused by performance variations of parts in the brake assembly from changes in the working parameters. The FEM was used to examine both the direct and indirect effects to investigate the influence of working parameters on the brake noise.

3.2. Direct Influence of Working Parameters on Brake Noise. Some working parameters of the braking system could affect braking noise directly. Based on the major brake pressure and initial brake speed parameters in the SAE J2521 test procedure, nine parameter combinations were selected with a brake pressure of 1.0, 2.0, or 3.0 MPa and an initial brake speed of 3, 5, or 10 km/h. The finite element model was

modified depending on the values of the above parameters and recalculated. To analyze the instability of each model, the tendency of instability (TOI) was introduced as a stability evaluation index for the brake system [25]:

$$\text{TOI} = \sum_i \left(\frac{\sigma_i}{\omega_i} \cdot 1000 \right), \quad \sigma_i > 0. \quad (6)$$

The negative ratio of σ_i and ω_i is the damping ratio of the corresponding mode. Therefore, TOI is positively correlated with system instability according to equation (6). Table 4 presents the TOI values of different working conditions.

Table 4 indicates that the TOI is not significantly changed by direct modification of the brake pressure or initial brake speed of the finite element model. However, the dynamometer test results in Figures 4 and 5 indicate that the occurrence of brake noise was clearly affected by variations in the brake pressure and initial brake speed. Therefore, direct changes to the working parameters are not the major cause of changes in the brake noise.

3.3. Indirect Influence of Working Parameters on the Brake Noise. In the present study, nonasbestos organic (NAO) material was selected as the friction material of the brake lining. The friction coefficient of an NAO material is affected by variations in the brake speed, brake pressure, and temperature. Meanwhile, the elastic modulus of an NAO material also changes with the temperature. Therefore, the indirect influence of working parameters on the brake noise is mainly from variations in the performance parameters of the friction material and can be examined by modifying the friction coefficient and elastic modulus of the brake material in the finite element model. For the performance parameters of the brake material, a friction coefficient range of 0.3–0.5 and an elastic modulus range of 550–950 MPa were chosen. Then, 81 parameter combinations were considered to modify the finite element model. Figure 7 shows the relationship between TOI and the performance parameters.

TOI showed significant changes when the friction coefficient or elastic modulus of the friction material was changed. The indirect influence of variations in the working conditions on the brake noise was found to be much more significant than the direct influence. This may be an important factor that affects the brake noise. In addition, the variation in TOI with changes to the friction coefficient and elastic modulus showed significant nonlinear and non-monotonic properties. This reflects the interaction between the above parameters. Therefore, in order to reduce the occurrence of brake noise under general working conditions, the indirect influence of the working parameters should be considered in the design of a brake system.

4. Optimization of Brake Lining Parameters Based on Monte Carlo Sampling and PSO

4.1. Parameter Selection and Structure Description of the Optimization Method. In the present study, the friction coefficient and elastic modulus were chosen as the material optimization parameters. The chamfer dimension (i.e.,

TABLE 3: Noisy modes in the finite element analysis.

Modal order	Frequency (Hz)	Eigenvalue (real part)	Damping ratio (10^{-2})
64	3880.3	192.20	-0.7883
99	7809.3	307.12	-0.6259
104	8201.8	698.59	-1.3555
110	8837.3	534.63	-0.9628
134	10885.0	609.76	-0.8915

FIGURE 3: Brake assembly and dynamometer.

FIGURE 4: Absolute noise occurrence for different average pressures.

FIGURE 5: Absolute noise occurrence for different brake speeds.

FIGURE 6: Absolute noise occurrence for different initial rotor temperatures.

TABLE 4: Direct effects of working conditions on TOI.

No.	Brake pressure (MPa)	Initial speed (km/h)	TOI
1	1.0	3.0	53.51
2	1.0	5.0	53.64
3	1.0	10.0	50.47
4	2.0	3.0	56.65
5	2.0	5.0	56.20
6	2.0	10.0	55.60
7	3.0	3.0	58.00
8	3.0	5.0	57.32
9	3.0	10.0	56.35

width of the chamfer bottom edge) was also used for the structural optimization parameter (Figure 8). The optimization ranges of the friction coefficient μ , elastic modulus E , and chamfer dimension S were set to 0.37–0.42, 550–750 MPa, and 0–21 mm, respectively.

The response surface method was used to establish the functional relationship between the optimization parameters and TOI [26]. Because changing the parameters induced obvious nonmonotonic and nonlinear characteristics in TOI, as shown in Figure 7, the response surface was divided into blocks to improve the fitting precision. The Monte Carlo method was used to randomly sample the search space to reflect the influence of random working conditions on the brake noise. Finally, PSO was used to obtain the optimal parameter combination in the search space.

4.2. Establishment of Blocked Response Surfaces. The Box–Behnken design (BBD) method was used to build the response surface [26]. To consider the nonmonotonic and nonlinear characteristics shown in Figure 7, the response surface was divided as follows: the range of each parameter was divided into three sections of the same size, and the whole response surface was divided into 27 subresponse surfaces. Figure 9 shows the corresponding experimental points for each parameter.

Based on the corresponding combinations of optimization parameters presented in Figure 9, the finite element model was modified and submitted for analysis. Design Expert 10 software was used to build the subresponse

surfaces [27]. The input variables included the elastic modulus, friction coefficient, and chamfer dimension, while the response variable was TOI. Analysis of variance was performed to validate the significance of each subresponse surface, and F-value and P-value were used to indicate the statistical significance and hypothesis test result of the functions [26]. The results showed that all functions satisfied the condition of significant ($F > F_{0.05} = 3.86$). Over 90% of the response surfaces satisfied the condition of very significant ($F > F_{0.01} = 6.99$). Moreover, the P-value of each function was less than 0.01. The results showed that the above subresponse surfaces had good statistical significance. To achieve the requirements of parameter optimization, all 27 subresponse surfaces were patched into a whole response surface through the construction of a piecewise function.

4.3. Monte Carlo Sampling Based on a Probability Density Function. Establishing a perfect mathematical model between the working parameters and brake noise is difficult because of the complex composition and random regularity. Because of the randomness of the working parameters, the Monte Carlo sampling method was introduced [28]. Random sampling was performed on the response surface according to the probability density function of the material parameters, and the probability density function was obtained from the results of the dynamometer test. This approach allows the indirect effects of the working parameters on the brake noise to be considered and solves the problem of discontinuity between the subresponse surfaces.

In the present study, a brake assembly was equipped with a pair of brake linings using an NAO friction material with a nominal friction coefficient of $\mu_n = 0.40$. Based on SAE J2521, a dynamometer test was carried out. Figure 10 shows the measured average friction coefficient. The probability distribution of the friction coefficient showed strong features of a normal distribution.

The average friction coefficient of each effective brake was recorded and fitted to the function of a normal distribution. The fitting results showed that the friction coefficient of the NAO material under various working conditions was subjected to the normal distribution N

FIGURE 7: Indirect effects of working conditions on TOI.

FIGURE 8: Chamfer optimization parameter for the brake lining.

FIGURE 9: Block of input parameters for the response surface.

FIGURE 10: Probability distribution of the friction coefficient from the SAE J2521 test.

TABLE 5: Test results for the compressive strain of the brake lining.

Recorded pressure (MPa)	Thickness variation (μm)		Compressive strain (10^{-3})	
	Room temp	High temp	Room temp	High temp
2.0	14	21	0.78	1.18
4.0	30	43	1.68	2.41
10.0	71	90	3.97	5.04
16.0	110	132	6.16	7.39

FIGURE 11: Sampling of the elastic modulus and friction coefficient.

$(0.3976, 0.0432^2)$. The probability density function is expressed in Eq. (8). A range of $\mu \pm 0.06$ was selected as the sampling interval, which corresponded to a confidence level of 83.6%.

$$f(x) = \frac{1}{0.0432 \cdot \sqrt{2\pi}} \exp\left(-\frac{(x - 0.3976)^2}{2 \cdot 0.0432^2}\right). \quad (7)$$

The temperature was found to have a major influence among the working conditions on the elastic modulus of the friction material. The elastic modulus decreased with increasing temperature. The average thickness variation and compressive strain of the brake linings at room temperature and high temperature were measured according to ISO6310: 2009 (Table 5) [29]. The test results were used to modify the elastic modulus of the friction material under different temperature conditions.

The elastic modulus of the friction material at different temperatures was calculated as follows: 709 MPa at room temperature (25°C) and 591 MPa at high temperature (400°C). Because the variation in the elastic modulus showed only a small amplitude within the experimental range, a linear function was established to represent the relationship between the temperature t and elastic modulus E :

$$E(t) = 709 - 0.3147 \cdot (t - 25), \quad 25 \leq t \leq 400. \quad (8)$$

The maximum initial brake temperature as per SAE J2521 was 300°C except for the after-fade section. The corresponding

elastic modulus was 622 MPa, which is 87.7% of the value at room temperature. Because the occurrence frequency of different initial temperatures is similar according to SAE J2521, a uniform distribution was used to sample the elastic modulus of the friction material. Based on the nominal elastic modulus E , the elastic modulus of friction material for computing was sampled within the range of $0.85E$ – $1.0E$. And the friction coefficient for computing was sampled within the range of $\mu \pm 0.06$, corresponding to the normal distribution shown as $N(\mu, 0.04322)$. Figure 11 shows an example of the sampling range and probability distributions of the friction coefficient and elastic modulus of the friction material: if the nominal friction coefficient is 0.39 and the nominal elastic modulus is 700 MPa, the sampling range would be a rectangular area in the coordinate system, and the color depth in the figure indicates the probability of sampling.

4.4. PSO and Validation of the Results. PSO was proposed in 1995, which comes from the simulation of simplified social model [30]. A program based on PSO algorithm was carried out by MATLAB, and it was used to find the optimal value on the response surface. The population size and number of iterations were set to 40 and 2000, respectively. Each particle was sampled 1000 times in the search space, and the average value of the samples was calculated as the fitness of algorithm. In order to avoid the effect of random sampling on the optimization results, the algorithm was executed repeatedly and independently

FIGURE 12: Distribution of parameter optimization results.

FIGURE 13: Brake lining after optimization.

TABLE 6: Noisy stops with the original and optimized schemes.

Frequency range	Threshold	Noisy stops	
		Original	Optimized
2–16 kHz	≥ 70 dB	456	21
	≥ 80 dB	329	11
2–4 kHz	≥ 70 dB	45	1
	≥ 80 dB	34	0
4–6 kHz	≥ 70 dB	32	0
	≥ 80 dB	25	0
6–10 kHz	≥ 70 dB	147	20
	≥ 80 dB	127	11
10–14 kHz	≥ 70 dB	232	0
	≥ 80 dB	143	0
14–16 kHz	≥ 70 dB	0	0
	≥ 80 dB	0	0

according to Bernoulli's experimental method [31]. Optimized points with TOI values of less than 8.0 were selected as good results. The algorithm was performed repeatedly until 100 good points were obtained. Figure 12 shows the 100 combinations of optimal parameters with low TOI values.

The optimal friction coefficient was concentrated in the range of 0.37–0.39, while the optimal chamfer dimension was concentrated in the range of 19–21 mm. However, particles showed good fitness when the elastic modulus was within either

600–650 MPa or 750–800 MPa. An NAO brake material with a smaller elastic modulus is less wear-resistant; because part of the friction material is removed when the chamfers are set, this would reduce the service life of the brake linings. Therefore, a friction material with an elastic modulus of 780 MPa and friction coefficient of 0.38 was selected, and chamfers with a bottom width of 20.5 mm were set on both sides of the brake linings. Figure 13 shows the produced samples with the optimized brake lining.

FIGURE 14: Noise distribution during the dynamometer test after optimization.

A dynamometer test based on SAE J2521 was carried on a brake assembly equipped with the optimized brake linings shown in Figure 13. Table 6 presents the occurrence of brake noise before and after optimization, and Figure 14 shows the brake noise distribution. Compared with the original scheme with 456 noisy stops, the optimized scheme had only 21 noisy stops. The proportion of the occurrence of brake noise to the total number of brakes decreased from 19.2% to 1.4%, which represents a significant decrease in the probability of occurrence.

The brake noise acceptability of the optimized scheme was evaluated based on SAE J2521. The overall noise index was AA, and the noise index of Schedule A (standard) and Schedule C (after fade) reached the highest level of AAA. Compared with the noise index B before optimization, the optimization scheme significantly suppressed the brake noise. Therefore, the optimized brake lining design scheme can effectively reduce noise during the braking process, and the optimization method of reducing brake noise based on Monte Carlo sampling and PSO was found to be effective.

5. Conclusion

A dynamometer test was used to study the influence of working parameters on the brake noise, and the FEM was used to study the direct and indirect effects of the working parameters. An optimization method of reducing the brake noise based on Monte Carlo sampling and PSO was used to optimize the brake linings. The results of the study are summarized as follows:

- (1) During the braking process, working parameters including the brake pressure, brake speed, and temperature have a significant impact on the occurrence of brake noise.
- (2) Working parameters mainly have an indirect effect on the brake noise by changing the properties of the components in the brake assembly. Parameters affected by the working conditions mainly include the elastic modulus and friction coefficient of the friction material.
- (3) The effect of random working conditions on the material parameters during the braking process can be simulated with Monte Carlo sampling. An optimization method for reducing brake noise based on Monte Carlo sampling and PSO was presented that considers the influence of working conditions. The results of a dynamometer test showed the effectiveness of the optimization method.

Data Availability

The noise data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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