Research Article

Quantized Feedback Control of Active Suspension Systems Based on Event Trigger

Jinwei Sun ¹, Jingyu Cong ², Weihua Zhao,¹ and Yonghui Zhang¹

¹*School of Vehicle Engineering, Xi'an Aeronautical University, Xi'an 710077, China*

²*Marine Resource Utilization in South China Sea and School of Information and Communication Engineering, Hainan University, Haikou 570228, China*

Correspondence should be addressed to Jingyu Cong; jingyuconghnu@126.com

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As the deviation error will accumulate during the data acquisition and transcoding process of an active suspension system, this paper presents a sliding-mode-based quantized feedback control method. The aim of the controller is to improve the vertical performance of vehicles in the presence of external interferences. A 7-DOF suspension model with nonlinear springs and actuator dynamics is built for the control purpose. Firstly, a static quantizer on the uplink channel and a dynamic quantizer on the downlink channel are considered in the sliding mode controller to reduce the cumulative error and suppress the sprung mass motions. Secondly, an event trigger mechanism is introduced in the controller design process to reduce energy consumption and operation frequency of the actuator. The overall stability of the designed controller is proved by the Lyapunov functions. Finally, numerical simulations are carried out to evaluate the efficacy of the proposed controller. Different quantitative and trigger conditions are discussed, and the random road excitation is considered as the external disturbance input. The results of the control method indicate that the designed controller can improve the riding comfort with little loss of handling stability compared with the passive system. In addition, the trigger mechanism can improve the working efficiency of actuators effectively.

1. Introduction

As the passive suspension system can hardly provide a better compromise between riding comfort and handling stability, the controllable suspension system has received extensive attention in recent years [1, 2], and the control technology can be classified into a semiactive and active suspension system [3]. Despite the high cost and power requirement, an active suspension system can provide desired control force and improve the vehicle's dynamic performance significantly [2].

Many active suspension control approaches such as optimal control [4], adaptive control [5], and linear quadratic regulator control have been proposed. An adaptive backstepping controller with hard constraints for the active suspension system is designed in [6] to stabilize the attitude of vehicle and improve ride comfort. A saturated adaptive robust controller to improve suspension performance is

proposed in [7], while in [8], backstepping control to compensate actuator lag in the nonlinear full vehicle model is designed. As the accurate dynamic model of the suspension system is difficult to determine, many different control techniques have been proposed for active suspension systems. A fuzzy logic controller has been used to provide desired actuation force for the capacity of resisting external disturbances and parameter uncertainty [9]. An output feedback finite-time controller was designed in [10] for active suspension systems to compensate the unknown external disturbance and improve the suspension performance, and a bioinspired dynamics-based adaptive tracking active suspension controller was presented in [11] to cancel vibration energy transmitted by suspension inherent nonlinearity. Another well-known control method which can exert robustness on unmeasured disturbances and parameter uncertainties for nonlinear systems is the sliding mode control [12]. The traditional sliding mode scheme can hardly

be implemented on complicated systems with unknown parameters and the system model must be known. Hence, many research studies focused on the combination of control algorithms for uncertain systems to achieve better performance [13]. An enhanced adaptive self-organizing fuzzy sliding mode controller for an active suspension system is proposed in [14], and a hybrid self-organizing fuzzy and radial basis-function neural-network controller is used to reduce the sprung acceleration in [15]. The model-free adaptive sliding controller proposed in [16] uses a functional approximation and online learning technique to improve the control performance.

With respect to the existing results, most of conventional control algorithms tend to ignore the performance degradation caused by quantization errors, which may cause a certain deviation from the original data after decoding at the receiving end. Therefore, quantization error should be considered in the active suspension design process. In addition, for active suspension systems, a certain amount of energy will be generated during the working process of the controller. The time-triggered mechanism [17] can be utilized to save energy and reduce the actuator action frequency while extending the life of the actuator.

This study proposes an event-trigger-based quantitative feedback active suspension system with quantization error, both the static quantizer and dynamic quantizer are utilized, and the sliding mode controller is also adopted. The objective of the proposed controller is to reduce the influence of accumulated sensor signal errors, enhance the actuator efficiency, and improve active suspension system properties. Compared with the recent active suspension control systems, the main contributions of this study are as follows:

- (1) The connection channels between the suspension model and the controller are considered, and robust quantization control theory is designed for the active suspension system to reduce the signal transmission during data acquisition and transcoding
- (2) The event trigger mechanism is introduced in the controller design process to reduce the operation frequency of the active suspension actuator

The rest of the paper is organized as follows. The 7-DOF suspension model with actuator dynamics is presented in the upcoming section. The procedure for the design of the quantized feedback controller and event trigger mechanism is illustrated next. Simulation and discussion of the results are summarized later, and concluding remarks are given in the last section.

2. Vehicle Suspension Model

A full vehicle suspension model with heave, pitch, and roll motions of the sprung mass and four unsprung mass vertical motions is utilized to describe the dynamics of the vehicle., as shown in Figure 1. The terms m_s and m_{ui} represent the sprung and unsprung masses, respectively. θ and φ are the pitch and roll angle of the vehicle sprung mass. The linear damping and spring coefficients are denoted as c_i and k_i , respectively, and k_{ti} represents the tire stiffness. The external road disturbance is denoted as z_{ri} . The suspension deflection can be expressed as

$$z_{sui} = z_{si} - z_{ui}, \quad (i = 1, 2, 3, 4). \quad (1)$$

Then, the spring and damping forces can be given by

$$\begin{aligned} F_{si} &= k_i z_{sui} + k_{ni} z_{sui}^3, \quad (i = 1, 2, 3, 4), \\ F_{di} &= c_i \dot{z}_{sui}, \quad (i = 1, 2, 3, 4), \end{aligned} \quad (2)$$

where k_{ni} is the nonlinear spring coefficient. z_{si} and z_{ui} are the sprung and unsprung mass displacements of each corner, respectively. Assuming the pitch and roll angle of the sprung mass are small, the following approximation can be obtained:

$$\begin{aligned} z_{s1} &= z_s + a \sin(\theta) + c \sin \varphi, \\ z_{s2} &= z_s + a \sin(\theta) - d \sin \varphi, \\ z_{s3} &= z_s - b \sin(\theta) + c \sin \varphi, \\ z_{s4} &= z_s - b \sin(\theta) - d \sin \varphi, \end{aligned} \quad (3)$$

where a, b, c , and d are the distances of the suspensions to the center of the vehicle. Then, the vehicle suspension model with sprung and unsprung mass dynamics can be given as

$$\begin{aligned} m_s \ddot{z}_s + \sum (F_{si} + F_{di} - F_{ai} + F_{ati1} + F_{ati2} + F_{ati3} + F_{ati4}) &= 0, \\ I_y \ddot{\theta} + a(F_{s1} + F_{d1} + F_{s2} + F_{d2} + F_{ati1} + F_{ati2}) - b(F_{s3} + F_{d3} + F_{s4} + F_{d4} + F_{ati3} + F_{ati4}) - F_\theta &= 0, \\ I_x \ddot{\varphi} - d(F_{s2} + F_{d2} + F_{ati2} + F_{s4} + F_{d4} + F_{ati4}) + c(F_{s1} + F_{d1} + F_{ati1} + F_{s3} + F_{d3} + F_{ati3}) - F_\varphi &= 0, \\ m_{ui} \ddot{z}_{ui} - F_{si} - F_{di} + F_{ti} - F_{ai} &= 0, \quad (i = 1, 2, 3, 4), \end{aligned} \quad (4)$$

where I_x and I_y are the roll and pitch moment inertia of the sprung mass; $F_{ti} = k_{ti}(z_{ui} - z_{ri})$ is the dynamic tire load. The antiroll bar forces are

FIGURE 1: Active suspension model.

$$\begin{aligned}
 F_{ati1} &= \frac{k_{af}}{c+d} \left(\varphi - \frac{z_{u2} - z_{u1}}{c+d} \right), \\
 F_{ati2} &= \frac{k_{af}}{c+d} \left(\varphi - \frac{z_{u2} - z_{u1}}{c+d} \right), \\
 F_{ati3} &= \frac{k_{rf}}{c+d} \left(\varphi - \frac{z_{u3} - z_{u4}}{c+d} \right), \\
 F_{ati4} &= \frac{k_{rf}}{c+d} \left(\varphi - \frac{z_{u3} - z_{u4}}{c+d} \right),
 \end{aligned} \tag{5}$$

where k_{af} and k_{rf} are the antiroll bar stiffness.

Active force F_{ai} is generated by the actuator located between the sprung and unsprung masses. The control input of the vertical, pitch, and roll motions can be expressed as

$$\begin{aligned}
 F_z &= F_{a1} + F_{a2} + F_{a3} + F_{a4}, \\
 F_\varphi &= c(F_{a1} + F_{a3}) - d(F_{a2} + F_{a4}), \\
 F_\theta &= a(F_{a1} + F_{a2}) - b(F_{a3} + F_{a4}).
 \end{aligned} \tag{6}$$

The electrohydraulic actuator model can be represented as

$$\dot{F}_{ai} = -\frac{C_d}{A_p} F_{ai} - \alpha A_p^2 \dot{z}_{sui} + \lambda U_i \sqrt{P_s - \text{sign}(U_i) \frac{F_{ai}}{A_p}}, \tag{7}$$

where $i = 1, 2, 3, 4$, C_d is the discharge coefficient, A_p is the actuator piston area, P_s is the hydraulic supply pressure, α is a function of the bulk modulus of the fluid, w is the spool valve area gradient, $\lambda = A_p C_d w \alpha / \sqrt{\rho}$, ρ is the hydraulic fluid

density, and U_i is the spool valve input that can be the controlled voltage or current.

3. Controller Design

A quantize-event-trigger-based sliding mode controller is designed in this section for the active suspension system, and the control diagram of the proposed controller is shown in Figure 2.

3.1. Robust Quantization Feedback Controller. Robust quantization control theory contains control problems based on the quantizer, in which the dynamic quantizer expands the quantization range by adjusting the quantization level to achieve asymptotic stability of the system. This section mainly focuses on the connection channels between the suspension model and the controller. Both static quantizer and dynamic quantizer are utilized, and a sliding mode controller is also adopted. The main feature of the dynamic quantizer is that it can increase the quantization range of the quantizer by adjusting the quantization level. It can also reduce the size of the limit cycle when the system is in a steady state and achieve asymptotic stability. Compared with the static quantizer, its control effect is better, but the structure is more complicated. For practical consideration, we utilize a static quantizer on the uplink channel and a dynamic quantizer on the downlink channel. The quantization system is shown in Figure 3.

Assuming the suspension system considered in this paper in bounded-state stable, the road profile and its derivate are totally bounded. Selecting the state variables as $[x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T = [z_s \ \dot{z}_s \ \theta \ \dot{\theta} \ \varphi \ \dot{\varphi}]^T$, the suspension system model including heave, pitch, and roll motions can be expressed as

FIGURE 2: Diagram block of the proposed controller.

FIGURE 3: Flow chart of the quantization system.

$$\begin{aligned}
 \dot{x}_1 &= x_2, \\
 \dot{x}_2 &= f_z + G_z F_z, \\
 f_z &= -\frac{1}{m_s} \sum (F_{si} + F_{di}), \quad G_z = \frac{1}{m_s}, \\
 \dot{x}_3 &= x_4, \\
 \dot{x}_4 &= f_\theta + G_\theta F_\theta, \\
 f_\theta &= -\frac{1}{I_y} [a(F_{s1} + F_{d1} + F_{s2} + F_{d2}) - b(F_{s3} + F_{d3} + F_{s4} + F_{d4})], \quad G_\theta = \frac{1}{I_y}, \\
 \dot{x}_5 &= x_6, \\
 \dot{x}_6 &= f_\varphi + G_\varphi F_\varphi, \\
 f_\varphi &= -\frac{1}{I_x} [c(F_{s1} + F_{d1} + F_{s3} + F_{d3}) - d(F_{s2} + F_{d2} + F_{s4} + F_{d4})], \quad G_\varphi = \frac{1}{I_x}.
 \end{aligned} \tag{8}$$

Then, the abovementioned functions can be written as [18]

$$\mathbf{x}(t) = A_0 \dot{\mathbf{x}}(t) + B_j [\mathbf{U}_{\text{force}} + \mathbf{d}_j(t)], \tag{9}$$

where $\mathbf{x}(t) = [x_i(t); x_{i+1}(t)]$; $i = 1, 3, 5$, $j = 1, 2, 3$, and $\mathbf{U}_{\text{force}} = [F_z; F_\theta; F_\varphi]$ represent the vertical, pitch, and roll force generated by the active suspension, respectively.

$A_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$; $B_1 = [0; G_z]$, $B_2 = [0; G_\theta]$, $B_3 = [0; G_\varphi]$; and $d_1(t) = [0; (f_z/G_z)]$, $d_2(t) = [0; (f_\theta/G_\theta)]$, and $d_3(t) = [0; (f_\varphi/G_\varphi)]$.

Assumption 1. $\mathbf{d}_j(t)$ satisfies the inequality $|\mathbf{d}_j(t)| \leq d_1 + d_2 |\mathbf{x}(t)| + d_3 |\mathbf{U}_{\text{force}}| \leq d_4$, where $d_1 \geq 0$, $d_2 \geq 0$, $d_4 > 0$, $0 \leq d_3 \leq 1$, and d_1, d_2, d_3, d_4 are constants.

As shown in Figure 3, a static quantizer $q_{\mu_s}(\mathbf{v})$ is used in the uplink channel, where $\mathbf{v}$ is the control law designed by the quantized state, μ_s is a fixed number; the down channel utilized a dynamic quantizer $q_{\mu}(\mathbf{x})$ to transfer system quantized states, and μ is the quantization parameter adjusted statically. Defining a function that takes the nearest integer, the uniform quantizer with the quantization level parameter μ is

$$q_{\mu}(\mathbf{x}) \stackrel{\text{def}}{=} \mu_j \text{round}\left(\frac{\mathbf{x}}{\mu_j}\right), \quad \mu_j > 0; j = 1, 2, 3. \quad (10)$$

The uniform quantization error is

$$\mathbf{e}_{\mu} \stackrel{\text{def}}{=} q_{\mu}(\mathbf{x}) - \mathbf{x}. \quad (11)$$

Then,

$$\mathbf{e}_{\mu} \stackrel{\text{def}}{=} |q_{\mu}(\mathbf{x}) - \mathbf{x}| \leq \Gamma \mu_j, \quad (12)$$

where $\Gamma = \sqrt{p}/2$, p is the dimension of the vector $\mathbf{x}$.

For the dynamic quantizer, a special quantization level parameter $\mu_j = 0$ is given when the system runs to the sliding surface.

$$q_{\mu}(\mathbf{x}) \stackrel{\text{def}}{=} 0, \quad \mu_j = 0. \quad (13)$$

The linear sliding surface is defined as follows:

$$s(\mathbf{x}(t)) = C\mathbf{x}(t) = 0, \quad (14)$$

where C is a given vector to ensure that the system has stable eigenvalues and $CB_j \neq 0$. The object of sliding mode control is to force the system state trajectory to reach the sliding surface and maintain the sliding mode in the presence of uncertainties and disturbances.

Lemma 1 (see [19]). *There is a fixed constant $\tau > 0$, and we assume $\mu > 0$ which satisfies*

$$\mu \leq \frac{|C\mathbf{x}|}{(\tau + 1)|C|\Gamma}. \quad (15)$$

Then, the following equation can be obtained:

$$|C\mathbf{e}_{\mu}| \leq |C|\Gamma\mu \leq \frac{1}{\tau} |Cq_{\mu}(\mathbf{x})|. \quad (16)$$

By considering the uplink and down channel quantize, the following control law can be designed [19]:

$$\begin{aligned} \mathbf{v} = & -\frac{\tau + 1}{\tau - \chi} (CB_j)^{-1} \left[|CAq_{\mu}(\mathbf{x}(t))| + |CA|\Gamma\mu_j + |CB_j|\Gamma_1\mu_s + |CB_j| \left[(d_1 + d_2|q_{\mu}(\mathbf{x}(t))| + d_2\Gamma\mu_j + d_3\Gamma_1\mu_s) \right] \text{sgn}[Cq_{\mu}(\mathbf{x}(t))] \right] \\ & - \frac{\tau\rho}{\tau - \chi} (CB_j)^{-1} \text{sgn}[Cq_{\mu}(\mathbf{x}(t))], \end{aligned} \quad (17)$$

$$\mathbf{U}_{\text{force}} = q_{\mu_s}(\mathbf{v}), \quad (18)$$

where $\chi = 1 + (\tau + 1)d_3$, $\tau > ((1 + d_3)/(1 - d_3))$, $\rho > 0$. The control law can keep the system states reach and maintain on the sliding surface $s(\mathbf{x}(t)) = 0$.

A simple and effective design of the adjustment law is as follows [20]:

$$\left\{ \begin{array}{ll} \mu_j = \frac{|C\mathbf{x}(t)|}{(\tau + 1)|C|\Gamma}, & |C\mathbf{x}(t)| \geq 1, \\ \mu_j = \frac{\alpha^{\lambda}}{(\tau + 1)|C|\Gamma}, & 0 < |C\mathbf{x}(t)| < 1, \\ \mu_j = 0, & |C\mathbf{x}(t)| = 0, \end{array} \right. \quad (19)$$

where $\alpha \in (0, 1)$, $\alpha^{\lambda} < |C\mathbf{x}| < \alpha^{\lambda-1}$.

3.2. Event Trigger Mechanism. In recent years, event triggers have been used to reduce computational costs and energy

consumption, resulting in a wide range of applications in control systems [21, 22]. The event trigger mechanism is based on the physical quantity signal of the sensor; when the semaphore reaches a certain condition, sampling is performed. In this way, the average sampling period is longer than the fixed sampling period, so that the sampling frequency of the system can be effectively reduced. As the next sample instant is dependent on the previous sampling information, the control law is held constant between sampling instants. Defining the event trigger instant as t_k , the system maintains the control output at time t_k in the time interval $[t_k, t_{k+1}]$. The discretization state error of the system is

$$\mathbf{e}(t) = \mathbf{x}(t) - \mathbf{x}(t_k). \quad (20)$$

The sampling is not periodic; hence, $t_{k+1} - t_k$ is not constant. For computational purposes, the deviation from the system state to the desired state is given by

$$\mathbf{e}_T = \mathbf{x} - \mathbf{x}_d, \quad (21)$$

where $\mathbf{x}_d$ is the set reference. As the error variable is required to vanish or settle in close vicinity of zero, the reference state is set as $\mathbf{x}_d = 0$.

The triggering instant t_k is completely characterized by a triggering rule, so a triggering rule is introduced [23].

$$\kappa = \|\zeta \mathbf{e}_T + \xi \dot{\mathbf{e}}_T^2\| - \psi(m_1 + m_2 e^{-\zeta t}), \quad (22)$$

where $\zeta > 0, \xi > 0, \psi \in (0, 1), \zeta \in (0, 1), m_1 \geq 0, m_2 \geq 0, m_1 + m_2 > 0$. The term $\|\zeta \mathbf{e}_T + \xi \dot{\mathbf{e}}_T^2\|$ represents the overall error; the system is triggered when the error is small, guaranteeing the control output. When $\kappa \leq 0$, the system is triggered at t_k , and the control output is maintained until t_{k+1} . The following relation determines the triggering instants in an iterative manner:

$$t_{k+1} = \inf\{t \in [t_k, \infty), \kappa > 0\}. \quad (23)$$

The event trigger time is $T_k = t_{k+1} - t_k$. For $T_k \leq t_{k+1} - t_k$, and lower bound on the event trigger time is ensured. T_k is bounded by [23]

$$T_k \geq \frac{1}{\|A\|} \ln \left(\frac{\|A\| \cdot \|\mathbf{e}\|_\infty}{\|A\| \bar{\omega} + \|B_j\| (q_{\mu_s}(\mathbf{v}) + d_4)} + 1 \right), \quad (24)$$

where $\bar{\omega} > 0$.

Proof: Considering system (9),

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B_j \mathbf{U}_{\text{force}}(t) + B_j \mathbf{d}_j(t). \quad (25)$$

At time interval $[t_k, t_{k+1}]$, discretization state error rises from 0 to the final value at T_k instant, so

$$\begin{aligned} \frac{d}{dt} \|\mathbf{e}(t)\| &\leq \frac{d}{dt} \|\bar{\mathbf{e}}(t)\| \leq \frac{d}{dt} \|\mathbf{x}(t)\| \\ &\leq \|A\mathbf{x}(t) + B_j \mathbf{U}_{\text{force}}(t) + B_j \mathbf{d}_j(t)\|, \end{aligned} \quad (26)$$

by inserting the control law (17) into abovementioned equation.

$$\begin{aligned} \frac{d}{dt} \|\mathbf{e}(t)\| &\leq \|A\mathbf{x}(t) + B_j q_{\mu_s}(\mathbf{v}) + B_j \mathbf{d}_j(t)\| \\ &\leq \|A\mathbf{x}(t)\| + \|B_j q_{\mu_s}(\mathbf{v})\| + \|B_j \mathbf{d}_j(t)\| \\ &\leq \|A(\mathbf{e}(t) + t\mathbf{x}(t_k))\| + \|B_j q_{\mu_s}(\mathbf{v})\| + \|B_j \mathbf{d}_j(t)\| \\ &\leq \|A\mathbf{e}(t)\| + \|A\mathbf{x}(t_k)\| + \|B_j q_{\mu_s}(\mathbf{v})\| + \|B_j \mathbf{d}_j(t)\| \\ &\leq \|A\| \|\mathbf{e}(t)\| + \|A\| \bar{\omega} + \|B_j\| (q_{\mu_s}(\mathbf{v}) + d_4). \end{aligned} \quad (27)$$

$\forall t \in [t_k, t_{k+1}]$, with the initial condition $\mathbf{e}(t) = 0$, and the solution of inequality (27) can be understood by utilizing the comparison lemma [24] and is given as

$$\|\mathbf{e}(t)\| \leq \frac{\|A\| \bar{\omega} + \|B_j\| (q_{\mu_s}(\mathbf{v}) + d_4)}{\|A\|} \left(e^{\|A\| (t-t_k)} - 1 \right). \quad (28)$$

Similarly, for triggering time instant t_{k+1} ,

$$\|\mathbf{e}_\infty\| = \|\mathbf{e}(t_{k+1})\| \leq \frac{\|A\| \bar{\omega} + \|B_j\| (q_{\mu_s}(\mathbf{v}) + d_4)}{\|A\|} (e^{\|A\| T_k} - 1), \quad (29)$$

and then, inequality (24) is satisfied.

Discretized state error after quantization is

$$\mathbf{e}_q(t) = q_\mu(\mathbf{x}(t)) - q_\mu(\mathbf{x}(t_k)). \quad (30)$$

Event is triggered at discrete moments $\forall t \in [t_k, t_{k+1}]$, when $s(t) > 0$ or $s(t) < 0$, $s(t) = s(t_k)$. So, the following equation is satisfied:

$$\text{sgn}(Cq_\mu(\mathbf{x}(t))) = \text{sgn}(Cq_\mu(\mathbf{x}(t_k))), \quad t \in [t_k, t_{k+1}]. \quad (31)$$

Therefore, the triggered control law applied to the system is

$$\begin{aligned} \mathbf{F}_{ai} = \mathbf{U}_{\text{force}} &= -\frac{\tau+1}{\tau-\chi} (CB_j)^{-1} \left[|CAq_\mu(\mathbf{x}(t_k))| + |CA|\Gamma\mu_j + |CB_j|\Gamma_1\mu_s + |CB_j| \left((d_1 + d_2|q_\mu(\mathbf{x}(t_k))| + d_2\Gamma\mu_j + d_3\Gamma_1\mu_s) \right) \right] \\ &\quad \cdot \text{sgn}[Cq_\mu(\mathbf{x}(t_k))] - \frac{\tau\rho}{\tau-\chi} (CB_j)^{-1} \text{sgn}[Cq_\mu(\mathbf{x}(t_k))]. \end{aligned} \quad (32)$$

□

3.3. Stability Analysis. In order to illustrate the reliability of the proposed algorithm, the stability of the quantize-event-trigger-based sliding mode controller needs to be considered. From (11) and (18), $\mathbf{e}_{\mu_s} = q_{\mu_s}(\mathbf{v}) - \mathbf{v} = \mathbf{U}_{\text{force}} - \mathbf{v}$; then, $\mathbf{U}_{\text{force}} = q_{\mu_s}(\mathbf{v}) = \mathbf{v} + \mathbf{e}_{\mu_s}$. Combining with $\mathbf{x}(t) = q_{\mu}(\mathbf{x}(t)) - \mathbf{e}_{\mu}$, the derivative of the sliding surface is written as

$$\begin{aligned}\dot{s}(\mathbf{x}(t)) &= C\dot{\mathbf{x}}(t) = C[A(q_{\mu}(\mathbf{x}(t)) - \mathbf{e}_{\mu}) + B_j(\mathbf{v} + \mathbf{e}_{\mu_s} + \mathbf{d}_j)] \\ &= C[Aq_{\mu}(\mathbf{x}(t)) - A\mathbf{e}_{\mu} + B_j\mathbf{v} + B_j\mathbf{e}_{\mu_s} + B_j\mathbf{d}_j].\end{aligned}\quad (33)$$

From inequalities $\mathbf{e}_{\mu} = q_{\mu}(\mathbf{x}) - \mathbf{x} \leq \Gamma\mu_j$ and $C\mathbf{e}_{\mu} \leq |C|\Gamma\mu_j \leq |1/\tau|Cq_{\mu}(\mathbf{x})$,

$$\begin{aligned}s(\mathbf{x}(t))\dot{s}(\mathbf{x}(t)) &= Cq_{\mu}(\mathbf{x}(t))\{C[Aq_{\mu}(\mathbf{x}(t)) - A\mathbf{e}_{\mu} + B_j\mathbf{v} + B_j\mathbf{e}_{\mu_s} + B_j\mathbf{d}_j]\} \\ &\quad - C\mathbf{e}_{\mu}\{C[Aq_{\mu}(\mathbf{x}(t)) - A\mathbf{e}_{\mu} + B_j\mathbf{v} + B_j\mathbf{e}_{\mu_s} + B_j\mathbf{d}_j]\} \\ &\leq \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x}(t))||CAq_{\mu}(\mathbf{x}(t))| + \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x}(t))||CA|\Gamma\mu_j + \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x}(t))||CB_j|\Gamma_1\mu_s \\ &\quad + \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x}(t))||CB_j||\mathbf{d}_j| + \frac{1}{\tau}|Cq_{\mu}(\mathbf{x}(t))||CB_j\mathbf{v}| + Cq_{\mu}(\mathbf{x}(t))CB_j\mathbf{v}.\end{aligned}\quad (34)$$

If Assumption 1 is satisfied, equation (34) can be written as

$$\begin{aligned}s(\mathbf{x}(t))\dot{s}(\mathbf{x}(t)) &\leq \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x}(t))||CAq_{\mu}(\mathbf{x}(t))| + \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x}(t))||CA|\Delta\mu_j + \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x}(t))||CB_j|\Gamma_1\mu_s \\ &\quad + \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x}(t))||CB_j|(d_1 + d_2|q_{\mu}(\mathbf{x}(t))| + d_2\Gamma\mu_j + d_3|\mathbf{U}_{\text{force}}|) \\ &\quad + \frac{1}{\tau}|Cq_{\mu}(\mathbf{x}(t))||CB_j\mathbf{v}| + Cq_{\mu}(\mathbf{x}(t))CB_j\mathbf{v}.\end{aligned}\quad (35)$$

As $|\mathbf{U}_{\text{force}}| = |q_{\mu_1}(\mathbf{v})| = |\mathbf{v} + \mathbf{e}_{\mu_s}| \leq |\mathbf{v}| + \Gamma_1\mu_s$ and $\chi = 1 + (\tau+1)d_3$, equation (35) can be written as

$$\begin{aligned}s(\mathbf{x})\dot{s}(\mathbf{x}) &\leq \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x})||CAq_{\mu}(\mathbf{x})| + \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x})||CA|\Gamma\mu_j + \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x})||CB_j|\Gamma_1\mu_s \\ &\quad + \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x})||CB_j|(d_1 + d_2|q_{\mu}(\mathbf{x})| + d_2\Gamma\mu_j + d_3\Gamma_1\mu_s) \\ &\quad + \frac{\chi-1}{\tau}|Cq_{\mu}(\mathbf{x})||CB_j||\mathbf{v}| + \frac{1}{\tau}|Cq_{\mu}(\mathbf{x})||CB_j\mathbf{v}| + Cq_{\mu}(\mathbf{x})CB_j\mathbf{v}, \\ &= \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x})||CAq_{\mu}(\mathbf{x})| + \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x})||CA|\Gamma\mu_j + \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x})||CB_j|\Gamma_1\mu_s \\ &\quad + \frac{\tau+1}{\tau}|Cq_{\mu}(\mathbf{x})||CB_j|(d_1 + d_2|q_{\mu}(\mathbf{x})| + d_2\Gamma\mu_j + d_3\Gamma_1\mu_s) \\ &\quad + \frac{\chi}{\tau}|Cq_{\mu}(\mathbf{x})||CB_j||\mathbf{v}| + \frac{\tau-\chi}{\tau}Cq_{\mu}(\mathbf{x})CB_j\mathbf{v} + \frac{\chi}{\tau}Cq_{\mu}(\mathbf{x})CB_j\mathbf{v}.\end{aligned}\quad (36)$$

For $\tau > ((1 + d_3)/(1 - d_3))$, $d_3 < ((\tau - 1)/(\tau + 1))$, $\chi = 1 + (\tau + 1)d_3 < \tau$. By substituting the control law (17), the following inequality can be obtained:

$$s(\mathbf{x}(t))\dot{s}(\mathbf{x}(t)) \leq -\rho|Cq_\mu(\mathbf{x}(t))| - \frac{\tau+1}{\tau}|Cq_\mu(\mathbf{x}(t))|\left(|CA\mathbf{e}_q| + |CB_j|d_2|\mathbf{e}_q|\right). \quad (37)$$

Defining $\sigma = \rho + ((\tau + 1)/\tau)(|CA\mathbf{e}_q| + |CB_j|d_2|\mathbf{e}_q|)$, (37) can be written as

$$s(\mathbf{x}(t))\dot{s}(\mathbf{x}(t)) \leq -\sigma|Cq_\mu(\mathbf{x}(t))|. \quad (38)$$

According to $|C\mathbf{e}_\mu| \leq |C|\Gamma\mu_j \leq (1/\tau)|Cq_\mu(\mathbf{x})|$ and triangle inequality, one can obtain

$$\begin{aligned} |C\mathbf{x}(t)| &= |Cq_\mu(\mathbf{x}(t)) - C\mathbf{e}_\mu| \leq |Cq_\mu(\mathbf{x}(t))| + |C\mathbf{e}_\mu| \\ &\leq \frac{\tau+1}{\tau}|Cq_\mu(\mathbf{x}(t))| \Rightarrow \frac{\tau}{\tau+1}|C\mathbf{x}(t)| \leq |Cq_\mu(\mathbf{x}(t))|. \end{aligned} \quad (39)$$

Then, (38) it can be observed as

$$\begin{aligned} s(\mathbf{x}(t))\dot{s}(\mathbf{x}(t)) &\leq -\rho|Cq_\mu(\mathbf{x}(t))| \leq -\frac{\tau}{\tau+1}|C\mathbf{x}(t)|, \\ &= -\frac{\tau\rho}{\tau+1}|s(\mathbf{x}(t))|. \end{aligned} \quad (40)$$

Hence, the state trajectories of the plant can be driven to the sliding surface, and the overall stability of the proposed suspension controller holds.

4. Numerical Simulation and Discussion

The effectiveness of the proposed controller is discussed in this section through the simulation results of the MATLAB/Simulink. The model and controller parameters are listed in Tables 1 and 2, respectively.

When the vehicle is driven in a straight line, road roughness is an important factor that affects the riding comfort and handling stability of the vehicle. Pavement roughness is often expressed in terms of statistical characteristic parameters in the frequency domain, such as power spectral density or variance. However, the power spectral density expression can hardly meet the research needs of vehicle dynamics control, and it is necessary to simulate the road roughness in the time domain. The power spectral density of signal wheel road disturbance can be expressed as [25]

$$G_q(n_0) = G_q(n_0)\left(\frac{n}{n_0}\right)^{-W}. \quad (41)$$

Road roughness $q(t)$ can be seen as the first-order response of unite white noise in the filtered white noise method, and through introducing a low cutoff frequency, the system frequency response is

$$H_{q \sim w}(\omega) = \frac{2\pi n_0 \sqrt{G_q(n_0)}v}{j\omega + 2\pi n_0 v}. \quad (42)$$

The time-domain model of signal wheel excitation can be obtained by writing (42) into a differential form.

$$\dot{q}(t) = 2\pi n_0 \sqrt{G_q(n_0)}v w(t) - 2\pi n_0 v q(t). \quad (43)$$

The definition of specific symbols can be seen in [25]. The four-wheel excitation model can be obtained by the time delay between front and rear axles and the coherence between left and right tracks [26]. Assuming that the vehicle is traveling on the B-level road with constant speed of 60 km/h, the time domain road profiles of four wheels are plotted in Figure 4.

The purpose of the active suspension controller is to improve the riding comfort while keeping the wheels in constant contact with the road surface. The riding comfort is evaluated by the movement of the sprung mass motion, and the handling stability is evaluated through the dynamic displacement of the tire. To verify the effectiveness of the quantized feedback controller, different quantitative and trigger conditions are considered, and the road disturbance input is the B-level road.

Figure 5–Figure 7 reflect the control effect of the sprung mass motions with the quantized feedback controller. Sprung mass vertical acceleration, pitch acceleration, and roll acceleration with the combined quantizer, static quantizer, and uncontrolled condition are plotted. As can be seen from the figures, the passive system reported significantly more amplitude than the other two controlled systems. Partial enlarged detail of the figures indicates that compared with the static quantizer, a combined quantizer composed of the static and dynamic quantizer can achieve better performance.

Figure 8 represents the simulation contrast curves of the suspension deflection of the vehicle. It can be seen from Figure 8 that the proposed combined quantizer in this study outperforms the static quantizer and passive system in time domain. The time responses of the vehicle's tire deflection of four corners are set out in Figure 9. As shown in the figure, although the peak value of the controlled system is slightly greater than that of the passive system, the root mean square values are nearly consistent with the uncontrolled model. The partial enlarged details also show that the tire deflections with the static and combined quantizer are little changed. These results suggest that the quantized feedback controller is able to improve riding comfort effectively while maintaining the wheels in contact with the road surface.

TABLE 1: Suspension model parameters.

Parameter	Value
Sprung mass m_s	1592 kg
Unsprung mass m_{ui}	120/150 kg
Gravity to front axle distance a	1.18 m
Gravity to rear axle distance b	1.77 m
Center of gravity to the left side c	0.7875 m
Center of gravity to the right side d	0.7875 m
Pitch moment of inertia I_y	2488 kg•m ²
Roll moment of inertia I_x	614 kg•m ²
Spring stiffness k_i	146000 N/m
Damping coefficient b_i	3000 Nm/s
Tire stiffness k_{ti}	230000 N/m
Hydraulic supply pressure P_s	1.034×10^7 N/m ²
Piston area A_p	3.35×10^{-4}
Discharge coefficient C_d	0.61
Spool valve area gradient w	1.436×10^{-2} m
Oil density ρ	858 kg/m ³
α	1.19143×10^{13}

TABLE 2: Controller parameters.

Parameter	Value
d_1	0.1
d_2	0.1
d_3	0.01
α	0.5
ρ	4
μ_s	0.05
p_1	0.8
p_2	0.8
p_3	0.05
p_4	$1e-5$
p_5	0
p_6	0.97

FIGURE 4: Four-wheel excitation input.

The results of the trigger signals are shown in Figure 10, and the quantization parameters are summarized in Figure 11. From the data in Figures 10 and 11, it is apparent that

FIGURE 5: Sprung mass vertical acceleration.

FIGURE 6: Sprung mass pitch acceleration.

the trigger moments are corresponding to the change of quantization parameters. Actuator force with the event trigger mechanism is presented in Figure 12. The figure illustrates that compared with the control law without the trigger mechanism, the output force frequency with the trigger mechanism is significantly reduced, which has a certain effect on reducing power consumption and reducing fatigue damage of the actuator. Therefore, it is necessary to introduce the trigger mechanism to the controller.

Table 3 compares the summary statistics for sprung mass accelerations, suspension deflection, and tire deflection with and without the trigger mechanism. Besides, the nonsingular fast terminal sliding mode (NFTSM) control method [27] is

FIGURE 7: Sprung mass roll acceleration.

FIGURE 8: Continued.

FIGURE 8: Suspension deflection.

FIGURE 9: Continued.

FIGURE 9: Tire deflection.

FIGURE 10: Trigger signals.

FIGURE 11: Quantization parameters.

FIGURE 12: Continued.

FIGURE 12: Actuator forces with the event trigger mechanism.

TABLE 3: Comparison of root mean square values.

	Vertical acceleration	Pitch acceleration	Roll acceleration	FL suspension deflection	FL tire deflection
With trigger	0.2437	0.3576	0.4565	0.0027	0.0036
Without trigger	0.2422	0.3559	0.4548	0.0027	0.0036
NFTSM	0.2330	0.3506	0.4509	0.0026	0.0037
Passive	0.4764	0.6427	0.8187	0.0032	0.0031

also added to verify the effectiveness of the proposed method. As Table 3 shows, the introduction of the trigger mechanism has very little effect on the vehicle's riding comfort, and the designed controller has performance similar to that of the NFTSM controller in improving the sprung mass characteristics.

5. Conclusions

The aim of the present research was to deal with the quantization error and improve the actuator efficiency of the control system. A sliding-mode-based quantized feedback controller was designed for the active suspension system to improve the riding comfort with external disturbances. Then, an event trigger mechanism was introduced in the design process of the quantization controller to reduce the operation frequency of the actuator. Some conclusions can be drawn as follows:

- (1) Compared with the static quantizer, the combined quantizer can achieve better performance
- (2) The introduction of the trigger mechanism can effectively reduce the operating frequency of the actuator while maintaining the control effect of the active system
- (3) The overall controller of the active suspension can improve riding comfort while maintaining handling stability without deterioration in the presence of external disturbances

The performance of the proposed controller has been validated under random road excitation in the MATLAB/Simulink environment.

In the future works, the influence of an in-wheel motor on vehicle vertical vibration will be considered, and critical test conditions will be applied. Besides, the longitude and lateral influence will also be considered.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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