

## Research Article

# Overlapping Decentralized Control Strategies of Building Structures' Vibration with Time Delay Based on $H_\infty$ Control Algorithms under Seismic Excitation

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A decentralized control strategy can effectively solve the control problem of the large-scale time delayed structures. In this paper, combining the overlapping decentralized control method, linear matrix inequality (LMI) method, and  $H_\infty$  control algorithm, overlapping decentralized  $H_\infty$  control approach of the time delayed structures has been established. The feedback gain matrixes of all subsystems are obtained by this method based on genetic algorithm optimization tools and the specific goal of optimization control. The whole vibration control system of the time delayed structures is divided into a series of overlapping subsystems by overlapping decentralized control strategy. The feedback gain matrixes of each subsystem can be obtained by using  $H_\infty$  control algorithm to calculate each subsystem. The vibration control of a twenty layers' antiseismic steel structure Benchmark model was analyzed with the numerical method. The results show that the proposed method can be applied to control system with time delay. The overlapping decentralized control strategies acquire the similar control effects with that of the centralized control strategy. Moreover, the flexibility of the controller design has been enhanced by using overlapping decentralized control strategies.

## 1. Introduction

With the construction of super-high, long-span, and other complex large-scale civil structures, the traditional passive control strategy has some limitations in its application, which is difficult to meet the requirements. In 1972, Yao had put forward the concept of active control technology and applied it to the structural vibration control system [1]. There are inevitably time-delay problems in the operation of the active control system, such as the actuator process, sensor signal transmission and communication, and controller data processing [2–4]. These phenomena will lead to the time delay of control force and the decline of control effect and even lead to the failure of the control system [5–7]. Cai and Huang [8] proposed a traditional discrete LQR

control method for the building structure vibration control system under earthquake excitation. The obtained controller not only contains the state feedback of the current step but also includes the linear combination of some previous control steps. Udawadia et al. [9] derived the design principle of the time-delay controller for single degree-of-freedom structure active control and developed the theory to multi-degree-of-freedom structure time-delay control. The theory is applied to the design of time-delay controller for the vibration of multistorey building structures. Chen [10] combined control  $H_\infty$  algorithm with the Tagagi–Sugeno (T-S) fuzzy control method, and a fuzzy robust control method for nonlinear structural systems with time delay was proposed. Du et al. [11] studied the parameter uncertainty and time delay of active vibration control of building

structures and proposed a robust saturation control method. Using genetic algorithm as optimization tool, Xue et al. [12] solved the corresponding control gain according to the specific optimization control objective and proposed a structural vibration control method with time delay based on genetic algorithm.

The vibration control system of large-scale civil structures under the action of earthquake and wind is a complex multi-degree-of-freedom system, and the centralized control method has some defects, such as large amount of calculation and unstable control performance [13]. Decentralized control strategy can effectively solve the problem of building structure vibration control. Intelligent optimization algorithm and decentralized control strategy are used to solve the vibration control problem of building structures under earthquake excitation [14]. The decentralized control strategy based on the homotopy method is used to transform a typical centralized controller into multiple decoupled decentralized controllers [15]. The benchmark model of twenty-story steel structure is used to verify the stability of decentralized control strategy [16]. The time-delay problem of control systems has been widely studied. Decentralized control strategy is also applied to solve complex time-delay control systems [17–23]. Fallah and Taghikhany [24] proposed an overlapping decentralized  $H_2/LQG$  control method for cable-stayed bridge with time delay and divided the cable-stayed bridge into two overlapping subsystems for control. Wang and Law [25] proposed a time-delay decentralized  $H_\infty$  control method for vibration of building structures under seismic excitation, and a six-story building structure was taken as an example to verify the method. The contraction principle and overlapping dispersion strategy are applied to the vibration control system of intelligent buildings to minimize the damage caused by earthquake [26]. Karimi et al. [27] and Palacios-Quiñonero et al. [28] proposed a multioverlapping LQR control method for high-rise building structures based on LQR control algorithm and inclusion principle. This method only needs the control information of adjacent floors to realize control. A good control effect is obtained in the example. However, there is little research on overlapping decentralized control methods to solve the vibration control problem of structures with time delay. The overlapping decentralized control strategy divides the whole structural vibration control system into a series of overlapping subsystems according to certain rules, and each subsystem uses local information to control independently.

In order to study the control effect of overlapping decentralized control method to solve the vibration problem of structures with time delay based on  $H_\infty$  control algorithm, first, in this study, the overlapping decentralized control strategy, linear matrix inequality (LMI) method, and  $H_\infty$  control algorithm are combined to propose an overlapping decentralized control method for structures with time delay based on  $H_\infty$  control algorithm. Secondly, based on the inclusion principle and decomposition principle, the whole vibration control system of building structure with time delay is divided into a series of subsystems, and each subsystem is controlled by  $H_\infty$  control algorithm. In the process of solving

the feedback gain matrix of each subsystem, the  $H_\infty$  control algorithm is transformed into linear matrix inequality (LMI). In this paper, genetic algorithm is used to solve the linear matrix inequality (LMI) of each subsystem, so as to obtain the feedback gain matrix of each subsystem. Finally, the feedback gain matrix of each subsystem is reduced to an overlapping decentralized controller according to the contraction principle. In this study, five kinds of control strategies, including  $H_\infty$  centralized control and overlapping decentralized  $H_\infty$  control, are designed for the vibration time-delay control problem of twenty-story seismic steel structure benchmark model under seismic excitation, and the corresponding calculation results are given. The overlapping decentralized control method provides a new control approach to solve the vibration control problem of structures with time delay based on  $H_\infty$  control algorithm, and the control strategy reduces the calculation cost and increases the flexibility of the controller design.

## 2. Building Structure Model with Time Delay

Consider a vibration control system of an  $n$ -story building structure, as shown in Figure 1. Under the action of earthquake load, the motion equation of building structure with time delay can be described as follows:

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = T_u u(t - \tau) + T_w w(t), \quad (1)$$

where  $q(t) = [q_1(t), q_2(t), \dots, q_n(t)]^T$ ,  $q_i(t)$  is the displacement of the layer  $i$  floor structure,  $u(t) = [u_1(t), u_2(t), \dots, u_r(t)]^T$ ,  $u_r(t)$  is the control force matrix of the  $r$ th controller,  $T_u \in R^{n \times r}$  is the position matrix of the  $r$ th controller,  $w(t)$  is the direction of external load interference (i.e., seismic load acceleration),  $T_w$  is the interference input matrix, and  $M, C, K \in R^{n \times n}$  are the mass, damping, and stiffness matrices of the building structure, respectively.

$a_i (i = 1, 2, \dots, n)$  are the drive control devices in Figure 1. By using the state variable  $x_I(t) = [q^T(t) \dot{q}^T(t)]^T$  for equation (1), the system in equation (1) can be transformed into the following spatial state form:

$$S_I: \dot{x}_I(t) = A_I x_I(t) + B_I u(t - \tau) + E_I w(t), \quad (2)$$

where  $A_I = \begin{bmatrix} [0]_{n \times n} & [I]_{n \times n} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$ ,  $B_I = \begin{bmatrix} [0]_{n \times n} \\ M^{-1}T_u \end{bmatrix}$ , and  $E_I = \begin{bmatrix} [0]_{n \times 1} \\ -[1]_{n \times 1} \end{bmatrix}$ .

Now, we define a new state vector:

$$x(t) = \Gamma x_I(t), \quad (3)$$

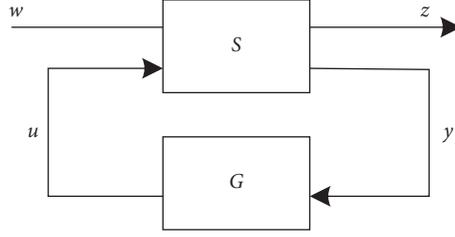
where  $\Gamma$  is the transformation matrix.

Therefore, the system of equation (2) can be converted into

$$S: \dot{x}(t) = Ax(t) + Bu(t - \tau) + Ew(t), \quad (4)$$

where  $A = \Gamma A_I \Gamma^{-1}$ ,  $B = \Gamma B_I$ ,  $E = \Gamma E_I$ ,  $x(t) = [x_1(t), x_2(t), \dots, x_{2n}(t)]$ , and  $x_i(t)$  satisfy the following equation:



FIGURE 2: The diagram of standard H $\infty$  control algorithm.

By substituting the above formula into formula (9), we can obtain

$$\dot{x}(t) = (A + BGC_y)x(t) - BGC_y \int_{t-\tau}^t \dot{x}(\theta)d\theta + Ew(t). \quad (11)$$

Now, considering the following Lyapunov function,

$$V(x(t)) = V_1 + V_2 + V_3, \quad (12)$$

where  $V_1 = x^T(t)Px(t)$ ,  $V_2 = \int_{-\tau}^0 \int_{t+\beta}^t \dot{x}^T(\alpha)Z\dot{x}(\alpha)d\alpha d\beta$ ,  $V_3 = \int_{t-\tau}^0 \int_{t+\beta}^t x^T(\alpha)Q_c x(\alpha)d\alpha d\beta$ , and  $P$ ,  $Z$ , and  $Q_c$  are symmetric positive definite matrices.

According to the bounded real lemma and [29], it can be concluded that

$$\begin{aligned} \dot{V}(x(t)) &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \\ &\leq x^T(t)(A^T P + PA + \bar{\tau}X + Y + Y^T)x(t) \\ &\quad + 2x^T(t)(PBGC_y - Y)x(t - \tau) \\ &\quad + \bar{\tau}[Ax(t) + Ew(t) + BGC_y x(t - \tau)]^T Z [Ax(t) + Ew(t) + BGC_y x(t - \tau)] \\ &\quad + x^T(t)Q_c x(t) - x^T(t - \tau)Q_c x(t - \tau) + w^T(t)E^T Px(t) + x(t)PEw(t). \end{aligned} \quad (13)$$

Assuming the initial condition  $\varphi(t) = 0, \forall t \in [-\tau, 0]$ , we can get  $V(x(t))|_{t=0} = 0$ .

Considering the following performance indicators,

$$J_{zw} = \int_0^{\infty} [z^T(t)z(t) - \gamma^2 w^T(t)w(t)]dt. \quad (14)$$

For any nonzero perturbation  $w(t) \in L_2[0, \infty)$ , we can obtain that

$$\begin{aligned} J_{zw} &\leq \int_0^{\infty} [z^T(t)z(t) - \gamma^2 w^T(t)w(t)]dt + V(x(t))|_{t=\infty} - V(x(t))|_{t=0} \\ &= \int_0^{\infty} [z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(x(t))]dt \\ &= \int_0^{\infty} \eta^T(t)\Pi\eta(t)dt, \end{aligned} \quad (15)$$

where  $\eta(t) = [x(t) \ x(t - \tau) \ w(t)]^T$  and

$$\Pi = \begin{bmatrix} \Phi & PBGC_y - Y + \bar{\tau}A^T ZBGC_y + C_z^T D_z GC_y & \bar{\tau}A^T ZE + PE \\ * & -Q_c + \bar{\tau}(GC_y)^T B^T ZBGC_y + (GC_y)^T D_z^T D_z GC_y & \bar{\tau}(GC_y)^T B^T ZE \\ * & * & -\gamma^2 I + \bar{\tau}E^T ZE \end{bmatrix} < 0, \quad (16)$$

where

$$\Phi = A^T P + PA + \bar{\tau}X + Y + Y^T + Q_c + \bar{\tau}A^T ZA + C_z^T C_z. \quad (17)$$

According to the Schurz complement theorem, equation (16) can be equivalent to equation (19). Given a scalar  $\bar{\tau} > 0$ , for any constant time delay  $\tau (0 \leq \tau \leq \bar{\tau})$ , if there exists a

matrix  $G, P > 0, Q_c > 0, Z > 0$ , and  $X$  and  $Y$  satisfy equations (18) and (19); the closed-loop system equation (9) with  $H_\infty$  performance index  $\gamma (\gamma > 0)$  is asymptotically stable:

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0, \quad (18)$$

$$\begin{bmatrix} A^T P + PA + \bar{\tau} X + Y + Y^T + Q_c & PBGC_y - Y & PE & \bar{\tau} A^T Z & C_z^T \\ * & -Q_c & 0 & \bar{\tau} (GC_y)^T B^T Z & (GC_y)^T D_z^T \\ * & * & -\gamma^2 I & \bar{\tau} E^T Z & 0 \\ * & * & * & -\bar{\tau} Z & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \quad (19)$$

where the symbol \* denotes symmetry.

**3.2. Genetic Algorithm for Solving Linear Matrix Inequality (LMI).** Genetic algorithm is a probabilistic search process algorithm based on natural selection and natural genetic mechanism, which has been applied to solve the global optimization problems of various controllers. Arfiadi and Hadi [30] use genetic algorithm to design static output feedback controllers for  $H_2$  or  $H_\infty$  norm optimization problems. When MATLAB Robust control toolbox is used to calculate the output feedback controller, it is found that the optimization problem of bilinear matrix inequality (BLMI) (equations (18) and (19)) is not solvable. Therefore, this section uses the random search function of genetic algorithm to solve the following problems:

$$\max_{G \in R^{r \times n_u}} \bar{\tau} \text{ subject to equations (18) and (19),} \quad (20)$$

where the state variable of the controller is  $n_u$ ,  $r$  is the input vector, and the genetic algorithm generates a feedback gain matrix  $G \in R^{r \times n_u}$  at random, and the evolution is carried out according to the condition of equation (20).

If the feedback gain matrix  $G$  after evolution satisfies equation (20) and the maximum value  $\bar{\tau}$  can be obtained, then the design of the controller can be completed.

According to the following strategy, genetic algorithm solves the corresponding linear matrix inequality (LMI):

Step 1: the feedback gain matrix is encoded with a binary string.

Step 2: an initial population of  $N_p$  chromosomes was randomly generated.

Step 3: evaluate goals and assign fitness values. The initial value of Step 2 is fed back to the actual population control matrix  $G_j, j = 1, 2, \dots, N_p$ , in each decoding step. For each  $G_j$  dichotomy is used to search

the maximum delay  $\tau_j$  so that  $\tau_j$  and  $G_j$  are feasible in equation (20). According to the permutation-based allocation method, each delay  $\tau_j$  and the corresponding target value  $G_j$  are obtained, and each group  $(\tau_j, G_j)$  obtained is substituted into step 4. For  $G_j$ , there is no feasible delay  $\tau_j$  in equation (20). In order to reduce its survival chance in the next generation, the target value in line with  $G_j$  will be assigned to a larger value.

Step 4: the natural selection of Darwin's theory of biological evolution and the biological evolution process of genetic mechanism are used to select offspring.

Step 5: perform consistent crossover probability  $p_c$  and generate new offspring.

Step 6: there is a small mutation possibility  $p_m$  in chromosome population.

Step 7: select the optimal chromosome population and put it into the relevant program.

The evolutionary process of genetic algorithm will repeat Step 3 to Step 7 for  $N_g$  times. The optimal chromosome is decoded into the actual value, and the feedback gain matrix is generated here.

**3.3. Overlapping Decentralized  $H_\infty$  Control Approach.** In order to design the overlapping decentralized controller for structural vibration with time delay, the steps of the overlapping decentralized  $H_\infty$  control algorithm can be described as follows:

- (1) The motion equation of the shear model of  $n$ -story building structure with time delay can be described as equation (4). According to the inclusion principle in the overlapping decentralized control method [31], the system  $S$  in equation (4) can be extended and decoupled into a series of overlapping subsystems  $\tilde{S}_D^{(i)} (i = 1, 2, \dots, L)$ :

$$\tilde{\mathbf{S}}_D^{(i)}: \begin{cases} \dot{\tilde{\mathbf{x}}}_i(t) = \tilde{\mathbf{A}}_{ii}\tilde{\mathbf{x}}_i(t) + \tilde{\mathbf{B}}_{ii}\tilde{\mathbf{u}}_i(t - \tau) + \tilde{\mathbf{E}}_i\mathbf{w}_i(t), \\ \tilde{\mathbf{z}}_i(t) = (\tilde{\mathbf{C}}_z)_{ii}\tilde{\mathbf{x}}_i(t) + (\tilde{\mathbf{D}}_z)_{ii}\tilde{\mathbf{u}}_i(t - \tau), \\ \tilde{\mathbf{y}}_i(t) = (\tilde{\mathbf{C}}_y)_{ii}\tilde{\mathbf{x}}_i(t), \end{cases} \quad (i = 1, 2, \dots, L; 2 \leq L \leq n - 1), \quad (21)$$

where  $\tilde{\mathbf{A}}_{ii}$ ,  $\tilde{\mathbf{B}}_{ii}$ ,  $\tilde{\mathbf{E}}_i$ , and  $(\tilde{\mathbf{C}}_y)_{ii}$  are the corresponding matrices of subsystem  $\tilde{\mathbf{S}}_D^{(i)}$  ( $i = 1, 2, \dots, L$ ), respectively, and  $(\tilde{\mathbf{C}}_z)_{ii}$  and  $(\tilde{\mathbf{D}}_z)_{ii}$  are the constant matrices of appropriate dimension of subsystem  $\tilde{\mathbf{S}}_D^{(i)}$  ( $i = 1, 2, \dots, L$ ), respectively.

- (2) A series of overlapping  $\tilde{\mathbf{S}}_D^{(i)}$  ( $i = 1, 2, \dots, L$ ) subsystems in equation (21) are designed with the H $\infty$  controller. The genetic algorithm in Section 3.2 is used to solve linear matrix inequality (18) and equation (19), and the optimal feedback gain matrix  $\tilde{\mathbf{G}}^{(i)}$  ( $i = 1, 2, \dots, L$ ) of each subsystem can be obtained after several times of genetic algorithm search and evolution process.
- (3) The control law of the extended decoupled overlapping control system  $s$  can be expressed as  $\tilde{\mathbf{u}}_D(t) = \tilde{\mathbf{G}}_D\tilde{\mathbf{x}}(t)$ :

$$\tilde{\mathbf{G}}_D = \text{diag}\{\tilde{\mathbf{G}}^{(1)}, \tilde{\mathbf{G}}^{(2)}, \dots, \tilde{\mathbf{G}}^{(L)}\}, \quad (22)$$

where  $\tilde{\mathbf{G}}^{(i)}$  ( $i = 1, 2, \dots, L$ ) is the feedback gain matrix of subsystem  $\tilde{\mathbf{S}}_D^{(i)}$  ( $i = 1, 2, \dots, L$ ).

- (4) According to the contraction principle and the corresponding linear transformation [31], the feedback gain matrix  $\tilde{\mathbf{G}}_D$  after extended decoupling can be reduced to the original state overlapping control:

$$\mathbf{G}_o = \mathbf{Q}\tilde{\mathbf{G}}_D\mathbf{V}, \quad (23)$$

where  $\mathbf{Q}$  and  $\mathbf{V}$  are contraction matrices and expansion matrices, respectively.

#### 4. Controller Design and Example Analysis

In order to verify the proposed overlapping decentralized H $\infty$  control method for building structures with time delay, the benchmark model of a twenty-story steel frame structure in Los Angeles, USA, is used in this section. The structural parameters are [32]  $m_1 = 215.2 \times 10^3$  kg,  $m_2 = 209.2 \times 10^3$  kg,  $m_3 = 207 \times 10^3$  kg,  $m_4 = 204.8 \times 10^3$  kg,  $m_5 = 266.1 \times 10^3$  kg,  $k_1 = 147 \times 10^3$  kN/m,  $k_2 = 113 \times 10^3$  kN/m,  $k_3 = 99 \times 10^3$  kN/m,  $k_4 = 89 \times 10^3$  kN/m, and  $k_5 = 84 \times 10^3$  kN/m. The structural damping rate is 5%. In this section, the overlapping decentralized control method, linear matrix inequality (LMI) method, and H $\infty$  control algorithm are combined to propose the overlapping decentralized H $\infty$  control method for structures with time delay. In this section, five control strategies are set according to the overlapping information sharing mode of different floor structures, as shown in Figure 3. The seismic excitation adopts El Centro (N-S, 1940) wave and the sampling step is 0.02 s (see Figure 4). The energy distribution of the input time-history on the natural

frequency-time plane can be obtained by the principle of wavelet transform (see Figure 5). The genetic algorithm parameters' set in this section are population chromosome number,  $N_p = 100$ , crossover probability,  $p_c = 0.9$ , mutation probability,  $p_m = 0.02$ , and maximum random search times,  $N_g = 500$ .

Centralized control is to control the whole building structure as a control system (Figure 3(a)). When MATLAB robust control toolbox is used to calculate the output feedback controller, it is found that the linear matrix inequality (LMI) of the system is not solvable. According to the parameters of the genetic algorithm set in this section and using the genetic algorithm to solve equation (20), the feedback gain matrix  $\mathbf{G}_c$  of the H $\infty$  controller can be obtained.

The overlapping decentralized control scheme is shown in Figure 3(b) to Figure 3(e). Among them, Figure 3(b) shows that the control information of layer 11 structure is shared by subsystems  $S^{(1)}$  and  $S^{(2)}$ . In Figure 3(c), the control information of layer 7 structure is shared by subsystems  $S^{(1)}$  and  $S^{(2)}$  and the control information of layer 14 structure is shared by subsystems  $S^{(2)}$  and  $S^{(3)}$ . In Figure 3(d), subsystems  $S^{(1)}$  and  $S^{(2)}$  share the control information of layer 5 structure. Subsystems  $S^{(2)}$  and  $S^{(3)}$  share the control information of the 10th layer structure. Subsystems  $S^{(3)}$  and  $S^{(4)}$  share the control information of layer 15 structure. In Figure 3(e), the control information of layer  $i$  structure is shared by subsystems  $S^{(i-1)}$  and  $S^{(i)}$  ( $i = 2, 3, \dots, 19$ ). In the above different overlapping decentralized control schemes, the H $\infty$  control algorithm based on linear matrix inequality is used to solve the subsystems of each scheme. In this section, the genetic algorithm in Section 3.2 and the set genetic algorithm parameters are used to solve the linear matrix inequality (LMI) of the corresponding subsystem. After several times of random search function of genetic algorithm, the subsystem feedback gain matrix of each scheme can be obtained, and the control gain matrix  $G_{ODC1}$ ,  $G_{ODC2}$ ,  $G_{ODC3}$ , and  $G_{ODC4}$  of each overlapping decentralized control scheme can be obtained by using the feedback gain matrix of each subsystem through formulas (22) and (23).

According to formula (14) and  $\gamma_G = \|T_{zw}\|_\infty = \max \bar{\sigma}[T_{zw}(j\omega)]$ , we can obtain the maximum singular values. From the gain matrix under different control strategies, the maximum singular values under different time delays can be obtained, as shown in Figures 6 and 7. The maximum interlayer displacement and maximum control force are shown in Figures 8–11.

As can be seen from Figure 6, when the time delay  $\tau = 20$ ms, the maximum singular value  $\gamma_{G_c}$  of H $\infty$  centralized control is 0.0511, the maximum singular value  $\gamma_{G_{ODC1}}$  of overlapping decentralized H $\infty$  controller 1 is

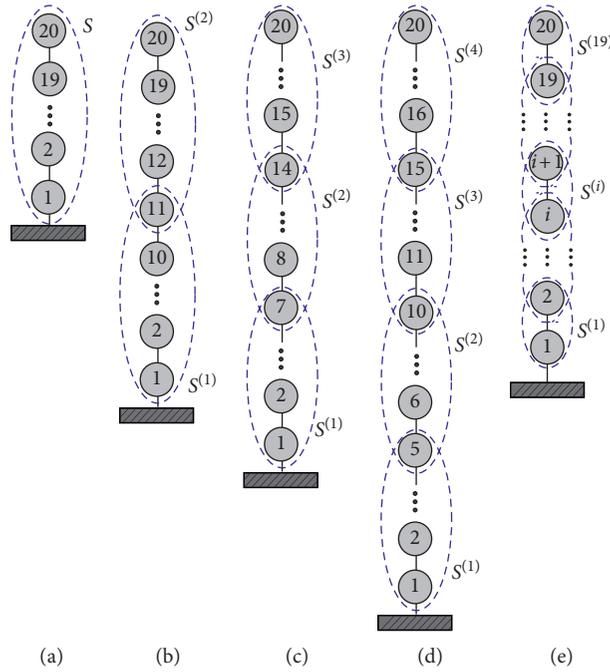


FIGURE 3: The design of controllers. (a) Centralized control. (b) ODC1 (c) ODC2. (d) ODC3. (e) ODC4.

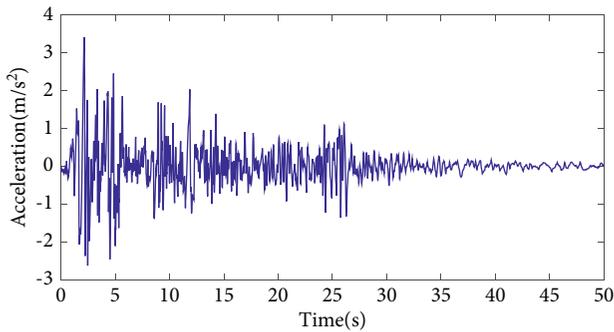


FIGURE 4: El Centro (1940) earthquake N-S component of ground motion acceleration record.

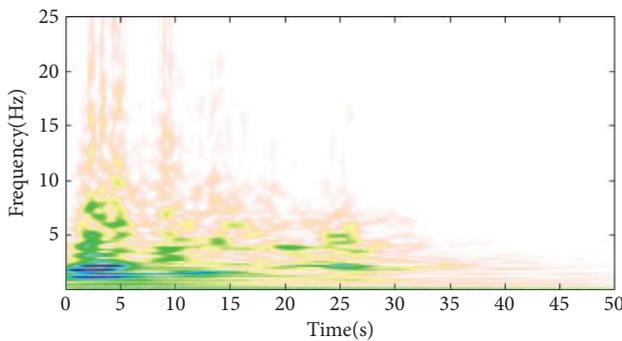


FIGURE 5: The energy distribution of input time-history on natural frequency-time plane.

0.0570, the maximum singular value  $\gamma_{G_{ODC_2}}$  of overlapping decentralized H $\infty$  controller 2 is 0.0748, the maximum singular value  $\gamma_{G_{ODC_3}}$  of overlapping decentralized H $\infty$

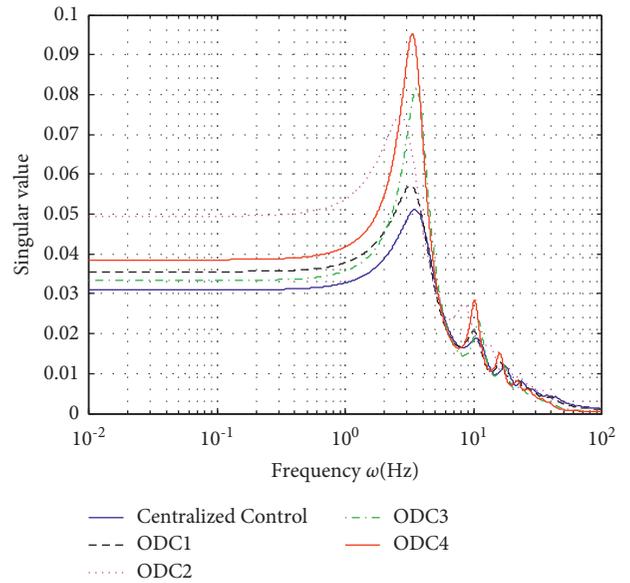


FIGURE 6: The maximum singular values of time delay  $\tau = 20\text{ms}$ .

controller 3 is 0.0814, and the maximum singular value  $\gamma_{G_{ODC_4}}$  of overlapping decentralized H $\infty$  controller 4 is 0.0951.

It can be seen from Figure 7 that when the time delay  $\tau = 40\text{ms}$ , the maximum singular value  $\gamma_{G_C}$  of H $\infty$  centralized control is 0.0756, the maximum singular value  $\gamma_{G_{ODC_1}}$  of overlapping decentralized H $\infty$  controller 1 is 0.0840, the maximum singular value  $\gamma_{G_{ODC_2}}$  of overlapping decentralized H $\infty$  controller 2 is 0.1048, the maximum singular value  $\gamma_{G_{ODC_3}}$  of overlapping decentralized H $\infty$  controller 3 is 0.1150, and the maximum singular value

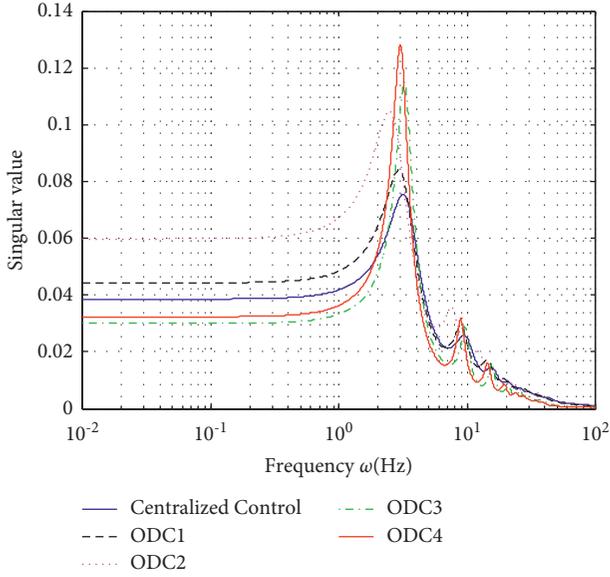


FIGURE 7: The maximum singular values of time delay  $\tau = 40\text{ms}$ .

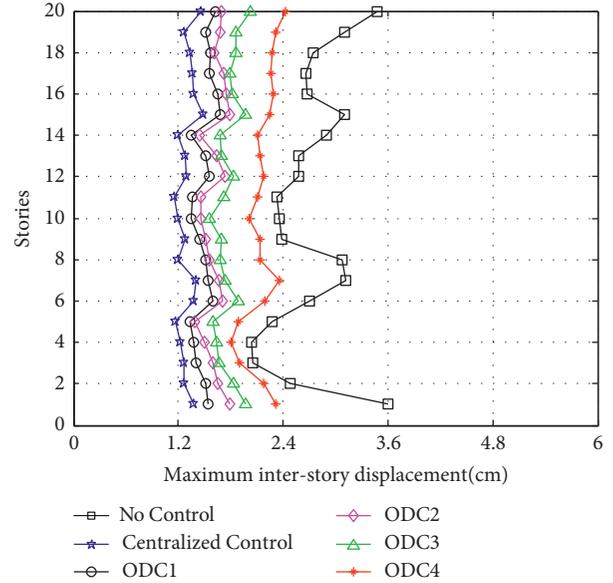


FIGURE 9: The maximum interstory displacements of time delay  $\tau = 40\text{ms}$ .

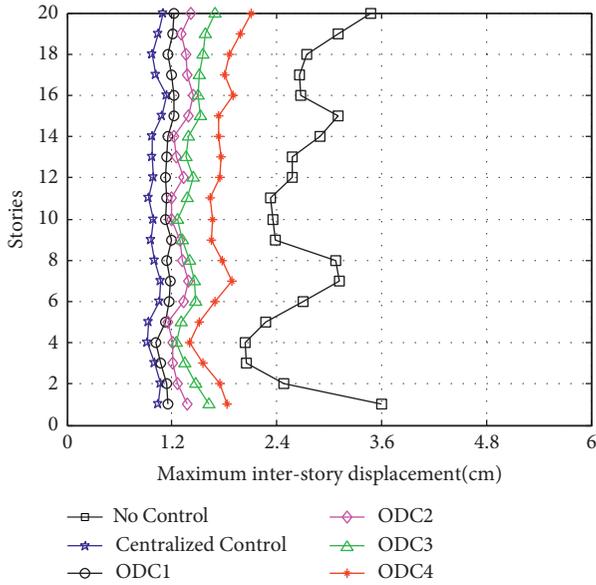


FIGURE 8: The maximum interstory displacements of time delay  $\tau = 20\text{ms}$ .

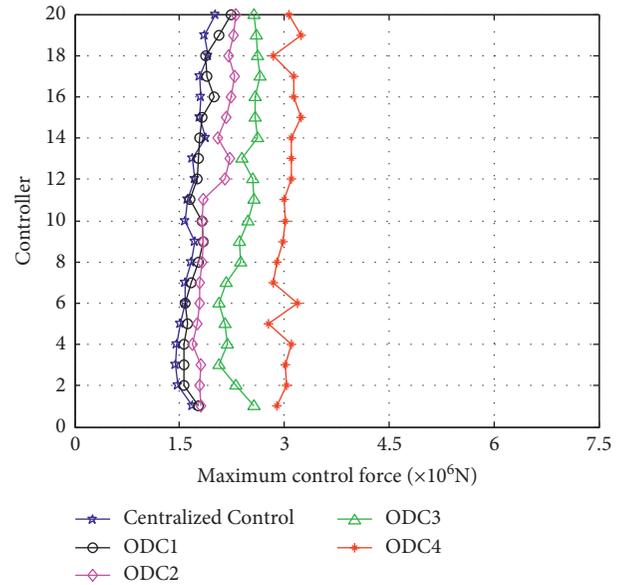


FIGURE 10: The maximum control force of time delay  $\tau = 20\text{ms}$ .

$\gamma_{G_{ODC4}}$  of overlapping decentralized  $H_{\infty}$  controller 4 is 0.1280.

According to the maximum interstory displacements of time delay shown in Figure 8, when the time delay  $\tau = 20\text{ms}$ , the story drift response value of the structure in the uncontrolled state is the largest, and the control effect of  $H_{\infty}$  centralized control is the best. The overlapping decentralized  $H_{\infty}$  controller 1 to the overlapping decentralized  $H_{\infty}$  controller 4 also gets better control effect. With the deepening of the overlapping degree of building structure, the control effect of overlapping decentralized controller is getting worse and worse. Among them, the overlapping decentralized  $H_{\infty}$  controller 4 has the worst control effect,

but it also obtains better control effect. As shown in Figure 9, the maximum interstory displacement of the building structure with time delay  $\tau = 40\text{ms}$  can be obtained. The control effect of the centralized controller is the best, and the control effect of the overlapping decentralized controller is also very good. The control effect of each overlapping decentralized controller is similar to that of  $\tau = 20\text{ms}$ .

Centralized control has some defects in solving the vibration control problems of multi-degree-of-freedom structures, such as large amount of calculation, poor reliability, and unstable control performance. It is difficult to be widely used in specific large-scale building structure vibration control system. The overlapping decentralized

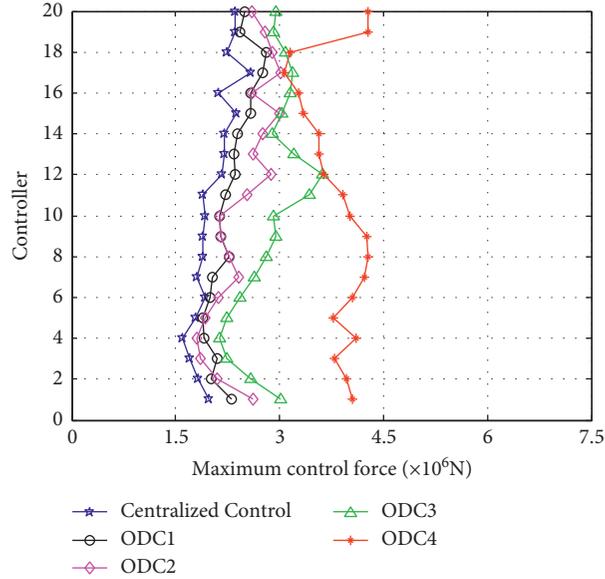


FIGURE 11: The maximum control force of time delay  $\tau = 40$ ms.

control strategy is more operable and can solve this problem. Therefore, for this kind of structural vibration control system, it is necessary to adopt an overlapping and decentralized control theory to control it. The overlapping decentralized  $H_{\infty}$  control strategies for structures with time delay are control methods based on overlapping decentralized control theory, which can solve the problems of centralized control strategy. The control strategy proposed in this paper provides a new way to solve the vibration control of complex multi-degree-of-freedom structures.

It can be seen from the maximum control force in Figure 10 and Figure 11 that the control force of the structure in the centralized control state is the smallest when the hysteresis are  $\tau = 20$ ms and  $\tau = 40$ ms. With the deepening of building structure overlap, the control force of overlapping decentralized controller is increasing. Among them, the overlapping decentralized  $H_{\infty}$  controller 4 has the largest control force. Therefore, it can be considered that with the deepening of the degree of building structure overlap, and the energy required by the overlapping decentralized controller is also more and more.

In order to describe the maximum displacement and the maximum control force of the structure under the action of time delay in more detail, limited to space, only the maximum displacement of the bottom layer of the structure (Figure 12) and the maximum control force (Figure 13) are given.

From the maximum displacement of the bottom layer of the structure under the action of time delay shown in Figure 12, it can be seen that the time delay effect should be given enough attention in the decentralized control system. Even if the time delay is very small, it may lead to the unstable state of the controlled structure. It can be seen from Figure 13 that the control force at the bottom layer of the structure increases with the time delay and finally tends to infinity.

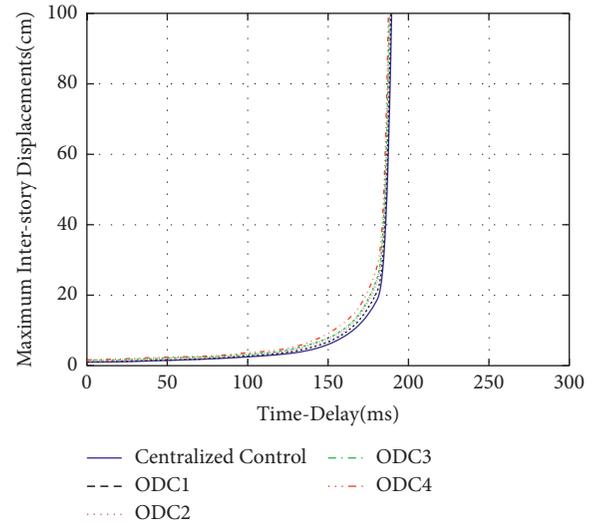


FIGURE 12: The maximum displacement value of first floor with time delay.

## 5. Discussion

- (1) In this paper, a new path is provided to solve the vibration control problem of structures with time delay based on  $H_{\infty}$  control algorithm: an overlapping decentralized control method, and the corresponding maximum singular values  $\gamma$  are calculated according to different overlapping decentralized control strategies. As the degree of overlap and decentralization increases, the corresponding maximum single values  $\gamma$  become larger and larger. However, when solving the time-delay problem of vibration control of practical engineering structures, the controller needs to be designed according to the time delay and the frequency of the structure itself.

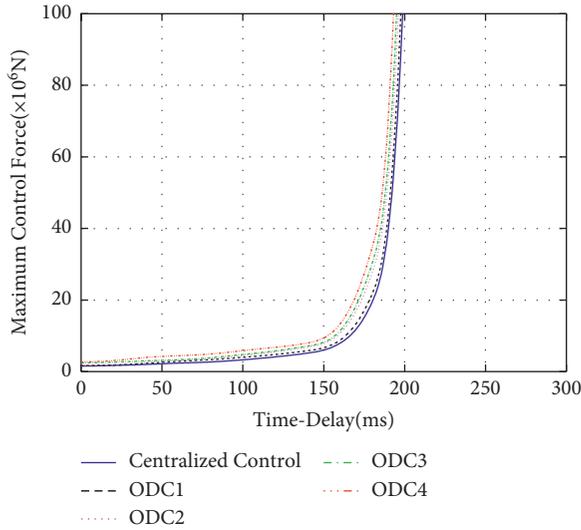


FIGURE 13: The maximum control force of first floor with time delay.

- (2) When the building structure vibration control system adopts the same control strategy, the maximum singular values  $\gamma$  are also increasing with the increase of hysteresis  $\tau$ .
- (3) Matlab R2016b software is used to program and calculate different control strategies of vibration control system of twenty-story building structure with time delay. The running time of the centralized control strategy (Figure 3(a)) is 1.5 hours. The operation time of overlapping decentralized controller 1 (ODC1) is 40 minutes. The running time of overlapping decentralized controller 2 (ODC2) is 30 seconds. The operation time of overlapping decentralized controller 3 (ODC3) is 20 seconds. The running time of overlapping decentralized controller 4 (ODC4) is 8 seconds. As the degree of overlap and decentralization increases, the cost of computer computing is also decreasing.

## 6. Conclusions and Directions

When a large number of intelligent wireless sensors and controllers are applied to the vibration control system of high-rise buildings, there will inevitably be the problem of time delay. Even if the time delay is very small, it may cause the control effect to decline and even lead to the unstable state of the controlled structure. Centralized control strategy collects and transmits data by a single central processor. Once a failure occurs, the whole control system will be paralyzed. However, decentralized control strategy can better solve this problem.

In this paper, the linear matrix inequality (LMI) method,  $H_\infty$  control algorithm, and overlapping decentralized control strategy are combined to propose the overlapping decentralized  $H_\infty$  control method for building structures with time delay. The proposed control method is used to design and calculate the controller for the benchmark model

of 20-story steel frame structure. Genetic algorithm is used to solve the corresponding linear matrix inequality (LMI) to obtain the feedback gain matrix of each subsystem, and then, the feedback gain matrix is reduced to the corresponding overlapping decentralized controller according to the overlapping decentralized control strategy. The results show that

- (1) Similar to  $H_\infty$  centralized control, overlapping decentralized  $H_\infty$  control also obtains good control effect.
- (2) Time delay overlapping decentralized control system can effectively reduce the vibration response of building structure under seismic excitation and improve the robust performance of feedback time delay of the control system.
- (3) Overlapping decentralized  $H_\infty$  control provides a new way to solve the vibration control problem of complex multi-degree-of-freedom structures with time delay. Time-delay effect should be paid enough attention in the decentralized control system, and it is worthy of further study.
- (4) In this paper, the computational efficiency of structural vibration control with time delay based on  $H_\infty$  control algorithm under earthquake is studied, and the interlayer actuator with control time-delay effect is designed. This theory may be helpful to solve the problem of vibration control of buildings under seismic excitation.
- (5) When the building structure is subjected to seismic load, its control system will produce time-delay phenomenon. However, the overlapping decentralized control method is developed based on the inclusion principle and contraction principle. The whole system is divided into a series of subsystems, and the subsystems can share the control information with overlapping structure. The number of degrees of freedom of the subsystem can be divided according to the load condition and the influence degree of time delay. Therefore, the overlapping decentralized control method increases the flexibility of the controller design.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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