Research Article
Dynamic Multiprojectile Attack and Killing Effects of Detonation Warheads

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Abstract
Dynamic spatial distribution and killing characteristics of warhead fragments are important topics in the field of weapon effectiveness and protection. However, there is little research on the fragment distribution formed by continuous dynamic attacks of multiple projectiles that explode above the ground. This study analyzes spatial distributions of warhead fragments using witness boards in a rectangular target test. The results show that the fragment distribution of multiple projectiles in continuous dynamic attacks demonstrates a spatial superposition characteristic. The superimposed distribution is the sum of the distributions of two independent fragment distributions. The distribution characteristics are consistent with fragment scattering behavior. Therefore, they can be used to analyze the killing effects of multiple projectiles conveniently. The effects of falling speed, falling angle, and explosion height on the damage range of fragments were explored by a fragment spatial distribution model obtained from experiments. Analysis indicates that the prefabricated fragment distribution conformed to a spatial superposition relationship under dynamic multiprojectile continuous attacks, and the superposition obeyed fragment scattering characteristics. As the projectile falling angle increased at the explosion center, or as the falling height decreased, the positive pressure duration increased gradually. The falling speed had the greatest impact on the specific impulse of overpressure. The falling angle had the greatest impact on the peak value of overpressure. Both the falling angle and the explosion height had the greatest impact on the positive pressure acting time.

1. Introduction
Detonation warheads are the most common type of warheads. When a warhead explodes, it disperses many high-speed fragments radially, causing damage to living targets and the environment, including structures and machinery. Warhead parameters and the combined damage capability of multiple projectiles determine damage efficiency. Many researchers have been interested in this topic for decades and have developed many models to study the maximum warhead fragment speed [1–3]. The Taylor formula [4] and the Shapiro formula are two commonly used models to solve axial fragment speed and fragment spatial dispersion.

2. Study on Dynamic Spatial Distribution of Multiple Projectiles

2.1. Analysis of Dynamic Fragment Distribution. Before the spatial distribution of prefabricated fragments formed by multiple projectiles is calculated, the spatial fragment distribution formed by a single projectile can be first analyzed via a probability estimate on the number of fragments. When the prefabricated-fragment warhead explodes, each fragment is projected normal to the surface of the warhead. Taylor presented the basic idea to predict static fragment distribution characteristics, and Shapiro applied it. Shapiro assumed that the warhead consisted of a series of rings with their centers located on the symmetric axis of the projectile. The detonation wave starts from the explosion center and propagates outwards in the form of a spherical wavefront. The fragment dispersion angle is the main research focus in such an approach. The angle \( \phi_1 \) is formed between the normal line of the prefabricated fragment and the symmetric axis of the projectile body and \( \phi_1 = \pi/2 \) for the spherical prefabricated fragment with cylindrical charge. The angle \( \phi_2 \) is formed between the normal line of the detonation wave and the symmetric axis of the projectile body. The deviation angle \( \theta \) of the fragment speed vector relative to the normal line of the shell is given by the Shapiro formula as follows:

\[
\tan(\theta) = \frac{V_0}{2D_z} \cos\left(\frac{\pi}{2} + \phi_2\right). \tag{1}
\]

After the warhead explodes, the fragment dispersion angle and flying speed are affected by the projectile body’s tractive speed. Assuming the tractive speed along the projectile axis direction as \( V_c \), the fragment speed after the static explosion as \( V_0 \), and the dispersion angle as \( \phi_0 \), the following equations can calculate the dynamic fragment dispersion angle \( \varphi \) and the initial dynamic speed \( V \) with the consideration of tractive speed \( V_c \):

\[
\varphi = \arctan\left(\frac{V_0 \sin \phi_0}{V_0 \sin \phi_0 + V_c}\right) \tag{2}
\]

\[
V = \sqrt{V_0^2 + V_0^2 + 2V_0 V_c \cos \varphi}. \tag{3}
\]

Assuming the number of fragments in the spherical zone from the spatial angle \( \theta_0 \) to \( \theta_0 + d\theta_0 \) as \( dN \) and the number of fragments generated by the entire projectile as \( N_0 \), then the quantity probability density of fragments in the space is given by

\[
\rho_N(\theta_0) = \frac{dN}{N_0 d\theta_0}. \tag{4}
\]

The unit spherical solid angle is given by \( d\Omega = \sin \theta_1 d\theta_1 d\Psi \), where \( d\Psi \) is the unit angle in the circumferential direction of the projectile. The quantity density of the unit spherical solid angle of the fragment is given by

\[
g_N(\theta_1) = \frac{dN}{d\Omega}. \tag{5}
\]

Moreover,

\[
\rho_N(\theta_1) = \frac{2\pi \sin \theta_1}{N_0} g_N(\theta_1). \tag{6}
\]

Previous experimental results show that the fragment spatial distribution curve is similar to the normal distribution curve given by

\[
\rho_N(\theta_1) = \frac{1}{\sqrt{2\pi} \sigma} \exp\left(-\frac{(\theta_1 - \theta_0)^2}{2\sigma^2}\right), \tag{7}
\]

where \( \theta_1 \) is the fragment space angle at the radius \( R \) and \( \theta_0 \) is the mathematical expectation of the space angle \( \theta_1 \) with a value usually close to \( \pi/2 \). The parameter \( s \) is the mean square deviation of the space angle \( \theta_1 \) and (5) is a function related to \( \theta_1 \). Therefore, the number of fragments in a flying region \( (\theta_i \sim \theta_{i+1}) \) can be determined by the following formula:

\[
N(\theta_i \sim \theta_{i+1}) = N_0 \int_{\theta_i}^{\theta_{i+1}} \frac{1}{\sqrt{2\pi} \sigma} \exp\left(-\frac{(\theta_1 - \theta_0)^2}{2\sigma^2}\right) d\theta_1. \tag{8}
\]

Assuming that the relative coordinates of the centroid of multiple projectiles are \( (\Delta x, \Delta y, \Delta z) \), and the projectile axis direction is \( y \), the Cartesian coordinate system can be transformed into a polar coordinate system by using

\[
\Delta x = \rho \cos \theta, \tag{9}
\]

\[
\Delta y = \rho \sin \theta, \tag{9}
\]

\[
\Delta z = \Delta z.
\]

When the relative polar coordinate distance of the mass center of multiple projectiles along the projectile axis direction is greater than the covering range of the dispersion angle of a single projectile, the multiple-projectile fragment distribution cannot be obtained by superposition, and the spatial density distribution of multiple-projectile fragments is given by
\[ N(\theta_i \sim \theta_{i+1}) = N_1 \int_{\theta_i}^{\theta_{i+1}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\theta - \theta_0)^2}{2\sigma^2}} \, d\theta, \quad (10) \]

\[ N(\theta_j \sim \theta_{j+1}) = N_2 \int_{\theta_j}^{\theta_{j+1}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\theta - \theta_0)^2}{2\sigma^2}} \, d\theta. \quad (11) \]

When the relative polar coordinate distance of the mass center of multiple projectiles along the projectile axis direction is less than the covering range of the dispersion angle of a single projectile, the multiple-projectile fragment distribution can be superimposed within a certain range of the dispersion angle. The polar coordinate system of the first projectile is assumed as a reference system. The spatial density distribution of multiple-projectile fragments can be obtained as follows:

\[ N(\theta_i \sim \theta_{i+1}) = N_1 \int_{\theta_i}^{\theta_{i+1}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\theta - \theta_0)^2}{2\sigma^2}} \, d\theta, \]

\[ + N_2 \int_{\theta_j}^{\theta_{j+1}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\theta - \theta_0)^2}{2\sigma^2}} \, d\theta, \]

\[ j + 1 = i + 1 - \arctan \left( \frac{\rho \sin \theta \cos \phi}{\rho \cos \theta} \right)^2 + \left( \frac{\rho \cos \theta}{\rho \cos \theta} \right)^2, \]

\[ j = i - \arctan \left( \frac{\rho \sin \theta \cos \phi}{\rho \cos \theta} \right)^2 + \left( \frac{\rho \cos \theta}{\rho \cos \theta} \right)^2. \quad (12) \]

2.2. Analysis of Explosion Shock Wave Overpressure. Assuming that a spherical or similar shaped charge of TNT explodes in an infinite air medium, and the resulting spherical shock wave is not influenced by other interfaces, the peak value of overpressure \( \Delta P_m \) at any distance \( R \) from the explosion center can be calculated by the following formulae:

\[ \Delta P_m = 0.0981 \times \left( \frac{14.0717}{R} + 5.5397 \times \frac{1}{R^2} - 0.3572 \times \frac{1}{R^3} + 0.00625 \times \frac{1}{R^4} \right) \]

\[ (\text{MPa}) 0.05 \leq R \leq 0.3, \]

\[ \Delta P_m = 0.0981 \times \left( \frac{6.1938}{R} + 0.3262 \times \frac{1}{R^2} + 2.1324 \times \frac{1}{R^3} \right) \]

\[ (\text{MPa}) 0.3 \leq R \leq 1, \]

\[ \Delta P_m = 0.0981 \times \left( \frac{0.662}{R} + 4.05 \times \frac{1}{R^2} + 3.288 \times \frac{1}{R^3} \right) \]

\[ (\text{MPa}) 1 \leq R \leq 10. \]

where \( R \) is the relative distance, determined by

\[ R = \sqrt{\frac{m}{m_w g^{1/3}}}. \quad (14) \]

where \( R \) is the distance to the explosion center (m) and \( m_w \) is TNT explosive mass (kg).

The explosive mass can be converted to a TNT equivalent in the above formula for other explosives.

The above calculation assumes an explosion condition in an infinite air medium. It is generally believed that when the explosion height \( H \) satisfies the following relationship:

\[ \frac{H}{\sqrt{m_w}} \geq 0.35 (m/kg^{1/3}). \quad (15) \]

Because the warhead is simplified as a cylindrical symmetric shell charge, the equivalent explosive charge mass can be expressed as

\[ m_w = \frac{m}{2 - \alpha} \left[ \alpha + 2(1 - \alpha) \left( \frac{r_0}{r_m} \right)^{2(\gamma - 1)} \right]. \quad (16) \]

where \( m \) is charge mass (kg), \( m_w \) is the equivalent explosive of detonation products (kg), \( \alpha \) is the loading coefficient, \( r_0 \) is the charge radius (m), and \( r_m \) is the radius where the fragment reaches its maximum speed (m). \( r_m \) of the steel shell is usually 1.5r0 to 2.1r0, according to test data.

According to the similarity law for an explosion, when a warhead explodes in air,

\[ \frac{t_r}{\omega} = 1.35 \times 10^{-3} \left( \frac{R}{\sqrt{m_w}} \right)^{1/2}. \quad (17) \]

The magnitude of a specific impulse directly determines the degree of shock wave damage. Theoretically, the specific impulse is determined by a time integral of the air shock wavefront overpressure:

\[ i = \int_0^{t_i} \Delta P(t) \, dt. \quad (18) \]

3. Experiment Design

In order to study the fragment dynamic spatial distribution under multiple-projectile continuous attacks and verify the spatial distribution formulae in Section 2, an experimental test using rectangular targets under continuous dynamic attacks were designed. As shown in Figures 1 and 2, two rectangular targets were arranged on both sides of the explosion center, respectively, and one rectangular target was arranged in front of the explosion center with the target board thickness 3 mm made of Q235 steel. The radial distance between the rectangular side target and the explosion center was 10 m, and each side target was located along the 45° direction from the ballistic trajectory line. The left-side rectangular target had a size of 6 × 3 m². The width of a single component of the target is 1 m, with a height of 3 m and a recovery angle \( \Delta \varphi = 65° \). The right-side rectangular target had a size of 5 × 3 m² and a recovery angle \( \Delta \varphi = 49° \). The front rectangular target was 13 × 3 m² and 30 m away from the explosion center. A Model 105 cannon launched the warhead specimen with flat firing. The warhead was detonated with a center fuse installed at one end. The firing
distance from the cannon to the explosion center was 200 m. The explosion height was 1.5 m above the ground.

Four measurement sensors for shock wave overpressure signals were arranged along the line of the firing direction, and four more were arranged perpendicular to the firing direction its perpendicular line. The sensors were placed at 2 m, 3 m, 4 m, and 5 m away from the explosion center along both axes. The angle between the two columns of the sensors was 90°. The sensor sampling frequency was 1 MHz, the pressure sensing range was 1.5 MPa, and the triggering method was internally triggered. Figure 3 shows the layout of the sensors. The distances of the sensors are shown in Table 1. Moreover, a high-speed camera was placed 50 m away from the explosion center. The high-speed camera system was a Fastcamultima APX high-speed camera produced by the Photron Company. The photo shooting rate was set at 24,000 frames per second during the test to capture the instantaneous fragment speed when the fragments penetrated the target. The fragment trajectory, the initial speed, and the flight attitude of the warhead at the explosion center were also traced. The overpressure data, the initial warhead speed and attitude, and the target perforation status under multiple-projectile continuous attacks were obtained during the test.

In order to identify the number of holes on the rectangular targets during the process of prefabricated fragment penetration and find out the corresponding fragment spatial distribution, the Q235 steel target was painted before the test. The holes on the targets made by prefabricated fragments and natural fragments could be easily distinguished.

The warhead used in the test was a 105 mm grenade, as shown in Figure 4. The charge diameter was 105 mm, and the charge length was 280 mm. The charge was a polyblack aluminum-2 press-fit charge with a density of 1.71 g/cm³, detonation pressure of 29.5 GPa, and detonation velocity of 8425 m/s. The charge mass was 1.672 kg. The shell material was 58SiMnVB with 12.18 kg mass. The material properties of the warhead are shown in Table 2. The shell wall thickness was 20 mm. A tungsten ball cap with a diameter of 3.3 mm and mass of 1.56 kg, made by injection molding, was installed at one end of the shell. A thread connection fixed the shell of the tungsten ball cap and the hood. A dynamic explosion test was repeated three times.
Figure 3: Dynamic explosion test layout of shock wave overpressure.

Table 1: Testing radius and device number of the first projectile.

<table>
<thead>
<tr>
<th>Theoretical distance to explosion center (m)</th>
<th>Number of measurement locations</th>
<th>Device number</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>C1, C3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>Y5, Y6</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>Y4, Y3</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>Y2, Y1</td>
</tr>
</tbody>
</table>

Figure 4: Schematic diagram of warhead: (a) warhead simplified diagram, (b) projectile body, and (c) tungsten ball fragment.
4. Analysis and Discussion of Test Results

4.1. Analysis of Warhead Attitude at Explosion. The warhead flying attitude at the explosion and the evolution of the shell fragment flight trajectories recorded by high-speed photography are shown in Figure 5. The test results show that there was obvious fire after the charge exploded. In time, the fire grew stronger (larger and brighter) and then faded away. The initial moment of the fire was set as 0 ms. The duration of the fire was 131 ms. Each high-speed photograph in Figure 5 was adjusted with the same scale. The size of the Q235 target in the background was used as a reference to measure the fragment speed and the flying direction at different moments.

The initial warhead speed, fragment speed, and flight trajectory were calculated using the time markers on the photos after the explosion and the target penetration events filmed by the high-speed camera. The warhead flight direction at the explosion was parallel to the ground and was 45° away from the target board. At the explosion center, the initial warhead speeds were 540 m/s, 563 m/s, and 523 m/s for the three projectiles. The fireball and the fragments began to separate at 1.394 ms after the explosion, and the fragments moved faster than the fireball.

The fragment data within the angle is collected by the witness plate. It can be found that after the explosion drive, the bright spot caused by fragment breakdown appeared in the witness plate at 30.8 ms, and then gradually reached 38.4 ms. Fractures spread and increased from the middle to both sides of the witness plate. It can be seen that the flying velocity of fragments at the warhead first increased and then decreased from the initiation end to the detonation center to the tail end. To calculate the fragment motion, it was assumed that the trajectory was a straight line. The influences of air lift and fragment gravitational force were ignored. The aerodynamic resistance effect was considered in the calculation. The initial fragment speed was calculated using (2) and (3) based on the measured speeds at the measurement locations. In the initial speed calculation, the camera shooting errors were removed to prevent using excessively low fragment speeds. The fragment speeds measured by the experiments of three specimens were compared. The spatial distributions of the fragment speed and dispersion angle under dynamic explosion conditions were obtained. The distribution of the fragment axial perforation speed within the angular range covered by the target boards was analyzed, as shown in Figure 6.

4.2. Analysis of Rectangular Target Perforation. After the experiments of continuous attacks, the fragment perforation conditions of the left Q235 steel target were analyzed. The images were segmented to extract hole shape information based on the abrupt change of the pixel grayscale value at the edge of the hole area on the witness board. The Image-Pro Plus and ImageJ image analysis software packages were used to distinguish the perforation hole contours on the witness boards. The holes made by natural fragment perforation and prefabricated fragment perforation were classified and counted. Figure 7 shows the fragment distribution on the left witness board after the tests of three projectiles. The grayscale bitmaps of fragment perforation were obtained by Image-Pro processing of the target boards. The holes made by prefabricated fragment perforation were counted according to the pre-divided angular regions on the witness boards for target-shooting conditions under continuous dynamic attacks with three projectiles.

Due to the differences in the explosion position and initial launch velocity of continuous dynamic attacks and the fact that the witness boards were placed at only 10 m away from the explosion center, there were not many fragments hitting the target boards. The test results of the fragments hitting the witness boards with three projectiles are shown in Figure 8. The spatial distribution of fragment quantity on the witness board was consistent with the normal distribution curve of the static fragment dispersion. The fragments are concentrated in regions 9–12 on the witness board. Many fragments hit the witness board in the overlapping area within regions 6–14 under multiple-projectile explosion conditions. This area is the superimposed spatial area created by continuous dynamic attacks of multiple projectiles.

4.3. Analysis on Damage Distribution under Multiple-Projectile Continuous Attacks. The fragment spatial distribution characteristics in Figure 8 were analyzed using (7) in Section 1 to calculate the dynamic multiple-projectile fragment distribution. The fragments were mainly concentrated in the angular range of 23°–31° on the Q235 target located 10 m away from the explosion center. The distribution illustrates the fragment dispersion characteristics of the prefabricated fragment warheads. The explosion centers of multiple projectiles had an interval of 0.91 m. The detonation direction was the same for all the projectiles. The fragments created by the multiple projectiles were mainly concentrated in the angular range of 12°–51° at 10 m away from the explosion center with a superimposed spatial area located at 19°–36°. These results are consistent with the experimental data of the rectangular targets shown in Figure 8. Therefore, it can be concluded that the spatial distribution of multiple dynamic projectiles is the superimposed result of the spatial distributions of every single projectile, and the superposition characteristics satisfy the normal distribution, meeting the requirement of (11). These data help analyze the effects of continuous attacks of multiple projectiles.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g/cm³)</th>
<th>Yield strength (MPa)</th>
<th>Elastic modulus (GPa)</th>
<th>Hardness</th>
<th>Poisson ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell</td>
<td>7.83</td>
<td>770</td>
<td>235</td>
<td>200 HB</td>
<td>0.22</td>
</tr>
<tr>
<td>Tungsten ball</td>
<td>19.2</td>
<td>832</td>
<td>265</td>
<td>198 HB</td>
<td>0.19</td>
</tr>
</tbody>
</table>
Figure 5: Evolutions of warhead explosion attitude and fragment flying trajectory: 0 ms, 0.287 ms, 1.394 ms, 2.009 ms, 2.911 ms, and 3.403 ms.

Figure 6: Dynamic fragment axial distribution.

Figure 7: Fragment distribution on the left witness board: (a) target hitting picture (left) on witness board #1, (b) target hitting picture (left) on witness board #2, and (c) target hitting picture (left) on witness board #3.
Table 3 presents the results of three projectiles obtained by the high-speed camera, as shown in Figure 5, including the warhead attitude at the explosion and initial falling angle. The dispersion angle and speed data of the natural fragments and the prefabricated fragments were integrated by computer programming to obtain the ground fragment distribution maps of three projectiles, as shown in Figure 9. Then, considering different locations, falling speeds, and falling attitudes of multiple warheads, the falling locations were calculated to obtain the combined fragment distribution of the multiple projectiles, as shown in Figure 10.

4.4. Analysis of Dynamic Shock Wave Field Created by Explosion. In the explosion-in-air test, the ambient pressure of the pressure sensor is the initial air pressure $p_0$ before the air shock wave generated by the explosion reaches the free-field pressure sensor. The overpressure of the air shock wave is defined as $\Delta p = p - p_0$. When the air shock wave reaches the sensor, the air pressure rapidly increases to $p$, and then the overpressure slowly decays to the initial air pressure. The pressure measured by the sensor in this test is the overpressure of the air shock wave, $\Delta p$. When the pressure is measured at the distance $R$, the time-dependent $\Delta p(t)$ curve of the air shock wave overpressure can be obtained. The shock wave overpressure curves measured in the warhead explosion test are shown in Figure 11. It is observed that the pressure curves have the same trend with respect to time. As the shock wave propagates, the parameters, including air pressure, decrease rapidly since the energy per unit area on the shock wave front decreases rapidly as the shock wave front expands with an increase in the propagation distance. Figure 11(b) shows that the initial oscillating shock wave is caused by the expanded and broken shell. The shock wave overpressure curve obtained in the test rapidly decays in the initial stage and then slowly decays. Shock waves with
smaller amplitudes are found immediately after the positive pressure zone or the negative pressure zone. These smaller waves are believed to be secondary shock waves.

Peak and valley pressures and propagation speed can be used to characterize the air shock wave caused by warhead explosion. In addition, wave arrival time, peak pressure, and shock wave positive pressure acting time can quantify the intensity of the instantaneous energy release of the explosive. The positive pressure acting time \( t^+ \) is a characteristic parameter of the explosion air shock wave, and it is an important parameter indicating the target damage effect. When the air shock wave reaches the pressure sensor, the air pressure suddenly rises to a certain peak value, called the overpressure peak of the air shock wave. Then, the air pressure slowly decays to the ambient pressure with time \( t^+ \). Therefore, the time duration when the air pressure is greater than the ambient pressure is the positive pressure acting time \( t^+ \). The overpressure peak \( \Delta p \), the positive pressure acting time \( t^+ \), and the specific impulse \( i \) at different locations were measured in the warhead explosion test, as shown in Table 4.

Figure 12 also shows the overpressure peak, the positive pressure acting time, and the specific impulse measured at different locations. In general, these parameters at different explosion heights and falling angles demonstrate certain trends. For the overpressure signals measured at distances less than 4 m away from the explosion center, the overpressure peak of the second projectile decayed faster than those of the other two projectiles. For the overpressure signals measured beyond 4 m, the overpressure peaks of the three projectiles decayed at similar rates. At a distance of 2 m away from the explosion center, the secondary shock wave appeared behind the negative pressure zone. At a distance of 5 m away from the explosion center, the secondary shock wave appeared behind the positive pressure zone (Y1, Y2), indicating that the arrival time of the secondary shock wave was related to the overpressure amplitude and the distance from the explosion center. At distances less than 3.5 m away

<table>
<thead>
<tr>
<th>ID</th>
<th>Falling speed (m/s)</th>
<th>Falling angle (°)</th>
<th>Explosion height (m)</th>
<th>Weighted killing area (m²)</th>
<th>Ground killing area (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>538</td>
<td>41</td>
<td>12</td>
<td>149</td>
<td>3364</td>
</tr>
<tr>
<td>#2</td>
<td>562</td>
<td>46</td>
<td>8.2</td>
<td>168</td>
<td>3005</td>
</tr>
<tr>
<td>#3</td>
<td>523</td>
<td>50</td>
<td>7.5</td>
<td>175</td>
<td>3116</td>
</tr>
</tbody>
</table>
Figure 11: Continued.
Figure 11: Time-dependent variation curves of shock wave overpressure: (a) witness board #1, (b) witness board #2, and (c) witness board #3.

Table 4: Measurement results of shock wave overpressure and positive pressure acting time.

<table>
<thead>
<tr>
<th>ID</th>
<th>Δp (MPa)</th>
<th>( t^+ ) (ms)</th>
<th>I (Pa·s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 m</td>
<td>3 m</td>
<td>4 m</td>
<td>5 m</td>
</tr>
<tr>
<td>#1</td>
<td>0.174</td>
<td>0.08</td>
<td>0.055</td>
</tr>
<tr>
<td>#2</td>
<td>0.274</td>
<td>0.135</td>
<td>0.065</td>
</tr>
<tr>
<td>#3</td>
<td>0.09</td>
<td>0.045</td>
<td>0.034</td>
</tr>
</tbody>
</table>

Figure 12: Continued.
from the explosion center, the specific impulse of the shock wave of the second projectile decayed faster than those of the other two projectiles. Beyond 3.5 m, the specific impulses of the three projectiles decayed at similar rates. These phenomena are mainly due to different explosion locations of different projectiles. As the falling angle at the explosion center increased, or as the falling height decreased, the positive pressure acting time gradually increased from 2 m to 5 m, with an increased amplitude of 57.2%.

At the distances of 2 m to 4 m (i.e., in the near field), the air shock wave’s specific impulse and overpressure peak decreased significantly due to the influence of falling speed. The test result showed that the falling speed had the greatest impact on the specific impulse of overpressure; the falling angle had the greatest impact on the peak value of overpressure; both the falling angle and the explosion height had the greatest impact on the positive pressure acting time. Thus, properly increasing the falling angle, falling height, and falling speed of the projectile may lead to an optimal damage effect in terms of increased overpressure peak and specific impulse. This topic is worth exploring in future research on dynamic damage effects.

5. Results and Discussion

This paper explores the performance and damage effects of continuous dynamic attacks of multiple grenade warheads, including dynamic warhead speed, fragment dispersion distribution, and dynamic overpressure distribution. Conclusions are drawn as follows:

(1) The warhead fragments mainly concentrated in the angular range of 23°–31°, with a superimposed spatial area located in the range of 19°–36°. These findings were consistent with the test data on the rectangular target boards. It is concluded that the dynamic fragment spatial distribution of multiple projectiles is the superposition of the spatial distributions of every single projectile, and the superposition obeys the normal distribution. Thus, the experimental data conforms to the theoretical model of fragment distribution of multiple projectiles under continuous attacks.

(2) The secondary shock wave appeared behind the negative pressure zone for the distances in the near field away from the explosion center. When the distance increased to the far-field, the secondary shock wave appeared behind the positive pressure zone. The arrival time of the secondary shock wave was related to the overpressure amplitude and the distance. As the falling angle increased at the explosion center, or as the falling height decreased, the positive pressure acting time gradually increased.

(3) The falling speed had the greatest impact on the specific impulse of overpressure. The falling angle had the greatest impact on the peak value of overpressure. Both the falling angle and explosion height had the greatest impact on the positive pressure acting time. Therefore, properly increasing the warhead falling angle, falling height, and falling speed can result in an optimal damage effect due to increased overpressure peak and specific impulse.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.
Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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