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*Research Article*

# Effect of Bed Roughness and Negative Step on Characteristics of Hydraulic Jump in Rectangular Stilling Basin

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Water flow often releases excess energy after passing through gates and spillways. This energy should be reduced to avoid the destruction of downstream structures. The hydraulic jump, a natural phenomenon, inevitably reduces incoming flow energy. The experiments were carried out in this study to investigate the effect of bed roughness and abrupt negative drops on the characteristics of the hydraulic jump. Also, to investigate the effect of geometric and hydraulic parameters on energy dissipation and location of the hydraulic jump, there was a change in the height of the abrupt drop and roughness for different discharges between 30 and 50 L/s and the Froude numbers were ranging from 4.9 to 9.5. The results showed that increasing the bed roughness causes a reduction of the sequent depth ratio and the relative length of the jump by 16.6% and 20.7%, respectively, and increases the relative energy loss and the bed shear force coefficient by 10% and 31%, respectively. In contrast, increasing the step height causes an increase in the sequent depth ratio and the relative length of the jump by 6.5% and 7%, respectively, and increases the relative energy loss and the bed shear force coefficient by 11% and 3.2%, respectively.

## 1. Introduction

A hydraulic leap occurs when a flow transitions rapidly from supercritical to subcritical, with significant disturbance and energy loss guiding the transition. The stability of the hydraulic jump has to be controlled under all possible flow conditions. The hydraulic jump's roller, on the other hand, must be inside the basin to avoid deterioration downstream. This can be accomplished by the construction of stilling basins. Sometimes, an abrupt drop (negative step) is applied to the stilling basin to stabilize the hydraulic jump location. The presence of a hydraulic jump at a sharp descent might cause the design of the stilling basins to change (Moore and Morgan [1]; Kawagoshi and Hager [2]; Ohtsu and Yasuda [3]; Chanson and Toombes [4]. Hager and Bretz [5] classified hydraulic jumps in the vicinity of a negative stage into five flow behavior based on input and tailwater flow factors (Figure 1). Whenever the tailwater depth is reduced from the A-jump condition (Figure 1(a)), the A-jump transforms into a stable waveform at a given point, which is known as the maximum wave (Figure 1(b)). The creation of a big eddy downstream is defined by the passage of the upward curved jet into the downward curved jet (Figure 1(c)) to convert into a B-jump (Figure 1(d) when the tailwater depth is reduced from the highest wave state. Because severe undulations propagate far downstream, the formation of upward and downward bent jets is undesirable (Ohtsu and Yasuda [3]).

Due to the expensive expense of stilling basins, researchers have been urged to minimize the subsequent depth and jump length, as well as enhance the energy loss (Pourabdollah, et al. [6]). Moreover, to increase the hydraulic jump energy loss, raising the bed roughness is one strategy for reducing the subsequent depth and jump duration. Using semitheoretical and empirical methodologies, the influence of the rough bed on hydraulic jump characteristics (e.g., sequent depth ratio, roller length, jump length, etc.) in horizontal channels has been investigated. Spheres

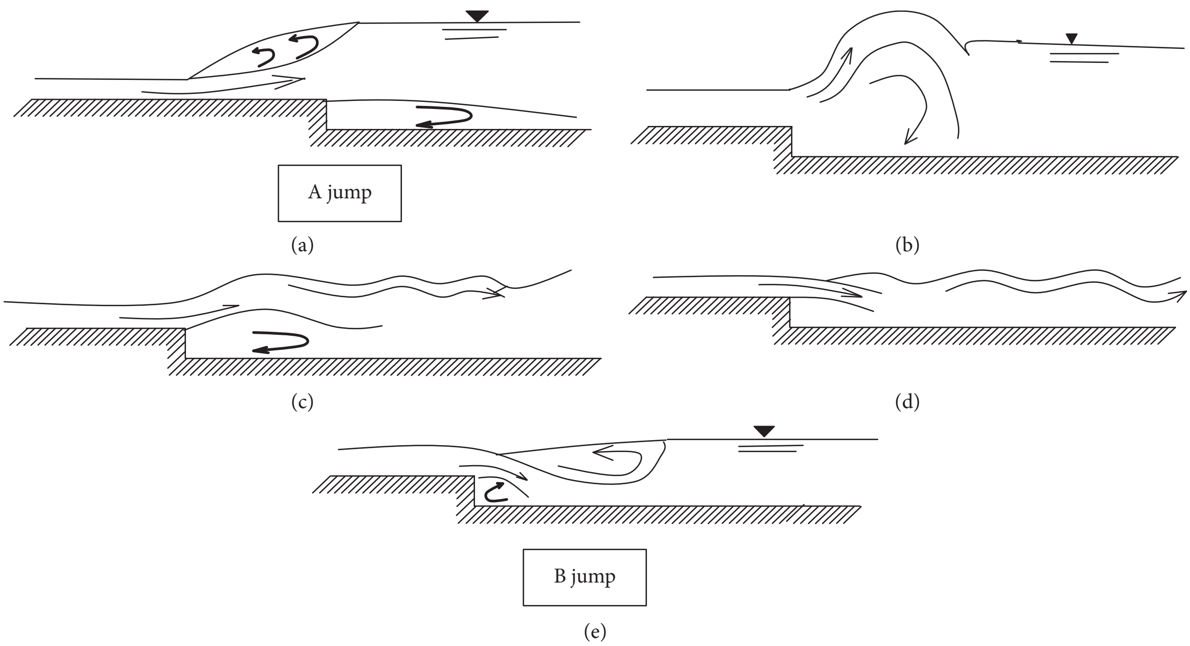


Figure 1: Flow conditions (from [3]): (a) A-jump, (b) wave jump, (c) wave train, (d) B-jump (maximum plunging condition), and (e) minimum B-jump (limited jump).

and strips were used by Leutheusser and Schiller [7] to investigate the characteristics of hydraulic leaps on roughened horizontal beds. Hughes and Flack [8] investigated several rough-bed arrangements that included strip roughness materials and gravels of varying sizes. Hydraulic leaps on a corrugated bed were also studied by Ead and Rajaratnam [9]. Carollo et al. [10] also looked at the hydraulic jump features on horizontal rough-bed patterns, finding that the subsequent depth ratio was influenced by the incoming flow Froude number as well as relative roughness. Pagliara et al. [11] also looked at the influence of nonuniform bed material, offering empirical relationships to predict hydraulic jump characteristics.

The literature and guidelines were improved on the design of a stilling basin over a rough bed, taking into account both the Froude number and bottom roughness. The formulas in the literature consider the effects of gravel roughness only, neglecting the effects due to the incident Froude number in the evaluation of integrated bottom shear stress (Ardiclioglu et al., [12]). The comparison of the presented results with experimental measurements in the literature highlights the reliability and accuracy of this novel method. The results of this study allowed the design of hydraulic jump stilling basins over rough beds.

The stilling basin with a sudden downward descent is particularly important among the several efficient energy dissipators (Hager and Bretz [5]). An abrupt drop is applied to prevent tailwater consequences and to maintain the jump location (De Padova et al., [13]). When a bed rough is accompanied by a negative step, it may alter the influence of rough-bed effects on the hydraulic jump features. The literature information concerning the effects of bed roughness and abrupt drops on hydraulic jumps is still incomplete. In particular, differences in the characterization parameters dimensionless of roughness and height of negative steps, and also the role of the Froude number are detected in hydraulic jump characteristics such as sequent depth ratio, relative energy loss, length of jump, and the evaluation of the bed shear stress. For this reason, this paper is devoted to the development of a new equation that can be useful in calculating the ratio of Sequent depths in rectangular channels with bed roughness and abrupt drops.

## 2. Theoretical Analysis

*2.1. Sequent Depth Ratio.* The following formula (1) may be constructed by using the momentum equation for sections 1 and 2 in Figure 2:

$$F_1 - F_2 + F_s - F_\tau = \frac{\gamma}{g} q \left(\beta_2 V_2 - \beta_1 V_1\right), \tag{1}$$

where $q$ shows the discharge per unit width; $\gamma$ represents the unit weight of water; $g$ indicates the acceleration due to gravity; $V_1$ and $V_2$ are the mean velocities in sections 1 and 2, respectively; $\beta_1$and $\beta_2$ are the momentum correction factors in sections 1 and 2, respectively; $F_1$and $F_2$ are the total pressures in sections 1 and 2; $F_\tau$ is the shear stress force; and $F_s$ is the total pressure on the face of the drop (Ohtsu and Yasuda [3]). $F_1$ and $F_2$ can be written as follows (equations (2) and (3)):

$$F_1 = \frac{1}{2}\gamma d_1^2, \tag{2}$$

$$F_2 = \frac{1}{2}\gamma d_2^2. \tag{3}$$

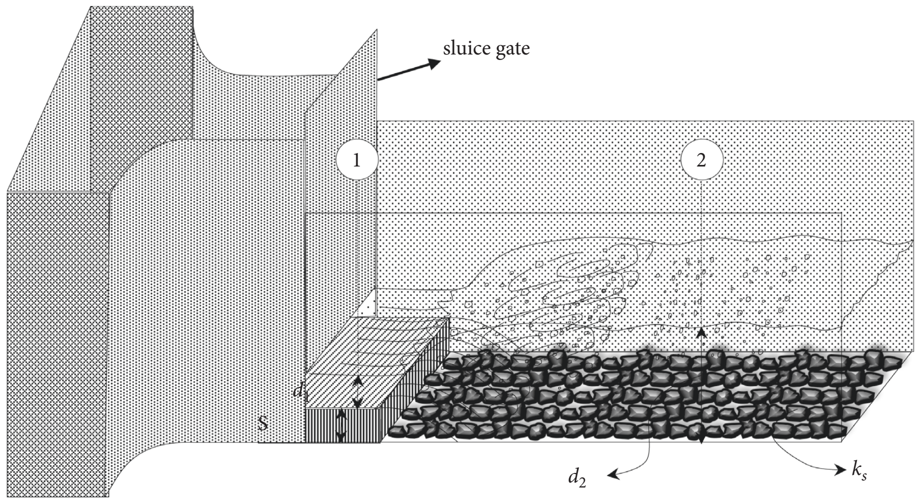


FIGURE 2: Definition sketch for the free jump.

The influence of the curvature of the streamline of the main flow going over the step is properly considered in F $s$, as described by Ohtsu and Yasuda [3] (equation (4)):

$$F_s = k\gamma s(d_1 + s/2). \quad (4)$$

The ratio of the actual pressure on the drop's face to the hydrostatic pressure is denoted by $k$. For calculating shear stress, Carollo and Ferro [14] suggest the following calculation (equation (5)):

$$F_\tau = \beta(M_1 - M_2) = \beta\frac{\gamma}{g}q(V_1 - V_2). \quad (5)$$

By combining equations (4) and (5), equation (6) can be presented as follows:

$$F_{ex} = F_s - F_\tau = k\gamma s(d_1 + s/2) - \beta\frac{\gamma}{g}q(V_1 - V_2). \quad (6)$$

After simplifying and replacing $V_1 = q/d_1$ and $Fr_1^2 = q^2/gd_1^3$:

$$\begin{aligned} F_{ex} &= k\gamma s(d_1 + s/2) - \beta\frac{\gamma}{g}q^2\left(\frac{1}{d_1} - \frac{1}{d_2}\right) \\ &= k\gamma s(d_1 + s/2) - \beta\gamma Fr_1^2 d_1^2\left(\frac{d_2 - d_1}{d_2}\right). \end{aligned} \quad (7)$$

After simplifying and replacing $D = d_2/d_1$:

$$\begin{aligned} F_{ex} &= \gamma Fr_1^2 d_1^2\left(\frac{k}{Fr_1^2}\frac{s}{d_1}\left(1 + \frac{s}{2d_1}\right) - \beta\left(\frac{d_2 - d_1}{d_2}\right)\right) \\ &= \frac{\gamma Fr_1^2 d_1^2}{D}\left(\frac{k\ D}{Fr_1^2}\frac{s}{d_1}\left(1 + \frac{s}{2d_1}\right) - \beta(D - 1)\right). \end{aligned} \quad (8)$$

By replacing $\delta = ((k\ D/Fr_1^2)(s/d_1)(1 + (s/2d_1)) - \beta(D - 1))$,

$$F_{ex} = \frac{\gamma Fr_1^2 d_1^2}{D}\delta. \quad (9)$$

Equations (2), (3), and (9) are substituted into equation (1) and equation (10) was obtained as follows:

$$\frac{1}{2}\gamma d_1^2 - \frac{1}{2}\gamma d_2^2 + \frac{\gamma Fr_1^2 d_1^2}{D}\delta = \gamma Fr_1^2 d_1^2\left(\frac{1 - D}{D}\right). \quad (10)$$

After simplifying,

$$\begin{aligned} D - D^3 &= 2Fr_1^2((1 - D) - \delta) \longrightarrow D^3 - D \\ &+ 2Fr_1^2((1 - D) - \delta) = 0. \end{aligned} \quad (11)$$

*2.2. Energy Loss.* A mathematical formula (equation (12)) was devised to estimate relative wasted energy (Rajaratnam [15]):

$$\frac{\Delta H}{H_1} = \frac{H_1 - H_2}{H_1}, \quad (12)$$

where $H_1$ and $H_2$ are the upstream and downstream energy heads of the hydraulic jump, respectively. $H_1$ and $H_2$ in a hydraulic jump with the abruptly negative step can be obtained using the following equations:

$$H_1 = d_1 + \frac{V_1^2}{2g} + Z_1 = d_1 + \frac{Fr_1^2 d_1}{2} + s, \quad (13)$$

$$H_2 = d_2 + \frac{V_2^2}{2g} + Z_2 = d_2 + \frac{Fr_2^2 d_2}{2}. \quad (14)$$

The relative energy loss equation may be stated by replacing $H_1$ and $H_2$ in equation (12) and applying mathematical simplification (12):

$$\begin{aligned} \frac{\Delta H}{H_1} = \frac{H_1 - H_2}{H_1} &= \frac{\left(1 + Fr_1^2/2 + s/d_1\right) - \left(\mathrm{D} + Fr_1^2/2\mathrm{D}^2\right)}{\left(1 + Fr_1^2/2 + s/d_1\right)} \\ &= 1 - \frac{\left(\mathrm{D} + Fr_1^2/2\mathrm{D}^2\right)}{\left(1 + Fr_1^2/2 + s/d_1\right)}. \end{aligned} \quad (15)$$

## 3. Pi-Theorem of Dimensional Analysis

In a horizontal and abruptly negative step with bed roughness, the following equation relates the relevant variables as follows:

$$f_1\left(g, s, d_1, d_2, V_1, \rho, \mu, k_s\right) = 0. \tag{16}$$

Here, $f_1$ is the functional symbol, $\rho\,(kg/m^3)$ shows water density, $\mu\,(N.s/m^2)$ represents water viscosity, $d_1\,(m)$ depicts the initial depth of the jump, $V_1\,(m/s)$ is the average velocity at the initial jump section, $g\,(m/s^2)$ is gravity acceleration, and $s$ (m) is the negative step height. A dimensionless functional connection may be derived using just five dimensionless groups, according to the $\pi$ theorem of dimensional analysis (Barenblatt [16]):

$$f_1\left(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5\right) = 0. \tag{17}$$

Choosing $V_1$, $d_1$, and $\rho$ as the repeating variables and rearranging the dimensionless group results in

$$f_1\left(\frac{d_2}{d_1}, \frac{V_1^2}{d_1 g}, \frac{V_1 d_1}{\vartheta}, \frac{k_s}{d_1}, \frac{s}{d_1}\right) = 0. \tag{18}$$

$V_1 d_1/\vartheta$ is the Reynolds number $(Re_1)$ based on $d_1$. The values of the Reynolds number were in the range of 75,000–250,000, and viscous effects could be neglected (Rajaratnam [17] and Hager and Bremen [18]). As a result, the subsequent depth ratio, as well as the relative duration of the hydraulic jump and relative energy loss, may be calculated:

$$\frac{d_2}{d_1} = f_1\left(\frac{V_1^2}{d_1 g}, \frac{k_s}{d_1}, \frac{s}{d_1}\right), \tag{19}$$

$$\frac{L_j}{d_1} = f_2\left(\frac{V_1^2}{d_1 g}, \frac{k_s}{d_1}, \frac{s}{d_1}\right), \tag{20}$$

$$\frac{\Delta H}{H_1} = f_3\left(\frac{V_1^2}{d_1 g}, \frac{k_s}{d_1}, \frac{s}{d_1}\right). \tag{21}$$

## 4. Materials and Methods

The trials were carried out in a rectangular laboratory flume with Plexiglas side walls, a length of 8.0 m, a width of 0.40 m, and a height of 0.60 m; which was related to a hydraulic circuit that allowed for discharge recirculation. The water supply into the flume was controlled by a valve installed in the supply line. The upstream sluice gate was in charge of the flow's supercritical depth, while the downstream sluice gate was in charge of the flow's tail water depth. Measured parameters include discharge, $Q$; inflow depth, $d_1$; length of jump, $L_j$; and tailwater depth, $d_2$. As a result, the subsequent depth ratio, as well as the relative duration of the hydraulic jump and relative energy loss, was calculated ±5%. Depths of flow, $d_1$ and $d_2$, were measured with a point gage of ±1 mm accuracy. A graduated device used for measuring the distance between the toe of the jump and the point when the constantly varying flow began.

The rough bed was simulated by pasting a layer of closely packed crushed gravel particles on a galvanized iron plate located on the bed flume. Two rough beds were established. A sample of 100 particles was used to determine the grain-size distribution of each gravel bed. Each object had three axial dimensions evaluated, with the diameter functioning as the mean value Hariri-Ardebili et al. [19]. The grain-size distributions were defined by $d50 = 1.25$ and 2.2 cm, where $d50$ is the diameter of the bed particle that is 50% finer. The roughness height $(k_s)$ was used to determine the gravel particles' median diameter (Figure 3). The abrupt drop heights were $s = 0.03$ and 0.06 m, respectively. A sluice gate was used to create the supercritical approach flow (Figure 2). V 1(gd 1), the Froude numbers, varied from 4.5 to 9.5. The discharge ranged from 30 to 50 litres (s). The tailwater level was regulated using a downstream gate for a specific flow so that the toe of the jump was set at the drop area.

## 5. Results and Discussion

In this study, various characteristics of the hydraulic jump such as length, sequent depth, bed shear stress, and energy loss in the B-jump on the abruptly negative step with the roughened bed were evaluated. The findings are provided in the section below.

*5.1. Sequent Depth Ratio.* The sequent depth ratio $(d_2/d_1)$, considering equation (19), is dependent on the inflow Froude number $(Fr_1)$, the relative height of the roughness elements $(k_s/d_1)$, and the relative height of the negative step $(s/d_1)$. The values of $d_2/d_1$ were shown against the inflow Froude number in order to analyze the influence of dissipative factors on the hydraulic jump's subsequent depth ratio, as shown in Figure 4. The subsequent depth ratio in the hydraulic jump on the rough bed was also compared to Belanger's [20] equation, as well as Pagliara's [24] and Carollo's 2015 equations (2007). As shown in Figure 4, in all heights of roughness elements, the sequent depth values were decreased as compared with the classical hydraulic jump, which was directly related to the inflow Froude number. A good agreement can be observed between the results obtained from the study by Carollo (Carollo, 2007) for $(k_s = 2.36 cm \text{ and } s = 0)$ with those obtained from this study for $(k_s = 2.2 cm \text{ and } s = 0)$. This agreement can be seen between (Pagliara, 2015) for $(k_s = 1.056 cm \text{ and } s = 0)$ with $(k_s = 1.25 cm \text{ and } s = 0)$ obtained from this study. From Figure 4, the sequent depth ratio is decreased as the height roughness is raised. By increasing the dissipative elements height, the flow separation and recirculation vortex was developed, which results in reduction of the sequent depth ratio. As Figure 4 shows, the sequent depth ratio $(d_2/d_1)$ was increased as the height of the negative step was raised.

Considering equation (18), the relationship between the sequent depth ratio $(d_2/d_1)$ and the inflow Froude number $(Fr_1)$, the height of negative steps, and the roughness elements could be described by the following regression-based

(a) (b)

Figure 3: Graver bed arrangement for (a) $k_s = 1.25$ cm and (b) $k_s = 2.2$ cm.

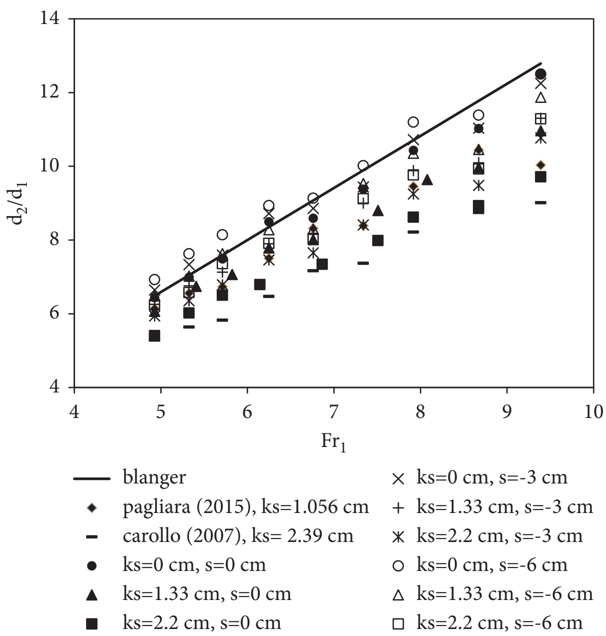


Figure 4: Variation of $d_2/d_1$ with $Fr_1$ for different heights of roughness and negative steps.

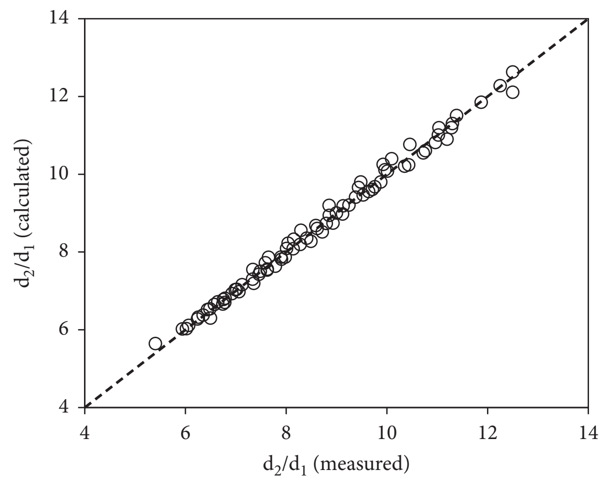


Figure 5: Comparison of the measured and calculated (equation (22)) values of $d_2/d_1$.

equation, with the coefficient of the determination ($R^2$) being equal to 0.98.

$$\frac{d_2}{d_1} = 0.9 + 1.15Fr_1 + 0.34\frac{s}{d_1} - 1.47\frac{k_s}{d_1}, \quad R^2 = 0.98. \tag{22}$$

Equation (22) shows that raising the inflow Froude number and height of the negative steps increases the sequent depth ratio. In addition, when the height of the roughness components increases, the subsequent depth ratio decreases (22). A comparison of all measured ($d_2/d_1$) values in the present study and those calculated from equation (22) is presented in Figure 5. This figure showed a good agreement in calculating the sequent depth ratio by equation (22).

By applying the momentum, equation (11) was developed to calculate the sequent depth ratio. The results showed that $\delta$ is a function of the inflow Froude number, the relative height of the negative step, and relative roughness. By using equation (11) and the experimental data, a regression-based equation was developed for $\delta$:

$$\delta = 1.97 - 0.37Fr_1 + 0.42\frac{s}{d_1} - 1.9\frac{k_s}{d_1}, \quad R^2 = 0.9. \tag{23}$$

Figure 6 illustrates a comparison of the observed ($D = d_2/d_1$) and calculated values, revealing that the highest error was $\pm$ 10% (equation (11)).

The dimensionless depth deficit parameter, $Y$, is defined as $Y = (d_2^* - d_2)/d_2^*$ (Ead and Rajaratnam [9]), representing the reduction of the sequent depth. $d_2^*$ is the subsequent depth of the hydraulic leap on a smooth bed with identical upstream circumstances in this calculation. For the experimental data, Figure 7 illustrates the fluctuation of $Y$ vs the input Froude number. By increasing the height of the negative step, the value of the parameter $Y$ became negative indicating that the negative step increased the secondary depth of the hydraulic jump.

The average dimensionless depth deficit parameter ($Y$%) for each experiment is shown in Table 1. As it can be seen from this table the secondary depth increases with the negative step and decreases in the rough bed.

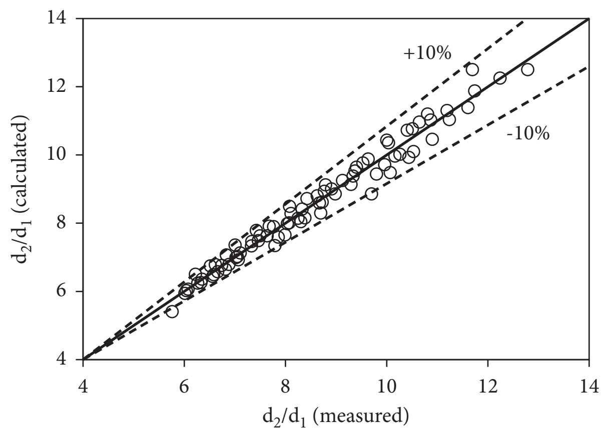


Figure 6: Comparison of the measured and calculated (equation (11)) values of $d_2/d_1$.

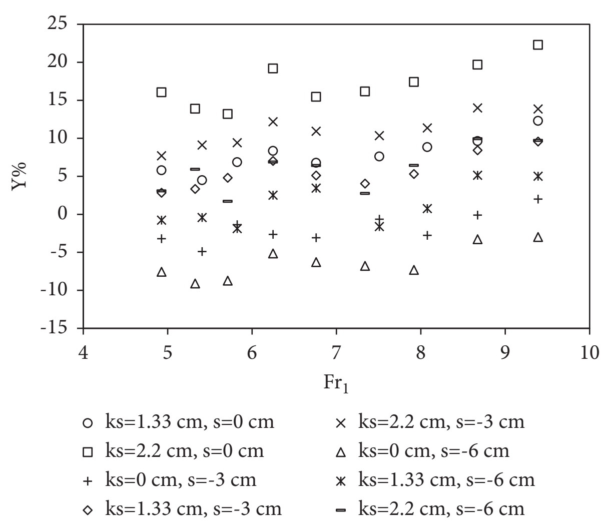


Figure 7: Dimensionless depth deficit parameter ($Y$) vs. the inflow Froude number.

Table 1: The average dimensionless depth deficit parameter ($Y$%).

| Experiments | $Y$% |
|---|---|
| $k_s = 0$ cm, $s = 0$ cm | 0 |
| $k_s = 1.33$ cm, $s = 0$ cm | 7.40 |
| $k_s = 2.2$ cm, $s = 0$ cm | 16.64 |
| $k_s = 0$ cm, $s = -3$ cm | −2.35 |
| $k_s = 1.33$ cm, $s = -3$ cm | 5.15 |
| $k_s = 2.2$ cm, $s = -3$ cm | 10.98 |
| $k_s = 0$ cm, $s = -6$ cm | −6.52 |
| $k_s = 1.33$ cm, $s = -6$ cm | 0.88 |
| $k_s = 2.2$ cm, $s = -6$ cm | 5.87 |

*5.2. Relative Energy Loss.* As shown in Figure 8, for certain values of the inflow Froude number, the relative energy loss was increased as the bed roughness was enhanced. The same results were obtained by Rajaratnam [15]. The pattern of the observed data for each model in this figure illustrates that

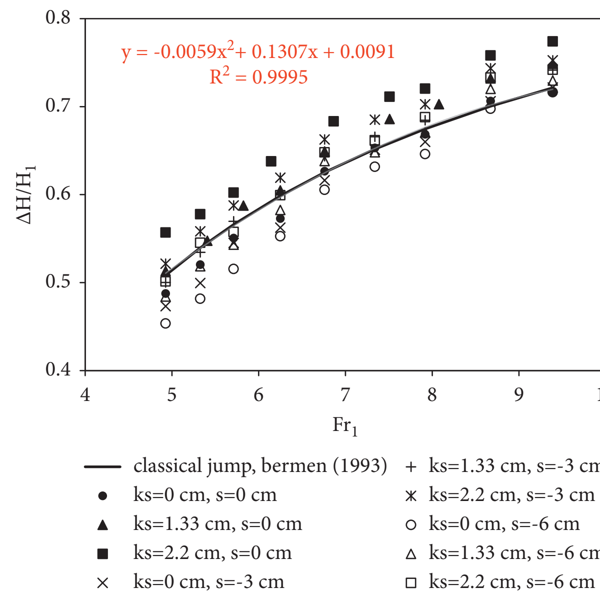


Figure 8: Variation of $\Delta H/H_1$ with $\mathrm{Fr}_1$ for different roughness.

increasing the height of steps can reduce relative energy loss for the same incoming Froude number.

$$\frac{\Delta H}{H_1} = 0.25 + 0.05Fr_1 - 0.012\frac{s}{b_1} + 0.052\frac{k_s}{d_1}, \quad R^2 = 0.96. \tag{24}$$

According to equation (21), the relationship between the relative energy loss ($\Delta H/H_1$), the inflow Froude number ($Fr_1$), the relative height of roughness, and the relative height of steps could be obtained by nonlinear regression.

The relative energy loss rose as the input Froude number and relative roughness ratio were raised, as can be observed in equation (24). In addition, raising the relative height of the stairs reduced relative energy loss. A comparison of all measured $(\Delta H)/H_1$ values in the present study and those calculated from equation (24) is presented in Figure 9. Also, the values calculated from equation (15) are presented in Figure 10. The results showed a good agreement which clearly showed the maximum error was ± 8%.

*5.3. The Length of Jump.* As shown in Figure 11, the relative length of the hydraulic jump ($L_j/d_1$) values was increased with the rise of the inflow Froude number ($Fr_1$) in all heights of roughness elements and step heights. The results obtained by USBR (USBR, 1955) for classical jump have been inserted in this figure for the sake of comparison. The roughness reduced the duration of the jump values as compared to a traditional hydraulic jump, according to the data. The relative length of jump decreases as the bed roughness increases, as seen in Figure 11 for given values of $Fr$ and $s$. As compared to a traditional hydraulic jump, the relative length values of the jump were reduced (Figure 11). In addition, as the step height is raised, the relative length of the leap increases.

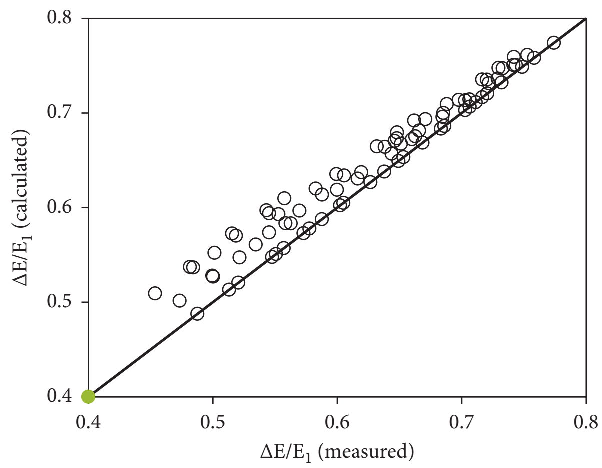


FIGURE 9: Measured and computed values $\Delta H/H_1$ (equation (24)).

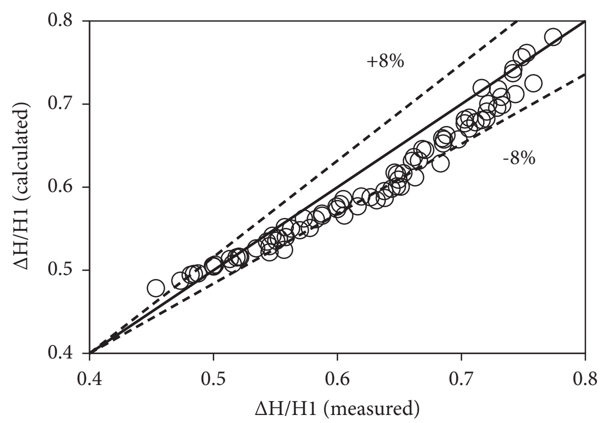


FIGURE 10: Measured and computed values of $\Delta E/E_1$ (equation (15)).

The mentioned regression-based equation could also be used to explain the relationship between the relative length of the hydraulic jump $(L_j/d_1)$, the inflow Froude number $(Fr_1)$, the relative height of roughness $(k_s/d_1)$, and the relative height of steps $(s/d_1)$, with the coefficient of determination $(R^2)$ equal to 0.97.

$$\frac{L_j}{d_1} = 4.1 + 7.33Fr_1 + 1.36\frac{s}{b_1} - 14.36\frac{k_s}{d_1}, \quad R^2 = 0.97. \tag{25}$$

According to equation (25), the relative length of the hydraulic jump was raised with increasing the inflow Froude number and height of the negative steps; also, the relative length of the hydraulic jump was decreased with increasing the roughness elements ratio. The relative length ratios for all trials were calculated using equation (25) and compared to the observed values in Figure 12 agreement, which easily demonstrated that the greatest error was ± 10%.

*5.4. Bed Shear Stresses.* The main explanation for the decrease in subsequent depth and jump length is an increase in bed shear stress. As a result, it is crucial to look at the shear stress on the bed. The shear stress coefficient $(\varepsilon)$ is commonly

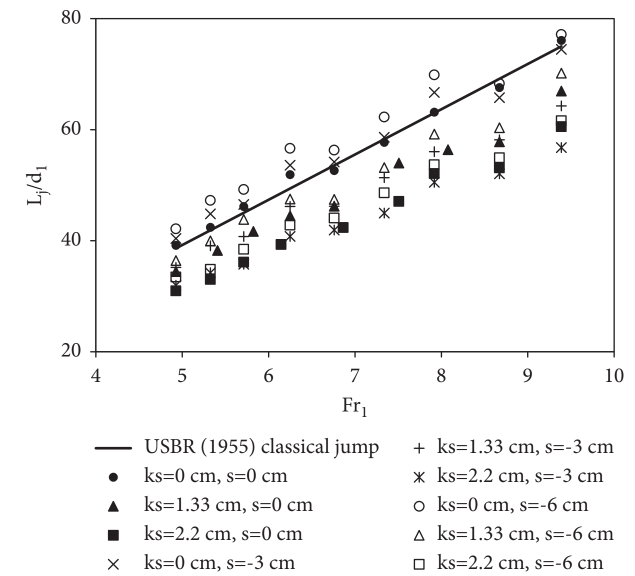


FIGURE 11: Variations of the Jump length ratio $(L_j/d_1)$ as a function of $Fr_1$ for different negative steps and heights of roughness.

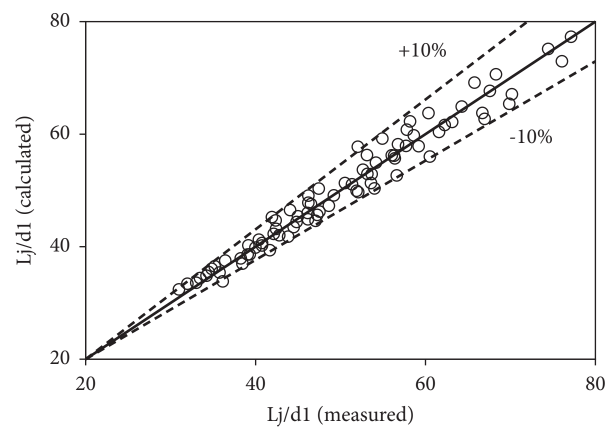


FIGURE 12: Measured and computed values of $L_j/d_2$ are compared (equation (25)).

used to characterise it. Ead and Rajaratnam [9] and Rajaratnam [21] provided the following shear stress equations for smooth and rough beds, respectively, as well as Izadjoo and Shafai-Bejestan [22] and Samadi-Boroujeni et al [23] for corrugated beds:

$$\varepsilon = 0.16Fr_1^2 - 0.8Fr_1 + 1, R^2 = 1, \tag{26}$$

$$\varepsilon = \left(Fr_1 - 1\right)^1, R^2 = 1, \tag{27}$$

$$\varepsilon = 0.058Fr_1^{3.035}, R^2 = 0.9433, \tag{28}$$

$$\varepsilon = 0.428Fr_1^{2.256}, R^2 = 0.93. \tag{29}$$

The shear force coefficient, as introduced by Rajaratnam [17], is defined as

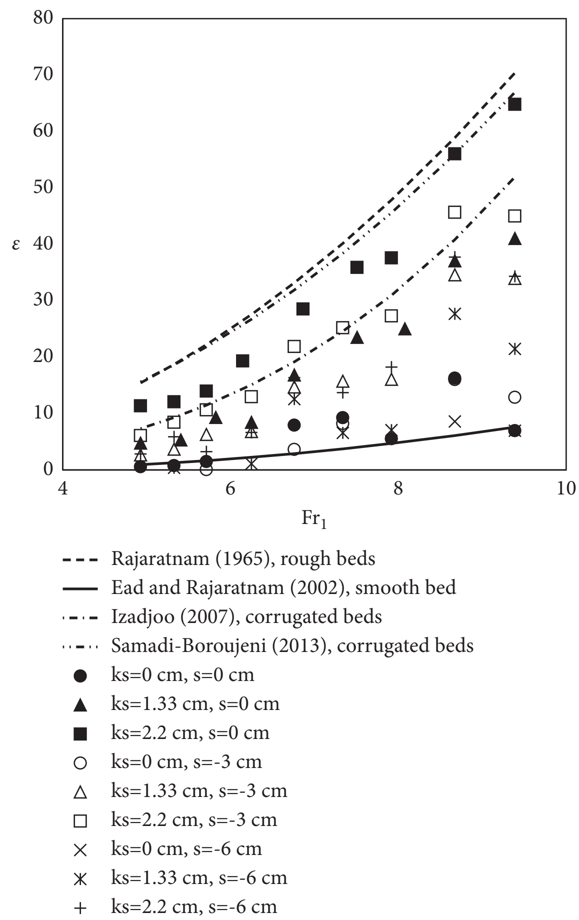


Figure 13: The bed shear stress coefficient as a function of the inflow Froude number.

$$\varepsilon = \frac{F_\tau}{0.5\gamma d_1^2}, \tag{30}$$

where $\gamma$ is the specific weight of water and $F_\tau$ is integrated bed shear stress over the hydraulic jump length. If $F_\tau$ is the integrated bed shear stress on the horizontal plane, it can be found using the integral momentum equation:

$$F_\tau = (P_1 - P_2) + (M_1 - M_2), \tag{31}$$

Where $P_1, P_2$, and $M_1$, $M_2$ are the integrated pressures and momentum fluxes at the sections just before and after the jump. To show the effect of the roughness elements and heights of the steps on the shear stress coefficient, the values of $\varepsilon$ were computed by equations (30) and (31), using the experimental data, and plotted as a function of the upstream Froude number, as can be seen in Figure 13. The data obtained from this study were compared with equations (26)–(29) to clarify the effect of the roughness elements and height of the negative steps on the shear stress coefficient.

The findings of this research were in strong conformity with those of Ead and Rajaratnam [9]; Rajaratnam [21]; Izadjoo and Shafai-Bejestan [22]; and Samadi-Boroujeni et al., as shown in Figure 13. (2013). The development of massive eddies within the jump led the bed shear force index to grow nonlinearly when the upstream Froude number increased for all tests, as seen in this figure. Furthermore, Figure 13 shows that increasing the bed shear stress coefficient increased the hydraulic leap on rough beds in all negative step heights when compared to smooth beds. Table 2 displays the average value of the bed shear coefficient on the rough bed for various step heights and roughness feature heights. The bed shear coefficient dropped as the height of the steps increased, as shown in Table 2. However, when the size was increased, the roughness diminished. In addition, the roughness height of 2.2 cm was more successful in generating significant turbulence and force, as well as raising the bed shear force coefficient.

Table 2: The average value of the bed shear coefficient.

| Experiments | Bed shear coefficient |
|---|---|
| $k_s = 0$ cm, $s = 0$ cm | 5.158 |
| $k_s = 1.33$ cm, $s = 0$ cm | 19.02 |
| $k_s = 2.2$ cm, $s = 0$ cm | 31.05 |
| $k_s = 0$ cm, $s = -3$ cm | 3.203 |
| $k_s = 1.33$ cm, $s = -3$ cm | 14.94 |
| $k_s = 2.2$ cm, $s = -3$ cm | 22.56 |
| $k_s = 0$ cm, $s = -6$ cm | −3.2 |
| $k_s = 1.33$ cm, $s = -6$ cm | 8.469 |
| $k_s = 2.2$ cm, $s = -6$ cm | 15.43 |

## 6. Conclusion

The hydraulic leap on a sudden descent over a rocky bed was explored in this study. According to research observations, installing an abrupt drop stabilizes the jump and roughness components by forming eddies, which raises bed shear stress and leads to energy dissipation in the supercritical flow. This study used the momentum equation to establish a relationship for the successive depths ratio, as well as regression to introduce certain connections for hydraulic jump characteristics. On the rough bed size 2.2 cm, the largest decrease in subsequent depth (16.6 percent) and relative length of leap (20.8 percent) were recorded. Moreover, when the step height was raised, the sequent depth ratio and relative length of leap were enhanced by putting an abrupt drop on the rough bed. In the presence of an abrupt drop and a rough bed, the largest reduction in subsequent depth was 5.87 percent and the relative duration of the leap was reduced by 16.8 percent when compared to a classical jump. As the roughness of the bed was increased, the relative energy loss rose. As the step height was raised, the relative energy loss was reduced. The bed shear force coefficient was increased to 31.05 percent as the bed roughness was increased; at the same time, the shear force coefficient and bed shear force coefficient were increased when the step height continued to increase.

## List of notations

$d_1$: Effective upstream depth of the hydraulic jump
$d_2$: Effective downstream depth of the hydraulic jump
$D$: $d_2/d_1$ sequent depth ratio

$L_j$: Hydraulic jump length
$H_1, H_2$: Specific energy heads at Sections 1 and 2, respectively
$\Delta H$: Relative energy dissipation
$f_1, f_2, f_3$: Functional symbol
$Fr_1$: Upstream Froude number
$k_s$: Bed roughness height
$V_1$: Section 1's average velocity
$V_2$: Section 2's average velocity
$g$: Gravity acceleration
$\vartheta$: Kinematic viscosity
$\gamma$: Specific weigh
$\rho$: Water density
$\mu$: Dynamic viscosity
$F_1$: Section 1 hydrostatic force per unit width
$F_2$: Section 2 hydrostatic force per unit width
$F_\tau$: Shear stress force
$q$: Discharge per unit width
$\beta_1$: Momentum correction factors in section 1
$\beta_2$: Momentum correction factors in section 2
$\varepsilon$: Bed shear stress coefficient
$M_1$: Momentum flux, per unit width, at the beginning of jump
$M_2$: Momentum flux, per unit width, at the end of jump.

## Data Availability

Requests for access to these data should be made to the corresponding author.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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