Dynamic Reliability Evaluation on Foundation of Stone Cutting Machine under Ambient Excitation

Jianjun Liu,1 Qiang Fu,2 Jiarui Shi,2,3 Xiao Li,2,3 Xueji Cai,4 Zilong Meng,2 and Jiyi Mi2

1Hunan Technical College of Railway High-speed, Hengyang, Hunan 421001, China
2School of Civil Engineering, Central South University, Changsha, Hunan 410075, China
3National Engineering Laboratory for High Speed Railway Construction, Changsha 410075, China
4School of Architectural Engineering, Sanming University, Sanming, Fujian 365004, China

Correspondence should be addressed to Qiang Fu; fuqiangmail@csu.edu.cn

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Dynamic reliability evaluation on foundation of stone cutting machine is usually affected by ambient background noise. In this paper, a nested evaluation approach of dynamic reliability is conducted for handling this complex situation. After getting the detected data, a fast identified process is available for fastly identifying the dynamic parameter foundation of stone cutting machine by using the peak-picking method, in which the natural frequency of foundation is able to be firstly identified by the equivalent frequency response function for the self-power spectrum of the foundation; subsequently, the damping ratio of the foundation, on the basis of the previous result of natural frequency, is identified by the half-power broadband technology; eventually, after obtaining these dynamic parameters identified, the failure probability of the foundation and its corresponding dynamic reliability index, under ambient excitation, is capable to be calculated based on the first-passage probability of the maximum response. An example of stone cutting machine foundation with engineering background is investigated in detail, and its result diagrams calculated by the proposed approach can be demonstrated that the foundation of stone cutting machine is safe and reliable in the current environment. The proposed method, in this paper, provides an effective path for evaluating the foundation reliability of stone cutting machine under ambient excitation.

1. Introduction

Stone is one of the fundamental materials in engineering practices, and the demands for stone cutting machine are continuously increasing with the development of society [1, 2]. However, the vibrations of the machine and its foundation appeared in the process of stone cutting easily result in the occurrence of safety accidents. Therefore, the evaluation on dynamic reliability refers to the foundation of stone cutting machine has become a key topic in optimizing the structural designing of its foundation, resisting the vibrations and increasing the accuracy of the failure diagnosis in both the academic and engineering field [3, 4].

Specifically, due to the complex work environment for the foundation of stone cutting machine in service, the foundation of stone cutting machine is commonly subjected to both the excitation generated by upper machine and ambient factors, such as the saw web, bogie, reducer, and spray. Therefore, when researchers are attempting to conduct dynamic analysis for the foundation of stone cutting machine, due to the uncertainty and diversity yield by those excitation sources, it is troublesome to obtain relevant dynamic parameters and consequently resulted in difficulties for proceeding relative dynamic reliability assessment [5, 6]. However, during the preceding decades, a number of approaches for identifying the dynamic parameters based on ambient excitation are proposed and applied successfully in a variety of engineering fields such as bridge, railway, and dam [7–9].

Generally, these methods are mainly based on the framework of time domain and frequency domain. In detail, for the framework of time domain, the approaches can be
concluded as (a) Ibrahim time domain technique (ITD),
Shang and Guan [10] introduced the geometric least square
method into the ITD and proposed the double least square
method (DLS), which significantly promotes its accuracy on
the identification of the damping ratio. Yang et al. [11]
adopted the Toeplitz matrix yield by stochastic subspace
method as the input source of the ITD, further improving the
accuracy of both the characteristic frequency and the
damping ratio; (b) Eigensystem realization algorithm (ERA),
Peterson and Alvin [12] verified the accuracy and feasibility
of ERA by conducting a case study of dynamic parameters
identification for a four-story frame structure. Siringoringo
and Fujino [13] adopted the ERA into the dynamic response
of a large-span suspension bridge based on the ambient excitation, by comparing with the results of the numerical simulation, it revealed that the accuracy of the ERA is reliable;
(c) random decrement technique (RDT), Morsy et al. [14]and
Gul and Catbas [15] utilized the RDT method to analyze a
reinforced concrete beam and a large-span bridge, which
further gained dynamic parameters and the location of the
damage; (d) stochastic subspace identification (SSI), Liu [16]
introduced the singular value decomposition method into SSI
and conducted a verification research towards to structural
acoustics and dynamic parameters. Li et al. [17] developed a
new type Toeplitz matrix to dimension reduction, by con-
ducting the dynamic parameters identifying of a steel truss
bridge, the time is reduced to 10.6% of original SSI.

In addition, in the framework of frequency domain,
relevant approaches are able to be traced as (a) peak-picking
method (PPM), the dynamic parameter was identified by the
peak of response power spectrum replaced the frequency
response function [18] and (b) frequency domain decom-
position (FDD), Brincker et al. [19] utilized Fourier in-
vention into single degree of freedom power spectrum and
proposed an enhanced FDD method, which improves the
identification accuracy of the damping significantly. Hzal
[20], based on FDD, proposed a modified frequency and
space domain decomposition method (MFSDD), which
obviously increased the efficiency and accuracy for FDD
with large environmental noise and model error. (c) Least
square complex frequency method (LSCF), Gorski [21]
adopted the RDT and Poly MAX for identifying the intrinsic
frequency and damping ratio of a high rise industrial
chimney through the filtering signal modified from GPS
monitor signals by using Chebyshev filter.

Among the aforementioned approaches, the peak-picking
method is in advantages of identifying dynamic parameters
of structure on account of its simplicity, high efficiency, and wide
feasibility. Therefore, during the past decades, the peak-
picking method is widely accepted and implemented in dif-
f erent engineering practices. For instances, Lacanna et al. [22]
adopted peak-picking method and FDD to estimate the in-
trinsic frequency and modal shape of a bell tower. Shi et al. [23]
conducted a comparison study between the peak-picking
method and Hilbert-Huang converter technique on the pre-
dictions of frequency and damping ratio for a super-tall tower.
Qiao et al. [24] used the peak-picking method to identify the
dynamic parameters of Shuikou concrete gravity dam.
Moreover, the peak-picking method is also broadly applied in
solving relevant problems and refers to modal parameters for
the identification of bridge engineering practices [25].

As one of the most important purposes for the dynamic
parameter identification of the stone cutting machine foun-
dation, the quantitative dynamic reliability assessment
attracted much attention in academic and engineering field
for years, and the most common method used is based on the
first-passage probability of the maximum response. Specifi-
cally, Rice [26] first established the expression of the numbers
for random processes transcend boundaries within specified
value interval and proposed an internal and external series
formula and a multiple integral formula for the first-passage
probability of the maximum response. Coleman [27] obtained
the analytical solution of the transition numbers and the first-
passage failure probability in B and D boundaries, by as-
suming the numbers of transitions between the response and
the threshold obeyed Poisson distribution. However, in later
researching, Li et al. [28] pointed out that such assumption is
inadequate due to the fact that the interleaved events are
trending to the crowds. Therefore, several modified methods
were continuously developed by a number of researchers,
such as Zhao et al. [29]. In those methods, the crossover
events were presumed to experience the Markoff process, and
the dynamic reliability formula suited for bilateral symmetric
boundary was proposed. Wen [30] developed an exponential
decay model for the probability density of the first-passage
failure and successfully applied it into relevant analysis in
single-degree linear system. Other modified methods were
also included the first-order perturbation method, difference
method, and the extreme probability density method [31,32].

For the single-degree nonlinear system, Roberts [33] devel-
oped Kolmogorov’s forward equations of the transfer prob-
ability density function based on energy envelope method.
For the multidegree nonlinear system, an amount of cele-
brated works were able to be traced, such as Foliente et al.
[34], Zhu et al. [35], and Chen and Li [36]. Moreover, in order
to verify the accuracy of the results yielded by above-men-
tioned approaches applied in the dynamic reliability assess-
ment, the Monte Carlo simulation (MCS) is usually utilized.
However, the traditional calculation process for this method is
usually more time consuming; hence, Bayer and Christian
[37] developed the importance sampling method for dynamic
reliability assessment, which significantly improved the effi-
ciency for MCS.

Currently, although there is a large number of a report
on the topic of reliability evaluation for kinds of structure
styles under ambient excitation in different engineering
practices, the researches of it on the foundation of stone
cutting machine are rarely traced. Therefore, conducting
relevant risk analysis utilizing the nested evaluation ap-
proach of dynamic reliability is of great significance for
improving relative designing, failure diagnosis, and main-
tenance of foundation of stone cutting machine.

Hence, based on above assertions, the main objective of the
current research is thus to develop a dynamic reliability eval-
uation methodology for the foundation of stone cutting machine
under ambient excitation. With this objective, the remainder
of this work is organized as follows. In Section 2, the procedure
of dynamic parameters identification for foundation of stone
cutting machine is established based on PPM. In Section 3, the analytic expression of dynamic reliability evaluation of foundation of stone cutting machine is yielded based on first-passage probability of the maximum response. In Section 4, the flow-chart and main calculative procedure for reliability evaluation on foundation of stone cutting machine are illustrated. It is then followed by Section 5, where a dynamic reliability evaluation is applied for the foundation of stone cutting machine with engineering background under ambient excitation. The conclusions of this work are finally given in Section 6.

2. Dynamic Parameters Identification for Foundation of Stone Cutting Machine

2.1. Modal Test of Foundation of Stone Cutting Machine. In order to evaluate the foundation reliability of stone cutting machine, its dynamic parameters, i.e. system nature frequency and damping ratio, are necessarily identified first. For this identification procedure, the modal test is an efficient and simple method to test the dynamic parameters of stone cutter foundation. After acquiring the fundamental detecting data of stone cutting machine by utilizing the spectrum analyzer, the nature frequency of foundation can be identified by peak-picking method; subsequently, the damping ratios are able to be identified by adopting half-power bandwidth method.

The first step of modal test is to input ambient simulation excitation into the foundation of stone cutter machine, so as to detect the relevant identification data for identifying dynamic parameters. The stone cutting machine foundation system, as shown in Figure 1, generally consists of motor, pillar, guide rail, trolley, monitoring point (point 1 to 4 in Figure 1), saw, and a reinforced concrete block foundation, etc. Aimed at providing fundamental data for structural health monitoring and dynamic reliability evaluation, modal test, which is shown as Figure 2, is usually used to identify the dynamic characteristics for foundation of stone cutting machine.

2.2. Collection for Monitoring Data of Foundation of Stone Cutting Machine. In order to collect and extract the data of foundation of stone cutting machine, modal test is usually applied to identifying dynamic parameters. The modal test is composed of four main steps, i.e., excitation input, noise sensing, signal acquisition, and data analysis, as shown in Figure 3. Its main operation procedure can be summarized as follows [38]:

1. Four acceleration sensors are first fixed at the measuring point (see Figure 1), and then the mechanical vibration acceleration signal generated by the stone cutter foundation can be picked up by the sensor, under the ambient excitation.
2. The vibration signal is amplified by charge amplifier; next, the signal is analyzed and processed by the data acquisition instrument on the computer; finally, the results are transformed into the corresponding self-power spectrum diagram.
3. Based on the peak-picking method, the nature frequency \( \omega_1 \) for foundation of stone cutting machine is capable to be identified from the self-power spectrum diagram.

(4) The damping \( \xi_1 \) of foundation of stone cutting machine is acquired by using half-power bandwidth technology.

2.3. Peak-Picking Method (PPM) for Identifying the Nature Frequency of System. Analyzing the stochastic response before, the foundation of stone cutting machine is usually ideal as a two-degree of freedom system, as shown in Figure 4, according to introduction for foundation of stone cutting machine in Section 2.1.

Where \( m_1, y_1(t) \) denote the mass and displacement of the upper machine, respectively; similarly, \( m_2, y_2(t) \) denote the mass and displacement of the lower foundation, respectively; \( c_1 \) and \( k_1 \) denote the coefficient of damping and elastic between the machine and the foundation; and \( c_2 \) and \( k_2 \) denote the coefficient of damping and elastic between the ground and the foundation.

Based on simplified diagram of calculation model in Figure 4, the system vibration equation of foundation of stone cutting machine can be written as [39]

\[
[M] \{ \ddot{Y}(t) \} + [C] \{ \dot{Y}(t) \} + [K] \{ Y(t) \} = [F(t)],
\]

where \( \{ Y(t) \} \) denotes the displacement matrix of system, \( \{ F(t) \} \) generalized ambient excitation matrix of system, and

\[
[M] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix},
\]

\[
[C] = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix},
\]

\[
[K] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix},
\]

\[
[F] = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix}.
\]

The damping in this project is able to be approximated as Rayleigh damping that it is capable to be represented as a
linear relationship between mass and stiffness. Thus, the equation (1) can be decoupled in the following form:

\[ \ddot{q}_i(t) + 2\xi_i\omega_i\dot{q}_i(t) + \omega_i^2 q_i(t) = f_i(t), \]  

(3)

where \( q_i(t) \) is representing as the displacement of a particle in generalized coordinates, \( i = 1, 2 \) and \( \omega_i \) is the nature frequency identified of the structural system under ambient excitation.

Based on random vibration theory, the multidegree of freedom frequency domain response function is as follows:

\[ H_i = \frac{1}{\omega_i^2 \left[ 1 - (\omega/\omega_i)^2 + 2i\xi_i\omega/\omega_i \right]} , \quad (i = 1, 2). \]

(4)

Meanwhile, the relationship between self-power spectral density of the ambient excitation and the response of foundation of stone cutting machine can be expressed as follows:

\[ \begin{bmatrix} S_{yy}(\omega) \end{bmatrix} = [H(\omega)]^* \begin{bmatrix} S_{xx}(\omega) \end{bmatrix} [H(\omega)]^T, \]

(5)

where \( \omega \) is the operating frequency of the excitation force, \( H(\omega) \) is the frequency response function matrix, \( S_{xx}(\omega) \) is the
self-power spectral density matrix of the input $f(t)$, $S_{yy}(\omega)$ is the self-power spectral density matrix of the output $y(t)$, and superscript $*$ and $T$ denote complex conjugate and transpose, respectively.

On the basis of white noise assumption, self-power spectral density of the input is a constant, i.e., $S_{xx}(\omega) = S_0$. Then, the frequency response function can be quantitatively identified as

$$S_{yy}(\omega) = |H(\omega)|^2 S_0(\omega).$$  \hfill (6)

According to specification, which number is GB 50040-2020, in China [40] and equation (3), the foundation can be decoupled into a single degree of freedom model. Then, the $S_{yy}(\omega)$ can be rewritten as follows:

$$|H(\omega)| = \frac{1}{\sqrt{m^2(\omega_n^2 - \omega^2)^2 + c^2 \omega^2}}.$$  \hfill (7)

Substituting equation (5) into equation (6), $S_{yy}(\omega)$ is expressed as follows:

$$S_{yy}(\omega) = \frac{S_0}{m^2(\omega_n^2 - \omega^2)^2 + c^2 \omega^2}. \hfill (8)$$

where $\omega_n$ denotes the system natural frequency and $\omega$ denotes the frequency of ambient excitation.

After gaining the equation (8), the peak-picking method can be conducted for identifying dynamic parameters of system, and its main principle is when the nature frequency of foundation of stone cutting machine is consistent with the ambient excitation, i.e., $\omega = \omega_n$, the self-power spectrum reaches its maximum value. In other word, these frequencies at this time are the real nature frequency of foundation of stone cutting machine that we need to identify.

2.4. Half-Power Bandwidth Method for Identifying the Damping of System. After getting the nature frequencies of foundation of stone cutting machine, the half-power bandwidth method is able to be utilized for identifying the damping [41]: first, finding the half-power point, i.e. the peak value of 0.7 times, on the diagram of ordinate power spectrum, and then, a horizontal line is made based on this point, the positions that this line intersect with curve of self-power spectrum are referred to as $\omega_u$ and $\omega_d$, eventually, the damping $\xi$ can be calculated by the following equation:

$$\xi = \frac{|\omega_u - \omega_d|}{2\omega_p}, \hfill (9)$$

where $\omega_u$ and $\omega_d$ denote the frequency corresponding to two half-power points and $\omega_p$ denotes the frequency corresponding to the peak of the power spectrum.

3. Dynamic Reliability Evaluation on Foundation of Stone Cutting Machine

For assessing dynamic reliability on stone cutting machine foundation under ambient excitation, the most commonly used method is based on the first-passage probability of the maximum response. In detail, it is required to complete a full analysis on the random response for the foundation before the formal reliability assessment.

3.1. Stochastic Response Analysis for Foundation of Stone Cutting Machine. In Section 2, the stone cutting foundation is usually ideal as a two-degree of freedom system as shown in Figure 4. According to the working power of the upper machine, the random response analysis of the foundation of stone cutting machine can be divided into linear system and nonlinear system. If the upper power is small, the stochastic response can be analyzed as a linear system. Generally, the damping in practical engineering is able to be approximated as Rayleigh damping, which can be represented as a linear relationship between mass $M$ and stiffness $K$.

$$[C] = C_M + C_K = a_0 [M] + a_1 [K], \hfill (10)$$

where $C_M$ and $C_K$ denote mass proportional damping and stiffness proportional damping and $a_0$ and $a_1$ denote their corresponding proportional factor.

After converting the equation (10) to the frequency domain, the modal superposition method is applicable to the random response analysis of the structural system, and its calculative principle is shown in Figure 5.

Figure 5 represents when the linear system for the foundation of stone cutting machine is subjected to the stationary random excitation $\{x(t)\}$, whose power spectrum matrix is $[S_{xx}(\omega)]$, then, the power spectrum matrix of system response $\{y(t)\}$ can be expressed as $[S_{yy}(\omega)]$ and its mutual power spectrum matrix is able to be expressed as $[S_{yx}(\omega)]$ and $[S_{xy}(\omega)]$, respectively. Specifically, if the upper machine power is large, the foundation of stone cutter should be regarded as a weak nonlinear system, then modal superposition method is no longer suitable for solving the random response of the system, at this time, the
perturbation method can usually meet the needs of engineering accuracy [42].

3.2. Dynamic Reliability Evaluation Based on the First-Passage Probability of the Maximum Response. Under the influence of random excitation, the vertical amplitude displacement $Y_R(t)$ of the foundation is able to be treated as stationary random vibration response. It can be seen from the graph that the event for $Y_R(t)$ exceeds the threshold $a$ is a rare event, as shown in Figure 6.

According to Rice formula, the average number of times $N_a^+$ of the normal stationary process $Y_R(t)$ with zero mean exceeds the level of the threshold limit of stone cutting machine $a$ per unit time can be obtained [26].

$$N_a^+ = \frac{1}{2\pi} \frac{\sigma_y}{\sigma_{yi}} \exp \left[ \frac{a^2}{2\sigma_y^2} \right],$$

where $\sigma_{yi}$ and $\sigma_y$ denote the standard deviation of the maximum displacement response for foundation of stone cutting machine and its derivative, respectively.

While the threshold limit $a$ is large, according to the Poisson distribution assumption [43], the probability mass function (PMF) for the number of peaks exceeding $y = a$ in time period $T$ can be expressed as

$$p(n, T) = \frac{(N_a^+)^n}{n!} \exp[-N_a^+ T].$$

Therefore, according to the two equations above, the cumulative distribution function (CDF) $F(a)$, in which the maximum number of peak values in the specified time period $T$ does exceed threshold value $y = a$, can be obtained:

$$F(a) = p(Y \geq a, n = 0, T) = \exp(-N_a^+ T) = \exp \left\{ -\frac{1}{2\pi} \frac{\sigma_{yi}}{\sigma_y} T \exp \left[ -\frac{1}{2} \left( \frac{a}{\sigma_{yi}} \right)^2 \right] \right\}.$$  

Therefore, in the specified time period $T$, the failure probability function $P_f$ as well as its dynamic reliability index $\beta$ for foundation of foundation of stone cutting machine is as follows:

$$P_f = 1 - P[Y < a] = 1 - \exp \left\{ -\frac{1}{2\pi} \frac{\sigma_{yi}}{\sigma_y} T \exp \left[ -\frac{1}{2} \left( \frac{a}{\sigma_{yi}} \right)^2 \right] \right\},$$

$$\beta = \Phi^{-1}(1 - P_f).$$

(14)

where $\Phi^{-1}$ denotes the inverse of cumulative density function.

4. Reliability Analysis Procedure for Foundation of Stone Cutting Machine under Ambient Excitation

In this section, for the sake of intuitively realizing the process of dynamic reliability evaluation on foundation of stone cutting machine under ambient excitation, the main calculating procedures are described as follows, and its corresponding flowchart is illustrated in Figure 7.

(1) Providing an external incentive, the reliable detection data of foundation of stone cutting machine can be obtained by detecting instrument. This operation process is shown in Figure 3.

(2) The natural frequency $\omega_i$ for foundation of stone cutting machine is first identified by the equivalent frequency response function of the self-power spectrum of the foundation, and then the damping ratio $\xi_i$ of the foundation is identified by the half-power broadband technology.

(3) The foundation of stone cutting machine is idealized as two degree of freedom system, i.e., Figure 4, and its corresponding vibration equation of stone cutting machine-foundation-soil system, can be established.

(4) According to the equation above, the standard deviation of displacement responses $\sigma_{yi}$ for foundation of stone cutting machine is acquired based on random vibration theory, in which the modal superposition method is applied for linear system, and the perturbation method is applied for nonlinear system.

(5) According to these parameters obtained, the failure probability $P_f$ of the foundation and its corresponding dynamic reliability $\beta$ is capable to be
evaluated based on the first-passage probability of the maximum response under ambient excitation.

5. Engineering Example and Investigations

5.1. Description of Foundation of Stone Cutting Machine

In order to investigate proposed method for the foundation of stone cutting machine under ambient excitation, an engineering case is investigated. ADZQ-1600-D18 hydraulic lock pillar stone cutting machine is analyzed in this paper. The size of pillar, motor frame, and guide rail foundation are 2.5 m × 1.8 m × 4.0 m, 3.5 m × 3.5 m × 1.2 m and 8.0 m × 3.2 m × 0.6 m, respectively, and the calculated sketch of the stone cutting machine is shown in Figure 4. According to the known information about the parameter guide of stone cutting machine and design drawing of foundation, the following approximate relationship can be obtained: \( m_1 : m_2 = 0.08 \), and the damping coefficient of machine and foundation are 0.01 and 0.05, respectively, and other relevant parameters are provided in Tables 1 and 2. Meanwhile, the white noise excitation is applied to Point 1 (refer to Figure 1) by WGN function as follows:

\[
\tilde{a} = a + \sigma_a \times 20\% \times \text{rand}(N, 1),
\]

where \( a \) denotes the acceleration sequence of the point 1, \( \sigma_a \) is representing the standard deviation of acceleration, \( \text{rand}(N, 1) \) denotes that a random sequence, its mean and standard deviation are 0 and 1, respectively, can be produced by Wolfram Mathematica merchandise software. In this example, the acceleration signal is sampled by accelerometer, which has a sampling frequency spectrum \( S_0 = 38 \text{ cm}^2/\text{s}^2 \) and sampling time of 8.53 second.

According to procedure described in Section 2, the modal test is applied to the foundation of stone cutting machine. In general, the response for foundation of stone cutting machine is affected by various uncertain excitations sources nearby; however, among these excitations, the excitation generated by the machine has the greatest influence proportion on the system response. Therefore, for the no noise input in the example, it reflects the actual situation that the stone cutter machine working; similarly, the input of 60% white noise excitation is to enhance the influence of other excitation so as to simulate the response under special working environment. Their results are displayed in Figure 8, and the abscissa coordinate is representing the frequency of the foundation detected by the acceleration sensor; the ordinate coordinate is representing the system identified PSD result.

In Figure 8(b), the PSD of system response has reached several peak frequencies under noiseless input, and then its corresponding first two main frequencies are 47.221 Hz and 105.620 Hz, respectively. According to the principle of peak-picking method in Section 2, their theoretical nature frequencies of system can be identified, i.e., \( f_1 = 47.221 \text{ Hz} \), \( f_2 = 105.620 \text{ Hz} \); subsequently, by adopting the half-power bandwidth method, the identified damping is able to be obtained as \( \xi_1 = 0.0107 \) and \( \xi_2 = 0.0575 \), respectively.

Additionally, in Figure 8(b), it is also revealed that the PSD of system response has reached several peak frequencies under 60% white noise excitation simulation imputed and system theoretical nature frequencies and damping are capable to be identified, respectively, \( f_1 = 47.392 \text{ Hz} \), \( f_2 = 105.779 \text{ Hz} \), \( \xi_1 = 0.0107 \), and \( \xi_2 = 0.0575 \).
According to the characteristics equation for free vibration of undamped vibration system, the two natural frequencies of system theoretical modal can be steady acquired, i.e.,
\[ f_1' = \omega_1/2\pi = 47.591\text{Hz} \quad \text{and} \quad f_2' = \omega_1/2\pi = 105.013\text{Hz}. \]
And the empirical damping ratios for upper and lower masses approximately are 0.01 and 0.05, respectively. And all of results identified are summarized in Table 3.

As seen from Table 3, in the absence of noise, the maximum error between the identified natural vibration frequency and the theoretical model is only 4.797% and only 3.112% in the presence of noise. The damping ratios identified by equation (9) are also close to the empirical damping ratio. It is revealed that the peak-picking method adopted in this paper can stably and effectively identify the real dynamic parameter values for the stone cutting machine foundation.

5.2. Dynamic Reliability Evaluation on Foundation of Stone Cutting Machine. According to Figure 4 in Section 2.3, the system vibration equation for stone cutting machine-foundation-soil system can be written as

\[
\begin{align*}
  m\dddot{Y}_1(t) + c_1\ddot{Y}_1(t) - k_1Y_1(t) + c_2\ddot{Y}_2(t) - k_2Y_2(t) &= f_1(t), \\
  m\dddot{Y}_2(t) + c_2\ddot{Y}_2(t) + k_2Y_2(t) &= f_2(t).
\end{align*}
\]

(16)

This equation above is able to be simplified as matrix form as follows:
\[
[M]\dddot{Y}(t) + [C]\dddot{Y}(t) + [K]Y(t) = \{F(t)\},
\]
where \( \{Y(t)\} \) denotes the displacement matrix of system and \( \{f(t)\} \) denotes generalized ambient excitation matrix of system.

In this example, the modal superposition method, which is recommended to analyze linear vibration system stochastic response in Section 3, is used to calculate the standard deviation for displacement response of the foundation \( \sigma_{y_2} \). And its analytical expression is as follows:
\[
\sigma_y = \sqrt{E[\dddot{Y}(t)^2]} = \sqrt{\frac{\pi^2\delta^2 T_1^2 T_2^2}{A}} (T_1 + T_2),
\]
(18)
in which

| Table 1: Technical parameters of stone cutting machine. |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Motor power (kW) | Motor speed (r·min⁻¹) | Shaft speed (r·min⁻¹) | Gear ratio | Blade diameter (min) | Trolley power (kW) | Gross weight (t) |
| 75 | 2970 | 980 | 3 | 1600 | 2.2 | 60 |

| Table 2: Technical parameters of other relevant measurement equipment. |
|----------------|----------------|----------------|----------------|
| Name | Accelerometer | Amplifier | Data collector |
| Type | EA-YD-103 | DH5862 | ECON-premax-1000 |
| Parameter | Sensitivity: 20 (mV/pC) | Input resistance: >1011 (Ω) | Input channel: 24 |
| | Frequency response range: 0.5-12 (kHz) | Maximum input charge: 106 (pC) | Output channel: 2-16 |
| | | Dynamic range: 110 (dB) | DSP:32 (bit) |

Figure 8: The peak-picking method for identifying the nature frequency for the foundation of stone cutting machine: (a) the noiseless input on point 1; (b) 60% white noise excitation input on point 1.
Table 3: The results for system of the identified frequencies and the theoretical frequencies.

<table>
<thead>
<tr>
<th>Input noise (%)</th>
<th>Modal</th>
<th>Identified frequency $f_i$ (Hz)</th>
<th>Theory frequency $f'_i$ (Hz)</th>
<th>Error (%)</th>
<th>Identified damping ratio $\xi_i$</th>
<th>Empirical damping ratio $\xi'_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>46.511</td>
<td>47.591</td>
<td>2.269</td>
<td>0.012</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>99.975</td>
<td>105.013</td>
<td>4.797</td>
<td>0.039</td>
<td>0.050</td>
</tr>
<tr>
<td>60</td>
<td>1</td>
<td>49.032</td>
<td>47.591</td>
<td>3.027</td>
<td>0.008</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
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<td>108.281</td>
<td>105.013</td>
<td>3.112</td>
<td>0.035</td>
<td>0.055</td>
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</tbody>
</table>

For a simple limit state of a given threshold level $a$ of the foundation displacement is exceeded, the failure probability and its corresponding dynamic reliability index can be steady evaluated as follows:

$$P_f = 1 - \exp\left\{-\frac{1}{2\pi} \frac{\sigma_{y_i}}{\sigma_y} \exp\left[\frac{1}{2}\left(\frac{a}{\sigma_y}\right)^2\right]\right\},$$

$$\beta = \Phi^{-1}(1 - P_f).$$

In practical normal operation process of stone cutting machine, with considered limited threshold for foundation displacement $a$ varying from 0.010 mm to 0.050 mm, the evaluation result for both noiseless and 60 percent white noise excitation are provided in Figure 9, respectively.

From Figure 9, several conclusions are able to be drawn as follows: (I) 60% white noise excitation for the system, relative to noiseless input, is little influence on evaluating dynamic reliability of the foundation of stone cutting machine; that is to say, the system response, in ambient excitation, is mainly influenced by the machine excitation; (II) the larger the limited threshold of foundation displacement, the safer the system structure; (III) in the present case, the system structure will be ensured safety.
while its displacement does not exceed the limit threshold $a = 0.02 \text{mm}$, in which this limit threshold is corresponding to objective reliability index $\beta_T = 3.2$ in Chinese-relevant specification [44].

6. Conclusions

In this paper, a dynamic reliability evaluation method for the foundation of stone cutting machine under ambient excitation was proposed; subsequently, a practical engineering example by presented method is analyzed, and its relevant conclusions can be drawn as follows:

(1) It is reveals that the proposed approach in this paper can stably and effectively identify the real dynamic parameters of the foundation of stone cutting machine.

(2) Sixty percent of the white noise excitation for the system, relative to noiseless input, is little influence on evaluating dynamic reliability of the foundation of stone cutting machine; that is to say, the system response, in ambient excitation, is mainly influenced by the machine excitation.

(3) The larger the limited threshold of foundation displacement, the safer the system structure.

(4) The structure will be ensured safety while its displacement does not exceed the limit threshold $a = 0.02 \text{mm}$, and this conclusion is consistent with the relevant specification in China.

(5) The proposed method is providing an effective path for identifying the dynamic parameter and evaluating the dynamic reliability of foundation of stone cutting machine under ambient excitation.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors’ Contributions

Jiangjun Liu and Qiang Fu conceived, designed, and performed the study. Xiao Li and Xueji Cai collected and statistic sample dates of engineering example used in the paper, Zilong Meng, Jiyi Mi, and Jiarui Shi wrote and revised the paper together. The authors have read and approved the final published manuscript.

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