Laboratory-Scale Insight into Fractal Dimension Indicators for Seismic Hazard Assessment Associated with Rock Failures

Wu Cai

State Key Laboratory of Coal Resources and Safe Mining, China University of Mining and Technology, Xuzhou, Jiangsu 221116, China

Correspondence should be addressed to Wu Cai; caiwu@cumt.edu.cn

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Investigation of acoustic emission (AE) characteristics during laboratory tests of coal samples can provide useful guidance on the recognition of microseismicity (MS) precursors for seismic hazards in underground coal mining. In this study, a methodology, involving fractal dimension indices and a fuzzy comprehensive evaluation model, was developed and demonstrated using AE monitoring data in a uniaxial loading test of coal samples, which allows for a better and quantitative recognition of AE/MS precursors to seismic hazards associated with rock failures. In this methodology, the fractal dimension indices include six information dimension indices as well as three previously used capacity dimension indices. The assessment results were initially characterised as probabilities belonging to each of four risk levels (none, weak, moderate, and strong), and then output as a comprehensive result corresponding to one of these four risk levels. The results indicated that this developed methodology was able to recognise the AE precursors for rock failures.

1. Introduction

It has been widely accepted that microseismicity (MS) in the field is a large-scale manifestation of acoustic emission (AE) phenomenon in the laboratory [1–7]. Therefore, laboratory-scale AE experiments of coal samples being loaded to failure can be confidently utilised to demonstrate seismic hazard assessment associated with rock failures, which is hopefully to be applied in the field-scale MS monitoring.

Since first introduced by Mandelbrot [8] to describe geometric patterns in nature, fractals or fractal dimensions have been widely applied in crustal seismology and earthquake engineering (e.g., [9] and references therein), MS in underground mining and tunnelling [10–13], and AE in the laboratory [14–17], towards quantification of their clustering properties. However, most of the current studies tended to focus on only one of the attributes (i.e., time, space, or energy) of AE/MS information based on fractal capacity dimension. In addition, quantification of AE/MS clustering of attributes was largely limited to the level of qualitative trend analysis with respect to giving an indication of such “abnormal values” towards seismic hazards associated with rock failures. In this study, a comprehensive methodology was developed, which includes all three attributes and uses fractal information as well as capacity dimensions to quantify the clustering properties. The use of fractal information dimensions allows the integration of two or more attributes for an improved quantification of the AE/MS clustering properties. Furthermore, the methodology incorporates a fuzzy comprehensive evaluation (FCE) model for quantifying the likelihood of the occurrence of seismic hazards associated with rock failures. The developed methodology has been applied to analyse AE data during uniaxial tests of coal samples to failure, and the findings thus obtained could be applied to in situ MS monitoring for seismic hazards, such as rock/coal burst and coal and gas outburst.

2. Methodology

The developed methodology involves two consecutive stages. (1) Fractal information as well as capacity dimension indices are used to characterise AE/MS clustering properties, which includes three capacity dimension indices and six
2.1. Fractal Dimension Indices. Depending on the characteristics of AE/MS data, there are three kinds of calculation scales for the fractal capacity dimension (\(D_0\)) to be defined in space (\(D_{30}\)), time (\(D_{70}\)), and energy (\(D_{80}\)). The three scale values of \(D_0\) can be obtained by performing a regression analysis of the plot \(\log N(\varepsilon)\) vs \(\log(\varepsilon)\) as follows:

\[
D_0 = -\lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log(\varepsilon)},
\]

where \(N(\varepsilon)\) is the smallest number of hypothetical boxes which can cover all AE events, as diagonal hatching boxes shown in Figure 1. In this study, the box-counting method was adopted to estimate \(N(\varepsilon)\) corresponding to the three scales. The study areas are first discretised into small 2D boxes of \(\varepsilon\) in space length (Figure 1(a)), 1D line segments of \(\varepsilon\) in time axis length (Figure 1(b)), and 1D line segments of \(\varepsilon\) in energy axis length (Figure 1(c)), respectively. Then, a group of dataset (\(\varepsilon, N(\varepsilon)\)) is generated via changing the length \(\varepsilon\).

Capacity dimension \(D_0\) mentioned above does not consider the specific information about the number of AE/MS events or the amount of AE/MS energy in each box. Therefore, the capacity dimension \(D_0\) might not be the best way to fully extract information of AE/MS in a statistical sense. To address this issue, further statistical extraction should be considered with the assistance of the information dimension \(D_i\), which is based on the normalised probabilities \(P_i(\varepsilon)\) for measure in the \(i\)th box and defined as

\[
P_i(\varepsilon) = \frac{m_i}{\sum_{i=1}^{N(\varepsilon)} m_i},
\]

\[
I_1(\varepsilon) = \sum_{i=1}^{N(\varepsilon)} [P_i(\varepsilon) \cdot \log[P_i(\varepsilon)]],
\]

\[
D_i = -\lim_{\varepsilon \to 0} \frac{I_1(\varepsilon)}{\log(\varepsilon)} = -\lim_{\varepsilon \to 0} \frac{\sum_{i=1}^{N(\varepsilon)} [P_i(\varepsilon) \cdot \log[P_i(\varepsilon)]]}{\log(\varepsilon)}
\]

where \(I_1(\varepsilon)\) is the Shannon entropy, \(m_i\) is the number of AE/MS events or the amount of AE/MS energy in the \(i\)th box, and \(\sum_{i=1}^{N(\varepsilon)} m_i\) is the total number of AE/MS events or the total amount of AE/MS energy. Accordingly, these information dimensions are named as \(D_{1-N}\) and \(D_{1-E}\) respectively. Specifically, the information dimensions, in terms of different time-space-energy scales with box-counting measures, are further divided into \(D_{31-N}\) and \(D_{31-E}\) in the space scale, \(D_{71-N}\) and \(D_{71-E}\) in the time scale, and \(D_{81-N}\) and \(D_{81-E}\) in the energy scale.

In this study, capacity and information dimension indices were both selected as inputs, although the information dimensions would be more effective than mere extraction of capacity dimension for the quantitative recognition of AE/MS precursors. They include three capacity dimension indices (\(D_{30}, D_{70},\) and \(D_{80}\)) and six information dimension indices (\(D_{31-N}, D_{31-E}, D_{71-N}, D_{71-E}, D_{81-N},\) and \(D_{81-E}\)).

2.2. Comprehensive Assessment Model. Based on the FCE model developed in the previous study [18, 19], a comprehensive assessment model can be developed for the quantitative recognition of AE/MS precursors to seismic hazards associated with rock failures, using the fractal dimension indices as inputs. It includes three key sections to determine: the membership function of each fractal dimension index, the weight for each index, and the probabilistic and comprehensive assessments. More specifically, there are six steps as shown in Figure 2 and summarised below:

1. Build factor set \(W = \{D_{70}, D_{71-N}, D_{71-E}, D_{80}, D_{31-N}, D_{31-E}, D_{81-N}, D_{81-E}\}\).

2. Build alternative set \(V = \{v_1, v_2, v_3, v_4\} = \{\text{none, weak, moderate, strong}\}\).

3. Build single-factor matrix \(R\) (see equation (3), \(q = 9\)) using Gaussian membership function. The subset \(r_{ij}\) refers to the membership degree of the \(i\)th factor to the \(j\)th alternative subset. It should be noted that an anomaly index \(A_p\) needs to be defined first to non-dimensionalise the data for the Gaussian membership function, which is based on fractal dimensions for categorising risk levels by referring to the definition of \(b\) value anomaly [20] (see equation (4)):

\[
A_D = \frac{D_b - D}{D_b},
\]

where \(D\) is the fractal index, calculated in a given moving time window. \(D_b\) is the background value of \(D\) determined using the entire catalogue of AE/MS events previously recorded. The four risk levels in the alternative set \(\{v_1, v_2, v_3, v_4\}\) are applied to \(A_D\) ranges (0 to 0.25, 0.25 to 0.50, 0.50 to 0.75, and 0.75 to 1).

4. Determine the weight set \(A = \{a_1, a_2, \ldots, a_6\}\) using the performance metric \(F\) score derived from the confusion matrix [21]. In this paper, equal weights were applied for them just to demonstrate the application.

5. Determine the probabilistic assessment for each risk level \(B = A.R = \{b_1, b_2, b_3, b_4\}\).

6. Determine the comprehensive assessment using the following combination model:
Figure 1: Box-counting method for estimating fractal dimension in the (a) space scale, (b) time scale, and (c) energy scale of AE events. Filled circles indicate AE events and different colours denote different magnitudes.

Figure 2: Framework of the fractal dimension-based methodology for the assessment of seismic hazards associated with rock failure using AE/MS monitoring data.
Figure 3: Continued.
Figure 3: Evolution of AE performance, the associated fractal dimensions, and the probabilistic and comprehensive assessment along with the stress-strain curve when a coal sample is being loaded to failure. (a) AE performances along with stress-strain curve. (b) Fractal capacity dimension indices ($D_{S0}$, $D_{T0}$, and $D_{E0}$) in qualitative trend analysis. (c) Additional six fractal information dimension indices. (d) Probabilistic and comprehensive assessment.
\[ V_{\text{index}} = \begin{cases} \text{MMDP model:} & v_j b_j \rightarrow \max \{ b_j \} V_c \geq 0.5, \\ \text{VFPR model:} & \frac{\sum \{ b_j v_j \}}{\sum b_j} V_c < 0.5, \end{cases} \]  

where MMDP indicates maximum membership degree principle. VFPR indicates variable fuzzy pattern recognition. \( j = 1, 2, 3, 4 \). \( V_1 = 0.125, V_2 = 0.375, V_3 = 0.625, \) and \( V_4 = 0.875 \) in the alternative set were numeralised in accordance with the risk levels \{ none, weak, moderate, strong \}. \( V_c \) is the validity index defined as

\[ V_c = \frac{n_{V'} B_{\text{max}} - 1}{2.B_{\text{second}} (n_{V'} - 1)}, \]  

where \( n_{V'} = 4 \) is the number of alternative subsets, \( B_{\text{max}} = \max \{ b_j \} \), and \( B_{\text{second}} = \max_{j \neq i} \{ b_j \} \).

### 3. Application

Uniaxial loading tests using an MTS apparatus were performed on cylindrical coal samples (50 mm diameter and 100 mm height) collected from an underground coal mine in China, as depicted in the top left corner of Figure 3(a). The loading system is the electro-hydraulic servo rock mechanics testing system (MTS-C64.106). The displacement control was performed in this experiment, and a constant loading rate of 0.18 mm/min was applied until the specimen failed. The PCI-2 AE monitoring system was adopted to record AE signals in this experiment, where a total of eight AE sensors (Nano 30 with a frequency domain of 100 to 400 kHz) were uniformly attached to the upper and lower parts of the sample. The sampling rate of AE was set to 2 MHz to record the released strain energy during the test. These recorded AE signals were first converted to electric signals and then amplified by the preamplifier with a gain of 40 dB. The high-speed camera was used to capture digital images. In the experimental procedure, stress, strain, AE signals, and digital images were automatically collected.

Analysing stress, AE, and strain relationship from an unconfined compression test of a typical coal sample, as displayed in Figure 3(a), five phases are observed: compaction (OA), elastic deformation (AB), microcrack growth (BC), microcrack propagation and macrofracture formation (CD), and postfailure (DE). During the stable phase BC, AE events increase significantly, as well as the associated fractal indices (Figures 3(b) and 3(c)). After approaching the point C, the fractal dimensions first decrease for a while and then increase to a high value again and finally drop dramatically until the failure (point D) is reached. This same evolution can be also obtained from a prevailing indicator \( b \) value (see Figure 3(b)) derived from Gutenberg–Richter frequency-magnitude relationship in seismology [22] which will be further discussed in Section 4.

It can be concluded that the macrofractures are formed by joining of microcracks and then cause the final failure. In

\[ \Delta_D S_0 \quad \bullet \quad b \text{ value} \]

**Figure 4:** Comparisons between fractal dimension and \( b \) value in Gutenberg–Richter relationship.
response to this process from microcrack growth and propagation to macrofracture formation (around C) and finally to the failure occurrence (around D), there will be a precursor within low value anomaly in fractal dimension indices calculated on the basis of AE monitoring data. Therefore, this precursor can be quantified by the anomaly index $A_D$ (see equation (4)), together with a Gaussian membership function, for the failure assessment.

 Applying equal weights for the fractal dimension indices to demonstrate the model development, the comprehensive assessment result can be obtained, as displayed in Figure 3(d). There are two weak risk levels identified around points B and C, respectively, and then a moderate risk level developing as the yield point $D$ is approached, which progresses towards the strong risk level as the coal fails.

4. Discussion and Conclusion

It should be noted that the definition of capacity dimension ($D_{30}$) as illustrated in Figure 1(c) used in this study is very similar to that of $b$ value in Gutenberg–Richter frequency-magnitude relationship in seismology. They are both computed from the magnitude-frequency relationship. It has been widely accepted that $b$ value is strongly correlated to the fractal dimension of earthquake rupture size within $D_{30} = 2b$ [10, 23], which could also be roughly demonstrated in Figure 3(b). It is suggested that each AE/MS event corresponds to a fracture plane, and the size distribution of these fracture planes follows fractal characteristics. Therefore, the nine fractal dimension indices developed in this study do not only address all three attributes of AE/MS information but also reflect the Gutenberg–Richter relationship. Figure 4 shows the results of $D_{30}$ and $b$ values calculated using the same MS dataset from an European coal mine [19], which shows that $D_{30} +1.4470$ is approximated to the $b$ value +1.4360. This is because the analyses of $D_{30}$ in this paper were performed in the two-dimensional plane-scale of mines, and their values are in the range of 0.0–2.0. However, the earthquake rupture, used to correlate Gutenberg–Richter relation and fractal, refers to a fracture plane in the three-dimensional rock mass.

In addition to the fractal capacity dimension indices (Figure 3(b)) widely used in the previous studies, the information dimension indices developed in this study could extract other more distinct characteristics (Figure 3(c)), e.g., $D_{31,E}$, $D_{31,E}$, and $D_{31,E}$. Moreover, the analysis process demonstrated in Figures 3(b) and 3(c) can only focus on the qualitative trend analysis by recognising low value or decrease trend as a precursor for the occurrence of failure. By contrast, the comprehensive assessment model developed in this study could be more progressive, especially in quantitative analysis (Figure 3(d)) via integrating the whole capacity and information dimension indices.

In conclusion, a fractal dimension-based methodology has been developed and successfully demonstrated using AE monitoring data in a uniaxial loading test of coal samples, which can better and quantitatively recognise AE/MS precursors to seismic hazards associated with rock failures. Since microfracturing and associated dynamic instability and failures in rocks have been encountered at different scales, the findings obtained from investigation into AE precursors for rock failures may be applied to in situ MS monitoring for seismic hazards (e.g., rock bursts and coal and gas outbursts).

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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