Research Article

Impact Response Prediction Method of Packaging Systems with a Key Component considering Different Excitations and Cushioning Materials

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Nonlinear dynamic models of the packaging systems that consider different excitations and cushioning materials are proposed to analyze their impact responses in this paper. To solve the nonlinear dynamic equations of the models, Newmark method is combined with Newton-Arithmetic mean method to transfer nonlinear equations to the linear discretization equations. Then, the acceleration responses and dropping damage boundary curves (DDBCs) of different nonlinear cushioning packaging systems are calculated conveniently with initial conditions. The good agreements between the simulated and tested results prove the accuracy and effectiveness of the proposed method. Furthermore, the effects of excitations parameters on the DDBCs of the packaging systems are analyzed. It is found that the key parameter for introducing differences between simulated and measured DDBCs is inappropriate duration configuration in the acceleration pulse test.

1. Introduction

Cushioning materials are usually used to construct cushioning systems for electronic equipment and protect the key equipment component from damage induced by environmental excitations, such as the crash stops of land vehicles [1], the drop impact of landing gear [2], and the free drop of electronic products during transportation [3, 4]. Among these excitations, drop impact is the most common and destructive dynamic load, and is likely to make electronic equipment suffer fatal damage. To achieve the excellent cushioning effect of packaging systems and avoid the failure of the key component in the equipment, it is essential to introduce a sophisticated analytical model to determine the major physical properties and describe the dropping damage state of the systems [4, 5].

The packaging systems probably suffer various types of drop impacts in their use, including triangular pulses, half-sine pulses, rectangular pulses, and velocity impacts. Furthermore, various cushioning materials, such as polymeric foams [6], honeycomb corrugated paperboard [7, 8], and expanded polyethylene [9, 10], are also used to attenuate the energy of impacts in the packaging system. Their mechanical properties are varied and exhibit nonlinear characteristics. Therefore, these various situations could increase the difficulty of constructing an analytical model to understand the impact behavior of different systems and effectively estimate their dropping damage boundaries.

In previous studies, researchers have paid a considerable amount of attention to the prediction of the dynamic responses of packaging systems and the analysis of the dropping damage boundary curves (DDBCs) of key components in the system. For example, analytical models of packaging systems with different cushioning materials have been studied [11–16], the stiffnesses of which are cubic,
tangent, and hyperbolic tangent function. They have been simplified as two-degree nonlinear oscillator models. The corresponding DDBC’s have been calculated to reveal the effects of different cushioning materials on responses of the systems under the velocity impact. Additionally, acceleration pulses have been used as the input excitation to predict or measure the DDBC’s of the cushioned packaging systems. These researches have revealed the fracture mechanics and reliability of the key components in the packaging system [17–21].

However, an analytical model for the response prediction of nonlinear packaging systems, considering with various cushioning materials and different types of excitations together, has rarely been researched. Therefore, a dynamic model of packaging systems with key components under velocity impact and different acceleration pulses, which considers the nonlinear properties of different cushioning materials, is proposed in the present study. The Newmark method [22] is combined with the Newton-Arithmetic mean method [23] to transform the dynamic equation of the packaging system to numerical equations. The nonlinear response of the system can be efficiently obtained by solving these equations. The current method divided the solution into the discretization of the nonlinear dynamic equation and root solution of the discretization equations. The nonlinearity of second, first order differential elements and stiffness function in the equation can be solved by searching root of their discretization style conveniently, comparing with other numerical methods. Based on the universal model, the error between DDBC’s of numerical analysis and test are discussed to reveal its source, in which the velocity condition is used to analyze DDBC’s of the systems and the acceleration pulse is applied to measure the DDBC’s.

The remainder of this paper is organized as follows. In Section 2, the model of the nonlinear packaging system considering different excitations and cushioning material is described. In Section 3, the dynamic equation of the nonlinear packaging system is solved by the Newmark method and Newton-Arithmetic mean method. In Section 4, the accuracy of the proposed model is proven, comparing the DDBC’s of the packaging systems calculated by the current method with those reported in previous publications. In Section 5, the effect of the pulse duration on the DDBC of the packaging system is analyzed. It is used to explain the primary reason for the occurrence of errors between the simulated and measured DDBC’s of the packaging system. Finally, concluding remarks are made in Section 6.

2. Model of the Nonlinear Packaging System with a Key Component

The cushioned packaging system is presented in detail in Figure 1. The entire packaging system can be divided into two subsystems, namely the external cushioning system and the internal response system. The external system is composed of the cushioning layer and the equipment shell, in which the cushioning material is simplified as a nonlinear stiffness and a damping, and the shell is considered as a mass. The internal subsystem is also idealized as an oscillator consisting of a linear stiffness, a damping, and a mass, which are used to describe the dynamic properties of the key components in electronic equipment, such as the IC package and PCB board. Then, the entire packaging system can be simplified as a two-degree nonlinear oscillator. To present a universal description of the relationship between the deformation and restoring force, the stiffness of the external oscillator is defined as a function of stiffness $K(x)$. Considering the velocity impact and different acceleration pulses, two types of nonlinear two-degree oscillators of the packaging system can be proposed, as shown in Figure 2.

The dynamic equations of the nonlinear two-degree oscillators in Figure 2 are given by

$$
\begin{align*}
\dot{u}_1(t) + c_1 \dot{u}_1(t) + k_1 u_1(t) &= \ddot{x}_0(t) - m_1 \ddot{x}_0(t), \\
\dot{u}_2(t) + c_2 \dot{u}_2(t) + k_2 u_2(t) &= -m_2 \ddot{x}_0(t) - m_2 \ddot{x}_0(t),
\end{align*}
$$

where $u_1(t) = (x_2 - x_1)$, $u_2(t) = (x_1 - x_0)$, $m_1$ and $m_2$ are the weights of the equipment shell and the key component, respectively, $K(x)$ and $K_1$ are the stiffnesses of the cushioning material and the key component, respectively, and $c_1$ and $c_2$ are the corresponding damping coefficients. Additionally, $x_0(t)$, $x_1$, and $x_2$ are the respective displacements of the basement, the equipment shell, and the key component. $\ddot{x}_0(t)$ represents the acceleration pulse transferred from the basement. The value of $a(t) = \ddot{x}_0(t)$ is set, and its mathematical expressions are related to the excitation type, as shown in Table 1.

If the nonlinear two-degree oscillator undergoes velocity impact, its dynamic equation can be rewritten as (2) when $\ddot{x}_0(t)$ and $x_0$ are equal to 0.

$$
\begin{align*}
\dot{x}_2(t) + c_2 \dot{x}_2(t) + k_2 x_2 &= c_2 \dot{x}_1(t) + k_2 x_1, \\
\dot{x}_1(t) + (c_1 + c_2) \dot{x}_1(t) + K(x_1) &= c_2 \dot{x}_2(t) + k_2 x_2.
\end{align*}
$$

The velocity impact is used as an initial condition for the dynamic equation of the packaging system, which is written as

$$
\begin{align*}
x_1(0) &= 0, \quad \dot{x}_1(0) = v_0 = \sqrt{2gh}, \\
x_2(0) &= 0, \quad \dot{x}_2(0) = v_0 = \sqrt{2gh},
\end{align*}
$$

where $g$ is gravitational acceleration, $h$ represents the dropping height, and $v_0$ is the initial velocity when the
packaging system falls to the ground. According to the deformation behaviors of different cushioning materials, their stiffness properties in equations (1) and (2) are represented by linear, cubic, tangent, and hyperbolic tangent functions, as described in Table 2.

3. Solution Method

Due to the introduction of the nonlinear term from the function \( K(x) \) in equations (1) and (2), the solutions of these dynamic equations are complex and lengthy. In contrast, the Newmark method is characterized by high accuracy and fast convergence, and is suitable for the solution of the nonlinear equations of the proposed model. Thus, the relationships between acceleration, velocity, and displacement represented by the Newmark method [22] are used as a baseline tool for solving the problem, and are expressed as

\[
\begin{align*}
\dot{x}(t + \Delta t) &= -\frac{\beta}{\lambda \Delta t} x(t) - \left(\frac{\beta}{\lambda} - 1\right) \dot{x}(t) - \left(\frac{\beta}{\lambda^2} - 1\right) \Delta t \ddot{x}(t) + \frac{\beta}{\lambda \Delta t} x(t + \Delta t), \\
\ddot{x}(t + \Delta t) &= -\frac{\beta}{\lambda \Delta t^2} x(t) - \frac{1}{\lambda \Delta t} \ddot{x}(t) - \left(\frac{1}{\lambda^2} - 1\right) \dddot{x}(t) + \frac{1}{\lambda \Delta t^2} x(t + \Delta t).
\end{align*}
\]

Substitution of equation (4) into equation (2) yields

\[
\begin{align*}
B_2 x_2 (t + \Delta t) - B_4 x_1 (t + \Delta t) &= B_3 - B_1, \\
A_2 x_1 (t + \Delta t) - A_4 x_2 (t + \Delta t) + K(x_1 (t + \Delta t)) &= A_3 - A_1.
\end{align*}
\]
where $A_1 = -[m_1/(\lambda \Delta t^2 + \beta (c_1 + c_2)/\lambda \Delta t)] \dot{x}_1(t) - [m_1/(\lambda \Delta t^2 + (c_1 + c_2)(\beta/\lambda - 1)/\lambda \Delta t)] \ddot{x}_1(t)$, $A_2 = m_1/(\lambda \Delta t^2 + (c_1 + c_2)(\beta/\lambda - 1)/\lambda \Delta t) + k_2$, $A_3 = -\beta c_2/\lambda \Delta t \dot{x}_1(t) - c_2(\beta/\lambda - 1)\ddot{x}_1(t)$, $A_4 = \beta c_2/\lambda \Delta t + k_2$, $B_1 = -[m_2/(\lambda \Delta t^2 + c_2\beta/\lambda \Delta t)] \dot{x}_2(t)$ - $[m_2/(\lambda \Delta t^2 + c_2\beta/\lambda \Delta t)] \ddot{x}_2(t)$ - $[m_2/(\lambda \Delta t^2 + c_2\beta/\lambda \Delta t)] \dddot{x}_2(t)$ - $[m_2/(\lambda \Delta t^2 + c_2\beta/\lambda \Delta t)] \ddot{x}_2(t)$ - $[m_2/(\lambda \Delta t^2 + c_2\beta/\lambda \Delta t)] \ddot{x}_2(t)$ - $[m_2/(\lambda \Delta t^2 + c_2\beta/\lambda \Delta t)] \dddot{x}_2(t)$ - $[m_2/(\lambda \Delta t^2 + c_2\beta/\lambda \Delta t)] \dddot{x}_2(t)$ - $[m_2/(\lambda \Delta t^2 + c_2\beta/\lambda \Delta t)] \dddot{x}_2(t)$ - $[m_2/(\lambda \Delta t^2 + c_2\beta/\lambda \Delta t)] \dddot{x}_2(t)$.

If the response of the system at a previous time has already been determined, the response at the present moment can be calculated by solving the following equation.

$$\begin{aligned}
\begin{cases}
\dot{x}_2(t + \Delta t) = \frac{(B_3 - B_1)}{B_2} + \frac{B_1}{B_2} \dot{x}_1(t + \Delta t), \\
\left(A_2 - \frac{B_4}{B_2}\right) x_1(t + \Delta t) + K(x_1(t + \Delta t)) - \left[A_3 - A_1 + \frac{A_4(B_3 - B_1)}{B_2}\right] = 0.
\end{cases}
\end{aligned}$$

Equation (6) is a nonlinear equation according to the existence of the nonlinear function $K(x)$, and it cannot be solved by the linear inversion method. Hence, the Newton-Arithmetic mean method [23] is selected to search for the approximate root of the second term in equation (6) by achieving convergence with the initial value after iteration. The corresponding iteration step is given by

$$\begin{aligned}
\tau_{k+1} = \tau_k - \frac{2f(\tau_k)}{f'(\tau_k) + f''(\tau_k^*)}, \\
\tau_{k+1}^* = \tau_k - \frac{f(\tau_k)}{f'(\tau_k)},
\end{aligned}$$

where $f(\tau) = (A_3 - B_4/B_2)\tau + K(\tau) - [A_3 - A_1 + A_4(B_3 - B_1)/B_2]$, $\tau = x(t + \Delta t)$, and $f'(\tau) = (A_3 - (B_4/B_2)) + K'(\tau)$.

In the iterative process, the displacement at the previous time $t$ is known, and is set as the initial value to obtain the convergence result conveniently and rapidly. After the displacement of the internal oscillator at time $t + \Delta t$ is calculated, the displacement of the external oscillator at that time is calculated by equation (6). Then, the corresponding acceleration and velocity of the nonlinear model are calculated by substituting the displacement into equation (4).

When the initial condition at the starting time is determined, the responses of the two oscillators later can be calculated by this method.

The dynamic equation of a nonlinear cushioned packaging system under an acceleration pulse can be rewritten as equation (8) by the Newmark method, liking to solving of those with the initial velocity condition. The roots of the nonlinear equations can also be searched by the Newton-Arithmetic mean method via the previous known response. The displacement response at time $t + \Delta t$ can be obtained accordingly by the iterative process. Then, the displacement responses of the two subsystems are substituted into equation (4). The corresponding velocity and acceleration responses are calculated. Finally, the time-dependent response of the system can be predicted by continuous iteration when the initial condition is defined.

$$\begin{aligned}
\begin{cases}
B_1 + B_2 u_2(t + \Delta t) = B_3 + B_4 u_1(t + \Delta t), \\
A_1 + A_2 u_2(t + \Delta t) + K(u_1(t + \Delta t)) = A_3 + A_4 u_2(t + \Delta t),
\end{cases}
\end{aligned}$$

where $B_1 = m_2[-1/(\lambda \Delta t^2)u_4(t) - 1/\lambda \Delta t u_5(t) - (1/2\lambda - 1)\ddot{u}_2(t)] + c_2[-\beta/\lambda u_2(t) - (\beta/\lambda - 1)\dot{u}_2(t) - (\beta/2\lambda - 1)\Delta \dot{u}_2(t)]$, $B_2 = k_2 + m_2/\lambda \Delta t^2 + c_2\beta/\lambda \Delta t$, $B_3 = m_2[1/(\lambda \Delta t^2)u_4(t) + 1/\lambda \Delta t u_5(t) - (1/2\lambda - 1)u_2(t)]$.
\[
\lambda \Delta t \ddot{u}_i (t) + (1/2\lambda - 1) \dot{u}_i (t) - a(t) \cdot B_4 = -m_3/\lambda \Delta t^2, \quad A_1 = m_1 [-1/\lambda \Delta t^2 \dot{u}_i (t) - 1/\lambda \Delta t \dot{u}_i (t) - (1/2\lambda - 1) \ddot{u}_i (t)] + c_1 [-\beta/\lambda \Delta t \dot{u}_i (t) - (\beta/2\lambda - 1) \Delta t \ddot{u}_i (t)], \quad A_2 = m_1/\lambda \Delta t^2 + c_1 [\beta/\lambda \Delta t], \quad A_3 = c_2 [-\beta/\lambda \Delta t \dot{u}_i (t) - (\beta/\lambda - 1) \ddot{u}_i (t) - (\beta/2\lambda - 1) \Delta t \ddot{u}_i (t)] - m_1 a(t), \quad A_4 = k_2 + c_2/\lambda \Delta t.
\]

Via the described modeling and solution process, the response of the cushioned packaging system can be predicted while considering various cushioning materials and excitations. It can be used to calculate the DDBC of the packaging system and reveal the differences between the responses related to the velocity impact and various acceleration pulses.

### 4. Method Validation

To validate the accuracy of the proposed method, different types of cushioning materials are selected. They are named as linear, cubic, tangent, and hyperbolic tangent stiffnesses, respectively. Their displacement and acceleration responses are calculated accordingly. Because the responses of packaging systems using different cushioning materials should be normalized before the comparison [14, 15, 24], the responses calculated in the present study are normalized by the parameters and expressions introduced in previous publications [14, 15, 24]. The normalized formulas are provided in Table 3.

To conveniently study the effects of various parameters on the system responses, the mass ratio, frequency ratio, and damping ratio between the internal and external oscillators are introduced, and defined as

\[
\begin{align*}
\omega &= \frac{\omega_2}{\omega_1}, \\
\xi &= \frac{\xi_2}{\xi_1},
\end{align*}
\]

where \(\omega_1 = \sqrt{k_0/m_1}\), \(\omega_2 = \sqrt{k_2/m_2}\), \(\eta_1 = \xi_1/2\sqrt{k_0/m_1}\), and \(\eta_2 = \xi_2/2\sqrt{k_2/m_2}\).

The packaging system with cubic stiffness is selected as the first simulation example. Its linear stiffness \(k_0\) is set as 7.5 \times 10^8 \text{ N/m}, and its cubic stiffness is considered as \(-1.58 \times 10^8 \text{ N/m}\). The weight of the external oscillator \(m_1\) and damping parameter \(\xi_1\) are set as 1 kg and 0.1, respectively. Additionally, the mass ratio, frequency ratio, and damping ratio are respectively configured as 0.05, 5, and 1. The time-dependent responses of the key component in the internal oscillator induced by a 0.3-normalized velocity are calculated and normalized for the comparison by the equations provided in Table 3. The simulation results are then compared with the responses obtained by the Runge-Kutta method in a previous study [24], as presented in Figure 3. It is evident that the time-dependent displacement and acceleration responses of the packaging system with cubic stiffness, calculated by the current method and the Runge–Kutta method, are very similar. This verifies the accuracy of the proposed method for the prediction of the nonlinear response of the packaging system.

Based on the accuracy responses calculated by the proposed method, the DDBCs of the packaging system using different cushioning materials can be conveniently obtained. First, a cushioning material with linear stiffness is assumed to be used for the external subsystem. The linear stiffness \(k_0\) is considered as \(6.1 \times 10^8 \text{ N/m}\), the weight of the external oscillator \(m_1\) is assumed as 3.19 kg, and the corresponding damping parameter \(\xi_1\) is set as 0.01. The mass ratio and damping ratio are configured as 0.2 and 2, respectively. The DDBC of the key component of the packaging system can be determined by investigating the acceleration response at different frequency ratios and input velocities, which are set as equal to 2 \(g\) and 5 \(g\). These simulated DDBCs are compared with experimental results reported in a previous study [14], as presented in Figure 4. It is evident that the DDBC calculated by the proposed method is almost the same as that obtained from a previous experiment [14] when the cushioned packaging system used a cushioning material with linear stiffness.

It is then assumed that the cushioning material of the external subsystem has cubic stiffness. Its linear stiffness \(k_0\) is considered as \(4.03 \times 10^8 \text{ N/m}\) and its cubic stiffness \(d_3\) is considered as \(2.74 \times 10^8 \text{ N/m}\). Additionally, the mass \(m_1\) and damping parameters \(\xi_1\) of the external oscillator are respectively set as 3.01 kg and 0.18. The mass and damping ratios of the system are configured as 0.2 and 0.055, respectively. Figure 5 presents the DDBCs of the key component in the packaging system with cubic stiffness calculated by the proposed method at 2 \(g\) and 5 \(g\), which are compared with experimental results reported in a previous study [15].

The cushioning material of the external subsystem is then considered as having a tangent stiffness characteristic. The corresponding stiffness is considered as \(3.1 \times 10^8 \text{ N/m}\) and the limited displacement \(d\) is considered as 10.4 mm. The weight of the external system \(m_1\) is the same as that of the cubic stiffness system, and the relevant damping parameter \(\xi_1\) is assumed to be 0.15. The mass and damping ratios of the entire system are respectively set as 0.2 and 0.67. The DDBCs of the key component in the tangent stiffness packaging system at 2 \(g\) and 5 \(g\) are calculated by the proposed method, and are compared with experimental results reported in a previous study [14] in Figure 6.

Finally, the external system is considered as using cushioning material with hyperbolic tangent stiffness. The stiffness parameter \(k_0\) and \(F_0\) are respectively set as \(4.03 \times 10^8 \text{ N/m}\) and 150. The external subsystem mass \(m_1\) is kept unchanged, and the damping parameter of the subsystem is set as 0.2. The mass and damping ratios of the entire system are configured as 0.2 and 0.5, respectively. As presented in Figure 7, the DDBCs of the key component in the packaging system with hyperbolic tangent stiffness at 2 \(g\), 5 \(g\), and 10 \(g\) are calculated by the proposed method. They are compared with simulation results obtained using the Runge-Kutta method reported in a previous study [15].

From the comparisons of the DDBCs obtained by the proposed method and those reported in previous studies, it is evident that the DDBCs of different nonlinear packaging systems determined by the proposed method largely agree with previously reported experimental and simulation results. A partial difference existed in the comparison of the...
DDBCsofthe packagingsystems with tangent and cubic stiffnesses at 5 g. Thus, it is proven that the proposed method is suitable for calculating the response and predicting the DDBC of nonlinear packaging systems, when the system considers different excitations and cushioning materials.

<table>
<thead>
<tr>
<th>Stiffness type</th>
<th>Normalized time</th>
<th>Normalized velocity</th>
<th>Normalized acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>( t \sqrt{\frac{k_0}{m_1}} )</td>
<td>( v_0 \left( \frac{1}{g} \right) \sqrt{\frac{k_0}{m_1}} )</td>
<td>( \frac{a_i}{g} )</td>
</tr>
<tr>
<td>Cubic</td>
<td>( t \sqrt{\frac{k_0}{m_1}} )</td>
<td>( v_0 \left( \sqrt{\frac{m_1d_3^2}{k_0}} \right) )</td>
<td>( \frac{a_i m_i \sqrt{d_3/k_0}}{k_0} )</td>
</tr>
<tr>
<td>Tangent</td>
<td>( t \sqrt{\frac{k_0}{m_1}} )</td>
<td>( v_0 \left( \sqrt{\frac{m_1d_3^2}{2k_0}d_5} \right) )</td>
<td>( \frac{a_i m_i}{2d_5k_0} )</td>
</tr>
<tr>
<td>Hyperbolic tangent</td>
<td>( t \sqrt{\frac{k_0}{m_1}} )</td>
<td>( v_0 \left( \sqrt{\frac{m_1k_0}{F_0}} \right) )</td>
<td>( \frac{a_i m_i}{F_0} )</td>
</tr>
</tbody>
</table>

Figure 3: Responses of a packaging system with cubic stiffness predicted by the proposed method and the Runge–Kutta method [24]. (a) Displacement. (b) Acceleration.

**Table 3: Normalized formulas of different cushioned packaging systems.**

5. Discussion

In the validation of the current method, it is revealed that differences exist between calculated results and measured results. The errors are perhaps due to different excitation in the numerical analysis and tests. In previous experiment [14, 15]. A vibration test machine is to produce an acceleration pulse to simulate the velocity change caused by free drop, using the same equivalent velocity. While, the acceleration pulse is distinguished with the velocity damage due to the selection of different excitation parameters. For instance, an acceleration pulse is composed of amplitude and duration, while the velocity impact is only related to amplitude. Therefore, the difference of excitation in analysis and measurement could introduce the errors between them.

Furthermore, the ability of the vibration test machine has limits, such as its output power and action time, which restricts the amplitude and duration of the expected acceleration pulse. This introduces the problem of the perspective parameter of the acceleration pulse probably be failed to achieve, which could increase the difference between the simulated and measured DDBC results. Therefore, it is essential to study the effects of the parameters of the acceleration pulse on the errors between the simulated and measured results, considering different cushioning materials.

Therefore, the influences of the parameters of acceleration pulse on the response of the packaging system are
analyzed firstly in this section. It could reveal the key parameter that makes a difference in the responses related to the acceleration pulse and velocity impulse. Then, the effect of the key parameter of the acceleration pulse on the DDBC of the packaging systems is analyzed. It is revealed that the use of acceleration pulse and velocity impulse in measurement and calculation respectively could introduce errors between them.

5.1. Influences of Different Excitations on the Response of the Packaging System. The effects of the excitation properties on the responses of the nonlinear cushioned packaging systems are investigated firstly, and the packaging system with cubic stiffness is selected as an example. The linear stiffness of the cushioning material with cubic stiffness is considered as $4.03 \times 10^4$ N/m, and the corresponding cubic stiffness $d_3$ is set as $2.74 \times 10^6$ N/m. The mass weight $m_1$ and damping parameter $\eta_1$ of the external oscillator are considered as 3.01 kg and 0.2, respectively. Additionally, the mass ratio, frequency ratio, and damping ratio of the system are assumed to be 0.1, 2, and 0.5, respectively. To maintain the inputs of different excitations at the same level, three pulses with different durations are configured to have the same equivalent velocity by using the transformation relationship described in Table 4. The equivalent velocities of the acceleration pulses are also equal to the amplitude of the velocity impact, which is set as 0.3 m/s. Then, the displacement and acceleration responses of the key component in the packaging system, which is excited by half-sine, triangular, and rectangular pulses at durations of 2 and 20 ms, are calculated and drawn with the responses induced by the velocity impact, as presented in Figures 8 and 9. When the duration of the different pulse types is 2 ms, the displacements and accelerations of the packaging system with cubic stiffness are the same. When the pulse duration increased to 20 ms, there are obvious differences in the responses related to different types of excitations. In summary, different acceleration pulses and velocity impacts could be equivalent to each other in the case of a pulse duration of 2 ms, but this relationship may be invalid with the increase of the duration.

5.2. Effects of Different Excitations on the DDBC of the Packaging System. Because the duration of different types of excitations can affect the response of the packaging system at the same equivalent velocity, the differences between the simulated and measured DDBCs introduced by the impact duration are analyzed by the proposed method. In previous research [14, 15], a half-sine acceleration pulse was applied...
in experiments to validate the simulated DDBCsof different packaging systems. However, while the equivalent velocities of the acceleration pulse for different cases were clearly reported, the amplitude and duration configurations in the experiment were not explained. Therefore, in this study, the DDBCsof different packaging systems are calculated by considering half-sine pulses of various duration and are presented Figure 10 in Figure 11 detail in Figure 12. A zone, the upper and lower boundaries of which in the DDBC are respectively related to long and short durations, can be found and used to include all measured results of the DDBC. It is revealed that the phenomenon of the experimental results exceeding the simulated curves may be induced by using an unexpected duration in the acceleration pulse test.

<table>
<thead>
<tr>
<th>Excitation type</th>
<th>Rectangular</th>
<th>Half-sine</th>
<th>Triangular</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent velocity</td>
<td>$a_0 \tau$</td>
<td>$2a_0 \tau/\pi$</td>
<td>$a_0 \tau/2$</td>
</tr>
</tbody>
</table>

Figure 8: Response of a packaging system with cubic stiffness under different excitations at a duration of 2 ms. (a) Displacement. (b) Acceleration.

Figure 9: Response of a packaging system with cubic stiffness under different excitations at a duration of 20 ms. (a) Displacement. (b) Acceleration.
Figure 10: DDBC's of a packaging system with cubic stiffness under half-sine pulses for different durations. (a) 2 g. (b) 5 g.

Figure 11: DDBC's of a packaging system with tangent stiffness under half-sine pulses for different durations. (a) 2 g. (b) 5 g.

Figure 12: DDBC's of a packaging system with linear stiffness under half-sine pulses for different durations. (a) 2 g. (b) 5 g.
Therefore, a portion of the errors between the simulation and test results originates from the different properties of the input pulses and inappropriate duration configurations in the experiment.

It is further demonstrated that, when the system is considered to use a cushioning material with nonlinear stiffness, the variance between the simulated and measured results become higher, and the related duration used as the upper boundary of the zone become longer. This is likely because the vibrational energies of systems with nonlinear cushioning materials, which are used to drive them to a certain amplitude, are larger than that of a system with a linear cushioning material. Therefore, the nonlinear system requires an acceleration with a greater amplitude as the input excitation. Considering the limitations of the acceleration amplitude in vibrational equipment, a longer duration is required to meet the need of the equivalent velocity. Accordingly, the response of the nonlinear system will be lower than the expected value determined in the simulation. Therefore, the experimental DDBCs of the packaging systems containing nonlinear cushioning materials are higher than the calculated results of the systems.

6. Conclusion

In this paper, a universal model is proposed for the prediction of the response of nonlinear packaging systems with a key component, in which various cushioning materials and different excitations are considered simultaneously. To solve the nonlinear equations conveniently and effectively, the Newmark method is applied to transform the differential equations to nonlinear expressions, which are then solved by the Newton-arithmetic mean method to determine the response of the system conveniently.

To validate the proposed method, the time-dependent response of the key component in a packaging system with cubic stiffness is predicted. They exhibit a good agreement with simulation results calculated by the Runge-Kutta method reported in a previous study. Then, the DDBCs of the key component in respective packaging systems with different cushioning materials, presenting linear, cubic, tangent, and hyperbolic tangent stiffnesses, at 2 g and 5 g are calculated. The good agreement between results calculated by the current model and results reported in previous research proves the accuracy of the proposed method.

Based on the proposed model, the responses and the DDBCs of the key component, induced by an acceleration pulse with different durations and the velocity impact, are calculated and compared with each other. Some important conclusions are presented as follows:

1. The current method is proved to be suitable for analyzing response and calculating DDBC of the key component of the packaging systems, considering various cushioning materials and excitations.

2. Responses of key component in the packaging system produced by velocity impact and acceleration pulse are extremely close to each other, when their equivalent velocity is same and pulse duration is short. While, responses of the packaging system related to velocity impact and acceleration pulse are obviously different with each other when pulse duration grows up.

3. A portion of the errors between the simulation and test results originates from the different properties of the input pulses and inappropriate duration configurations in the experiment. The errors originate from the duration selection in the nonlinear packaging systems are large than that in the linear packaging system.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References


