

Research Article

Examination of Precast Concrete Movement Subjected to Vibration Employing Mass-Spring Model with Two Convective Masses

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This article investigates the movement of concrete subjected to vibration employing mass-spring model. For this purpose, two different prefabricated concrete-formworks, on which experimental work was done before, were analysed analytically. The theoretical modeling of precast formworks employed in experiments has been made by the three-dimensional finite element method employing the SAP2000 program. Modeling of mortar is performed to resolve the interaction between the fresh concrete and formwork using the mass-spring model. Considering the dynamic behavior of the fluid, it is possible to define multiple oscillation (convective) masses with different frequency values in addition to the impulse mass. Thus, one impulse mass and two convective masses were used in the mass-spring model. This study is essentially theoretical, and its accuracy has been strengthened by experimental work. The time-dependent results of concrete movement obtained from the dynamic mass-spring model were compared with the measured ones. The matches indicate that the findings are consistent.

1. Introduction

External vibrators (clamp-on) have been widely employed for the compaction of concrete in the production of prefabricated concrete members. The reason for compaction is to cast off the air void, which is trapped in unfastened concrete. Two types of vibrators are common in construction sites: internal vibrators and surface vibrators. Internal vibrator is the most effective in compacting fresh concrete. The reason for this is that the vibrator directly contacts it. The third type is clamp-on vibrators. Clamp-on vibrators contain an electrically or pneumatically worked motor with an eccentric piece. They operate by vibrating the pattern to which they are attached. The resulting vibrations are transferred to the concrete. This vibrator is commonly used in the production of prefabricated concrete, but it is also occasionally employed on the construction site where there is frequent reinforcement.

There are theoretical and/or experimental works in the limited number within the literature to identify the motion

of mortar subjected to vibration. In many works, it is expressed that mortar is a non-Newtonian fluid and applies the unvibrated Bingham model. Tattersall and Baker [1] described the flow motion of unvibrated fresh concrete with the Bingham model as follows:

$$\tau = \tau_o + \mu\dot{\gamma}, \quad (1)$$

where τ is the shear stress, τ_o is the yield stress, μ is the plastic viscosity, and $\dot{\gamma}$ is the shear rate. It is stated in [1] that yield stress loses its value via measurements, consequently; fresh concrete gained Newtonian fluid (0 yield stress) properties, and its plastic viscosity reduced.

Transportation Department of United States [2] defined the “Poisson rate” of mortar with an equation employing a program named HIPERPAV. Poisson’s rate is obtained to be between 0.40 and 0.42 in the plastic state. Poisson’s ratio as a function of time is defined using the following equation:

$$v(t) = -0.05 \ln(t + 1.11) + 0.425 \leq 0.42, \quad (2)$$

where t is the time in hours after the preparation of the fresh concrete.

The following articles can be given as examples of some recent studies on the properties and behavior of fresh concrete. Proske et al. [3] studied the shear behavior of fresh concrete. They investigated the shear behavior of concrete subjected to pressure by employing a special test setup employing a cylindrical pressure cell. Fresh concrete is cut, which is subjected to several pressure levels with an adapted concrete rheometer. In the work, varied shear experiments were performed on concretes with moderate and high fluidity at a steady water-binder rate of 0.4.

Yucel [4] stated that carrying mortar composes an important portion of the manufacture procedure. New advances in concrete technology and mineral and chemical additives have led to new advances in pumping techniques and the employment of various concrete mixes and equipment. Last advances needed more information on the action of concrete subjected to pressure.

The problem of the fresh concrete-mold interaction was inspired by the liquid-tank interaction in liquid storage tanks. In general, research has been performed on the earthquake reaction of liquid storage tanks in recent 50 years. A simple mass-spring model (MSM) still commonly performed with some changes in the settlement of rectangular and cylindrical tanks was suggested by Housner [5–8] to calculate the earthquake reaction of liquid storage tanks. His reduced MSM is a two-degree-of-freedom (DOF) system for a rigid tank. One DOF (impulsive mode) involves behavior of the tank-liquid system in which some of the contained liquid is rigidly attached to the tank wall, and the other DOF is for the behavior of the turbulent liquid on the tank wall (convective mode).

In the works that followed, Housner's clarified MSM was altered for explaining Young's module of the tank wall. Veletsos and Yang [9] employed one mass in the impulsive part and two masses in the convective part in their reduced MSM. Malhotra et al. [10] replaced the properties of the reduced MSM suggested by Veletsos and Yang [9] employing one convective mode.

The following works can be given as examples of current works on liquid storage tanks. Goktepe et al. [11] explained that the outcome of the area examination of vibrations caused by rail traffic is provided to investigate the acceptability levels of ground vibration and to interpret the serviceability of a liquid storage tank. Zhao and Zhou [12] stated that academic research studies on liquid storage tanks were reviewed to provide valuable references to engineering applications in the seismic design. A summary of the performance of tanks during past earthquakes is described in this paper.

Aktas [13] employed one mass in the impulsive part and one mass in the convective part in his theoretical MSM. The study was inspired by the works of Housner [5, 6] on liquid storage tanks. In the study, the movement of fresh concrete in the prefabricated formwork under vibration was investigated using MSM. The findings found from the theoretical study were matched with the test ones. From the comparison of the results, they were seen as compatible.

Although the results were compatible, close results have not been obtained at some dots.

Some recent studies performed in different engineering branches using the mass-spring model are available in the literature. Some of them are presented below.

El-Sayed and Farghaly [14] handled the breakdown of the natural frequencies, mode shapes of an axially loaded beam system wearing ends containing nonconcentrated tip masses, and three spring-two mass subsystems. They stated in their paper that the influence of material features, rotary inertia, and shear deformation of the beam system is included. Azizi et al. [15] handled the small fast impulse reaction and dynamic stresses of compound Sandwich truncated conical shells with a compressible or incompressible kernel. It was stated that the effects are accepted to take place usually on the top face sheet, and the interplay between the impactor and the structure is simulated employing a new equivalent three-degree-of-freedom spring-mass-damper pattern. The motion areas of the core and surface plates were handled by the higher order and first-order shear deformation theorem, respectively. Rong and Lu [16] expressed that one nonlinear gas spring was combined into the traditional tuned mass absorber, causing a novel gas spring tuned mass damper system, which can be employed to reduce the structural reactions. In the article, one symmetrical integrated gas spring (SCGS) related to a unique gas spring was presented to better integrate the tuned mass damping system and its mechanical features were examined with one status work. Also in the study, the pattern technique of the gas spring TMD was obtained and the full mode factors were computed.

There are studies in the literature using the mass-spring model in precast and other structural elements [17–20]. In addition, in recent years, dynamic studies in the fields of construction and geotechnical sciences are available in the literature [21, 22].

This study is the continuation of the article by Aktas [13] who is the author of this paper and aims to improve the previous model. The article investigates the motion of concrete subjected to vibration with the mass-spring model. As mentioned in the previous study [13], analysis was made using one mass in both the impulsive part and convective part. In this study, it was aimed to improve the results by using two masses in the convective part. Here, the use of two masses in the convective part of the model can be considered one new contribution. By comparing the results, it can be said that the target is approached. Theoretical analysis of the molds used in the experimental study was made using the mass-spring model. Analytical FEM analysis by SAP2000 [23] software is realized to imitate the motion of the full-scale formworks. Additionally, the CSI Analysis Reference Manual [24], which is the Theory Manual for the SAP2000 program, is available in the literature. The linear Link/Support object in SAP2000 software is employed for the pattern movement of concrete. In the work, the theoretical conclusions obtained from the motion of fresh concrete under vibration are compared with the test ones.

2. Three-Dimensional Modeling of the Empty Mold

Three-dimensional appearances of formworks (box culvert and column elements) employed in finite element method (FEM) are displayed in Figures 1 and 2. Two-dimensional appearances of formworks are displayed in Figures 3 and 4. Dots I and J are the measure dots in Figure 3, and dots K, L, and M are the measure dots in Figure 4. The length and height of the Column formwork are 9 m and 0.6 m, respectively. The prefabricated column is produced horizontally in the production workshop. The dimensions of the box culvert formwork with 200 mm thickness in the x-way and 250 mm in the y-way is 3.0×2.9 m. Its height is 0.97 m. Productions were made in the prefabricated concrete manufacture workplace with aforementioned formworks. The formworks employed in the experiment samples were built with 5 mm thick steel plates. For making the system strong, steel profiles of different dimensions were attached to the dies in the horizontal, vertical, and diagonal ways. External vibrators employed in test samples are attached to one steely sheet having sizes of 200×250 mm.

The measurement readings were received on the face of the formworks in two prefabricated concrete constructive elements by employing one computer-based data acquisition grid. The place of measure dots was chosen near and far from vibration dots to understand their differences and impact. The data collection layout including the instrument, hardware, and software employed in this paper has been given broadly [25, 26].

The thickness of the steel mold used in the experiment is 5 mm. Steel profiles that strengthen the formwork system at the bottom of the die were accepted as a simple support in the theoretical pattern. The boundary status of dies for the box culvert and column elements was presented [25].

The FEM network in formworks was composed of Isoparametric Linear Frame and Shell (Area) Quad4NL finite elements. The SAP2000 4-node linear interpolation quadrilateral element was used. Area finite elements are composed almost in size of 100×100 mm as a quadrilateral linear, described by the four nodes as square, rectangular, or trapezoidal members, and triangular, described by three joints, as transition purposed triangular members subjected to the formwork geometry. Frame members use the same nodes with Area elements.

In the analysis, the common measure for determining whether there is an adequate mode is the mass participation rate. The modes employed for the examination may be undamped free-vibration modes or load-dependent Ritz vectors. In this work, load-dependent Ritz vectors were employed in the dynamic investigation [27].

First, dynamic analysis was carried out only in the nonconcrete formwork to confirm the FEM network employed in the pattern. Modeling of the nonconcrete formwork is widely presented [25], and the boundary conditions of the formwork systems and the properties of the external vibrators are defined here. The modeling of the formwork full of fresh concrete is presented below.

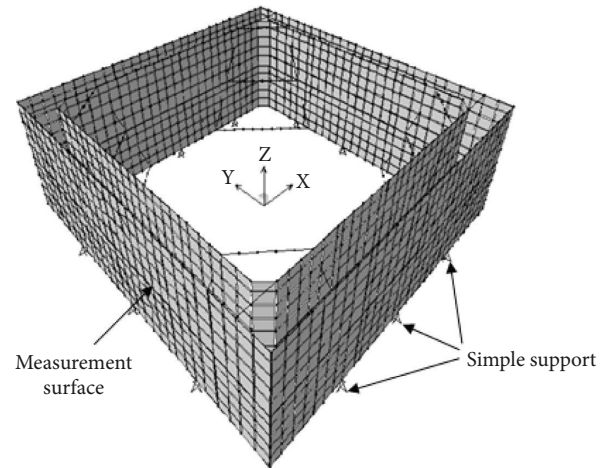


FIGURE 1: Three-dimensional FEM mesh view of box culvert formwork [13].

3. Three-Dimensional Modeling of the Full Mold

3.1. Dynamic Analysis: Mass-Spring Model (MSM)—Parametric Study. Theoretical analysis of molds used in the experimental study was performed using the mass-spring model. This work can be considered a continuation of a previous study [13]. The study is a theoretical study in which the conclusions were checked with the empirical conclusions to understand its correctness. As mentioned earlier, this study is based on the work of Housner [5, 6]. Housner's mass approximations contain two pieces. The first part of fluid mass is impulsive mass, which acts with the tank. The second portion of fluid mass is convective mass, which acts apart from the tank. Considering the dynamic behavior of the fluid, it is possible to define multiple oscillation (convective) masses with different frequency values in addition to the impulse mass. Thus, two convective masses were used in this study.

In accordance with Housner's formulas, the height of the impulse mass is three times the height of the fluid divided by eight, which is 37.5%. In the MSM, one impulsive mass and two convective masses were used.

In this manuscript, impulsive mass was enforced at the level of 400 mm for the box culvert element and 200 mm for the column. Two convective masses were enforced at the level of 600 mm and 800 mm for the box culvert element and 300 mm and 500 mm for the column. In assignment of the height of impulsive mass, 37.5% of the height of construction was based on nearly. For the determination of the height of two convective masses, around 70% of the height of the structure with 200 mm intervals was assumed. 60% of mass of concrete has been assigned to the impulse mass and 40% to the convective mass. The convective mass was also assumed as 30% and 10% from the bottom to top.

Link elements in SAP2000 software are employed particularly at the interplay of compound constructional members, the structure-soil load transfer pattern. The linear Link/Support object in SAP2000 is employed for modeling

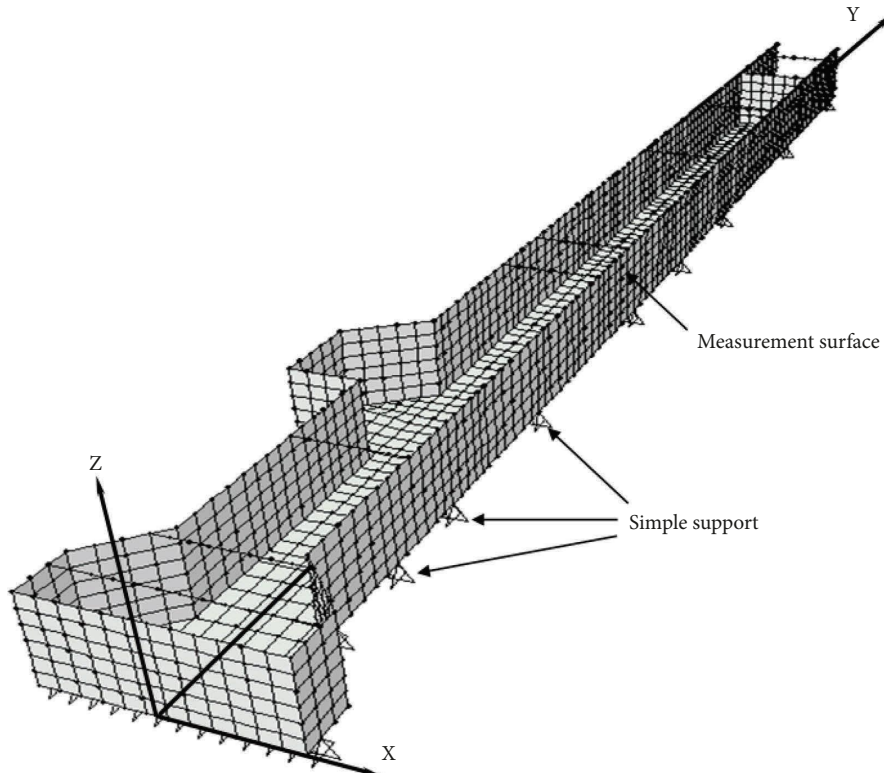


FIGURE 2: Three-dimensional FEM mesh view of the column formwork [13].

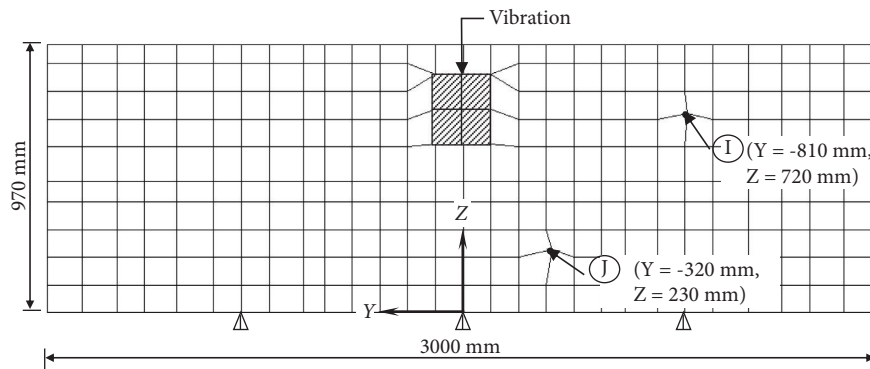


FIGURE 3: The surface measured and computed in the box culvert formwork [13].

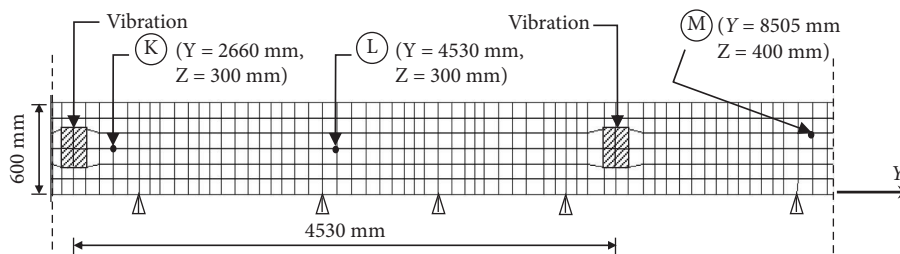


FIGURE 4: The surface measured and computed in the column formwork [13].

concrete in MSM. Whole oriented features—degree of freedom—are fastened in the impulsive portion. In the convective (sloshing) part, the way of the Link member is active, while the others are fastened. MSM is displayed for

the box culvert and column formworks in Figures 5 and 6. As shown in the figures, in this work, two masses have been used for the convective part (oscillating). The impulsive mass and convective mass numbers used in the mass-spring

model were formed depending on the geometric dimensions of two molds. According to this, 62 impulsive masses and 124 convective masses were used in the formwork of the box culvert. In the column mold, 91 impulsive masses and 182 convective masses were used.

4. Exercises

In this segment, the outcomes of the accomplished empirical and numerical works on two steely formworks are supplied and mentioned. In whole shapes of the exercises, the plus distance values are from the formwork face to the concrete. The distance values are perpendicular to the face of the die. In figures of examples, the comparisons are shown in the steady-state portion of motion. Although several irregularities in motion occur earlier, they disappear in a short time and the motion becomes stable. So, graphs have been given over a typical time period during which motion is regular. The outcomes acquired from the theoretical FEM analysis have been checked with the empirical outcomes. The damping ratio has been accepted as 0.05 in whole modes, and the mass participation rate has been accepted according to the Turkish Earthquake Code (TEC) to be minimum 90% of the overall mass of the system [28]. While determining the count of vibration modes in the analysis, it is aimed that the modal mass attendance rates of the construction in the global axis exceed 90%. In the analysis, the time step, the number of time steps, and the time interval have been chosen as 0.5 ms, 8192, and 4.096 sec, respectively, being the same as in the empirical recordings [25, 26].

Formwork systems of box culvert and column members have been considered in exercises. The cyclical frequency of the vibrator has been chosen as 100 Hz, which is the same as in experimental recordings. One vibrator was employed in the box culvert formwork. Two vibrators were employed in the column formwork.

The values of steel (specific gravity, elasticity module, and Poisson's rate) have been considered equal to the values in SAP2000 software. The specific gravity of concrete was calculated in the Kambeton workplace. In this study, the impulsive mass, convective mass rates, and spring constant in the Link object are assumed at the values given below. Parameters used in the steel, concrete, and Link object are given in Table 1.

In a master's thesis conducted at Cukurova University (Turkey), the spring constant (k) value for water was computed as 0.97 N/mm [29]. Inspired by the above-mentioned study, the spring constant for fresh concrete has been supposed by considering its specific weight. Some properties of mass, spring, steel, and concrete have been presented above. The values of steel's specific weight, elasticity module, and Poisson rate are the default values in the SAP 2000 program. In the parametric study, the impulsive mass ratio and convective mass ratios m_{c1} and m_{c2} were accepted considering the volume they affect. Fresh concrete's specific gravity was measured.

4.1. Exercise 1. In this example, the theoretical concrete movement was calculated at two dots of the box culvert formwork, where measurements were taken before and

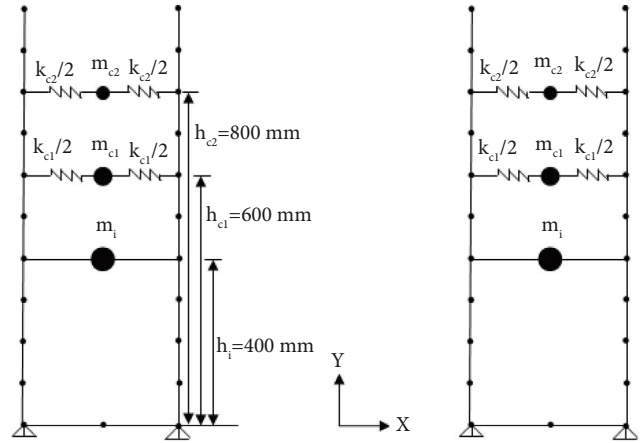


FIGURE 5: Modeling of the fresh concrete movement in the box culvert formwork with MSM.

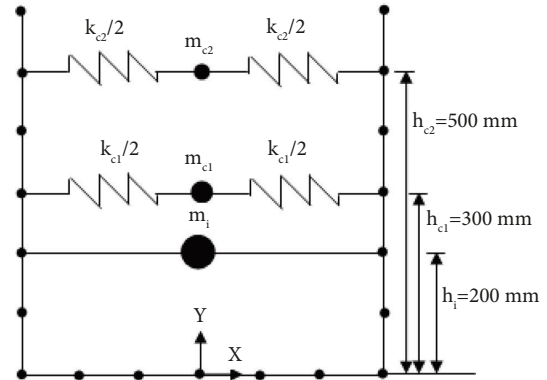


FIGURE 6: Modeling of the fresh concrete movement in the column formwork with MSM.

compared with the empirical movement. Comparisons were made at two dots, called I and J (Figure 3). In the time history analysis, the mass participation rate exceeded 90% with 90 modes. The calculated numerical conclusions are compared with the empirical ones in Figures 7 and 8.

4.2. Exercise 2. In this example, the theoretical concrete movement was calculated at three dots of the column formwork, where measurements were taken before and compared with the empirical movement. Comparisons are fulfilled at three dots named K, L, and M (Figure 6). By using 60 modes in the analysis, the mass participation rate exceeded 90%. The calculated numerical conclusions are compared with the empirical ones in Figures 9–11.

5. Results and Discussion

In this part of the article, the theoretical results obtained from the MSM model compared with the experimental results are examined and discussed. The developed MSM model was compared with the experimental data, and its accuracy was checked. A three-dimensional model of the formwork was improved by employing the finite element method. Firstly, a theoretic three-dimensional imitation

TABLE 1: Parameters used in the steel, concrete, and Link object.

Impulsive mass rate (m_i)	0.6
Convective mass rate (m_{c1})	0.3
Convective mass rate (m_{c2})	0.1
Spring constant (k_{ci} , rigidity)	2.4 N/mm
Steel's specific weight (γ_s)	7.682×10^{-5} N/mm ³
Steel's elasticity module (E_s)	199948 N/mm ²
Steel's Poisson rate (ν_s)	0.3
Fresh concrete's specific gravity (γ_c)	24×10^{-6} N/mm ³

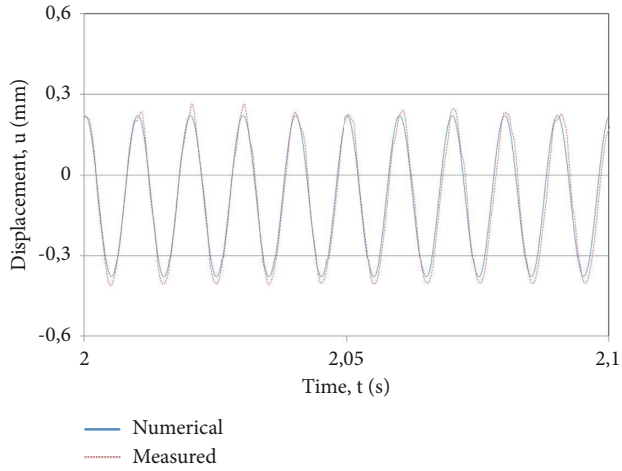


FIGURE 7: Numerical and measured results of concrete motion in the box culvert formwork (dot I).

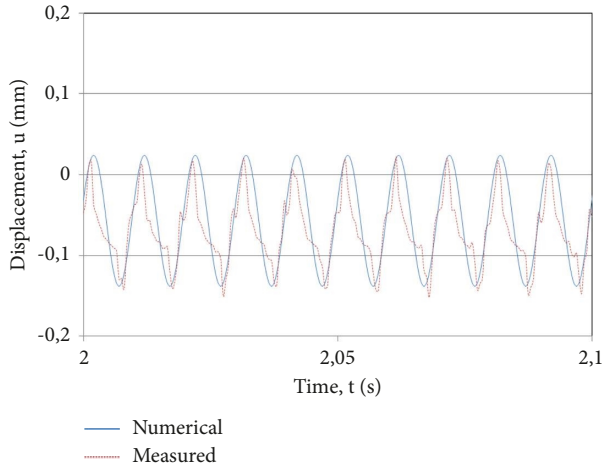


FIGURE 8: Numerical and measured results of concrete motion in the box culvert formwork (dot J).

pattern of the nonconcrete formwork was realized to see the competence of the FEM model [25]. Later, a three-dimensional model of the formwork full of concrete was realized employing FEM [26]. In the aforementioned study, fresh concrete was patterned employing the time-dependent function in a computer-aided formwork design.

In this work, the linear Link objects were added along the surface of the dies in MSM without making any changes to the finite element meshes of the empty formworks. By using

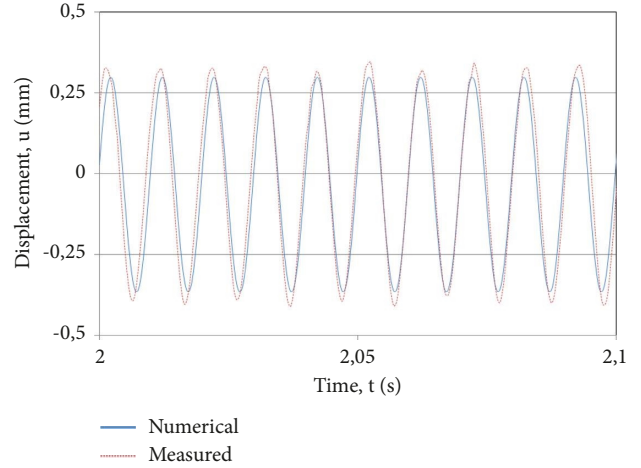


FIGURE 9: Numerical and measured results of concrete motion in the column formwork (dot K).

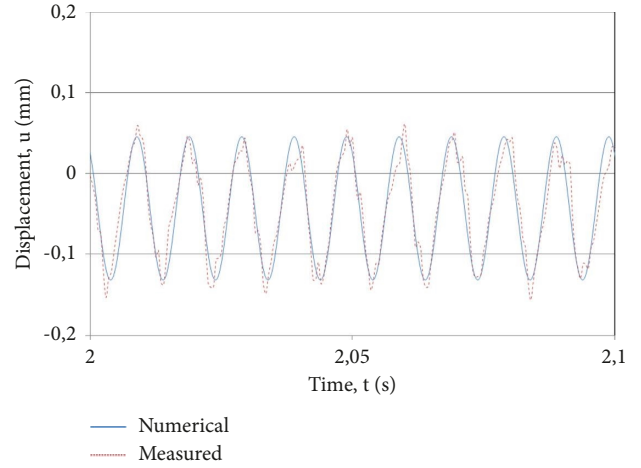


FIGURE 10: Numerical and measured results of concrete motion in the column formwork (dot L).

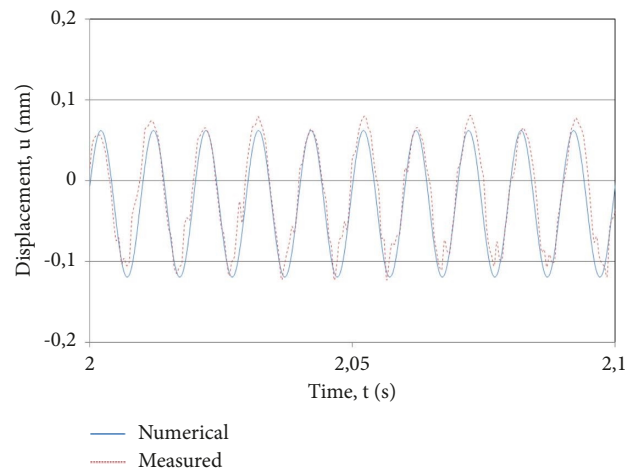


FIGURE 11: Numerical and measured results of concrete motion in the column formwork (dot M).

TABLE 2: Some statistical parameters at selected points.

Point	Root mean square error (RMSE)	Mean absolute error (MAE)	Correlation coefficient (r)	Coefficient of determination (R^2)	
				This study	Previous study [13]
I	0.036776	0.030376	0.98839	0.97691	0.97166
J	0.03221	0.025997	0.86492	0.74809	0.72917
K	0.069789	0.056091	0.96503	0.93128	0.90448
L	0.019234	0.015395	0.95186	0.90604	0.86967
M	0.022139	0.017512	0.96104	0.92360	0.92018

Improvement rate (%)

0.5

2.5

3.0

4.2

0.4

MSM, the interaction between the fresh concrete and the formwork was realized in two prefabricated concrete-formworks named box culvert and column. The numerical and measured results of the variation of the concrete movement with time were compared (Figures 7–11). From the examination of the figures, it is seen that the numerical and measured results are consistent.

As mentioned before, the aim of this study is to improve the results obtained by [13]. When the measured and calculated results are examined, it can be said that this target has been approached (Figures 7–11).

Experimental results are considered correct. Accordingly, it can be said that the closer the theoretical results are to the experimental results, the closer the target is approached. The most important criterion is the fit of the curves. The consistency between the computed and measured responses (Figures 7–11) can be understood by calculating some statistical parameters. The statistical parameters at points I, J, K, L, and M are given in Table 2.

When the table is examined, it is seen that the best fit is at point I and the worst fit is at point J. Since linear variable displacement transducer (LVDT) is a sensitive measuring instrument affected by ambient air, it could reach some measurement values even without starting the vibrator. As the measurement point moves away from the vibrator, the expected amplitude value decreases and the curves become irregular as the effect of the environment increases. This situation, as well as the fact that the J point was far from the vibrator, created the expected small and irregular amplitude. A similar situation is valid for the points L and M. This result can also be seen when looking at the graphs.

6. Conclusions

A theoretical model using FEM, the accuracy of which has been confirmed by the experimental results, has been realized in the nonconcrete formwork [23]. It is thought that modeling of fresh concrete may be performed more precisely by employing this model.

MSM was employed to pattern fresh concrete in the concrete-formwork interaction. The competence of the pattern was controlled by comparing numerical conclusions with the empirical ones. In the previous study by Aktas [13], some differences were observed between the theoretical and measurement displacement amplitudes in some points. In this study, it is seen that the results regarding the points mentioned in the previous study have become more consistent.

This work intends to show that various materials may be modeled using MSM. The mold design can be done by modeling fresh concrete with MSM.

Prefabricated building elements differ in form and size. Therefore, it is difficult to design each mold by the trial and error method. The most important issue in the mold design for prefabricated building elements is the location and number of vibrators. The mold design for the prefabricated building elements is largely done by the trial and error method in production workshops, which leads to the loss of time and economy. Therefore, it is important that the mold design is to be made with the aid of a computer. For this

purpose, first of all, the experimental study to be carried out by taking measurements in the mold with advanced experimental devices in a few molds must be seen to match the results with the mass-spring or another model. Then, the mold design can be made with that model.

In future studies, the compatibility of the theoretical study results with the experimental studies should be seen and the accuracy of the model should be tested. Then, using the appropriate model, a design can be made for fresh concrete or other materials. In the literature, the mass-spring model is used in different materials, some of which are given in the Introduction of this article.

Data Availability

The data used to support the findings of the study are included within the article.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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