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Research Article

Coefficient Estimates for Initial Taylor-Maclaurin Coefficients for a Subclass of Analytic and Bi-Univalent Functions Defined by Al-Oboudi Differential Operator

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We introduce and investigate an interesting subclass $\mathcal{NP}_{\Sigma}^{\lambda,\delta}(n,\beta;h)$ of analytic and bi-univalent functions in the open unit disk $\mathbb{U}$. For functions belonging to the class $\mathcal{NP}_{\Sigma}^{\lambda,\delta}(n,\beta;h)$, we obtain estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$.

1. Introduction

Let $\mathbb{R} = (-\infty,\infty)$ be the set of real numbers, $\mathbb{C}$ the set of complex numbers, and

$$\mathbb{N} := \{1,2,3,\ldots\} = \mathbb{N}_0 \setminus \{0\} \tag{1}$$

the set of positive integers.

Let $\mathcal{A}$ denote the class of all functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{2}$$

which are analytic in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}. \tag{3}$$

We also denote by $\mathcal{S}$ the class of all functions in the normalized analytic function class $\mathcal{A}$ which are univalent in $\mathbb{U}$.

For two functions f and g, analytic in $\mathbb{U}$, we say that the function f is subordinate to g in $\mathbb{U}$ and write

$$f(z) \prec g(z) \quad (z \in \mathbb{U}), \tag{4}$$

if there exists a Schwarz function ω, which is analytic in $\mathbb{U}$ with

$$\omega(0) = 0, \qquad |\omega(z)| < 1, \quad (z \in \mathbb{U}) \tag{5}$$

such that

$$f(z) = g(\omega(z)), \quad (z \in \mathbb{U}). \tag{6}$$

Indeed, it is known that

$$f(z) \prec g(z), \quad (z \in \mathbb{U}) \Longrightarrow f(0) = g(0), \quad f(\mathbb{U}) \subset g(\mathbb{U}). \tag{7}$$

Furthermore, if the function g is univalent in $\mathbb{U}$, then we have the following equivalence:

$$f(z) \prec g(z), \quad (z \in \mathbb{U}) \Longleftrightarrow f(0) = g(0), \quad f(\mathbb{U}) \subset g(\mathbb{U}). \tag{8}$$

For $f \in \mathcal{A}$, Al-Oboudi [1] introduced the following operator:

$$D_{\delta}^{0} f(z) = f(z), \tag{9}$$

$$D_{\delta}^{1} f(z) = (1-\delta) f(z) + \delta z f'(z) =: D_{\delta} f(z), \quad (\delta \geq 0), \tag{10}$$

$$D_{\delta}^{n} f(z) = D_{\delta}\left(D_{\delta}^{n-1} f(z)\right), \quad (n \in \mathbb{N}). \tag{11}$$

If f is given by (2), then from (10) and (11) we see that

$$D_\delta^n f(z) = z + \sum_{k=2}^{\infty} [1 + (k-1)\delta]^n a_k z^k, \quad (n \in \mathbb{N}_0), \tag{12}$$

with $D_\delta^n f(0) = 0$. When $\delta = 1$, we get Sălăgean's differential operator $D_1^n = D^n$, [2].

Since univalent functions are one-to-one, they are invertible and the inverse functions need not be defined on the entire unit disk $\mathbb{U}$. In fact, the Koebe one-quarter theorem [3] ensures that the image of $\mathbb{U}$ under every univalent function $f \in \mathcal{S}$ contains a disk of radius 1/4. Thus every function $f \in \mathcal{A}$ has an inverse f^{-1}, which is defined by

$$\begin{aligned} f^{-1}(f(z)) &= z \quad (z \in \mathbb{U}), \\ f(f^{-1}(w)) &= w \qquad \left(|w| < r_0(f);\ r_0(f) \geq \frac{1}{4}\right). \end{aligned} \tag{13}$$

In fact, the inverse function f^{-1} is given by

$$\begin{aligned} f^{-1}(w) = w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 \\ - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots . \end{aligned} \tag{14}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (2). For a brief history and interesting examples of functions in the class Σ, see [4] (see also [5, 6]). In fact, the aforecited work of Srivastava et al. [4] essentially revived the investigation of various subclasses of the bi-univalent function class Σ in recent years; it was followed by such works as those by Frasin and Aouf [7], Porwal and Darus [8], and others (see, e.g., [9–17]).

Motivated by the abovementioned works, we define the following subclass of function class Σ.

Definition 1. Let $h : \mathbb{U} \to \mathbb{C}$ be a convex univalent function such that

$$h(0) = 1, \qquad h(\bar{z}) = \overline{h(z)} \quad (z \in \mathbb{U}; \Re(h(z)) > 0). \tag{15}$$

A function f, defined by (2), is said to be in the class $\mathcal{NP}_\Sigma^{\lambda,\delta}(n,\beta;h)$ if the following conditions are satisfied:

$$\begin{aligned} & f \in \Sigma, \\ & e^{i\beta}\left((1-\lambda)\frac{D_\delta^n f(z)}{z} + \lambda\left(D_\delta^n f(z)\right)'\right) \prec h(z)\cos\beta + i\sin\beta \quad (z \in \mathbb{U}), \\ & e^{i\beta}\left((1-\lambda)\frac{D_\delta^n g(w)}{w} + \lambda\left(D_\delta^n g(w)\right)'\right) \\ & \qquad \prec h(w)\cos\beta + i\sin\beta \quad (w \in \mathbb{U}), \end{aligned} \tag{16}$$

where $\beta \in (-\pi/2, \pi/2)$, $\lambda \geq 1$, the function g is given by

$$\begin{aligned} g(w) = w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 \\ - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots , \end{aligned} \tag{17}$$

and D_δ^n is the Al-Oboudi differential operator.

Remark 2. If we set

$$h(z) = \frac{1 + Az}{1 + Bz} \quad (-1 \leq B < A \leq 1) \tag{18}$$

in Definition 1, then the class $\mathcal{NP}_\Sigma^{\lambda,\delta}(n,\beta;h)$ reduces to the class denoted by $\mathcal{NP}_\Sigma^{\lambda,\delta}(n,\beta;A,B)$ which is the subclass of the functions $f \in \Sigma$ satisfying

$$\begin{aligned} & e^{i\beta}\left((1-\lambda)\frac{D_\delta^n f(z)}{z} + \lambda\left(D_\delta^n f(z)\right)'\right) \\ & \qquad \prec \frac{1 + Az}{1 + Bz}\cos\beta + i\sin\beta \quad (z \in \mathbb{U}), \\ & e^{i\beta}\left((1-\lambda)\frac{D_\delta^n g(w)}{w} + \lambda\left(D_\delta^n g(w)\right)'\right) \\ & \qquad \prec \frac{1 + Aw}{1 + Bw}\cos\beta + i\sin\beta \quad (w \in \mathbb{U}), \end{aligned} \tag{19}$$

where $\beta \in (-\pi/2, \pi/2)$, $\lambda \geq 1$, the function g is defined by (17), and D_δ^n is the Al-Oboudi differential operator.

Remark 3. If we set

$$h(z) = \frac{1 + (1 - 2\alpha) z}{1 - z} \quad (0 \leq \alpha < 1) \tag{20}$$

in Definition 1, then the class $\mathcal{NP}_\Sigma^{\lambda,\delta}(n,\beta;h)$ reduces to the class denoted by $\mathcal{NP}_\Sigma^{\lambda,\delta}(n,\beta,\alpha)$ which is the subclass of the functions $f \in \Sigma$ satisfying

$$\begin{aligned} & \Re\left\{e^{i\beta}\left((1-\lambda)\frac{D_\delta^n f(z)}{z} + \lambda\left(D_\delta^n f(z)\right)'\right)\right\} > \alpha\cos\beta \\ & \qquad (z \in \mathbb{U}), \\ & \Re\left\{e^{i\beta}\left((1-\lambda)\frac{D_\delta^n g(w)}{w} + \lambda\left(D_\delta^n g(w)\right)'\right)\right\} > \alpha\cos\beta \\ & \qquad (w \in \mathbb{U}), \end{aligned} \tag{21}$$

where $\beta \in (-\pi/2, \pi/2)$, $\lambda \geq 1$, the function g is defined by (17), and D_δ^n is the Al-Oboudi differential operator.

Remark 4. If we set

$$\delta = 1, \qquad h(z) = \frac{1 + (1 - 2\alpha) z}{1 - z} \quad (0 \leq \alpha < 1) \tag{22}$$

in Definition 1, then the class $\mathcal{NP}_\Sigma^{\lambda,\delta}(n,\beta;h)$ reduces to the class denoted by $\mathcal{NP}_\Sigma^{\lambda}(n,\beta,\alpha)$ which is the subclass of the functions $f \in \Sigma$ satisfying

$$\begin{aligned} & \Re\left\{e^{i\beta}\left((1-\lambda)\frac{D^n f(z)}{z} + \lambda\left(D^n f(z)\right)'\right)\right\} > \alpha\cos\beta \\ & \qquad (z \in \mathbb{U}), \\ & \Re\left\{e^{i\beta}\left((1-\lambda)\frac{D^n g(w)}{w} + \lambda\left(D^n g(w)\right)'\right)\right\} > \alpha\cos\beta \\ & \qquad (w \in \mathbb{U}), \end{aligned} \tag{23}$$

where $\beta \in (-\pi/2, \pi/2)$, $\lambda \geq 1$, the function g is defined by (17), and D^n is the Sălăgean differential operator.

Remark 5. If we set

$$n = 0, \qquad h(z) = \frac{1 + (1 - 2\alpha) z}{1 - z} \qquad (0 \leq \alpha < 1) \tag{24}$$

in Definition 1, then the class $\mathcal{NP}_{\Sigma}^{\lambda,\delta}(n, \beta; h)$ reduces to the class denoted by $\mathcal{NP}_{\Sigma}^{\lambda}(\beta, \alpha)$ which is the subclass of the functions $f \in \Sigma$ satisfying

$$\begin{gathered} \Re \left\{ e^{i\beta} \left((1 - \lambda) \frac{f(z)}{z} + \lambda f'(z) \right) \right\} > \alpha \cos \beta \quad (z \in \mathbb{U}), \\ \Re \left\{ e^{i\beta} \left((1 - \lambda) \frac{g(w)}{w} + \lambda g'(w) \right) \right\} > \alpha \cos \beta \quad (w \in \mathbb{U}), \end{gathered} \tag{25}$$

where $\beta \in (-\pi/2, \pi/2)$, $\lambda \geq 1$, and the function g is defined by (17).

Remark 6. If we set

$$n = 0, \qquad \lambda = 1, \qquad h(z) = \frac{1 + (1 - 2\alpha) z}{1 - z} \qquad (0 \leq \alpha < 1) \tag{26}$$

in Definition 1, then the class $\mathcal{NP}_{\Sigma}^{\lambda,\delta}(n, \beta; h)$ reduces to the class denoted by $\mathcal{NP}_{\Sigma}(\beta, \alpha)$ which is the subclass of the functions $f \in \Sigma$ satisfying

$$\begin{gathered} \Re \left\{ e^{i\beta} f'(z) \right\} > \alpha \cos \beta \quad (z \in \mathbb{U}), \\ \Re \left\{ e^{i\beta} g'(w) \right\} > \alpha \cos \beta \quad (w \in \mathbb{U}), \end{gathered} \tag{27}$$

where $\beta \in (-\pi/2, \pi/2)$ and the function g is defined by (17).

We note that

$$\begin{gathered} \mathcal{NP}_{\Sigma}^{\lambda}(n, 0, \alpha) = \mathcal{H}_{\Sigma}(n, \alpha, \lambda) \quad (\text{see [8]}), \\ \mathcal{NP}_{\Sigma}^{\lambda}(0, \alpha) = \mathcal{B}_{\Sigma}(\alpha, \lambda) \quad (\text{see [7]}), \\ \mathcal{NP}_{\Sigma}(0, \alpha) = \mathcal{H}_{\Sigma}(\alpha) \quad (\text{see [4]}). \end{gathered} \tag{28}$$

Firstly, in order to derive our main results, we need the following lemma.

Lemma 7 (see [18]). *Let the function $h(z)$ given by*

$$h(z) = \sum_{n=1}^{\infty} B_n z^n \tag{29}$$

be convex in $\mathbb{U}$. Suppose also that the function $\varphi(z)$ given by

$$\varphi(z) = \sum_{n=1}^{\infty} c_n z^n \tag{30}$$

is holomorphic in $\mathbb{U}$. If $\varphi(z) \prec h(z)$ $(z \in \mathbb{U})$, then

$$|c_n| \leq |B_1| \quad (n \in \mathbb{N}). \tag{31}$$

The object of the present paper is to find estimates on the Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions in this new subclass $\mathcal{NP}_{\Sigma}^{\lambda,\delta}(n, \beta; h)$ of the function class Σ.

2. A Set of General Coefficient Estimates

In this section, we state and prove our general results involving the bi-univalent function class $\mathcal{NP}_{\Sigma}^{\lambda,\delta}(n, \beta; h)$ given by Definition 1.

Theorem 8. *Let the function $f(z)$ given by the Taylor-Maclaurin series expansion* (2) *be in the function class*

$$\mathcal{NP}_{\Sigma}^{\lambda,\delta}(n, \beta; h) \quad (\beta \in (-\pi/2, \pi/2), \lambda \geq 1, \delta \geq 0, n \in \mathbb{N}_0) \tag{32}$$

with

$$h(z) = 1 + B_1 z + B_2 z^2 + \cdots. \tag{33}$$

Then

$$|a_2| \leq \min \left\{ \frac{|B_1| \cos \beta}{(1 + \delta)^n (1 + \lambda)}, \sqrt{\frac{|B_1| \cos \beta}{(1 + 2\delta)^n (1 + 2\lambda)}} \right\}, \tag{34}$$

$$|a_3| \leq \frac{|B_1| \cos \beta}{(1 + 2\delta)^n (1 + 2\lambda)}. \tag{35}$$

Proof. It follows from (16) that

$$\begin{aligned} & e^{i\beta} \left((1 - \lambda) \frac{D_{\delta}^n f(z)}{z} + \lambda (D_{\delta}^n f(z))' \right) \\ & \quad = p(z) \cos \beta + i \sin \beta \quad (z \in \mathbb{U}), \end{aligned} \tag{36}$$

$$\begin{aligned} & e^{i\beta} \left((1 - \lambda) \frac{D_{\delta}^n g(w)}{w} + \lambda (D_{\delta}^n g(w))' \right) \\ & \quad = q(w) \cos \beta + i \sin \beta \quad (w \in \mathbb{U}), \end{aligned} \tag{37}$$

where $p(z) \prec h(z)$ and $q(w) \prec h(w)$ have the following Taylor-Maclaurin series expansions:

$$p(z) = 1 + p_1 z + p_2 z^2 + \cdots, \tag{38}$$

$$q(w) = 1 + q_1 w + q_2 w^2 + \cdots, \tag{39}$$

respectively. Now, upon equating the coefficients in (36) and (37), we get

$$e^{i\beta} (1 + \delta)^n (1 + \lambda) a_2 = p_1 \cos \beta, \tag{40}$$

$$e^{i\beta} (1 + 2\delta)^n (1 + 2\lambda) a_3 = p_2 \cos \beta, \tag{41}$$

$$-e^{i\beta} (1 + \delta)^n (1 + \lambda) a_2 = q_1 \cos \beta, \tag{42}$$

$$\begin{aligned} & e^{i\beta} \left[-(1 + 2\delta)^n (1 + 2\lambda) a_3 + 2(1 + 2\delta)^n (1 + 2\lambda) a_2^2 \right] \\ & \quad = q_2 \cos \beta. \end{aligned} \tag{43}$$

From (40) and (42), we obtain

$$p_1 = -q_1, \tag{44}$$

$$2 e^{2i\beta} (1 + \delta)^{2n} (1 + \lambda)^2 a_2^2 = \left(p_1^2 + q_1^2 \right) \cos^2 \beta. \tag{45}$$

Also, from (41) and (43), we find that

$$a_2^2 = \frac{e^{-i\beta}(p_2+q_2)\cos\beta}{2(1+2\delta)^n(1+2\lambda)}. \tag{46}$$

Since $p, q \in h(\mathbb{U})$, according to Lemma 7, we immediately have

$$\begin{aligned} |p_k| &= \left|\frac{p^{(k)}(0)}{k!}\right| \le |B_1| \quad (k \in \mathbb{N}), \\ |q_k| &= \left|\frac{q^{(k)}(0)}{k!}\right| \le |B_1| \quad (k \in \mathbb{N}). \end{aligned} \tag{47}$$

Applying (47) and Lemma 7 for the coefficients p_1, p_2, q_1, and q_2, from the equalities (45) and (46), we obtain

$$|a_2|^2 \le \frac{|B_1|^2\cos^2\beta}{(1+\delta)^{2n}(1+\lambda)^2}, \tag{48}$$

$$|a_2|^2 \le \frac{|B_1|\cos\beta}{(1+2\delta)^n(1+2\lambda)}, \tag{49}$$

respectively. So we get the desired estimate on the coefficient $|a_2|$ as asserted in (34).

Next, in order to find the bound on the coefficient $|a_3|$, we subtract (43) from (41). We thus get

$$\begin{aligned} &2(1+2\delta)^n(1+2\lambda)a_3 - 2(1+2\delta)^n(1+2\lambda)a_2^2 \\ &\quad = e^{-i\beta}(p_2-q_2)\cos\beta. \end{aligned} \tag{50}$$

Upon substituting the value of a_2^2 from (45) into (50), it follows that

$$a_3 = \frac{e^{-2i\beta}(p_1^2+q_1^2)\cos^2\beta}{2(1+\delta)^{2n}(1+\lambda)^2} + \frac{e^{-i\beta}(p_2-q_2)\cos\beta}{2(1+2\delta)^n(1+2\lambda)}. \tag{51}$$

So we get

$$|a_3| \le \frac{|B_1|^2\cos^2\beta}{(1+\delta)^{2n}(1+\lambda)^2} + \frac{|B_1|\cos\beta}{(1+2\delta)^n(1+2\lambda)}. \tag{52}$$

On the other hand, upon substituting the value of a_2^2 from (46) into (50), it follows that

$$a_3 = \frac{e^{-i\beta}(p_2+q_2)\cos\beta}{2(1+2\delta)^n(1+2\lambda)} + \frac{e^{-i\beta}(p_2-q_2)\cos\beta}{2(1+2\delta)^n(1+2\lambda)}. \tag{53}$$

And we get

$$|a_3| \le \frac{|B_1|\cos\beta}{(1+2\delta)^n(1+2\lambda)}. \tag{54}$$

Comparing the inequalities in (52) and (54) completes the proof of Theorem 8. □

3. Corollaries and Consequences

By setting

$$h(z) = \frac{1+Az}{1+Bz} \quad (-1 \le B < A \le 1) \tag{55}$$

in Theorem 8, we have the following corollary.

Corollary 9. *Let the function $f(z)$ given by the Taylor-Maclaurin series expansion (2) be in the function class*

$$\begin{aligned} &\mathcal{NP}_\Sigma^{\lambda,\delta}(n,\beta;A,B) \\ &(\beta \in (-\pi/2,\pi/2), \lambda \ge 1, \delta \ge 0, -1 \le B < A \le 1, n \in \mathbb{N}_0). \end{aligned} \tag{56}$$

Then

$$\begin{aligned} |a_2| &\le \min\left\{\frac{(A-B)\cos\beta}{(1+\delta)^n(1+\lambda)}, \sqrt{\frac{(A-B)\cos\beta}{(1+2\delta)^n(1+2\lambda)}}\right\}, \\ |a_3| &\le \frac{(A-B)\cos\beta}{(1+2\delta)^n(1+2\lambda)}. \end{aligned} \tag{57}$$

By setting

$$h(z) = \frac{1+(1-2\alpha)z}{1-z} \quad (0 \le \alpha < 1) \tag{58}$$

in Theorem 8, we have the following corollary.

Corollary 10. *Let the function $f(z)$ given by the Taylor-Maclaurin series expansion (2) be in the function class*

$$\begin{aligned} &\mathcal{NP}_\Sigma^{\lambda,\delta}(n,\beta,\alpha) \\ &(\beta \in (-\pi/2,\pi/2), \lambda \ge 1, \delta \ge 0, 0 \le \alpha < 1, n \in \mathbb{N}_0). \end{aligned} \tag{59}$$

Then

$$\begin{aligned} |a_2| &\le \min\left\{\frac{2(1-\alpha)\cos\beta}{(1+\delta)^n(1+\lambda)}, \sqrt{\frac{2(1-\alpha)\cos\beta}{(1+2\delta)^n(1+2\lambda)}}\right\}, \\ |a_3| &\le \frac{2(1-\alpha)\cos\beta}{(1+2\delta)^n(1+2\lambda)}. \end{aligned} \tag{60}$$

By setting

$$\delta = 1, \qquad h(z) = \frac{1+(1-2\alpha)z}{1-z} \quad (0 \le \alpha < 1) \tag{61}$$

in Theorem 8, we have the following corollary.

Corollary 11. *Let the function $f(z)$ given by the Taylor-Maclaurin series expansion (2) be in the function class*

$$\begin{aligned} &\mathcal{NP}_\Sigma^{\lambda}(n,\beta,\alpha) \\ &(\beta \in (-\pi/2,\pi/2), \lambda \ge 1, 0 \le \alpha < 1, n \in \mathbb{N}_0). \end{aligned} \tag{62}$$

Then

$$|a_2| \leq \min\left\{\frac{2(1-\alpha)\cos\beta}{2^n(1+\lambda)}, \sqrt{\frac{2(1-\alpha)\cos\beta}{3^n(1+2\lambda)}}\right\},$$
$$|a_3| \leq \frac{2(1-\alpha)\cos\beta}{3^n(1+2\lambda)}. \tag{63}$$

Remark 12. When $\beta = 0$, Corollary 11 is an improvement of the following estimates obtained by Porwal and Darus [8].

Corollary 13 (see [8]). *Let the function $f(z)$ given by the Taylor-Maclaurin series expansion* (2) *be in the function class*

$$\mathscr{H}_\Sigma(n, \alpha, \lambda) \quad (\lambda \geq 1,\ 0 \leq \alpha < 1,\ n \in \mathbb{N}_0). \tag{64}$$

Then

$$|a_2| \leq \sqrt{\frac{2(1-\alpha)}{3^n(1+2\lambda)}},$$
$$|a_3| \leq \frac{4(1-\alpha)^2}{2^{2n}(1+\lambda)^2} + \frac{2(1-\alpha)}{3^n(1+2\lambda)}. \tag{65}$$

By setting

$$n = 0, \qquad h(z) = \frac{1+(1-2\alpha)z}{1-z} \quad (0 \leq \alpha < 1) \tag{66}$$

in Theorem 8, we have the following corollary.

Corollary 14. *Let the function $f(z)$ given by the Taylor-Maclaurin series expansion* (2) *be in the function class*

$$\mathscr{N}\mathscr{P}_\Sigma^\lambda(\beta, \alpha) \quad (\beta \in (-\pi/2, \pi/2),\ \lambda \geq 1,\ 0 \leq \alpha < 1). \tag{67}$$

Then

$$|a_2| \leq \min\left\{\frac{2(1-\alpha)\cos\beta}{1+\lambda}, \sqrt{\frac{2(1-\alpha)\cos\beta}{1+2\lambda}}\right\},$$
$$|a_3| \leq \frac{2(1-\alpha)\cos\beta}{1+2\lambda}. \tag{68}$$

Remark 15. When $\beta = 0$, Corollary 14 is an improvement of the following estimates obtained by Frasin and Aouf [7].

Corollary 16 (see [7]). *Let the function $f(z)$ given by the Taylor-Maclaurin series expansion* (2) *be in the function class*

$$\mathscr{B}_\Sigma(\alpha, \lambda) \quad (\lambda \geq 1,\ 0 \leq \alpha < 1). \tag{69}$$

Then

$$|a_2| \leq \sqrt{\frac{2(1-\alpha)}{1+2\lambda}},$$
$$|a_3| \leq \frac{4(1-\alpha)^2}{(1+\lambda)^2} + \frac{2(1-\alpha)}{1+2\lambda}. \tag{70}$$

By setting

$$n = 0, \quad \lambda = 1, \quad h(z) = \frac{1+(1-2\alpha)z}{1-z} \quad (0 \leq \alpha < 1) \tag{71}$$

in Theorem 8, we have the following corollary.

Corollary 17. *Let the function $f(z)$ given by the Taylor-Maclaurin series expansion* (2) *be in the function class*

$$\mathscr{N}\mathscr{P}_\Sigma(\beta, \alpha) \quad (\beta \in (-\pi/2, \pi/2), 0 \leq \alpha < 1). \tag{72}$$

Then

$$|a_2| \leq \min\left\{(1-\alpha)\cos\beta, \sqrt{\frac{2(1-\alpha)\cos\beta}{3}}\right\},$$
$$|a_3| \leq \frac{2(1-\alpha)\cos\beta}{3}. \tag{73}$$

Remark 18. When $\beta = 0$, Corollary 17 is an improvement of the following estimates obtained by Srivastava et al. [4].

Corollary 19 (see [4]). *Let the function $f(z)$ given by the Taylor-Maclaurin series expansion* (2) *be in the function class*

$$\mathscr{H}_\Sigma(\alpha) \quad (0 \leq \alpha < 1). \tag{74}$$

Then

$$|a_2| \leq \sqrt{\frac{2(1-\alpha)}{3}},$$
$$|a_3| \leq \frac{(1-\alpha)(5-3\alpha)}{3}. \tag{75}$$

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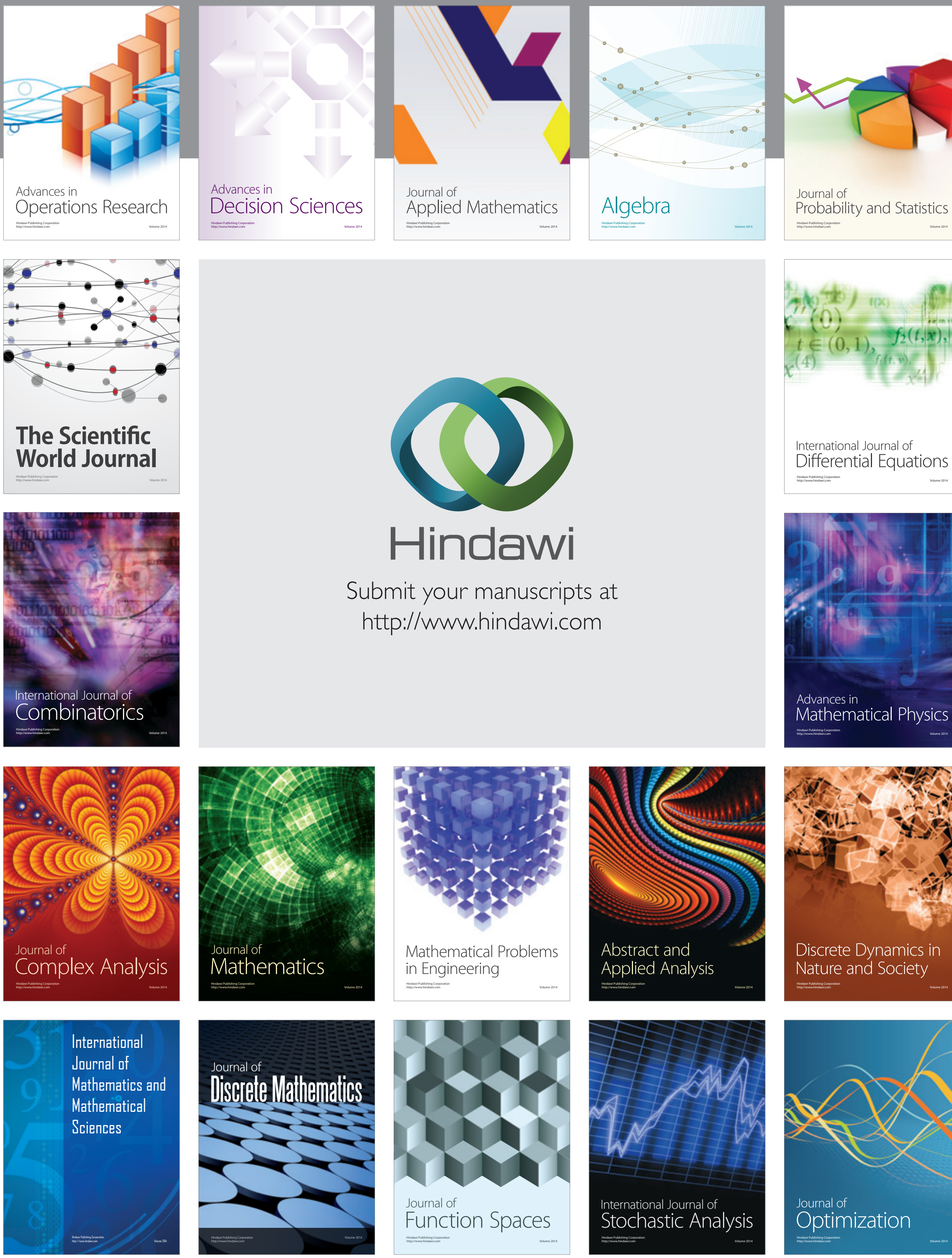
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