Hindawi Publishing Corporation
The Scientific World Journal
Volume 2014, Article ID 536709, 6 pages
http://dx.doi.org/10.1155/2014/536709

Research Article

Soft Translations and Soft Extensions of BCI/BCK-Algebras

Nazra Sultana,[1] Nazia Rani,[1,2] Muhammad Irfan Ali,[3] and Azhar Hussain[1]

[1] *Department of Mathematics, University of Sargodha, Sargodha 40100, Pakistan*
[2] *Department of Mathematics, University of Sargodha, Women Campus, Faisalabad, Pakistan*
[3] *Islamabad Model College for Girls F-6/2, Islamabad 44000, Pakistan*

Correspondence should be addressed to Azhar Hussain; hafiziqbal30@gmail.com

Received 12 June 2014; Accepted 8 August 2014; Published 14 September 2014

Academic Editor: Feng Feng

The concept of soft translations of soft subalgebras and soft ideals over BCI/BCK-algebras is introduced and some related properties are studied. Notions of Soft extensions of soft subalgebras and soft ideals over BCI/BCK-algebras are also initiated. Relationships between soft translations and soft extensions are explored.

1. Introduction

Recently soft set theory has emerged as a new mathematical tool to deal with uncertainty. Due to its applications in various fields of study researchers and practitioners are showing keen interest in it. As enough number of parameters is available here, so it is free from the difficulties associated with other contemporary theories dealing with uncertainty. Prior to soft set theory, probability theory, fuzzy set theory, rough set theory, and interval mathematics were common mathematical tools for dealing with uncertainties, but all these theories have their own difficulties. These difficulties may be due to lack of parametrization tools [1, 2]. To overcome these difficulties, Molodtsov [2] introduced the concept of soft sets. A detailed overview of these difficulties can be seen in [1, 2]. As a new mathematical tool for dealing with uncertainties, Molodtsov has pointed out several directions for the applications of soft sets. Theoretical development of soft sets is due to contributions from many researchers. However in this regard initial work is done by Maji et al. in [1]. Later Ali et al. [3] introduced several new operations in soft set theory.

At present, work on the soft set theory is progressing rapidly. Maji et al. [4] described the application of soft set theory in decision making problems. Aktaş and Çağman studied the concept of soft groups and derived their basic properties [5]. Chen et al. [6] proposed parametrization reduction of soft sets, and then Kong et al. [7] presented the normal parametrization reduction of soft sets. Feng and his colleagues studied roughness in soft sets [8, 9]. Relationship between soft sets, fuzzy sets, and rough sets is investigated in [10]. Park et al. [11] worked on notions of soft WS-algebras, soft subalgebras, and soft deductive system. Jun and Park [12] presented the notions of soft ideals, idealistic soft, and idealistic soft BCI/BCK-algebras. Further applications of soft sets can be seen in [13–25].

The study of BCI/BCK-algebras was initiated by Imai and Iseki [26] as the generalization of concept of set theoretic difference and propositional calculus. For the general development of BCI/BCK-algebras, the ideal theory and its fuzzification play an important role. Jun et al. [27–30] studied fuzzy trends of several notions in BCI/BCK-algebras. Application of soft sets in BCI/BCK is given in [12, 31].

Translations play a vital role in reducing the complexity of a problem. In geometry it is a common practice to translate a system to some new position to study its properties. In linear algebra translations help find solution to many practical problems. In this paper idea of translations is being extended to soft BCI/BCK algebras.

This paper is arranged as follows: in Section 2, some basic notions about BCI/BCK-algebra and soft sets are given. These notions are required in the later sections. Concept of translation is introduced in Section 3 and some properties of it are discussed here. Section 4 is devoted for the study of soft ideal translation in BCI/BCK-algera. In Section 5, concept of ideal extension is introduced and some of its properties are studied.

2. Preliminaries

First of all some basic concepts about BCI/BCK-algebra are given. For a comprehensive study on BCI/BCK-algebras [32] is a very nice monograph by Meng and Jun. Then some notions about soft sets are presented here as well.

An algebra $(X, *, 0)$ is called a BCI-algebra if it satisfies the following conditions:

(1) $(\forall x, y, z \in X)\ (((x * y) * (x * z)) * (z * y) = 0)$,

(2) $(\forall x, y \in X)\ ((x * (x * y)) * y = 0)$,

(3) $(\forall x \in X)\ (x * x = 0)$,

(4) $(\forall x, y \in X)\ (x * y = 0, y * x = 0 \Rightarrow x = y)$.

If a BCI-algebra satisfies the following identity:

(5) $(\forall x \in X)\ (0 * x = 0)$,

then X is called a BCK-algebra. Any BCK-algebra satisfies the following axioms:

(i) $(\forall x \in X)\ (x * 0 = x)$,

(ii) $(\forall x, y, z \in X)\ (x * y = 0 \Rightarrow (x * z) * (y * z) = 0, (z * y) * (z * x) = 0)$,

(iii) $(\forall x, y, z \in X)\ ((x * y) * z = (x * z) * y)$,

(iv) $(\forall x, y, z \in X)\ (((x * z) * (y * z)) * (x * y) = 0)$.

A subset S of a BCI/BCK-algebra X is called a subalgebra of X if $x * y \in S$, for all $x, y \in S$.

A subset A of a BCI/BCK-algebra X is called an ideal of X, denoted by $A \lhd X$, if it satisfies:

(1) $0 \in A$,

(2) $(\forall x, y \in X)\ (x * y \in A, y \in A \Rightarrow x \in A)$.

Now we recall some basic notions in soft set theory. Let U be a universe and E be a set of parameters. Let $P(U)$ denote the power set of U and let A, B be nonempty subsets of E.

Definition 1 (see [2]). A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2 (see [3]). Let U be a universe, let E be the set of parameters, and let $A \subseteq E$.

(a) (F, A) is called a relative null soft set (with respect to the parameters set A), denoted by $\emptyset_A$, if $F(a) = \emptyset$, for all $a \in A$.

(b) (G, A) is called a relative whole soft set (with respect to the parameters set A), denoted by U_A, if $G(e) = U$, for all $e \in A$.

Definition 3 (see [3]). The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \to P(U)$ is a mapping given by $F^c(a) = U - F(a)$, $\forall a \in A$. Clearly, $((F, A)^c)^c = (F, A)$.

Definition 4 (see [8]). A soft set (F, A) over U is called a full soft set if $\bigcup_{a\in A} F(a) = U$.

3. Soft Translations of Soft Subalgebras

Here notion of translations in soft BCI/BCK-algebra is initiated. Concept of soft extensions is introduced here also.

Let $F_A : X \to P(X)$ be set valued map defined as

$$F_A(x) \neq \emptyset \quad \text{if } x \in A, \tag{1}$$

where $A \subseteq X$. Then F_A also denotes a soft set over a BCI/BCK algebra X. From here onward a soft set will be denoted by symbols like F_A, unless stated otherwise.

A soft set F_A over a BCI/BCK-algebra X is called a soft subalgebra of X if it satisfies

$$(\forall x, y \in X) \quad (F_A(x * y) \supseteq F_A(x) \cap F_A(y)). \tag{2}$$

In what follows $X = (X, *, 0)$ denote a BCI/BCK-algebra, and for any soft set F_A over X, we denote $T := X - \cup\{F_A(x) \mid x \in \mathbf{X}\}$ unless otherwise specified.

That is $T = (\bigcup_{x\in X} F_A(x))^c = \bigcap_{x\in X} F_A^c(x)$.

It is easy to see that $T \cap F_A(x) = \emptyset$ for all $x \in X$. If F_A is a full soft set then T is an empty set. Therefore throughout this paper only those soft set are considered which are not full.

Definition 5. Let F_A be a soft set over X and let $U_1 \subseteq T$. A mapping $F^T_{U_1} : X \to P(X)$ is called a soft U_1-translation of F_A if, for all $x \in X$,

$$F^T_{U_1}(x) = F_A(x) \cup U_1. \tag{3}$$

Lemma 6. *Let $U_1 \subseteq T$ and F_A be a soft set over X, then $F_A(x) \cup U_1 \supseteq F_A(y) \cup U_1$ implies $F_A(x) \supseteq F_A(y)$, for all $x, y \in X$.*

Proof. Since $U_1 \subseteq T$, $F_A(x) \cap U_1 = \emptyset$ and $F_A(y) \cap U_1 = \emptyset$. Let $a \in F_A(y)$ then $a \in F_A(y) \cup U_1 \subseteq F_A(x) \cup U_1$ this implies $a \in F_A(x)$ or $a \in U_1$ but $a \notin U_1$ because $F_A(y) \cap U_1 = \emptyset$. So $a \in F_A(x)$ that is $F_A(x) \supseteq F_A(y)$, for all $x, y \in X$. □

Proposition 7. *Let F_A be a soft subalgebra of X and $U_1 \subseteq T$. Then the soft U_1-translation $F^T_{U_1}$ of F_A is a soft subalgebra of X.*

Proof. Let $x, y \in X$. Then

$$\begin{aligned} F^T_{U_1}(x * y) &= F_A(x * y) \cup U_1 \\ &\supseteq (F_A(x) \cap F_A(y)) \cup U_1 \\ &= (F_A(x) \cup U_1) \cap (F_A(y) \cup U_1) \\ &= (F^T_{U_1}(x)) \cap (F^T_{U_1}(y)). \end{aligned} \tag{4}$$

Hence $F^T_{U_1}$ is a soft subalgebra of X. □

Proposition 8. *Let F_A be a soft set over X such that the U_1-translation $F^T_{U_1}$ of F_A is a soft subalgebra of X for some $U_1 \subseteq T$. Then F_A is a soft subalgebra of X.*

Proof. Assume $F_{U_1}^{T}$ is a soft subalgebra of X for some $U_1 \subseteq T$. Let $x, y \in X$, we have

$$\begin{aligned} F_A(x*y)\cup U_1 &= F_{U_1}^{T}(x*y) \\ &\supseteq F_{U_1}^{T}(x)\cap F_{U_1}^{T}(y) \\ &= (F_A(x)\cup U_1)\cap(F_A(y)\cup U_1) \\ &= (F_A(x)\cap(y))\cup U_1. \end{aligned} \tag{5}$$

Now by Lemma 6 we have

$$F_A(x*y)\supseteq F_A(x)\cap F_A(y), \tag{6}$$

for all $x, y \in X$. Hence F_A is a soft subalgebra of X. □

From Propositions 7 and 8 we have the following.

Theorem 9. *A soft set F_A of X is a soft subalgebra of X if and only if U_1-translation $F_{U_1}^{T}$ of F_A is a soft subalgebra of X for some $U_1 \subseteq T$.*

Definition 10. Let F_A and G_B be two soft sets over X. If $F_A(x) \subseteq G_B(x)$ for all $x \in X$, then we say that G_B is a soft extension of F_A.

Example 11. Consider a BCI/BCK-algebra $X = \{0,1,2,3\}$ presented as follows:

$$\begin{array}{c|cccc} * & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 2 & 2 & 2 & 0 & 2 \\ 3 & 3 & 3 & 3 & 0 \end{array} \tag{7}$$

Define two soft sets F_A and G_B of X as in Table 1.

Here $F_A(0) \subseteq G_B(0)$, $F_A(1) \subseteq G_B(1)$, $F_A(2) \subseteq G_B(2)$, and $F_A(3) \subseteq G_B(3)$, which implies that G_B is a soft extension of F_A.

Next the concept of soft S-extension is being introduced.

Definition 12. Let F_A and G_B be two soft sets over X. Then G_B is called a soft S-extension of F_A, if the following conditions hold:

(1) G_B is a soft extension of F_A.

(2) If F_A is a soft subalgebra of X, then G_B is a soft subalgebra of X.

As we know $F_{U_1}^{T}(x) \supseteq F_A(x)$ for all $x \in X$. As a consequence of Definition 12 and Theorem 9, we have the following.

Theorem 13. *Let F_A be a soft subalgebra of X and $U_1 \subseteq$* **T**. *Then the soft U_1-translation $F_{U_1}^{T}$ of F_A is a soft S-extension of F_A.*

The converse of Theorem 13 is not true in general as seen in the following example.

Table 1

X	0	1	2	3
F_A	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{1,2\}$
G_B	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$	$\{0,1,2\}$

Example 14. Consider a BCI/BCK-algebra $X = \{0,1,2,3\}$ given as follows:

$$\begin{array}{c|cccc} * & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 2 & 2 & 2 & 0 & 2 \\ 3 & 3 & 3 & 3 & 0 \end{array} \tag{8}$$

Define a soft set F_A of X by Table 2.

Then F_A is a soft subalgebra of X. For soft set F_A, $T = \{3\}$. Let G_B be a soft set over X given by Table 3.

Then G_B is a soft S-extension of X. But it is not a soft U_1-translation of F_A for any nonempty $U_1 \subseteq T$.

For a soft set F_A of X, $U_1 \subseteq T$ and $U_2 \in P(X)$ with $U_2 \supseteq U_1$, let

$$U_{U_1}(F_A; U_2) := \{x \in X \mid F_A(x) \supseteq U_2 - U_1\}. \tag{9}$$

If F_A is a soft subalgebra of X, then it is clear that $U_{U_1}(F_A; U_2)$ is a subalgebra of X for all $U_2 \in P(X)$ with $U_2 \supseteq U_1$. But, if we do not give condition that F_A is a soft subalgebra of X, then $U_{U_1}(F_A; U_2)$ may not be a subalgebra of X as seen in the following example.

Example 15. Let $X = \{0,1,2,3,4\}$ be a BCI/BCK-algebra presented as follows:

$$\begin{array}{c|ccccc} * & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 1 & 0 & 0 & 0 \\ 3 & 3 & 1 & 1 & 0 & 0 \\ 4 & 4 & 3 & 3 & 1 & 0 \end{array} \tag{10}$$

Define a soft subset F_A of X by Table 4.

Then F_A is not a soft subalgebra of X with $T = \{1\}$. Since $F_A(3*4) = \{0\} \nsupseteq \{0,4\} = F_A(3)\cap F_A(4)$ For $U_2 = \{1,4\}$ and $U_1 = \{1\}$, we obtain $U_{U_1}(F_A; U_2) = \{3,4\}$ which is not a subalgebra of X since $3*3 = 0 \notin U_{U_1}(F_A; U_2)$.

In the following theorem, relationship between U_1-translations and $U_{U_1}(F_A; U_2)$ is studied in case of soft subalgebra.

Theorem 16. *Let F_A be a soft set over X and $U_1 \subseteq T$. Then the soft U_1-translation $F_{U_1}^{T}$ of F_A is a soft subalgebra of X if and only if $U_{U_1}(F_A; U_2)$ is a subalgebra of X for all $U_2 \in P(U)$ with $U_2 \supseteq U_1$.*

Proof. Assume that the soft U_1-translation $F_{U_1}^{T}$ of F_A is a soft subalgebra of X. Then by Theorem 9, F_A is a soft subalgebra

Table 2

X	0	1	2	3
F_A	$\{0,1,2\}$	$\{0,1\}$	$\{0,2\}$	$\{1,2\}$

Table 3

X	0	1	2	3
G_B	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,2\}$	$\{0,1,2\}$

Table 4

X	0	1	2	3	4
F_A	$\{0\}$	$\{0,2\}$	$\{0,2,3\}$	$\{0,3,4\}$	$\{0,4\}$

of X if $F^T_{U_1}$ is a soft subalgebra of X. Further let $a, b \in U_{U_1}(F_A; U_2)$, then $F_A(a) \supseteq U_2 - U_1$ and $F_A(b) \supseteq U_2 - U_1$ are subalgebras of X for all $U_2 \in P(U)$ with $U_2 \supseteq U_1$. Consider

$$F_A(a * b) \supseteq F_A(a) \cap F_A(b) \supseteq U_2 - U_1. \tag{11}$$

Therefore $a * b \in U_{U_1}(F_A; U_2)$, which shows that $U_{U_1}(F_A; U_2)$ is a subalgebra of X, for all $U_2 \subseteq P(U)$, with $U_2 \supseteq U_1$.

Conversely, suppose that $U_{U_1}(F_A; U_2)$ is a subalgebra of X for all $U_2 \subseteq P(U)$ with $U_2 \supseteq U_1$. Now assume that there exist $a, b \in X$ such that

$$F^T_{U_1}(a * b) \subset U_2 \subseteq F^T_{U_1}(a) \cap F^T_{U_1}(b). \tag{12}$$

Then $F_A(a) \supseteq U_2 - U_1$ and $F_A(b) \supseteq U_2 - U_1$ but $F_A(a * b) \subset U_2 - U_1$. This shows that $a, b \in U_{U_1}(F_A; U_2)$ and $a * b \notin U_{U_1}(F_A; U_2)$, which is a contradiction and so $F^T_{U_1}(a * b) \supseteq F^T_{U_1}(a) \cap F^T_{U_1}(b)$ for all $a, b \in X$. Hence $F^T_{U_1}$ is a soft subalgebra of X. □

Theorem 17. *Let F_A be a soft subalgebra of X and let $U_1, U_2 \subseteq T$. If $U_1 \supseteq U_2$, then the soft U_1-translation $F^T_{U_1}$ of F_A is a soft S-extension of the soft U_2-translation $F^T_{U_2}$ of F_A.*

Proof. Since $U_1 \supseteq U_2$, this implies $F^T_{U_1}(x) \supseteq F^T_{U_2}(x)$, for all $x \in X$. So U_1-translation is an extension of U_2-translation, and from Theorem 9, $F^T_{U_1}$ and $F^T_{U_2}$ are soft subalgebras of F_A. Hence soft U_1-translation $F^T_{U_1}$ of F_A is a soft S-extension of the soft U_2-translation $F^T_{U_2}$ of F_A. □

For every soft subalgebra F_A of X and $U_2 \subseteq T$, the soft U_2-translation $F^T_{U_2}$ of F_A is a soft subalgebra of X. If G_B is a soft S-extension of $F^T_{U_2}$ and then there exists $U_1 \subseteq T$ such that $U_1 \supseteq U_2$ and $G_B(x) \supseteq F^T_{U_1}(x)$, for all $x \in X$. Thus, we have the following theorem.

Theorem 18. *Let F_A be a soft subalgebra of X and $U_2 \subseteq T$. For every soft S-extension G_B of soft U_2-translation $F^T_{U_2}$ of F_A, there exists a $U_1 \subseteq T$ such that $U_1 \supseteq U_2$ and G_B are a soft S-extension of U_1-translation of F_A.*

Proof. For every soft subalgebra F_A of X and $U_2 \subseteq T$, the soft U_2-translation $F^T_{U_2}$ of F_A is a soft subalgebra of X. If G_B is a soft S-extension of $F^T_{U_2}$ and then there exists $U_1 \subseteq T$ such that $U_1 \supseteq U_2$ and $G_B(x) \supseteq F^T_{U_1}(x)$, for all $x \in X$. Then by Theorem 17, G_B is a soft S-extension of U_1-translation of F_A. □

Definition 19. A soft S-extension G_B of a soft subalgebra F_A of X is said to be normalized if there exists $x_0 \in X$ such that $G_B(x_0) = X$.

Definition 20. Let F_A be a soft subalgebra of X. A soft set G_B of X is called a maximal soft S-extension of F_A if it satisfies the following conditions:

(1) G_B is a soft S-extension of F_A,

(2) there does not exist another soft subalgebra of X which is a soft extension of G_B.

Example 21 (see [33]). Let $\mathbb{Z}^+$ be a set of positive integers and let "$*$" be a binary operation on $\mathbb{Z}^+$ defined by

$$x * y = \frac{x}{(x, y)}, \tag{13}$$

$\forall x, y \in \mathbb{Z}^+$, where (x, y) is the greatest common divisor of x and y. Then $(\mathbb{Z}^+; *, 1)$ is a BCK-algebra. Let F_A and G_B be soft sets of $\mathbb{Z}^+$ which are defined by $F_A(x) = \{1, 2, 3\}$ and $G_B(x) = \mathbb{Z}^+$ for all $x \in \mathbb{Z}^+$. Clearly, F_A and G_B are soft subalgebras of $\mathbb{Z}^+$. By using definition of maximal soft S-extension, then it is easy to see that G_B is a maximal soft S-extension of F_A.

Proposition 22. *If a soft set G_B of X is a normalized soft S-extension of a soft subalgebra F_A of X, then $G_B(0) = X$.*

Proof. Assume that G_B is a normalized soft S-extension of a soft subalgebra F_A of X then there exists $x_0 \in X$ such that $G_B(x_0) = X$, for some $x_0 \in X$. Consider

$$G_B(0) = G_B(x_0 * x_0) \supseteq G_B(x_0) \cap G_B(x_0) = X. \tag{14}$$

This implies $G_B(0) = X$. □

Theorem 23. *Let F_A be a soft subalgebra of X. Then every maximal soft S-extension of F_A is normalized.*

Proof. This follows from the definitions of the maximal and normalized soft S-extensions. □

4. Soft Translations of Soft Ideals in Soft BCI/BCK-Algebras

Now concept of translation of a soft ideal of a BCI/BCK-algebra is introduced.

Definition 24. A soft subset F_A of a BCI/BCK-algebra is called a soft ideal of X, denoted by $F_A \lhd_S X$, if it satisfies:

(1) $(\forall x \in X)\,(F_A(0) \supseteq F_A(x))$,

(2) $(\forall x, y \in X)\,(F(x) \supseteq (F_A(x * y) \cap F_A(y)))$.

Theorem 25. *If F_A is a soft subset of X, then F_A is a soft ideal of X if and only if soft U_1-translation $F^T_{U_1}$ of F_A is a soft ideal of X for all $U_1 \subseteq T$.*

Proof. Assume that $F_A \lhd_S X$ and let $U_1 \subseteq T$. Then $F^T_{U_1}(0) = F_A(0) \cup U_1 \supseteq F_A(x) \cup U_1 = F^T_{U_1}(x)$ and

$$\begin{aligned} F^T_{U_1}(x) &= F_A(x) \cup U_1 \supseteq (F_A(x * y) \cap F_A(y)) \cup U_1 \\ &= (F_A(x * y) \cup U_1) \cap (F_A(y) \cup U_1) \\ &= F^T_{U_1}(x * y) \cap F^T_{U_1}(y) \quad \forall x, y \in X. \end{aligned} \tag{15}$$

Hence $F^T_{U_1} \lhd_S X$.

Conversely, assume that $F^T_{U_1}$ is a soft ideal of X for some $U_1 \subseteq T$. Let $x, y \in X$. Then

$$\begin{aligned} F^T_{U_1}(0) \supseteq F^T_{U_1}(x) &\Longrightarrow F_A(0) \cup U_1 \supseteq F_A(x) \cup U_1 \\ &\Longrightarrow F_A(0) \supseteq F_A(x) \ \text{ by Lemma 6,} \end{aligned} \tag{16}$$

and so $F_A(0) \supseteq F_A(x)$. Next

$$\begin{aligned} F_A(x) \cup U_1 &= F^T_{U_1}(x) \\ &\supseteq F^T_{U_1}(x * y) \cap F^T_{U_1}(y) \\ &= (F_A(x * y) \cup U_1) \cap (F_A(y) \cup U_1) \\ &= (F_A(x * y) \cap F_A(y)) \cup U_1, \end{aligned} \tag{17}$$

which implies that $F_A(x) \supseteq F_A(x * y) \cap F_A(y)$ (by Lemma 6). Hence F_A is a soft ideal of X. □

5. Soft Extensions and Soft Ideal Extensions of Soft Subalgebras

In this section concept of soft ideal extension is being introduced and some of its properties are studied.

Definition 26. Let F_A and G_B be the soft subsets of X. Then G_B is called the soft ideal extension of F_A, if the following conditions hold:

(1) G_B is a soft extension of F_A.

(2) $F_A \lhd_S X \Rightarrow G_B \lhd_S X$.

For a soft subset F_A of X, $U_1 \subseteq T$ and $U_2 \in P(X)$ with $U_2 \supseteq U_1$, define $E_{U_1}(F_A; U_2) := \{x \in X \mid F_A(x) \cup U_1 \supseteq U_2\}$.

It is clear that if $F_A \lhd_S X$, then $U_{U_1}(F_A; U_2) \lhd X$ for all $U_2 \in P(U)$ with $U_2 \supseteq U_1$.

Theorem 27. *For $U_1 \subseteq T$, let $F^T_{U_1}$ be the soft U_1-translation of F_A. Then the following are equivalent:*

(1) $F^T_{U_1} \lhd_S X$.

(2) $(\forall U_2 \in P(U))\,(U_2 \supset U_1 \Rightarrow E_{U_1}(F_A; U_2) \lhd X)$.

Proof. (1) ⇒ (2) Consider $F^T_{U_1} \lhd_S X$ and let $U_2 \in P(U)$ be such that $U_2 \supset U_1$. Since $F^T_{U_1}(0) \supseteq F^T_{U_1}(x)$ for all $x \in X$, we have

$$F_A(0) \cup U_1 = F^T_{U_1}(0) \supseteq F^T_{U_1}(x) = F_A(x) \cup U_1 \supseteq U_2, \tag{18}$$

for $x \in E_{U_1}(F_A; U_2)$.

$$\text{Hence } 0 \in E_{U_1}(F_A; U_2). \tag{19}$$

Let $x, y \in X$ be such that $x * y \in E_{U_1}(F_A; U_2)$ and $y \in E_{U_1}(F_A; U_2)$. Then $F_A(x * y) \cup U_1 \supseteq U_2$ and $F_A(y) \cup U_1 \supseteq U_2$, that is, $F^T_{U_1}(x * y) = F_A(x * y) \cup U_1 \supseteq U_2$ and $F^T_{U_1}(y) = F_A(y) \cup U_1 \supseteq U_2$. Since $F^T_{U_1} \lhd_S X$, it follows that

$$F_A(x) \cup U_1 = F^T_{U_1}(x) \supseteq F^T_{U_1}(x * y) \cap F^T_{U_1}(y) \supseteq U_2, \tag{20}$$

that is, $F_A(x) \cup U_1 \supseteq U_2$ so that $x \in E_{U_1}(F_A; U_2)$. Therefore $E_{U_1}(F_A; U_2) \lhd X$.

(2) ⇒ (1) Suppose that $E_{U_1}(F_A; U_2) \lhd X$ for every $U_2 \in P(U)$ with $U_2 \supseteq U_1$. If there exists $x \in X$ with $U_3 \supseteq U_1$ such that $F^T_{U_1}(0) \subset U_3 \subseteq F^T_{U_1}(x)$ and then $F_A(x) \cup U_1 \supseteq U_3$ but $F_A(0) \cup U_1 \subset U_3$. This shows that $x \in E_{U_1}(F_A; U_2)$ and $0 \notin E_{U_1}(F_A; U_2)$. This is a contradiction, and so $F^T_{U_1}(0) \supseteq F^T_{U_1}(x)$, for all $x \in X$.

Now assume that there exist $a, b \in X$ such that $F^T_{U_1}(a) \subset U_4 \subseteq F^T_{U_1}(a * b) \cap F^T_{U_1}(b)$. Then $F_A(a * b) \cup U_1 \supseteq U_4$ and $F_A(b) \cup U_1 \supseteq U_4$, but $F_A(a) \cup U_1 \subset U_4$. Hence $a * b \in E_{U_1}(F_A; U_4)$ and $b \in E_{U_1}(F_A; U_4)$, but $a \notin E_{U_1}(F_A; U_4)$. This is impossible and therefore $F^T_{U_1}(x) \supseteq F^T_{U_1}(x * y) \cap F^T_{U_1}(y)$, for all $x, y \in X$. Consequently $F^T_{U_1} \lhd_S X$. □

Theorem 28. *Let $F_A \lhd_S X$ and $U_1, U_2 \subseteq T$. If $U_1 \supseteq U_2$, then the soft U_1-translation $F^T_{U_1}$ of F_A is a soft ideal extension of the soft U_2-translation $F^T_{U_2}$ of F_A.*

Proof. Since

$$F^T_{U_1}(x) = F_A(x) \cup U_1, \qquad F^T_{U_2}(x) = F_A(x) \cup U_2, \tag{21}$$

$U_1 \supseteq U_2$, this implies that $(F^T_{U_1}(x) \supseteq F^T_{U_2}(x))$ $(\forall x \in X)$. This shows that $F^T_{U_1}$ is a soft extension of $F^T_{U_2}$.

Now, let $F^T_{U_2}$ is a soft ideal of X, then $F^T_{U_1}(0) = F_A(0) \cup U_1 \supseteq F_A(x) \cup U_1 = F^T_{U_1}(x)$ for all $x \in X$, so we have $(F^T_{U_1}(0) \supseteq F^T_{U_1}(x))$. Consider

$$\begin{aligned} F^T_{U_1}(x) &= F_A(x) \cup U_1 \\ &\supseteq (F_A(x * y) \cap F_A(y)) \cup U_1 \\ &= (F_A(x * y) \cup U_1) \cap (F_A(y) \cup U_1) \\ &= F^T_{U_1}(x * y) \cap F^T_{U_1}(y) \ \text{ for all} x, y \in X. \end{aligned} \tag{22}$$

That is $(F^T_{U_1}(x) \supseteq F^T_{U_1}(x * y) \cap F^T_{U_1}(y))$ $(\forall x, y \in X)$ so $F^T_{U_1}$ is a soft ideal of X. Hence $F^T_{U_1}$ is a soft ideal extension of $F^T_{U_2}$. □

6. Conclusion

Soft set theory is a mathematical tool to deal with uncertainties. Translation and extension are very useful concepts in mathematics to reduce the complexity of a problem. These concepts are frequently employed in geometry and algebra. In this papers, we presented some new notions such as soft translations and soft extensions for BCI/BCK-algebras. We also examined some relationships between soft translations and soft extensions. Moreover, soft ideal extensions and translations have been introduced and investigated as well. It is hoped that these results may be helpful in other soft structures as well.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

Authors are grateful to referees and Professor Feng Feng, the Lead Guest Editor of this special issue, for his kind suggestions to improve this paper.

References

[1] P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," *Computers & Mathematics with Applications*, vol. 45, no. 4-5, pp. 555–562, 2003.

[2] D. Molodtsov, "Soft set theory: first results," *Computers & Mathematics with Applications*, vol. 37, no. 4-5, pp. 19–31, 1999.

[3] M. I. Ali, F. Feng, X. Liu, W. K. Min, and M. Shabir, "On some new operations in soft set theory," *Computers & Mathematics with Applications*, vol. 57, no. 9, pp. 1547–1553, 2009.

[4] P. K. Maji, A. R. Roy, and R. Biswas, "An application of soft sets in a decision making problem," *Computers & Mathematics with Applications*, vol. 44, no. 8-9, pp. 1077–1083, 2002.

[5] H. Aktaş and N. Çağman, "Soft sets and soft groups," *Information Sciences*, vol. 177, no. 13, pp. 2726–2735, 2007.

[6] D. Chen, E. C. C. Tsang, D. S. Yeung, and X. Wang, "The parameterization reduction of soft sets and its applications," *Computers & Mathematics with Applications*, vol. 49, no. 5-6, pp. 757–763, 2005.

[7] Z. Kong, L. Gao, and L. Wang, "Comment on "a fuzzy soft set theoretic approach to decision making problems"," *Journal of Computational and Applied Mathematics*, vol. 223, no. 2, pp. 540–542, 2009.

[8] F. Feng, C. Li, B. Davvaz, and M. I. Ali, "Soft sets combined with fuzzy sets and rough sets: a tentative approach," *Soft Computing*, vol. 14, no. 9, pp. 899–911, 2010.

[9] F. Feng, X. Liu, and V. Leoreanu-Fotea, "Soft sets and soft rough sets," *Information Sciences*, vol. 181, no. 6, pp. 1125–1137, 2011.

[10] M. I. Ali, "A note on soft sets, rough soft sets and fuzzy soft sets," *Applied Soft Computing Journal*, vol. 11, no. 4, pp. 3329–3332, 2011.

[11] C. H. Park, Y. B. Jun, and M. Öztürk, "Soft WS-algebras," *Communications of the Korean Mathematical Society*, vol. 23, no. 3, pp. 313–324, 2008.

[12] Y. B. Jun and C. H. Park, "Applications of soft sets in ideal theory of BCK/BCI-algebras," *Information Sciences*, vol. 178, no. 11, pp. 2466–2475, 2008.

[13] A. Aygünoğlu and H. Aygün, "Introduction to fuzzy soft groups," *Computers & Mathematics with Applications*, vol. 58, no. 6, pp. 1279–1286, 2009.

[14] N. Çağman and S. Enginoğlu, "Soft matrix theory and its decision making," *Computers & Mathematics with Applications*, vol. 59, no. 10, pp. 3308–3314, 2010.

[15] N. Çağman and S. Engino, "Soft set theory and uni-int decision making," *European Journal of Operational Research*, vol. 207, pp. 848–855, 2010.

[16] F. Feng, Y. Li, and N. Çağman, "Generalized *uni-int* decision making schemes based on choice value soft sets," *European Journal of Operational Research*, vol. 220, no. 1, pp. 162–170, 2012.

[17] F. Feng and Y. Li, "Soft subsets and soft product operations," *Information Sciences*, vol. 232, pp. 44–57, 2013.

[18] F. Feng, H. Fujita, Y. B. Jun, and M. Khan, "Decomposition of fuzzy soft sets with finite value spaces," *The Scientific World Journal*, vol. 2014, Article ID 902687, 10 pages, 2014.

[19] X. Liu, F. Feng, and Y. B. Jun, "A note on generalized soft equal relations," *Computers & Mathematics with Applications*, vol. 64, no. 4, pp. 572–578, 2012.

[20] X. Liu, F. Feng, and H. Zhang, "On some nonclassical algebraic properties of interval-valued fuzzy soft sets," *The Scientific World Journal*, vol. 2014, Article ID 192957, 11 pages, 2014.

[21] X. Ma and H. S. Kim, "(M;N)-soft intersection BL-algebras and their congruences," *The Scientific World Journal*, vol. 2014, Article ID 461060, 6 pages, 2014.

[22] G. Muhiuddin, F. Feng, and Y. B. Jun, "Subalgebras of BCK/BCI-algebras based on cubic soft sets," *The Scientific World Journal*, vol. 2014, Article ID 458638, 9 pages, 2014.

[23] A. S. Sezer, "A new view to ring theory via soft union rings, ideals and bi-ideals," *Knowledge-Based Systems*, vol. 36, pp. 300–314, 2012.

[24] A. S. Sezer, A. O. Atagün, and N. Çağman, "Soft intersection near-rings with its applications," *Neural Computing and Applications*, vol. 21, no. 1, pp. 221–229, 2012.

[25] X. Xin and W. Li, "Soft congruence relations over rings," *The Scientific World Journal*, vol. 2014, Article ID 541630, 9 pages, 2014.

[26] Y. Imai and K. Iseki, "On axiom systems of propositional calculi, XIV," *Proceedings of the Japan Academy*, vol. 42, no. 1, pp. 19–22, 1966.

[27] Y. B. Jun and S. Z. Song, "Fuzzy set theory applied to implicative ideals in BCK-algebras," *Bulletin of the Korean Mathematical Society*, vol. 43, no. 3, pp. 461–470, 2006.

[28] J. B. Jun and X. L. Xin, "Involutory and invertible fuzzy BCK-algebras," *Fuzzy Sets and Systems*, vol. 117, no. 3, pp. 463–469, 2001.

[29] Y. B. Jun and J. Meng, "Fuzzy commutative ideals in BCI-algebras," *Communications of the Korean Mathematical Society*, vol. 9, no. 1, pp. 19–25, 1994.

[30] J. Meng, Y. B. Jun, and H. S. Kim, "Fuzzy implicative ideals of BCK-algebras," *Fuzzy Sets and Systems*, vol. 89, no. 2, pp. 243–248, 1997.

[31] Y. B. Jun, "Soft BCI/BCK-algebras," *Computers & Mathematics with Applications*, vol. 56, no. 5, pp. 1408–1413, 2008.

[32] J. Meng and Y. B. Jun, *BCK-Algebras*, Kyung Moon, Seoul, South Korea, 1994.

[33] K. J. Lee, Y. B. Jun, and M. I. Doh, "Fuzzy translations and fuzzy multiplications of BCK/BCI-algebras," *Communications of the Korean Mathematical Society*, vol. 24, no. 3, pp. 353–360, 2009.