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Research Article

Stabilities for Nonisentropic Euler-Poisson Equations

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We establish the stabilities and blowup results for the nonisentropic Euler-Poisson equations by the energy method. By analysing the second inertia, we show that the classical solutions of the system with attractive forces blow up in finite time in some special dimensions when the energy is negative. Moreover, we obtain the stabilities results for the system in the cases of attractive and repulsive forces.

1. Introduction

The compressible nonisentropic Euler ($\delta = 0$) or Euler-Poisson ($\delta = \pm 1$) system for fluids can be written as

$$\begin{gathered} \rho_t + \nabla \cdot (\rho u) = 0, \\ (\rho u)_t + \nabla \cdot (\rho u \otimes u) + \nabla P + \beta \rho u = -\delta \rho \nabla \Phi, \\ S_t + u \cdot \nabla S = 0, \\ \Delta \Phi (t, x) = \alpha (N) \rho, \\ P = K \rho^{\gamma} e^{S}, \end{gathered} \tag{1}$$

where $\beta \geq 0$ is the frictional damping constant and $\alpha(N)$ is a constant related to the unit ball in $\mathbb{R}^N$. $\alpha(1) = 2$, $\alpha(2) = 2\pi$ and

$$\alpha (N) = N (N-2) V (N), \tag{2}$$

where $V(N)$ is the volume of the unit ball in $\mathbb{R}^N$. As usual, $\rho = \rho(t,x) \geq 0$, $u = u(t,x) \in \mathbb{R}^N$, and $S(t,x)$ are the density, the velocity, and the entropy, respectively. P is the pressure function, for which the constants $K \geq 0$ and $\gamma \geq 1$.

When $\delta = 1$, the system is self-attractive. The system (1) is the Newtonian description of gaseous stars [1]. When $\delta = -1$, the system comprises the Euler-Poisson equations with repulsive forces and can be used as a semiconductor model [2, 3]. When $\delta = 0$, the system comprises the compressible Euler equations and can be applied as a classical model in fluid mechanics [3]. For more classical and recent results in these systems, readers can refer to [1, 4–10].

It is well known that the solution for the Poisson equation $(1)_4$ can be written as

$$\Phi (t, x) = \alpha (N) \int_{\mathbb{R}^N} G (x - y) \rho (t, y) \, dy, \tag{3}$$

where G is the Green's function for the Poisson equation in the N-dimensional spaces defined by

$$G (x) := \begin{cases} |x|, & N = 1; \\ \log |x|, & N = 2; \\ \dfrac{-1}{|x|^{N-2}}, & N \geq 3. \end{cases} \tag{4}$$

Notation. In the following discussion, classical solutions (ρ, u, S) are C^1 solutions with compact support $\Omega = \Omega(t)$ for each fixed time t. We also denote the total mass by M, where

$$M = \int_{\Omega} \rho \, dx = \int_{\Omega(0)} \rho_0 \, dx, \tag{5}$$

where $\rho_0 = \rho_0(x) := \rho(0, x)$.

Lastly, we will denote

$$R (t) = \text{the diameter of } \Omega (t). \tag{6}$$

2. Lemmas

In this section, we establish some lemmas for the proof of the main results. The following lemma will be used to derive the energy functional for $\gamma > 1$; namely,

$$E(t) = \int_\Omega \left(\frac{1}{2}\rho|u|^2 + \frac{1}{\gamma-1}P\right)dx + \frac{\delta}{2}\int_\Omega \rho\Phi\,dx \quad (7)$$

is conserved in time if the system (1) is not damped.

Lemma 1. *For the classical solution (ρ, u, S) of system (1), we have*

$$\int_\Omega P_t dx = (\gamma-1)\int_\Omega u\cdot\nabla P\,dx, \quad (8)$$

where P is defined by $(1)_5$.

Proof. We have

$$\begin{aligned}
P_t &= K\left(\rho^\gamma e^S S_t + e^S\gamma\rho^{\gamma-1}\rho_t\right) \quad \text{by } (1)_5 \\
&= PS_t + Ke^S\gamma\rho^{\gamma-1}\left[-\nabla\cdot(\rho u)\right] \quad \text{by } (1)_5 \text{ and } (1)_1 \\
&= P(-u\cdot\nabla S) + Ke^S\gamma\rho^{\gamma-1}\left[-\nabla\cdot(\rho u)\right] \quad \text{by } (1)_3 \\
&= -P\left(\sum_{i=1}^N u_i\partial_i S\right) - K\gamma e^S\rho^{\gamma-1}\left[\sum_{i=1}^N(\rho\partial_i u_i + u_i\partial_i\rho)\right] \\
&= -\gamma P\sum_{i=1}^N\partial_i u_i - \sum_{i=1}^N\left[P\partial_i S + K\gamma e^S\rho^{\gamma-1}\partial_i\rho\right]u_i \quad \text{by } (1)_5 \\
&= -\gamma P\sum_{i=1}^N\partial_i u_i - \sum_{i=1}^N(\partial_i P)u_i \\
&= -\gamma P\nabla\cdot u - u\cdot\nabla P.
\end{aligned} \quad (9)$$

Note that, by Divergence Theorem,

$$-\int_\Omega P\nabla\cdot u\,dx = \int_\Omega u\cdot\nabla P\,dx. \quad (10)$$

Thus,

$$\int_\Omega P_t\,dx = (\gamma-1)\int_\Omega u\cdot\nabla P\,dx. \quad (11)$$

□

Next, the results of the following two lemmas will be used in the derivations of the energy functionals for both $\gamma > 1$ and $\gamma = 1$. It will be shown that in Section 3 the energy functional for $\gamma = 1$ is

$$E(t) = \int_\Omega\left(\frac{1}{2}\rho|u|^2 + K\rho e^S(\ln\rho - 1)\right)dx + \frac{\delta}{2}\int_\Omega\rho\Phi\,dx \quad (12)$$

which is conversed in time if the system (1) is not damped.

Lemma 2. *For the classical solution (ρ, u, S) of system (1), we have*

$$\int_\Omega\left(\frac{1}{2}\rho|u|^2\right)_t dx = -\int_\Omega u\cdot\nabla P - \delta\int_\Omega(\nabla\Phi)\cdot(\rho u)\,dx - \beta\int_\Omega\rho|u|^2\,dx, \quad (13)$$

where P is defined by $(1)_5$ and Φ is the solution of $(1)_4$.

Proof. We have

$$\left(\frac{1}{2}\rho|u|^2\right)_t \quad (14)$$

$$= (\rho u)_t\cdot u - \frac{1}{2}\rho_t|u|^2 \quad \text{(by product rule)} \quad (15)$$

$$\begin{aligned}
&= -(\delta\rho\nabla\Phi + \nabla\cdot(\rho u\otimes u) + \nabla P + \beta\rho u)\cdot u \\
&\quad + \frac{1}{2}(\nabla\cdot(\rho u))|u|^2 \quad \text{by } (1)_1 \text{ and } (1)_2.
\end{aligned} \quad (16)$$

One can check a detail proof of the following equality in the Appendix:

$$\int_\Omega\left(\frac{1}{2}|u|^2\nabla\cdot(\rho u) - u\cdot[\nabla\cdot(\rho u\otimes u)]\right)dx = 0. \quad (17)$$

Thus,

$$\int_\Omega\left(\frac{1}{2}\rho|u|^2\right)_t dx = -\int_\Omega(\delta\rho\nabla\Phi + \nabla P + \beta\rho u)\cdot u\,dx \quad (18)$$

by (16) and (17).

Thus, the proof is complete. □

Lemma 3. *For the classical solution (ρ, u, S) of system (1), we have*

$$\int_\Omega(\nabla\Phi)\cdot(\rho u)\,dx = \int_\Omega\Phi\rho_t dx = \frac{1}{2}\int_\Omega[\rho\Phi]_t\,dx, \quad (19)$$

where Φ is the solution of $(1)_4$.

Proof. We have

$$\begin{aligned}
&\int_\Omega(\nabla\Phi)\cdot(\rho u)\,dx \\
&= -\int_\Omega\Phi\nabla\cdot(\rho u)\,dx \quad \text{by Divergence Theorem} \\
&= \int_\Omega\Phi\rho_t dx \quad \text{by } (1)_1.
\end{aligned} \quad (20)$$

Thus, the first equality in (19) holds.

Next,

$$\begin{aligned}
\int_\Omega\Phi\rho_t dx &= \frac{1}{\alpha(N)}\int_\Omega\Phi\Delta\Phi_t dx \quad \text{by } (1)_4 \\
&= \frac{1}{\alpha(N)}\int_\Omega\Delta\Phi\Phi_t dx \quad \text{by Green's Formula} \\
&= \int_\Omega\rho\Phi_t dx \quad \text{by } (1)_4.
\end{aligned} \quad (21)$$

Thus, the second equality in (19) holds. □

The lemma below is crucial to obtaining the energy functional for $\gamma = 1$. Comparing the left hand sides of (8) and (22), we note that the left hand side of (22) (given in the next lemma), which contains the term $\ln\rho - 1$, is nontrivial to be found.

Lemma 4. *For the classical solution (ρ, u, S) of system (1) with $\gamma = 1$, we have*

$$\frac{d}{dt}\int_\Omega K\rho e^S(\ln\rho - 1)\,dx = \int_\Omega u\cdot\nabla P\,dx, \quad (22)$$

where P is defined by $(1)_5$.

Proof. Note that

$$\begin{aligned}
&[\rho e^S(\ln\rho - 1)]_t \\
&= e^S[\rho(\ln\rho - 1)]_t + [\rho(\ln\rho - 1)]e^S S_t \\
&= e^S[\rho_t + (\ln\rho - 1)\rho_t] + e^S\rho(\ln\rho - 1)S_t \\
&= e^S(\ln\rho)\rho_t + e^S\rho(\ln\rho - 1)S_t \\
&= -e^S(\ln\rho)(\nabla\cdot(\rho u)) - e^S\rho(\ln\rho - 1)(u\cdot\nabla S) \\
&\qquad \text{by } (1)_1 \text{ and } (1)_3.
\end{aligned} \quad (23)$$

Thus,

$$\begin{aligned}
&\frac{d}{dt}\int_\Omega K\rho e^S(\ln\rho - 1)\,dx \\
&= -\int_\Omega Ke^S(\ln\rho)[\nabla\cdot(\rho u)]\,dx \\
&\quad - \int_\Omega Ke^S\rho(\ln\rho - 1)(u\cdot\nabla S)\,dx \\
&= \int_\Omega (\rho u)\cdot\nabla[Ke^S(\ln\rho)]\,dx \\
&\quad - \int_\Omega Ke^S\rho(\ln\rho - 1)(u\cdot\nabla S)\,dx \\
&\qquad \text{by Divergence Theorem} \\
&= \int_\Omega u\cdot[K\rho\nabla(e^S\ln\rho) - Ke^S\rho(\ln\rho - 1)\nabla S]\,dx \\
&= \int_\Omega u\cdot[Ke^S\nabla\rho + Ke^S\rho\nabla S]\,dx \\
&= \int_\Omega u\cdot\nabla P\,dx \quad \text{by } (1)_5.
\end{aligned} \quad (24)$$

The proof is complete. □

3. Main Results

In this section, we find out the energy functionals for the system (1) in the case of $\gamma > 1$ (Proposition 5) and $\gamma = 1$ (Proposition 6). Moreover, we establish the stabilities results (Proposition 8) and a blowup result (Proposition 9) for system (1).

Proposition 5. *For the classical solution (ρ, u, S) of system (1) with $\gamma > 1$, let*

$$E(t) = \int_\Omega\left(\frac{1}{2}\rho|u|^2 + \frac{1}{\gamma - 1}P\right)dx + \frac{\delta}{2}\int_\Omega\rho\Phi\,dx. \quad (25)$$

Then,

$$\dot{E}(t) = -\beta\int_\Omega\rho|u|^2\,dx, \quad (26)$$

where $\dot{E}(t)$ is the devertive of $E(t)$ with respect to t.

Thus, $E(t)$ is a decreasing function and is conserved if the system is not damped.

Proof. By Lemma 1,

$$\frac{1}{\gamma - 1}\int_\Omega P_t dx = \int_\Omega u\cdot\nabla P\,dx. \quad (27)$$

By Lemma 2,

$$\begin{aligned}
\int_\Omega\left(\frac{1}{2}\rho|u|^2\right)_t dx = &-\int_\Omega u\cdot\nabla P - \delta\int_\Omega(\nabla\Phi)\cdot(\rho u)\,dx \\
&- \beta\int_\Omega\rho|u|^2\,dx.
\end{aligned} \quad (28)$$

By Lemma 3,

$$\int_\Omega(\nabla\Phi)\cdot(\rho u)\,dx = \frac{1}{2}\int_\Omega[\rho\Phi]_t\,dx. \quad (29)$$

Thus, the proof is complete. □

Proposition 6. *For the classical solution (ρ, u, S) of system (1) with $\gamma = 1$, let*

$$E(t) = \int_\Omega\left(\frac{1}{2}\rho|u|^2 + K\rho e^S(\ln\rho - 1)\right)dx + \frac{\delta}{2}\int_\Omega\rho\Phi\,dx. \quad (30)$$

Then,

$$\dot{E}(t) = -\beta\int_\Omega\rho|u|^2\,dx, \quad (31)$$

where $\dot{E}(t)$ is the derivative of $E(t)$ with respect to t.

Thus, $E(t)$ is a decreasing function and is conserved if the system is not damped.

Proof. By Lemma 2,

$$\begin{aligned}
\int_\Omega\left(\frac{1}{2}\rho|u|^2\right)_t dx = &-\int_\Omega u\cdot\nabla P - \delta\int_\Omega(\nabla\Phi)\cdot(\rho u)\,dx \\
&- \beta\int_\Omega\rho|u|^2\,dx.
\end{aligned} \quad (32)$$

By Lemma 3,

$$\int_\Omega(\nabla\Phi)\cdot(\rho u)\,dx = \frac{1}{2}\int_\Omega[\rho\Phi]_t\,dx. \quad (33)$$

By Lemma 4,

$$\frac{d}{dt}\int_\Omega K\rho e^S(\ln\rho - 1)\,dx = \int_\Omega u\cdot\nabla P\,dx. \quad (34)$$

Thus, the proof is complete. □

Proposition 7. *Let*

$$H(t) = \int_\Omega \rho |x|^2 \, dx. \tag{35}$$

We have

$$\begin{aligned} \ddot{H}(t) &= -\beta \dot{H}(t) + 2\int_\Omega \left(\rho |u|^2 + 2P\right) dx - 2\pi\delta M^2, \\ &\qquad\qquad \text{for } N = 2, \\ \ddot{H}(t) &= -\beta \dot{H}(t) + 2\int_\Omega \left(\rho |u|^2 + NP\right) dx \\ &\quad + (N-2)\,\delta \int_\Omega \rho\Phi \, dx, \quad \text{for } N \geq 3, \end{aligned} \tag{36}$$

where $\dot{H}(t) = (\mathrm{d}/\mathrm{d}t)H(t)$, $\ddot{H}(t) = (\mathrm{d}/\mathrm{d}t)\dot{H}(t)$, (ρ, u, S) *is a classical solution of system (1), and* M *is defined by (5).*

Proof.

$$\begin{aligned} \dot{H}(t) &= -\int_\Omega \nabla \cdot (\rho u) |x|^2 \, dx \quad \text{by } (1)_1 \\ &= \int_\Omega \nabla |x|^2 \cdot (\rho u) \, dx \quad \text{by Divergence Theorem} \\ &= \int_\Omega 2x \cdot (\rho u) \, dx, \\ \ddot{H}(t) &= 2\int_\Omega x \cdot \left(-\nabla \cdot (\rho u \otimes u) - \nabla P - \delta\rho\nabla\Phi - \beta\rho u\right) dx \\ &\qquad \text{by } (1)_2 \\ &= -\beta \dot{H}(t) + 2\int_\Omega x \cdot \left(-\nabla \cdot (\rho u \otimes u) - \nabla P - \delta\rho\nabla\Phi\right) dx. \end{aligned} \tag{37}$$

We split the last term of the above equality into three parts.

Firstly,

$$\begin{aligned} &-\int_\Omega x \cdot \left[\nabla \cdot (\rho u \otimes u)\right] dx \\ &\quad = -\sum_{j=1}^{N} \int_\Omega x_j \sum_{i=1}^{N} \partial_i \left(\rho u_i u_j\right) dx \quad \text{by definitions} \\ &\quad = \int_\Omega \rho |u|^2 \, dx \quad \text{by integration by parts.} \end{aligned} \tag{38}$$

Secondly,

$$\begin{aligned} -\int_\Omega x \cdot \nabla P \, dx &= \int_\Omega P \nabla \cdot x \, dx \quad \text{by Divergence Theorem} \\ &= \int_\Omega NP \, dx. \end{aligned} \tag{39}$$

Thirdly,

$$\begin{aligned} &-\delta \int_\Omega \rho x \cdot \nabla\Phi \, dx \\ &\quad = -\delta\alpha(N) \int_\Omega \int_\Omega \rho(x)\rho(y) \left[\nabla_x G(x-y) \cdot x\right] dy \, dx \\ &\qquad\qquad \text{by } (3) \\ &\quad =: -\delta\alpha(N) I, \end{aligned} \tag{40}$$

where ∇_x is the gradient operator with respect to the spatial variable x.

Note that

$$\begin{aligned} I &= \int_\Omega \int_\Omega \rho(x)\rho(y) \left[\nabla_x G(x-y) \cdot x\right] dy \, dx \\ &= \int_\Omega \int_\Omega \rho(x)\rho(y) \left[\nabla_x G(x-y) \cdot (x-y)\right] dy \, dx \\ &\quad + \int_\Omega \int_\Omega \rho(x)\rho(y) \left[\nabla_x G(x-y) \cdot y\right] dy \, dx. \end{aligned} \tag{41}$$

For $N = 2$,

$$\nabla_x G(x) = \nabla_x \log |x| = \frac{1}{|x|^2} x. \tag{42}$$

Thus,

$$\begin{aligned} I &= \int_\Omega \int_\Omega \rho(x)\rho(y) \, dy \, dx - I, \\ I &= \frac{1}{2} \int_\Omega \int_\Omega \rho(x)\rho(y) \, dy \, dx = \frac{M^2}{2}. \end{aligned} \tag{43}$$

The result for $N = 2$ is established.

For $N \geq 3$,

$$\nabla_x G(x) = -\nabla_x \frac{1}{|x|^{N-2}} = (N-2) \frac{1}{|x|^N} x. \tag{44}$$

Thus,

$$\begin{aligned} I &= \int_\Omega \int_\Omega \rho(x)\rho(y) \left[\nabla_x G(x-y) \cdot (x-y)\right] dy \, dx \\ &\quad + \int_\Omega \int_\Omega \rho(x)\rho(y) \left[\nabla_x G(x-y) \cdot y\right] dy \, dx \\ &= -\frac{(N-2)}{\alpha(N)} \int_\Omega \rho\Phi \, dx - I, \\ I &= -\frac{N-2}{2\alpha(N)} \int_\Omega \rho\Phi \, dx. \end{aligned} \tag{45}$$

The results for $N \geq 3$ are also established. □

Now, we are ready to present the stability results.

Proposition 8. *Considering the classical solutions of system (1), we have the following.*

Case 1. For $\delta = 1$, $\beta = 0$, $N = 3$ or 4, $\gamma \geq 2(N-1)/N$, and $E(0) \geq 0$, we have

$$\lim_{t\to\infty} \inf \frac{R(t)}{t} \geq \sqrt{\frac{(N-2)E(0)}{M}}. \tag{46}$$

Case 2. For $\delta = -1$, $\beta = 0$, $N \geq 4$, and $\gamma \geq (N+2)/N$, we have

$$\lim_{n\to\infty} \inf \frac{R(t)}{t} \geq \sqrt{\frac{2E(0)}{M}}. \tag{47}$$

Case 3. For $\delta = -1$, $\beta = 0$, $N = 2$, and $\gamma > 1$, we have

$$\lim_{n\to\infty} \inf \frac{R(t)}{t} \geq \sqrt{\pi M}. \tag{48}$$

Proof. First of all, by definitions (35), (6), and (5), we always have

$$H(t) = \int_{\Omega(t)} \rho |x|^2 \, dx \leq R^2(t) M. \tag{49}$$

Case 1. By Propositions 5 and 7, we have

$$\begin{aligned}
\ddot{H}(t) &= 2\int_\Omega \left(\rho |u|^2 + NP\right) dx + (N-2)\int_\Omega \rho\Phi \, dx \\
&= 2\int_\Omega \rho |u|^2 \, dx + 2N\int_\Omega P \, dx \\
&\quad + (N-2)\left[2E(t) - \frac{2}{\gamma-1}\int_\Omega P \, dx - \int_\Omega \rho |u|^2 \, dx\right] \\
&= [2-(N-2)]\int_\Omega \rho |u|^2 \, dx \\
&\quad + \left[2N - \frac{2(N-2)}{\gamma-1}\right]\int_\Omega P \, dx + 2(N-2)E(t) \\
&\geq 2(N-2)E(t) \\
&= 2(N-2)E(0).
\end{aligned} \tag{50}$$

Thus,

$$H(t) \geq (N-2)E(0)t^2 + \dot{H}(0)t + H(0). \tag{51}$$

In view of inequality (49), we have

$$\frac{R(t)}{t} \geq \sqrt{\frac{(N-2)E(0)}{M} + \frac{\dot{H}(0)}{Mt} + \frac{H(0)}{Mt^2}}. \tag{52}$$

Thus,

$$\lim_{t\to\infty} \inf \frac{R(t)}{t} \geq \sqrt{\frac{(N-2)E(0)}{M}}. \tag{53}$$

Case 2. By Propositions 5 and 7, we have

$$\begin{aligned}
\ddot{H}(t) &= 2\int_\Omega \left(\rho |u|^2 + NP\right) dx + (N-2)\int_\Omega \rho(-\Phi) \, dx \\
&= 2\left[2E(t) - \frac{2}{\gamma-1}\int_\Omega P \, dx - \int_\Omega \rho(-\Phi) \, dx\right] \\
&\quad + 2N\int_\Omega P \, dx + (N-2)\int_\Omega \rho(-\Phi) \, dx \\
&\geq 4E(t) \\
&= 4E(0).
\end{aligned} \tag{54}$$

Note that $-\Phi$ is a positive function for $N \geq 3$ by (3) and (4). Thus,

$$H(t) \geq 2E(0)t^2 + \dot{H}(0)t + H(0). \tag{55}$$

It follows that

$$\lim_{n\to\infty} \inf \frac{R(t)}{t} \geq \sqrt{\frac{2E(0)}{M}}. \tag{56}$$

Case 3. By Proposition 7, we have

$$\begin{aligned}
\ddot{H}(t) &= 2\int_\Omega \left(\rho |u|^2 + 2P\right) dx + 2\pi M^2 \\
&\geq 2\pi M^2.
\end{aligned} \tag{57}$$

It follows that

$$\lim_{n\to\infty} \inf \frac{R(t)}{t} \geq \sqrt{\pi M}. \tag{58}$$

□

Finally, we can give the blowup result.

Proposition 9. *If $\delta = 1$, $N \geq 4$, $1 < \gamma \leq 2(N-1)/N$, and $E(0) < 0$, then the classical solutions of (1) blow up in finite time.*

Proof.

Case 1 ($\beta = 0$). As before, we have, from Propositions 5 and 7, that

$$\begin{aligned}
\ddot{H}(t) &= (4-N)\int_\Omega \rho |u|^2 \, dx + \left[2N - \frac{2(N-2)}{\gamma-1}\right]\int_\Omega P \, dx \\
&\quad + 2(N-2)E(t) \\
&\leq 0 + 0 + 2(N-2)E(t) \\
&= 2(N-2)E(0).
\end{aligned} \tag{59}$$

It follows that

$$H(t) \leq (N-2)E(0)t^2 + \dot{H}(0)t + H(0). \tag{60}$$

Suppose the solutions exist globally; then for sufficient large t, we see that $H(t)$ is negative as the leading coefficient of the right hand side of (60) is negative. However, $H(t)$ is nonnegative by definition (35). This is a contradiction. As a result, the solutions blow up in finite time.

Case 2 ($\beta \neq 0$). Now (59) becomes

$$\ddot{H}(t) + \beta \dot{H}(t) \leq 2(N-2)E(0). \tag{61}$$

It follows by multipying an integral factor $e^{\beta t}$ on both sides and taking integration that

$$H(t) \leq A_1 + A_2 e^{-\beta t} + \frac{2(N-2)E(0)}{\beta} t \tag{62}$$

for some constants A_1 and A_2. Note that this implies that $H(t)$ is negative for sufficient large t as $\beta > 0$ and $(N-2)E(0) < 0$. Therefore, the solutions blow up in finite time. □

Appendix

We here complement the proof of Lemma 2 by proving the equality (17); namely,

$$\int_\Omega \left(\frac{1}{2} |u|^2 \nabla \cdot (\rho u) - u \cdot [\nabla \cdot (\rho u \otimes u)] \right) dx = 0. \tag{A.1}$$

Proof. Firstly, by divergence theorem,

$$\int_\Omega \frac{1}{2} |u|^2 \nabla \cdot (\rho u)\, dx = -\frac{1}{2} \int_\Omega (\rho u) \cdot \nabla |u|^2 \, dx. \tag{A.2}$$

Secondly, by definitions of the operations,

$$\begin{aligned} u \cdot [\nabla \cdot (\rho u \otimes u)] &= \sum_{j=1}^N u_j \left(\sum_{i=1}^N \partial_i (\rho u_i u_j) \right) \\ &= \sum_{j=1}^N u_j \left(\sum_{i=1}^N \left(u_j \partial_i (\rho u_i) + \rho u_i \partial_i u_j \right) \right) \\ &= \sum_{j=1}^N u_j^2 \sum_{i=1}^N \partial_i (\rho u_i) + \sum_{j=1}^N u_j \sum_{i=1}^N \rho u_i \partial_i u_j \\ &= |u|^2 \nabla \cdot (\rho u) + \sum_{i=1}^N \rho u_i \sum_{j=1}^N \frac{1}{2} \partial_i u_j^2 \\ &= |u|^2 \nabla \cdot (\rho u) + \frac{1}{2} \sum_{i=1}^N \rho u_i \partial_i |u|^2 \\ &= |u|^2 \nabla \cdot (\rho u) + \frac{1}{2} (\rho u) \cdot \nabla |u|^2 . \end{aligned} \tag{A.3}$$

Thus,

$$\begin{aligned} &\int_\Omega u \cdot [\nabla \cdot (\rho u \otimes u)]\, dx \\ &\quad = \int_\Omega \left(|u|^2 \nabla \cdot (\rho u) + \frac{1}{2} (\rho u) \cdot \nabla |u|^2 \right) dx \\ &\quad = \int_\Omega \frac{1}{2} |u|^2 \nabla \cdot (\rho u)\, dx. \quad \text{by (A.2)}. \end{aligned} \tag{A.4}$$

Thus, equality (A.1) is established. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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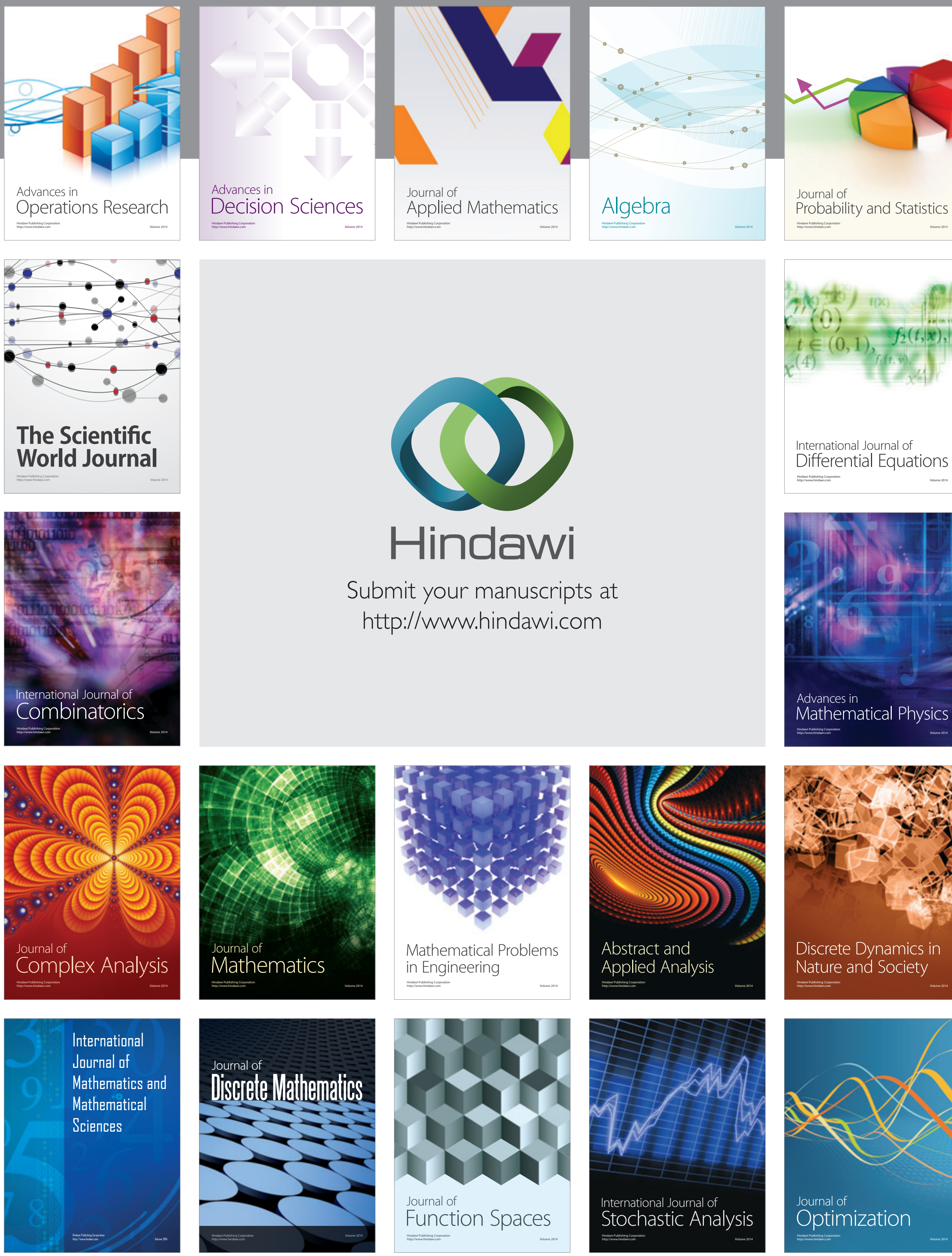
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