Hindawi
The Scientific World Journal
Volume 2020, Article ID 3192852, 24 pages
https://doi.org/10.1155/2020/3192852

*Research Article*

# Two-Parameter Modified Ridge-Type M-Estimator for Linear Regression Model

**Adewale F. Lukman**,[1] **Kayode Ayinde**,[2] **B. M. Golam Kibria**,[3] **and Segun L. Jegede**[1]

[1]*Department of Physical Sciences, Landmark University, Omu-Aran, Nigeria*
[2]*Department of Statistics, Federal University of Technology, Akure, Nigeria*
[3]*Department of Mathematics and Statistics, Florida International University, Miami, FL, USA*

Correspondence should be addressed to Adewale F. Lukman; adewale.folaranmi@lmu.edu.ng

Received 9 November 2019; Revised 18 March 2020; Accepted 13 April 2020; Published 15 May 2020

Academic Editor: Roberto Rivelino



The general linear regression model has been one of the most frequently used models over the years, with the ordinary least squares estimator (OLS) used to estimate its parameter. The problems of the OLS estimator for linear regression analysis include that of multicollinearity and outliers, which lead to unfavourable results. This study proposed a two-parameter ridge-type modified M-estimator (RTMME) based on the M-estimator to deal with the combined problem resulting from multicollinearity and outliers. Through theoretical proofs, Monte Carlo simulation, and a numerical example, the proposed estimator outperforms the modified ridge-type estimator and some other considered existing estimators.

## 1. Introduction

A multiple linear regression model can be defined mathematically as

$$y = X\beta + \varepsilon, \tag{1}$$

where $y$ is an $n \times 1$ vector of observations referred to as the dependent variable; $X$ is a known full column rank of $n \times p$standardized and centered explanatory variable matrix; $\beta$ is an $p \times 1$ vector of unknown parameters; and $\varepsilon$ is an $n \times 1$ vector of disturbances with $E(\varepsilon) = 0$ and dispersion matrix $\mathrm{Cov}(\varepsilon) = \sigma^2 I$, $I$ is the $n \times n$ identity matrix. The ordinary least squares estimator (OLSE) of $\beta$ is given as

$$\hat{\beta} = \left(X'X\right)^{-1} X'y. \tag{2}$$

According to Gauss–Markov theorem, OLS estimator is the best linear unbiased estimator (BLUE) possessing minimum variance in the class of all unbiased linear estimators [1, 2]. However, the performance of the estimator is imprecise in the presence of multicollinearity [3]. Biased estimators such as ridge regression estimator [4], Liu estimator [5] and Stein estimator [6], principal component estimator [7], modified ridge regression estimator [8], and others are often employed to tackle this problem. Another factor whose presence can negatively influence the regression coefficients of the OLS estimator is the outlier. The general practice in the literature is that one adopts robust estimators as an alternative to the OLS estimator. The M-estimator is popularly used to handle outlier in the $y$-direction [9].

Hoerl and Kennard [4] defined the ridge estimator (RE) as

$$\hat{\beta}(k) = \left(X'X + kI\right)^{-1} X'y = R_k \hat{\beta}, \tag{3}$$

where $R_k = \left(X'X + kI\right)^{-1} X'X$ and $k > 0$. However, RE can be sensitive to outliers in the $y$-direction; a remedial measure is the ridge M-estimator (RME) suggested by Silvapulle [10] as

$$\hat{\beta}_M(k) = R_k \hat{\beta}_M, \tag{4}$$

where $\hat{\beta}_M$ is the M-estimator of $\beta$ [11].

Dorugade [12] modified the ridge estimator and defined it as

$$\widehat{\beta}_D(k) = R_{kd1}\widehat{\beta}, \tag{5}$$

where $R_{kd1} = (X'X + kdI)^{-1}X'X$ and $d$ introduced as an additional basing parameter.

Following Dorugade [12], Lukman et al. [13] modified the ridge estimator in (3) and called it the modified ridge-type estimator (MRT). The estimator is defined as

$$\widehat{\beta}_{\mathrm{MRT}}(k,d) = \left(X'X + k(1+d)I\right)^{-1}X'Y = R_{kd}\widehat{\beta}, \tag{6}$$

where $R_{kd} = (X'X + k(1+d)I)^{-1}X'X$.

The organization of the paper is as follows. We proposed the new estimator in Section 2 and provided a theoretical comparison among the estimators in Section 3. We discussed the robust choice of the biasing parameters in Section 4 and conducted simulation studies in Section 5 to evaluate the performance of the proposed estimator. A real-life data set is analyzed in Section 6 to illustrate the findings in the paper, and Section 7 ends with some concluding remarks.

## 2. A New Estimator

The presence of outliers in the $y$-direction affects the performance of the MRT estimator. Therefore, we suggest a ridge-type modified M-estimator (RTMME). This is defined as

$$\widehat{\beta}_{\mathrm{RTMME}}(k,d) = R_{kd}\widehat{\beta}_M, \tag{7}$$

where $k>0$, $0<d<1$. It appears that $\widehat{\beta}_{\mathrm{RTMME}}(k,d)$ is a general estimator, which includes $\widehat{\beta}_M$ and $\widehat{\beta}_M(k)$:

$$\begin{aligned}\widehat{\beta}_{\mathrm{RTMME}}(0,0) &= \widehat{\beta}_M,\\ \widehat{\beta}_{\mathrm{RTMME}}(k,0) &= \widehat{\beta}_M(k).\end{aligned} \tag{8}$$

The canonical form of model (1) is written as

$$y = z\alpha + \varepsilon, \tag{9}$$

where $Z = XT$, $\alpha = T'\beta$, and $T$ is the orthogonal matrix whose columns contain the eigenvectors of $X'X$. Then,

$$Z'Z = T'X'XT = \Lambda = \mathrm{diag}\left(\lambda_1, \lambda_2, \ldots, \lambda_p\right), \tag{10}$$

where $\lambda_1, \lambda_2, \ldots, \lambda_p > 0$ are the ordered eigenvalues of $X'X$.

Let $\widehat{\alpha}_M$ be M-defined by the solution of the M-estimating equations $\sum \varphi(e_i/s)z_i = 0$, where $e_i = y_i - z_i\widehat{\alpha}_M$, $s$ is an estimator of scale for the errors and $\varphi(\cdot)$ is some suitably chosen function [14]. The estimators presented in equations (2)–(7) can be written in canonical form as follows:

$$\begin{aligned}\widehat{\alpha} &= \Lambda^{-1}Z'y,\\ \widehat{\alpha}(k) &= (\Lambda + \mathrm{kI})^{-1}Z'y = R_k^*\widehat{\alpha},\\ \widehat{\alpha}_M(k) &= R_k^*\widehat{\alpha}_M,\\ \widehat{\alpha}_{\mathrm{MRT}}(k,d) &= (\Lambda + k(1+d)I)^{-1}Z'y,\\ \widehat{\alpha}_{\mathrm{MRT}}(k,d) &= R_{kd}\widehat{\alpha},\\ \widehat{\alpha}_{\mathrm{RTMME}}(k,d) &= R_{kd}\widehat{\alpha}_M,\end{aligned} \tag{11}$$

where $R_{kd} = \Lambda(\Lambda + k(1+d)I)^{-1}$, $R_k^* = \Lambda(\Lambda + kI)^{-1}$, and $k>0$.

## 3. Superiority of the New Estimator

The mean square error (MSE) criterion is used to compare the performance of the estimators. The following conditions are imposed to present our main theorems:

(i) $\varphi$ is skew-symmetric and nondecreasing

(ii) The errors are symmetric

(iii) $\Omega$ is finite

Note that any estimator of $\alpha$ has a corresponding relation $\widetilde{\beta} = T'\widetilde{\alpha}$ such that $\mathrm{MSE}(\widetilde{\beta}) = \mathrm{MSE}(\widetilde{\alpha})$. Thus, it is sufficient to consider the canonical form only. The MSEs of the aforementioned estimators are derived to be

$$\begin{aligned}\mathrm{MSE}(\widehat{\alpha}) &= \sigma^2\sum_{i=1}^{p}\frac{1}{\lambda_i},\\ \mathrm{MSE}(\widehat{\alpha}_M) &= \sum_{i=1}^{p}\Omega_{ii},\\ \mathrm{MSE}[\widehat{\alpha}(k)] &= \sigma^2\sum_{i=1}^{p}\frac{\lambda_i}{(\lambda_i+k)^2} + \sum_{i=1}^{p}\frac{k^2\alpha_i^2}{(\lambda_i+k)^2},\\ \mathrm{MSE}[\widehat{\alpha}_M(k)] &= \sum_{i=1}^{p}\frac{\lambda_i^2}{(\lambda_i+k)^2}\Omega_{ii} + \sum_{i=1}^{p}\frac{k^2\alpha_i^2}{(\lambda_i+k)^2},\\ \mathrm{MSE}[\widehat{\alpha}_{\mathrm{MRT}}(k,d)] &= \sigma^2\sum_{i=1}^{p}\frac{\lambda_i}{(\lambda_i+k(1+d))^2} + \sum_{i=1}^{p}\frac{k^2(1+d)^2\alpha_i^2}{(\lambda_i+k(1+d))^2},\\ \mathrm{MSE}[\widehat{\alpha}_{\mathrm{RTMME}}(k,d)] &= \sum_{i=1}^{p}\frac{{\lambda_i}^2}{(\lambda_i+k(1+d))^2}\Omega_{ii} + \sum_{i=1}^{p}\frac{k^2(1+d)^2\alpha_i^2}{(\lambda_i+k(1+d))^2},\end{aligned} \tag{12}$$

where $\Omega_{ii} = \mathrm{Cov}(\widehat{\alpha}_M)$.

**Theorem 1.** *For $MSE(\widehat{\alpha}_{RTMME}(k,d)) < MSE(\widehat{\alpha}_{MRT}(k,d))$, then $\sum_{i=1}^{p}\Omega_{ii} < \sum_{i=1}^{p}\sigma^2\lambda_i^{-1}$ for every $k>0$ and $i = 1, 2, \ldots, p$, where $\Omega_{ii}$ are the diagonal elements of $\Omega$.*

*Proof.* After some algebraic manipulation, the difference between $\mathrm{MSE}(\widehat{\alpha}_{\mathrm{RTMME}}(k,d)) - \mathrm{MSE}(\widehat{\alpha}_{\mathrm{MRT}}(k,d))$ gives

$$\Delta_1 = \mathrm{MSE}\left(\widehat{\alpha}_{\mathrm{RTMME}}(k,d)\right) - \mathrm{MSE}\left(\widehat{\alpha}_{MRT}(k,d)\right), \tag{13}$$

$$\Delta_1 = \sum_{i=1}^{p}\frac{{\lambda_i}^2\Omega_{ii} - \sigma^2\lambda_i}{(\lambda_i+k(1+d))^2}. \tag{14}$$

For equation (14) to be less than zero, we should have $\Omega_{ii} - \sigma^2\lambda_i^{-1} < 0$, which also implies $\sum_{i=1}^{p}\Omega_{ii} < \sum_{i=1}^{p}\sigma^2\lambda_i^{-1}$ for $k > 0$, $0 < d < 1$ and for $i = 1, 2, \ldots, p$.

**Theorem 2.** *When $MSE(\hat{\alpha}_{RTMME}(k,d)) < MSE(\hat{\alpha}_M(k))$, there exists a positive constant $k > k_i > 0$, where*

$$k_{1i} = \frac{\lambda_i\left[\sqrt{\left((d+2)\left(\alpha_i^2 - \Omega_{ii}\right)\right)^2 + 8\alpha_i^2\Omega_{ii}(d+1)} - (d+2)\left(\alpha_i^2 - \Omega_{ii}\right)\right]}{4\alpha_i^2(d+1)}. \tag{15}$$

*Proof.* The difference $\text{MSE}(\hat{\alpha}_{\text{RTMME}}(k, d)) - \text{MSE}(\hat{\alpha}_M(k))$ is:

$$\Delta_2 = \sum_{i=1}^{p}\frac{\lambda_i^2\Omega_{ii} + k^2(1+d)^2\alpha_i^2}{\left(\lambda_i + k(1+d)\right)^2} - \sum_{i=1}^{p}\frac{\lambda_i^2\Omega_{ii} + k^2\alpha_i^2}{\left(\lambda_i + k\right)^2}. \tag{16}$$

The difference is strictly less than zero if and only if, after simplification, the following expression holds:

$$k^2\left[2\alpha_i^2\lambda_i d\right] + k\left[\lambda_i^2 d(d+2)\left(\alpha_i^2 - \Omega_{ii}\right)\right] - 2\Omega_{ii}\lambda_i^3 d < 0. \tag{17}$$

Solving inequality (17) for $k$, we get

$$k_{1i} = \frac{\lambda_i\left[\sqrt{\left((d+2)\left(\alpha_i^2 - \Omega_{ii}\right)\right)^2 + 8\alpha_i^2\Omega_{ii}(d+1)} - (d+2)\left(\alpha_i^2 - \Omega_{ii}\right)\right]}{4\alpha_i^2(d+1)}. \tag{18}$$

Notice that if $\lambda_i[\sqrt{((d+2)(\alpha_i^2 - \Omega_{ii}))^2 + 8\alpha_i^2\Omega_{ii}(d+1)} - (d+2)(\alpha_i^2 - \Omega_{ii})] > 0$; $k_{1i} > 0$, and there is, therefore, a positive constant $k > k_{1i} > 0$.

**Theorem 3.** *A necessary condition for $MSE(\hat{\alpha}_{RTMME}(k,d)) < MSE(\hat{\alpha}_M)$ is*

$$\sum_{i=1}^{p}\Omega_{ii} > \sum_{i=1}^{p}\frac{k(1+d)\alpha_i^2}{2\lambda_i + k(1+d)}. \tag{19}$$

*Proof.* The difference $\text{MSE}(\hat{\alpha}_{\text{RTMME}}(k,d)) - \text{MSE}(\hat{\alpha}_M)$ is

$$\Delta_3 = \sum_{i=1}^{p}\frac{\lambda_i^2\Omega_{ii} + k^2(1+d)^2\alpha_i^2}{\left(\lambda_i + k(1+d)\right)^2} - \sum_{i=1}^{p}\Omega_{ii}, \tag{20}$$

$$\Delta_3 = -\Omega_{ii}\left(2\lambda_i k(1+d) + k^2(1+d)^2\right) + k^2(1+d)^2\alpha_i^2. \tag{21}$$

To obtain $\text{MSE}(\hat{\alpha}_{\text{RTMME}}(k,d)) < \text{MSE}(\hat{\alpha}_M)$,

$$\sum_{i=1}^{p}\Omega_{ii} < \sum_{i=1}^{p}\frac{k(1+d)\alpha_i^2}{2\lambda_i + k(1+d)}. \tag{22}$$

Theorem 2 provided $k > k_{1i}$ such that $\text{MSE}(\hat{\alpha}_{\text{RTMME}}(k,d)) < \text{MSE}(\hat{\alpha}_M)$. Besides this, in the Theorem of Silvapulle [10] (part (i), pg. 321), it is indicated that there exists $k > 0$ such that $\text{MSE}(\hat{\alpha}_M(k)) - \text{MSE}(\hat{\alpha}_M) < 0$. Thus, we obtain the corollary as follows. □

**Corollary.** *There exists $k > 0$ such that $MSE(\hat{\alpha}_{RTMME}(k,d)) - MSE(\hat{\alpha}_M)$.*

## 4. Robust Choice of *k* and *d* for $\hat{\alpha}_{\text{RTMME}}(k,d)$

For the robust biasing parameters $k$ and $d$ for the modified two-parameter estimator, the optimal values can be determined by minimizing equation (23) with respect to each of the parameters:

$$f(k,d) = \sum_{i=1}^{p}\frac{\lambda_i^2}{\left(\lambda_i + k(1+d)\right)^2}\Omega_{ii} + \sum_{i=1}^{p}\frac{k^2(1+d)^2\alpha_i^2}{\left(\lambda_i + k(1+d)\right)^2}. \tag{23}$$

This can be obtained by solving

$$\frac{\partial f(k,d)}{\partial d} = 0, \qquad \frac{\partial f(k,d)}{\partial k} = 0. \tag{24}$$

By doing this, we have

$$d = \frac{\lambda_i\Omega_{ii}}{k\alpha_i^2} - 1, \tag{25}$$

$$k = \frac{\lambda_i\Omega_{ii}}{\alpha_i^2(1+d)}. \tag{26}$$

We substitute $\Omega_{ii}$ and $\alpha_i^2$ into equations (24) and (25) with their corresponding estimates. We assume that $\hat{\alpha}_M$ is normally distributed with mean $\alpha$ and covariance matrix $A^2\Lambda^{-1}$. This assumption holds since $n^{1/2}(\hat{\alpha}_M^2 - \alpha) \longrightarrow N(0, A^2\Lambda^{-1})$, where

$$A^2 = \frac{s_o^2 E\left(\varphi^2(\varepsilon/s_o)\right)}{\left(E\left(\varphi'(\varepsilon/s_o)\right)\right)^2}, \tag{27}$$

with the scale estimate $s_o$. Thus, the estimate of $\alpha_i^2$ is $\widehat{\alpha}_{Mi}^2$, and the unbiased estimator of $\Omega_{ii}$ is asymptotically $\widehat{A}^2/\lambda_i$, where $\widehat{A}^2$ is given by Huber [9] as

$$\widehat{A}^2 = \frac{s^2 (n-p)^{-1} \sum_{i=1}^{p} (\varphi(e_i/s))^2}{\left(\sum_{i=1}^{p} (1/n)\varphi'(e_i/s)\right)^2}. \quad (28)$$

We get the optimal estimator of $d$ and $k$ as

$$d = \frac{\widehat{A}^2}{k\widehat{\alpha}_{Mi}^2} - 1, \quad i = 1, 2, \ldots, p, \quad (29)$$

$$k = \frac{\widehat{A}^2}{(1+d)\widehat{\alpha}_{Mi}^2}, \quad i = 1, 2, \ldots, p. \quad (30)$$

Following Kibria [15], the arithmetic and geometric mean version of $k$ is obtained, respectively, as

$$\widetilde{k}_{\mathrm{GMR}} = \left[\prod_{i=1}^{p} \frac{\widehat{A}^2}{(1+d)\widehat{\alpha}_{Mi}^2}\right]^{1/p}, \quad (31)$$

$$\widetilde{k}_{\mathrm{AMR}} = \frac{1}{p}\sum_{i=1}^{p} \frac{\widehat{A}^2}{(1+d)\widehat{\alpha}_{Mi}^2} \quad (32)$$

The harmonic mean version is generally preferred to other versions [3]. Hence, the robust harmonic mean version of the proposed $d$ and $k$ from (31) and (32) is obtained as

$$\widetilde{k}_{\mathrm{HMR}} = \frac{p\widehat{A}^2}{\sum_{i=1}^{p} (1+d)\widehat{\alpha}_{Mi}^2}, \quad (33)$$

$$\widetilde{d}_{\mathrm{HMR}} = \frac{p}{\sum_{i=1}^{p} (1/d)}. \quad (34)$$

The selection of the estimators of the parameters $d$ and $k$ can be obtained iteratively as follows:

Step 1: use $\widehat{d} = \min(\widehat{A}^2/\widehat{\alpha}_{Mi}^2)$ to obtain an initial estimate for $d$

Step 2: from (33), get $\widetilde{k}_{\mathrm{HMR}}$ using $d$ in Step 1

Step 3: calculate $\widetilde{d}_{\mathrm{HMR}}$ in (34) by using $\widetilde{k}_{\mathrm{HMR}}$ in Step 2

Step 4: use $\widehat{d}$ in Step 1 if $\widetilde{d}_{\mathrm{HMR}} < 0$

## 5. Monte Carlo Simulation Study

We adopted the simulation design by McDonald and Galarneau [16], Kibria [15], and Lukman et al. [17]. The explanatory variables are generated using the following equation:

$$x_{ij} = \left(1-\rho^2\right)^{1/2} z_{ij} + \rho z_{i,p+1}, \quad i = 1, 2, \ldots, n, \; j = 1, 2, \ldots, p, \quad (35)$$

where $\rho^2$ denotes the correlation between explanatory variables and $z_{ij}$ are pseudo-random numbers from the standard normal distribution. The coefficients $\beta_1, \beta_2, \ldots, \beta_p$ are selected as the normalized eigenvectors corresponding to the largest eigenvalue of $X'X$ so that we have $\beta'\beta = 1$, which is a common restriction in simulation studies of this type ([3]). The dependent variable is then determined using

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} + \varepsilon_i, \quad i = 1, 2, \ldots, n, \quad (36)$$

where the error term $\varepsilon_i' s$ is generated with mean and variance 0 and $\sigma^2$, respectively. We fixed the number of explanatory variables to three and seven ($p = 3, 7$), and other parameters such as $\rho$, $\sigma$, and $n$ were varied; their values considered in this study are given as follows:

$\rho$ = 0.7, 0.8, 0.9 and 0.99

The standard deviation ($\sigma$) of the error term in this simulation study is 1, 5, 10

$n$ = 20, 50, 100

We considered three different cases in this study:

Case I: no outlier

Case II: one outlier

Case III: two outliers

In the case of no outliers, equation (36) is taken into consideration. For the case of one outlier, the tenth observation is changed as $y_{10}^* = y_{10} + 20\sigma$. For the case of two outliers, the fifth and the tenth observations are changed as $y_5^* = y_5 + 20\sigma$ and $y_{10}^* = y_{10} - 20\sigma$, respectively. The experiment is replicated 2,000 times by generating new pseudo-random numbers, and the estimated MSE is calculated as

$$\mathrm{mse}(\widehat{\alpha}) = \frac{1}{2000}\sum_{j=1}^{2000} \left(\widehat{\alpha}_{ij} - \alpha_i\right)'\left(\widehat{\alpha}_{ij} - \alpha_i\right). \quad (37)$$

The results of the simulation are presented in Tables 1–18. As expected, the OLSE is observed to have the least performance. The following observations are also made:

(i) As the error standard deviation ($\sigma$) and the degree of multicollinearity ($\rho$) increase, the MSEs of the estimators ($\alpha, \widehat{\alpha}_M, \widehat{\alpha}(k), \widehat{\alpha}_M(k), \widehat{\alpha}_{\mathrm{MRT}}$, and $\widehat{\alpha}_{\mathrm{RTMME}}$) increase.

(ii) As the biasing parameters $k$ and $d$ increase, the MSEs of the estimators ($\alpha, \widehat{\alpha}_M, \widehat{\alpha}(k), \widehat{\alpha}_M(k), \widehat{\alpha}_{\mathrm{MRT}}$, and $\widehat{\alpha}_{\mathrm{RTMME}}$) also decrease.

(iii) The MSEs of the estimators ($\alpha, \widehat{\alpha}_M, \widehat{\alpha}(k), \widehat{\alpha}_M(k)$, $\widehat{\alpha}_{\mathrm{MRT}}$, and $\widehat{\alpha}_{\mathrm{RTMME}}$) decrease as the sample size increases. However, as the number of outliers increases, the MSEs also increase.

(iv) Finally, just as in the outcome of Lukman et al. [13], the MRT estimator outperforms other estimators considered in the case of no outlier. However, when outliers were introduced, the RTMME outperforms other considered estimators.

TABLE 1: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 20$ and $p = 3$ when there is no outlier.

| $k$ | Sigma | Rho | $d = 0.2$ | | | | | | $d = 0.5$ | | | | | | $d = 0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\widehat{\alpha}$ | $\widehat{\alpha}_M$ | $\widehat{\alpha}(k)$ | $\widehat{\alpha}_M(k)$ | $\widehat{\alpha}_{\text{MRT}}$ | $\widehat{\alpha}_{\text{RTMME}}$ | $\widehat{\alpha}$ | $\widehat{\alpha}_M$ | $\widehat{\alpha}(k)$ | $\widehat{\alpha}_M(k)$ | $\widehat{\alpha}_{\text{MRT}}$ | $\widehat{\alpha}_{\text{RTMME}}$ | $\widehat{\alpha}$ | $\widehat{\alpha}_M$ | $\widehat{\alpha}(k)$ | $\widehat{\alpha}_M(k)$ | $\widehat{\alpha}_{\text{MRT}}$ | $\widehat{\alpha}_{\text{RTMME}}$ |
| 0.3 | 1 | 0.7 | 8.973 | 4.898 | 8.208 | 4.483 | 8.069 | 4.408 | 8.973 | 4.898 | 8.208 | 4.483 | 7.867 | 4.299 | 8.973 | 4.898 | 8.208 | 4.483 | 7.675 | 4.195 |
| | | 0.8 | 13.374 | 7.518 | 11.664 | 6.544 | 11.366 | 6.375 | 13.374 | 7.518 | 11.664 | 6.544 | 10.943 | 6.135 | 13.374 | 7.518 | 11.664 | 6.544 | 10.547 | 5.911 |
| | | 0.9 | 0.945 | 0.980 | 0.930 | 0.966 | 0.681 | 0.707 | 0.945 | 0.980 | 0.930 | 0.966 | 0.635 | 0.659 | 0.945 | 0.980 | 0.930 | 0.966 | 0.595 | 0.618 |
| | | 0.99 | 11.197 | 11.806 | 9.351 | 10.000 | 1.344 | 1.404 | 11.197 | 11.806 | 9.351 | 10.000 | 1.050 | 1.094 | 11.197 | 11.806 | 9.351 | 10.000 | 0.858 | 0.892 |
| | 5 | 0.7 | 0.789 | 0.623 | 0.785 | 0.620 | 0.784 | 0.619 | 0.789 | 0.623 | 0.785 | 0.620 | 0.783 | 0.618 | 0.789 | 0.623 | 0.785 | 0.620 | 0.782 | 0.617 |
| | | 0.8 | 1.161 | 0.893 | 1.152 | 0.886 | 1.151 | 0.884 | 1.161 | 0.893 | 1.152 | 0.886 | 1.148 | 0.882 | 1.161 | 0.893 | 1.152 | 0.886 | 1.145 | 0.880 |
| | | 0.9 | 0.945 | 0.980 | 0.944 | 0.979 | 0.931 | 0.965 | 0.945 | 0.980 | 0.944 | 0.979 | 0.927 | 0.962 | 0.945 | 0.980 | 0.944 | 0.979 | 0.924 | 0.958 |
| | | 0.99 | 11.197 | 11.806 | 11.112 | 11.723 | 9.377 | 9.882 | 11.197 | 11.806 | 11.112 | 11.723 | 8.998 | 9.481 | 11.197 | 11.806 | 11.112 | 11.723 | 8.644 | 9.107 |
| | 10 | 0.7 | 0.179 | 0.115 | 0.179 | 0.115 | 0.179 | 0.115 | 0.179 | 0.115 | 0.179 | 0.115 | 0.179 | 0.114 | 0.179 | 0.115 | 0.179 | 0.115 | 0.179 | 0.114 |
| | | 0.8 | 0.259 | 0.170 | 0.259 | 0.170 | 0.259 | 0.170 | 0.259 | 0.170 | 0.259 | 0.170 | 0.258 | 0.170 | 0.259 | 0.170 | 0.259 | 0.170 | 0.258 | 0.170 |
| | | 0.9 | 0.945 | 0.980 | 0.944 | 0.980 | 0.941 | 0.976 | 0.945 | 0.980 | 0.944 | 0.980 | 0.940 | 0.975 | 0.945 | 0.980 | 0.944 | 0.980 | 0.939 | 0.974 |
| | | 0.99 | 11.197 | 11.806 | 11.176 | 11.785 | 10.690 | 11.270 | 11.197 | 11.806 | 11.176 | 11.785 | 10.569 | 11.142 | 11.197 | 11.806 | 11.176 | 11.785 | 10.450 | 11.016 |
| 0.7 | 1 | 0.7 | 8.973 | 4.898 | 7.352 | 4.020 | 7.089 | 3.878 | 8.973 | 4.898 | 7.352 | 4.020 | 6.724 | 3.682 | 8.973 | 4.898 | 7.352 | 4.020 | 6.391 | 3.503 |
| | | 0.8 | 13.374 | 7.518 | 9.900 | 5.545 | 9.388 | 5.256 | 13.374 | 7.518 | 9.900 | 5.545 | 8.700 | 4.869 | 13.374 | 7.518 | 9.900 | 5.545 | 8.095 | 4.529 |
| | | 0.9 | 0.945 | 0.980 | 0.930 | 0.966 | 0.493 | 0.511 | 0.945 | 0.980 | 0.930 | 0.966 | 0.441 | 0.457 | 0.945 | 0.980 | 0.930 | 0.966 | 0.400 | 0.415 |
| | | 0.99 | 11.197 | 11.806 | 9.351 | 10.000 | 0.542 | 0.559 | 11.197 | 11.806 | 9.351 | 10.000 | 0.444 | 0.455 | 11.197 | 11.806 | 9.351 | 10.000 | 0.386 | 0.395 |
| | 5 | 0.7 | 0.789 | 0.623 | 0.780 | 0.616 | 0.778 | 0.614 | 0.789 | 0.623 | 0.780 | 0.616 | 0.775 | 0.612 | 0.789 | 0.623 | 0.780 | 0.616 | 0.772 | 0.610 |
| | | 0.8 | 1.161 | 0.893 | 1.141 | 0.876 | 1.137 | 0.873 | 1.161 | 0.893 | 1.141 | 0.876 | 1.131 | 0.868 | 1.161 | 0.893 | 1.141 | 0.876 | 1.125 | 0.864 |
| | | 0.9 | 0.945 | 0.980 | 0.944 | 0.979 | 0.912 | 0.947 | 0.945 | 0.980 | 0.944 | 0.979 | 0.905 | 0.939 | 0.945 | 0.980 | 0.944 | 0.979 | 0.897 | 0.931 |
| | | 0.99 | 11.197 | 11.806 | 11.112 | 11.723 | 7.617 | 8.022 | 11.197 | 11.806 | 11.112 | 11.723 | 7.013 | 7.384 | 11.197 | 11.806 | 11.112 | 11.723 | 6.484 | 6.826 |
| | 10 | 0.7 | 0.179 | 0.115 | 0.178 | 0.114 | 0.178 | 0.114 | 0.179 | 0.115 | 0.178 | 0.114 | 0.178 | 0.114 | 0.179 | 0.115 | 0.178 | 0.114 | 0.178 | 0.114 |
| | | 0.8 | 0.259 | 0.170 | 0.258 | 0.170 | 0.258 | 0.169 | 0.259 | 0.170 | 0.258 | 0.170 | 0.257 | 0.169 | 0.259 | 0.170 | 0.258 | 0.170 | 0.257 | 0.169 |
| | | 0.9 | 0.945 | 0.980 | 0.944 | 0.980 | 0.936 | 0.971 | 0.945 | 0.980 | 0.944 | 0.980 | 0.934 | 0.969 | 0.945 | 0.980 | 0.944 | 0.980 | 0.932 | 0.967 |
| | | 0.99 | 11.197 | 11.806 | 11.176 | 11.785 | 10.071 | 10.615 | 11.197 | 11.806 | 11.176 | 11.785 | 9.818 | 10.348 | 11.197 | 11.806 | 11.176 | 11.785 | 9.576 | 10.093 |
| 0.9 | 1 | 0.7 | 8.973 | 4.898 | 6.981 | 3.820 | 6.674 | 3.655 | 8.973 | 4.898 | 6.981 | 3.820 | 6.257 | 3.431 | 8.973 | 4.898 | 6.981 | 3.820 | 5.885 | 3.232 |
| | | 0.8 | 13.374 | 7.518 | 9.182 | 5.140 | 8.609 | 4.817 | 13.374 | 7.518 | 9.182 | 5.140 | 7.858 | 4.396 | 13.374 | 7.518 | 9.182 | 5.140 | 7.213 | 4.035 |
| | | 0.9 | 0.945 | 0.980 | 0.930 | 0.966 | 0.434 | 0.450 | 0.945 | 0.980 | 0.930 | 0.966 | 0.385 | 0.399 | 0.945 | 0.980 | 0.930 | 0.966 | 0.348 | 0.361 |
| | | 0.99 | 11.197 | 11.806 | 9.351 | 10.000 | 0.434 | 0.445 | 11.197 | 11.806 | 9.351 | 10.000 | 0.368 | 0.376 | 11.197 | 11.806 | 9.351 | 10.000 | 0.331 | 0.337 |
| | 5 | 0.7 | 0.789 | 0.623 | 0.777 | 0.614 | 0.775 | 0.612 | 0.789 | 0.623 | 0.777 | 0.614 | 0.771 | 0.609 | 0.789 | 0.623 | 0.777 | 0.614 | 0.767 | 0.606 |
| | | 0.8 | 1.161 | 0.893 | 1.135 | 0.872 | 1.130 | 0.868 | 1.161 | 0.893 | 1.135 | 0.872 | 1.122 | 0.862 | 1.161 | 0.893 | 1.135 | 0.872 | 1.115 | 0.856 |
| | | 0.9 | 0.945 | 0.980 | 0.944 | 0.979 | 0.904 | 0.938 | 0.945 | 0.980 | 0.944 | 0.979 | 0.894 | 0.927 | 0.945 | 0.980 | 0.944 | 0.979 | 0.884 | 0.918 |
| | | 0.99 | 11.197 | 11.806 | 11.112 | 11.723 | 6.933 | 7.299 | 11.197 | 11.806 | 11.112 | 11.723 | 6.278 | 6.608 | 11.197 | 11.806 | 11.112 | 11.723 | 5.720 | 6.019 |
| | 10 | 0.7 | 0.179 | 0.115 | 0.178 | 0.114 | 0.178 | 0.114 | 0.179 | 0.115 | 0.178 | 0.114 | 0.178 | 0.114 | 0.179 | 0.115 | 0.178 | 0.114 | 0.178 | 0.114 |
| | | 0.8 | 0.259 | 0.170 | 0.258 | 0.169 | 0.257 | 0.169 | 0.259 | 0.170 | 0.258 | 0.169 | 0.257 | 0.169 | 0.259 | 0.170 | 0.258 | 0.169 | 0.257 | 0.169 |
| | | 0.9 | 0.945 | 0.980 | 0.944 | 0.980 | 0.934 | 0.969 | 0.945 | 0.980 | 0.944 | 0.980 | 0.931 | 0.966 | 0.945 | 0.980 | 0.944 | 0.980 | 0.929 | 0.964 |
| | | 0.99 | 11.197 | 11.806 | 11.176 | 11.785 | 9.783 | 10.311 | 11.197 | 11.806 | 11.176 | 11.785 | 9.476 | 9.986 | 11.197 | 11.806 | 11.176 | 11.785 | 9.184 | 9.678 |

Table 2: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 20$ and $p = 3$ when there is one outlier.

| $k$ | Sigma | Rho | $d = 0.2$ | | | | | | $d = 0.5$ | | | | | | $d = 0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{\mathrm{MRT}}$ | $\hat{\alpha}_{\mathrm{RTMME}}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{\mathrm{MRT}}$ | $\hat{\alpha}_{\mathrm{RTMME}}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{\mathrm{MRT}}$ | $\hat{\alpha}_{\mathrm{RTMME}}$ |
| 0.3 | 1 | 0.7 | 14.015 | 6.603 | 12.842 | 6.066 | 12.628 | 5.968 | 14.015 | 6.603 | 12.842 | 6.066 | 12.319 | 5.827 | 14.015 | 6.603 | 12.842 | 6.066 | 12.023 | 5.691 |
| | | 0.8 | 21.072 | 9.888 | 18.352 | 8.633 | 17.879 | 8.414 | 21.072 | 9.888 | 18.352 | 8.633 | 17.207 | 8.104 | 21.072 | 9.888 | 18.352 | 8.633 | 16.576 | 7.813 |
| | | 0.9 | 608.818 | 1.152 | 592.046 | 1.140 | 410.694 | 0.853 | 608.818 | 1.152 | 592.046 | 1.140 | 376.593 | 0.800 | 608.818 | 1.152 | 592.046 | 1.140 | 346.658 | 0.753 |
| | | 0.99 | 8184.099 | 14.132 | 5874.695 | 12.202 | 574.320 | 1.601 | 8184.099 | 14.132 | 5874.695 | 12.202 | 411.088 | 1.239 | 8184.099 | 14.132 | 5874.695 | 12.202 | 308.936 | 1.004 |
| | 5 | 0.7 | 1.125 | 0.924 | 1.120 | 0.920 | 1.119 | 0.919 | 1.125 | 0.924 | 1.120 | 0.920 | 1.118 | 0.918 | 1.125 | 0.924 | 1.120 | 0.920 | 1.116 | 0.917 |
| | | 0.8 | 1.639 | 1.292 | 1.628 | 1.284 | 1.626 | 1.282 | 1.639 | 1.292 | 1.628 | 1.284 | 1.623 | 1.279 | 1.639 | 1.292 | 1.628 | 1.284 | 1.620 | 1.277 |
| | | 0.9 | 122.561 | 1.152 | 122.425 | 1.152 | 120.476 | 1.136 | 122.561 | 1.152 | 122.425 | 1.152 | 119.964 | 1.133 | 122.561 | 1.152 | 122.425 | 1.152 | 119.454 | 1.129 |
| | | 0.99 | 1646.049 | 14.129 | 1622.592 | 14.043 | 1331.117 | 11.784 | 1646.049 | 14.129 | 1622.592 | 14.043 | 1266.599 | 11.296 | 1646.049 | 14.129 | 1622.592 | 14.043 | 1206.688 | 10.841 |
| | 10 | 0.7 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 |
| | | 0.8 | 0.306 | 0.190 | 0.306 | 0.190 | 0.306 | 0.190 | 0.306 | 0.190 | 0.306 | 0.190 | 0.305 | 0.190 | 0.306 | 0.190 | 0.306 | 0.190 | 0.305 | 0.190 |
| | | 0.9 | 61.768 | 1.152 | 61.751 | 1.152 | 61.506 | 1.148 | 61.768 | 1.152 | 61.751 | 1.152 | 61.441 | 1.147 | 61.768 | 1.152 | 61.751 | 1.152 | 61.376 | 1.146 |
| | | 0.99 | 828.724 | 14.128 | 825.755 | 14.106 | 784.255 | 13.474 | 828.724 | 14.128 | 825.755 | 14.106 | 773.701 | 13.318 | 828.724 | 14.128 | 825.755 | 14.106 | 763.361 | 13.165 |
| 0.7 | 1 | 0.7 | 14.015 | 6.603 | 11.526 | 5.464 | 11.120 | 5.279 | 14.015 | 6.603 | 11.526 | 5.464 | 10.558 | 5.022 | 14.015 | 6.603 | 11.526 | 5.464 | 10.044 | 4.787 |
| | | 0.8 | 21.072 | 9.888 | 15.549 | 7.339 | 14.735 | 6.963 | 21.072 | 9.888 | 15.549 | 7.339 | 13.644 | 6.459 | 21.072 | 9.888 | 15.549 | 7.339 | 12.684 | 6.016 |
| | | 0.9 | 608.818 | 1.152 | 592.046 | 1.140 | 269.408 | 0.631 | 608.818 | 1.152 | 592.046 | 1.140 | 230.019 | 0.568 | 608.818 | 1.152 | 592.046 | 1.140 | 198.852 | 0.517 |
| | | 0.99 | 8184.099 | 14.132 | 5874.695 | 12.202 | 148.737 | 0.616 | 8184.099 | 14.132 | 5874.695 | 12.202 | 101.143 | 0.496 | 8184.099 | 14.132 | 5874.695 | 12.202 | 73.364 | 0.425 |
| | 5 | 0.7 | 1.125 | 0.924 | 1.114 | 0.914 | 1.112 | 0.912 | 1.125 | 0.924 | 1.114 | 0.914 | 1.108 | 0.910 | 1.125 | 0.924 | 1.114 | 0.914 | 1.105 | 0.907 |
| | | 0.8 | 1.639 | 1.292 | 1.614 | 1.272 | 1.610 | 1.268 | 1.639 | 1.292 | 1.614 | 1.272 | 1.603 | 1.262 | 1.639 | 1.292 | 1.614 | 1.272 | 1.596 | 1.256 |
| | | 0.9 | 122.561 | 1.152 | 122.425 | 1.152 | 117.781 | 1.116 | 122.561 | 1.152 | 122.425 | 1.152 | 116.631 | 1.108 | 122.561 | 1.152 | 122.425 | 1.152 | 115.498 | 1.099 |
| | | 0.99 | 1646.049 | 14.129 | 1622.592 | 14.043 | 1035.101 | 9.523 | 1646.049 | 14.129 | 1622.592 | 14.043 | 935.995 | 8.750 | 1646.049 | 14.129 | 1622.592 | 14.043 | 850.550 | 8.075 |
| | 10 | 0.7 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.202 | 0.127 |
| | | 0.8 | 0.306 | 0.190 | 0.305 | 0.190 | 0.305 | 0.190 | 0.306 | 0.190 | 0.305 | 0.190 | 0.304 | 0.189 | 0.306 | 0.190 | 0.305 | 0.190 | 0.304 | 0.189 |
| | | 0.9 | 61.768 | 1.152 | 61.751 | 1.152 | 61.160 | 1.143 | 61.768 | 1.152 | 61.751 | 1.152 | 61.009 | 1.141 | 61.768 | 1.152 | 61.751 | 1.152 | 60.859 | 1.138 |
| | | 0.99 | 828.724 | 14.128 | 825.755 | 14.106 | 730.360 | 12.676 | 828.724 | 14.128 | 825.755 | 14.106 | 708.525 | 12.351 | 828.724 | 14.128 | 825.755 | 14.106 | 687.663 | 12.040 |
| 0.9 | 1 | 0.7 | 14.015 | 6.603 | 10.954 | 5.203 | 10.481 | 4.987 | 14.015 | 6.603 | 10.954 | 5.203 | 9.837 | 4.692 | 14.015 | 6.603 | 10.954 | 5.203 | 9.260 | 4.429 |
| | | 0.8 | 21.072 | 9.888 | 14.408 | 6.812 | 13.499 | 6.392 | 21.072 | 9.888 | 14.408 | 6.812 | 12.307 | 5.842 | 21.072 | 9.888 | 14.408 | 6.812 | 11.285 | 5.370 |
| | | 0.9 | 608.818 | 1.152 | 592.046 | 1.140 | 225.122 | 0.560 | 608.818 | 1.152 | 592.046 | 1.140 | 187.451 | 0.499 | 608.818 | 1.152 | 592.046 | 1.140 | 158.695 | 0.452 |
| | | 0.99 | 8184.099 | 14.132 | 5874.695 | 12.202 | 96.278 | 0.484 | 8184.099 | 14.132 | 5874.695 | 12.202 | 64.898 | 0.403 | 8184.099 | 14.132 | 5874.695 | 12.202 | 46.836 | 0.358 |
| | 5 | 0.7 | 1.125 | 0.924 | 1.111 | 0.912 | 1.108 | 0.909 | 1.125 | 0.924 | 1.111 | 0.912 | 1.104 | 0.905 | 1.125 | 0.924 | 1.111 | 0.912 | 1.100 | 0.902 |
| | | 0.8 | 1.639 | 1.292 | 1.608 | 1.266 | 1.602 | 1.261 | 1.639 | 1.292 | 1.608 | 1.266 | 1.593 | 1.254 | 1.639 | 1.292 | 1.608 | 1.266 | 1.584 | 1.246 |
| | | 0.9 | 122.561 | 1.152 | 122.425 | 1.152 | 116.468 | 1.106 | 122.561 | 1.152 | 122.425 | 1.152 | 115.018 | 1.095 | 122.561 | 1.152 | 122.425 | 1.152 | 113.596 | 1.085 |
| | | 0.99 | 1646.049 | 14.129 | 1622.592 | 14.043 | 923.015 | 8.648 | 1646.049 | 14.129 | 1622.592 | 14.043 | 817.497 | 7.812 | 1646.049 | 14.129 | 1622.592 | 14.043 | 729.190 | 7.100 |
| | 10 | 0.7 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.202 | 0.127 | 0.203 | 0.127 | 0.203 | 0.127 | 0.202 | 0.126 |
| | | 0.8 | 0.306 | 0.190 | 0.305 | 0.190 | 0.304 | 0.189 | 0.306 | 0.190 | 0.305 | 0.190 | 0.304 | 0.189 | 0.306 | 0.190 | 0.305 | 0.190 | 0.303 | 0.189 |
| | | 0.9 | 61.768 | 1.152 | 61.751 | 1.152 | 60.988 | 1.140 | 61.768 | 1.152 | 61.751 | 1.152 | 60.795 | 1.137 | 61.768 | 1.152 | 61.751 | 1.152 | 60.603 | 1.134 |
| | | 0.99 | 828.724 | 14.128 | 825.755 | 14.106 | 705.487 | 12.306 | 828.724 | 14.128 | 825.755 | 14.106 | 679.005 | 11.910 | 828.724 | 14.128 | 825.755 | 14.106 | 653.998 | 11.535 |

Table 3: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 20$ and $p = 3$ when there are two outliers.

| $k$ | Sigma | Rho | $d = 0.2$ | | | | | | $d = 0.5$ | | | | | | $d = 0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ |
| 0.3 | 1 | 0.7 | 21.041 | 16.798 | 19.134 | 15.122 | 18.786 | 14.819 | 21.041 | 16.798 | 19.134 | 15.122 | 18.282 | 14.381 | 21.041 | 16.798 | 19.134 | 15.122 | 17.800 | 13.964 |
| | | 0.8 | 29.866 | 24.389 | 26.044 | 20.913 | 25.374 | 20.312 | 29.866 | 24.389 | 26.044 | 20.913 | 24.420 | 19.459 | 29.866 | 24.389 | 26.044 | 20.913 | 23.522 | 18.661 |
| | | 0.9 | 875.795 | 2.113 | 832.443 | 2.065 | 697.950 | 1.534 | 875.795 | 2.113 | 832.443 | 2.065 | 662.429 | 1.430 | 875.795 | 2.113 | 832.443 | 2.065 | 629.686 | 1.336 |
| | | 0.99 | 8864.956 | 25.822 | 5151.316 | 19.581 | 1588.579 | 2.684 | 8864.956 | 25.822 | 5151.316 | 19.581 | 1227.830 | 1.984 | 8864.956 | 25.822 | 5151.316 | 19.581 | 978.504 | 1.530 |
| | 5 | 0.7 | 0.909 | 0.893 | 0.904 | 0.889 | 0.904 | 0.888 | 0.909 | 0.893 | 0.904 | 0.889 | 0.902 | 0.886 | 0.909 | 0.893 | 0.904 | 0.889 | 0.901 | 0.885 |
| | | 0.8 | 1.276 | 1.242 | 1.267 | 1.233 | 1.265 | 1.231 | 1.276 | 1.242 | 1.267 | 1.233 | 1.263 | 1.229 | 1.276 | 1.242 | 1.267 | 1.233 | 1.260 | 1.226 |
| | | 0.9 | 176.002 | 2.111 | 175.637 | 2.109 | 174.288 | 2.081 | 176.002 | 2.111 | 175.637 | 2.109 | 173.864 | 2.074 | 176.002 | 2.111 | 175.637 | 2.109 | 173.441 | 2.067 |
| | | 0.99 | 1779.071 | 25.812 | 1732.191 | 25.503 | 1575.743 | 21.596 | 1779.071 | 25.812 | 1732.191 | 25.503 | 1531.295 | 20.716 | 1779.071 | 25.812 | 1732.191 | 25.503 | 1489.009 | 19.893 |
| | 10 | 0.7 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.146 |
| | | 0.8 | 0.332 | 0.220 | 0.332 | 0.219 | 0.332 | 0.219 | 0.332 | 0.220 | 0.332 | 0.219 | 0.332 | 0.219 | 0.332 | 0.220 | 0.332 | 0.219 | 0.331 | 0.219 |
| | | 0.9 | 88.506 | 2.111 | 88.460 | 2.110 | 88.288 | 2.103 | 88.506 | 2.111 | 88.460 | 2.110 | 88.234 | 2.101 | 88.506 | 2.111 | 88.460 | 2.110 | 88.179 | 2.100 |
| | | 0.99 | 894.056 | 25.809 | 888.028 | 25.731 | 866.223 | 24.636 | 894.056 | 25.809 | 888.028 | 25.731 | 859.509 | 24.356 | 894.056 | 25.809 | 888.028 | 25.731 | 852.888 | 24.082 |
| 0.7 | 1 | 0.7 | 21.041 | 16.798 | 16.992 | 13.267 | 16.331 | 12.701 | 21.041 | 16.798 | 16.992 | 13.267 | 15.415 | 11.921 | 21.041 | 16.798 | 16.992 | 13.267 | 14.577 | 11.214 |
| | | 0.8 | 29.866 | 24.389 | 22.050 | 17.365 | 20.879 | 16.343 | 29.866 | 24.389 | 22.050 | 17.365 | 19.299 | 14.977 | 29.866 | 24.389 | 22.050 | 17.365 | 17.899 | 13.782 |
| | | 0.9 | 875.795 | 2.113 | 832.443 | 2.065 | 537.079 | 1.089 | 875.795 | 2.113 | 832.443 | 2.065 | 484.307 | 0.958 | 875.795 | 2.113 | 832.443 | 2.065 | 439.189 | 0.851 |
| | | 0.99 | 8864.956 | 25.822 | 5151.316 | 19.581 | 533.881 | 0.791 | 8864.956 | 25.822 | 5151.316 | 19.581 | 383.311 | 0.569 | 8864.956 | 25.822 | 5151.316 | 19.581 | 289.637 | 0.442 |
| | 5 | 0.7 | 0.909 | 0.893 | 0.899 | 0.883 | 0.897 | 0.881 | 1.125 | 0.924 | 1.114 | 0.914 | 1.108 | 0.910 | 0.909 | 0.893 | 0.899 | 0.883 | 0.891 | 0.875 |
| | | 0.8 | 1.276 | 1.242 | 1.255 | 1.221 | 1.251 | 1.217 | 1.639 | 1.292 | 1.614 | 1.272 | 1.603 | 1.262 | 1.276 | 1.242 | 1.255 | 1.221 | 1.240 | 1.205 |
| | | 0.9 | 176.002 | 2.111 | 175.637 | 2.109 | 172.045 | 2.043 | 176.002 | 2.111 | 175.637 | 2.109 | 171.078 | 2.026 | 176.002 | 2.111 | 175.637 | 2.109 | 170.119 | 2.010 |
| | | 0.99 | 1779.071 | 25.812 | 1732.191 | 25.503 | 1361.739 | 17.499 | 1779.071 | 25.812 | 1732.191 | 25.503 | 1283.348 | 16.089 | 1779.071 | 25.812 | 1732.191 | 25.503 | 1212.316 | 14.853 |
| | 10 | 0.7 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.145 |
| | | 0.8 | 0.332 | 0.220 | 0.331 | 0.219 | 0.331 | 0.219 | 0.332 | 0.220 | 0.331 | 0.219 | 0.331 | 0.219 | 0.332 | 0.220 | 0.331 | 0.219 | 0.330 | 0.218 |
| | | 0.9 | 88.506 | 2.111 | 88.460 | 2.110 | 87.999 | 2.093 | 88.506 | 2.111 | 88.460 | 2.110 | 87.872 | 2.089 | 88.506 | 2.111 | 88.460 | 2.110 | 87.747 | 2.085 |
| | | 0.99 | 894.056 | 25.809 | 888.028 | 25.731 | 831.465 | 23.202 | 894.056 | 25.809 | 888.028 | 25.731 | 817.035 | 22.618 | 894.056 | 25.809 | 888.028 | 25.731 | 803.045 | 22.056 |
| 0.9 | 1 | 0.7 | 21.041 | 16.798 | 16.061 | 12.470 | 15.290 | 11.816 | 21.041 | 16.798 | 16.061 | 12.470 | 14.240 | 10.930 | 21.041 | 16.798 | 16.061 | 12.470 | 13.300 | 10.145 |
| | | 0.8 | 29.866 | 24.389 | 20.407 | 15.933 | 19.089 | 14.797 | 29.866 | 24.389 | 20.407 | 15.933 | 17.348 | 13.315 | 29.866 | 24.389 | 20.407 | 15.933 | 15.845 | 12.054 |
| | | 0.9 | 875.795 | 2.113 | 832.443 | 2.065 | 477.431 | 0.941 | 875.795 | 2.113 | 832.443 | 2.065 | 421.825 | 0.812 | 875.795 | 2.113 | 832.443 | 2.065 | 375.665 | 0.711 |
| | | 0.99 | 8864.956 | 25.822 | 5151.316 | 19.581 | 367.250 | 0.546 | 8864.956 | 25.822 | 5151.316 | 19.581 | 260.049 | 0.404 | 8864.956 | 25.822 | 5151.316 | 19.581 | 195.002 | 0.326 |
| | 5 | 0.7 | 0.909 | 0.893 | 0.896 | 0.880 | 0.894 | 0.877 | 0.909 | 0.893 | 0.896 | 0.880 | 0.890 | 0.874 | 0.909 | 0.893 | 0.896 | 0.880 | 0.887 | 0.870 |
| | | 0.8 | 1.276 | 1.242 | 1.250 | 1.215 | 1.245 | 1.210 | 1.276 | 1.242 | 1.250 | 1.215 | 1.237 | 1.202 | 1.276 | 1.242 | 1.250 | 1.215 | 1.230 | 1.195 |
| | | 0.9 | 176.002 | 2.111 | 175.637 | 2.109 | 170.940 | 2.024 | 176.002 | 2.111 | 175.637 | 2.109 | 169.711 | 2.003 | 176.002 | 2.111 | 175.637 | 2.109 | 168.496 | 1.983 |
| | | 0.99 | 1779.071 | 25.812 | 1732.191 | 25.503 | 1272.776 | 15.902 | 1779.071 | 25.812 | 1732.191 | 25.503 | 1183.871 | 14.370 | 1779.071 | 25.812 | 1732.191 | 25.503 | 1104.875 | 13.061 |
| | 10 | 0.7 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.145 | 0.218 | 0.146 | 0.218 | 0.146 | 0.218 | 0.145 | 0.218 | 0.146 | 0.218 | 0.146 | 0.217 | 0.145 |
| | | 0.8 | 0.332 | 0.220 | 0.331 | 0.219 | 0.331 | 0.219 | 0.332 | 0.220 | 0.331 | 0.219 | 0.330 | 0.218 | 0.332 | 0.220 | 0.331 | 0.219 | 0.330 | 0.218 |
| | | 0.9 | 88.506 | 2.111 | 88.460 | 2.110 | 87.854 | 2.088 | 88.506 | 2.111 | 88.460 | 2.110 | 87.693 | 2.083 | 88.506 | 2.111 | 88.460 | 2.110 | 87.531 | 2.077 |
| | | 0.99 | 894.056 | 25.809 | 888.028 | 25.731 | 815.010 | 22.536 | 894.056 | 25.809 | 888.028 | 25.731 | 797.179 | 21.823 | 894.056 | 25.809 | 888.028 | 25.731 | 780.028 | 21.146 |

Table 4: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 50$ and $p = 3$ when there is no outlier.

| $k$ | Sigma | Rho | $d=0.2$ | | | | | | $d=0.5$ | | | | | | $d=0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ |
| 0.3 | 1 | 0.7 | 4.889 | 2.753 | 4.735 | 2.671 | 4.706 | 2.655 | 4.889 | 2.753 | 4.735 | 2.671 | 4.662 | 2.631 | 4.889 | 2.753 | 4.735 | 2.671 | 4.619 | 2.608 |
| | | 0.8 | 7.282 | 4.247 | 6.928 | 4.046 | 6.860 | 4.008 | 7.282 | 4.247 | 6.928 | 4.046 | 6.762 | 3.952 | 7.282 | 4.247 | 6.928 | 4.046 | 6.666 | 3.897 |
| | | 0.9 | 0.339 | 0.356 | 0.337 | 0.354 | 0.296 | 0.311 | 0.339 | 0.356 | 0.337 | 0.354 | 0.287 | 0.301 | 0.339 | 0.356 | 0.337 | 0.354 | 0.278 | 0.292 |
| | | 0.99 | 4.000 | 4.247 | 3.714 | 3.968 | 1.247 | 1.322 | 4.000 | 4.247 | 3.714 | 3.968 | 1.046 | 1.108 | 4.000 | 4.247 | 3.714 | 3.968 | 0.898 | 0.950 |
| | 5 | 0.7 | 0.242 | 0.156 | 0.241 | 0.156 | 0.241 | 0.156 | 0.242 | 0.156 | 0.241 | 0.156 | 0.241 | 0.156 | 0.242 | 0.156 | 0.241 | 0.156 | 0.241 | 0.156 |
| | | 0.8 | 0.362 | 0.233 | 0.361 | 0.232 | 0.361 | 0.232 | 0.362 | 0.233 | 0.361 | 0.232 | 0.361 | 0.232 | 0.362 | 0.233 | 0.361 | 0.232 | 0.361 | 0.232 |
| | | 0.9 | 0.339 | 0.356 | 0.339 | 0.356 | 0.337 | 0.354 | 0.339 | 0.356 | 0.339 | 0.356 | 0.337 | 0.354 | 0.339 | 0.356 | 0.339 | 0.356 | 0.336 | 0.353 |
| | | 0.99 | 4.000 | 4.247 | 3.988 | 4.235 | 3.719 | 3.949 | 4.000 | 4.247 | 3.988 | 4.235 | 3.654 | 3.879 | 4.000 | 4.247 | 3.988 | 4.235 | 3.591 | 3.813 |
| | 10 | 0.7 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 |
| | | 0.8 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 |
| | | 0.9 | 0.339 | 0.356 | 0.339 | 0.356 | 0.339 | 0.356 | 0.339 | 0.356 | 0.339 | 0.356 | 0.338 | 0.355 | 0.339 | 0.356 | 0.339 | 0.356 | 0.338 | 0.355 |
| | | 0.99 | 4.000 | 4.247 | 3.997 | 4.244 | 3.926 | 4.169 | 4.000 | 4.247 | 3.997 | 4.244 | 3.908 | 4.150 | 4.000 | 4.247 | 3.997 | 4.244 | 3.890 | 4.131 |
| 0.7 | 1 | 0.7 | 4.889 | 2.753 | 4.544 | 2.567 | 4.480 | 2.533 | 4.889 | 2.753 | 4.544 | 2.567 | 4.388 | 2.483 | 4.889 | 2.753 | 4.544 | 2.567 | 4.299 | 2.435 |
| | | 0.8 | 7.282 | 4.247 | 6.501 | 3.803 | 6.363 | 3.724 | 7.282 | 4.247 | 6.501 | 3.803 | 6.165 | 3.612 | 7.282 | 4.247 | 6.501 | 3.803 | 5.977 | 3.505 |
| | | 0.9 | 0.339 | 0.356 | 0.337 | 0.354 | 0.252 | 0.265 | 0.339 | 0.356 | 0.337 | 0.354 | 0.237 | 0.249 | 0.339 | 0.356 | 0.337 | 0.354 | 0.224 | 0.235 |
| | | 0.99 | 4.000 | 4.247 | 3.714 | 3.968 | 0.608 | 0.641 | 4.000 | 4.247 | 3.714 | 3.968 | 0.498 | 0.524 | 4.000 | 4.247 | 3.714 | 3.968 | 0.426 | 0.446 |
| | 5 | 0.7 | 0.242 | 0.156 | 0.241 | 0.156 | 0.241 | 0.155 | 0.242 | 0.156 | 0.241 | 0.156 | 0.241 | 0.155 | 0.242 | 0.156 | 0.241 | 0.156 | 0.240 | 0.155 |
| | | 0.8 | 0.362 | 0.233 | 0.360 | 0.232 | 0.360 | 0.231 | 0.362 | 0.233 | 0.360 | 0.232 | 0.359 | 0.231 | 0.362 | 0.233 | 0.360 | 0.232 | 0.359 | 0.231 |
| | | 0.9 | 0.339 | 0.356 | 0.339 | 0.356 | 0.335 | 0.351 | 0.339 | 0.356 | 0.339 | 0.356 | 0.333 | 0.350 | 0.339 | 0.356 | 0.339 | 0.356 | 0.332 | 0.349 |
| | | 0.99 | 4.000 | 4.247 | 3.988 | 4.235 | 3.394 | 3.603 | 4.000 | 4.247 | 3.988 | 4.235 | 3.267 | 3.468 | 4.000 | 4.247 | 3.988 | 4.235 | 3.148 | 3.341 |
| | 10 | 0.7 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 |
| | | 0.8 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 |
| | | 0.9 | 0.339 | 0.356 | 0.339 | 0.356 | 0.338 | 0.355 | 0.339 | 0.356 | 0.339 | 0.356 | 0.338 | 0.355 | 0.339 | 0.356 | 0.339 | 0.356 | 0.337 | 0.354 |
| | | 0.99 | 4.000 | 4.247 | 3.997 | 4.244 | 3.832 | 4.068 | 4.000 | 4.247 | 3.997 | 4.244 | 3.791 | 4.026 | 4.000 | 4.247 | 3.997 | 4.244 | 3.752 | 3.984 |
| 0.9 | 1 | 0.7 | 4.889 | 2.753 | 4.454 | 2.519 | 4.375 | 2.476 | 4.889 | 2.753 | 4.454 | 2.519 | 4.261 | 2.415 | 4.889 | 2.753 | 4.454 | 2.519 | 4.153 | 2.356 |
| | | 0.8 | 7.282 | 4.247 | 6.305 | 3.691 | 6.137 | 3.596 | 7.282 | 4.247 | 6.305 | 3.691 | 5.900 | 3.461 | 7.282 | 4.247 | 6.305 | 3.691 | 5.679 | 3.335 |
| | | 0.9 | 0.339 | 0.356 | 0.337 | 0.354 | 0.235 | 0.247 | 0.339 | 0.356 | 0.337 | 0.354 | 0.219 | 0.229 | 0.339 | 0.356 | 0.337 | 0.354 | 0.204 | 0.214 |
| | | 0.99 | 4.000 | 4.247 | 3.714 | 3.968 | 0.486 | 0.511 | 4.000 | 4.247 | 3.714 | 3.968 | 0.402 | 0.421 | 4.000 | 4.247 | 3.714 | 3.968 | 0.349 | 0.364 |
| | 5 | 0.7 | 0.242 | 0.156 | 0.241 | 0.155 | 0.240 | 0.155 | 0.242 | 0.156 | 0.241 | 0.155 | 0.240 | 0.155 | 0.242 | 0.156 | 0.241 | 0.155 | 0.240 | 0.155 |
| | | 0.8 | 0.362 | 0.233 | 0.360 | 0.231 | 0.359 | 0.231 | 0.362 | 0.233 | 0.360 | 0.231 | 0.359 | 0.230 | 0.362 | 0.233 | 0.360 | 0.231 | 0.358 | 0.230 |
| | | 0.9 | 0.339 | 0.356 | 0.339 | 0.356 | 0.333 | 0.350 | 0.339 | 0.356 | 0.339 | 0.356 | 0.332 | 0.349 | 0.339 | 0.356 | 0.339 | 0.356 | 0.330 | 0.347 |
| | | 0.99 | 4.000 | 4.247 | 3.988 | 4.235 | 3.249 | 3.449 | 4.000 | 4.247 | 3.988 | 4.235 | 3.099 | 3.290 | 4.000 | 4.247 | 3.988 | 4.235 | 2.960 | 3.142 |
| | 10 | 0.7 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 | 0.035 | 0.019 |
| | | 0.8 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 | 0.053 | 0.030 |
| | | 0.9 | 0.339 | 0.356 | 0.339 | 0.356 | 0.338 | 0.355 | 0.339 | 0.356 | 0.339 | 0.356 | 0.337 | 0.354 | 0.339 | 0.356 | 0.339 | 0.356 | 0.330 | 0.347 |
| | | 0.99 | 4.000 | 4.247 | 3.997 | 4.244 | 3.786 | 4.020 | 4.000 | 4.247 | 3.997 | 4.244 | 3.735 | 3.966 | 4.000 | 4.247 | 3.988 | 4.235 | 2.960 | 3.142 |

Table 5: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 50$ and $p = 3$ when there is one outlier.

| $k$ | Sigma | Rho | $d = 0.2$ | | | | | | $d = 0.5$ | | | | | | $d = 0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\widehat{\alpha}$ | $\widehat{\alpha}_M$ | $\widehat{\alpha}(k)$ | $\widehat{\alpha}_M(k)$ | $\widehat{\alpha}_{\text{MRT}}$ | $\widehat{\alpha}_{\text{RTMME}}$ | $\widehat{\alpha}$ | $\widehat{\alpha}_M$ | $\widehat{\alpha}(k)$ | $\widehat{\alpha}_M(k)$ | $\widehat{\alpha}_{\text{MRT}}$ | $\widehat{\alpha}_{\text{RTMME}}$ | $\widehat{\alpha}$ | $\widehat{\alpha}_M$ | $\widehat{\alpha}(k)$ | $\widehat{\alpha}_M(k)$ | $\widehat{\alpha}_{\text{MRT}}$ | $\widehat{\alpha}_{\text{RTMME}}$ |
| 0.3 | 1 | 0.7 | 5.240 | 2.879 | 5.081 | 2.795 | 5.051 | 2.779 | 5.240 | 2.879 | 5.081 | 2.795 | 5.005 | 2.755 | 5.240 | 2.879 | 5.081 | 2.795 | 4.961 | 2.731 |
| | | 0.8 | 7.955 | 4.487 | 7.574 | 4.277 | 7.501 | 4.237 | 7.955 | 4.487 | 7.574 | 4.277 | 7.395 | 4.179 | 7.955 | 4.487 | 7.574 | 4.277 | 7.292 | 4.122 |
| | | 0.9 | 37.725 | 0.362 | 37.365 | 0.360 | 32.709 | 0.318 | 37.725 | 0.362 | 37.365 | 0.360 | 31.629 | 0.309 | 37.725 | 0.362 | 37.365 | 0.360 | 30.609 | 0.300 |
| | | 0.99 | 450.876 | 4.394 | 398.785 | 4.144 | 127.316 | 1.381 | 450.876 | 4.394 | 398.785 | 4.144 | 104.515 | 1.161 | 450.876 | 4.394 | 398.785 | 4.144 | 87.850 | 1.000 |
| | 5 | 0.7 | 0.332 | 0.185 | 0.332 | 0.185 | 0.332 | 0.185 | 0.332 | 0.185 | 0.332 | 0.185 | 0.332 | 0.185 | 0.355 | 0.201 | 0.355 | 0.201 | 0.354 | 0.201 |
| | | 0.8 | 0.501 | 0.280 | 0.500 | 0.279 | 0.500 | 0.279 | 0.501 | 0.280 | 0.500 | 0.279 | 0.499 | 0.279 | 0.520 | 0.300 | 0.519 | 0.300 | 0.518 | 0.299 |
| | | 0.9 | 7.783 | 0.362 | 7.781 | 0.362 | 7.739 | 0.360 | 7.783 | 0.362 | 7.781 | 0.362 | 7.728 | 0.359 | 7.783 | 0.362 | 7.781 | 0.362 | 7.717 | 0.359 |
| | | 0.99 | 94.623 | 4.394 | 94.143 | 4.383 | 87.590 | 4.085 | 94.623 | 4.394 | 94.143 | 4.383 | 85.967 | 4.014 | 94.623 | 4.394 | 94.143 | 4.383 | 84.393 | 3.945 |
| | 10 | 0.7 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 |
| | | 0.8 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 |
| | | 0.9 | 4.049 | 0.362 | 4.049 | 0.362 | 4.043 | 0.361 | 4.049 | 0.362 | 4.049 | 0.362 | 4.042 | 0.361 | 4.049 | 0.362 | 4.049 | 0.362 | 4.041 | 0.361 |
| | | 0.99 | 49.779 | 4.394 | 49.716 | 4.391 | 48.813 | 4.313 | 49.779 | 4.394 | 49.716 | 4.391 | 48.576 | 4.293 | 49.779 | 4.394 | 49.716 | 4.391 | 48.341 | 4.273 |
| 0.7 | 1 | 0.7 | 5.240 | 2.879 | 4.883 | 2.690 | 4.818 | 2.655 | 5.240 | 2.879 | 4.883 | 2.690 | 4.722 | 2.604 | 5.240 | 2.879 | 4.883 | 2.690 | 4.629 | 2.555 |
| | | 0.8 | 7.955 | 4.487 | 7.115 | 4.024 | 6.966 | 3.942 | 7.955 | 4.487 | 7.115 | 4.024 | 6.753 | 3.824 | 7.955 | 4.487 | 7.115 | 4.024 | 6.551 | 3.713 |
| | | 0.9 | 37.725 | 0.362 | 37.365 | 0.360 | 27.579 | 0.274 | 37.725 | 0.362 | 37.365 | 0.360 | 25.749 | 0.258 | 37.725 | 0.362 | 37.365 | 0.360 | 24.117 | 0.245 |
| | | 0.99 | 450.876 | 4.394 | 398.785 | 4.144 | 55.219 | 0.684 | 450.876 | 4.394 | 398.785 | 4.144 | 42.745 | 0.563 | 450.876 | 4.394 | 398.785 | 4.144 | 34.339 | 0.484 |
| | 5 | 0.7 | 0.332 | 0.185 | 0.331 | 0.185 | 0.331 | 0.185 | 0.332 | 0.185 | 0.331 | 0.185 | 0.331 | 0.185 | 0.332 | 0.185 | 0.331 | 0.185 | 0.330 | 0.184 |
| | | 0.8 | 0.501 | 0.280 | 0.499 | 0.278 | 0.498 | 0.278 | 0.501 | 0.280 | 0.499 | 0.278 | 0.497 | 0.278 | 0.501 | 0.280 | 0.499 | 0.278 | 0.497 | 0.277 |
| | | 0.9 | 7.783 | 0.362 | 7.781 | 0.362 | 7.681 | 0.357 | 7.783 | 0.362 | 7.781 | 0.362 | 7.655 | 0.356 | 7.783 | 0.362 | 7.781 | 0.362 | 7.630 | 0.355 |
| | | 0.99 | 94.623 | 4.394 | 94.143 | 4.383 | 79.478 | 3.728 | 94.623 | 4.394 | 94.143 | 4.383 | 76.315 | 3.589 | 94.623 | 4.394 | 94.143 | 4.383 | 73.357 | 3.458 |
| | 10 | 0.7 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 |
| | | 0.8 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 |
| | | 0.9 | 4.049 | 0.362 | 4.049 | 0.362 | 4.036 | 0.361 | 4.049 | 0.362 | 4.049 | 0.362 | 4.033 | 0.361 | 4.049 | 0.362 | 4.049 | 0.362 | 4.029 | 0.360 |
| | | 0.99 | 49.779 | 4.394 | 49.716 | 4.391 | 47.572 | 4.209 | 49.779 | 4.394 | 49.716 | 4.391 | 47.046 | 4.165 | 49.779 | 4.394 | 49.716 | 4.391 | 46.529 | 4.122 |
| 0.9 | 1 | 0.7 | 5.240 | 2.879 | 4.790 | 2.640 | 4.708 | 2.597 | 5.240 | 2.879 | 4.790 | 2.640 | 4.591 | 2.534 | 5.240 | 2.879 | 4.790 | 2.640 | 4.479 | 2.475 |
| | | 0.8 | 7.955 | 4.487 | 6.904 | 3.908 | 6.723 | 3.808 | 7.955 | 4.487 | 6.904 | 3.908 | 6.468 | 3.668 | 7.955 | 4.487 | 6.904 | 3.908 | 6.231 | 3.536 |
| | | 0.9 | 37.725 | 0.362 | 37.365 | 0.360 | 25.505 | 0.256 | 37.725 | 0.362 | 37.365 | 0.360 | 23.472 | 0.239 | 37.725 | 0.362 | 37.365 | 0.360 | 21.702 | 0.225 |
| | | 0.99 | 450.876 | 4.394 | 398.785 | 4.144 | 41.346 | 0.550 | 450.876 | 4.394 | 398.785 | 4.144 | 31.540 | 0.457 | 450.876 | 4.394 | 398.785 | 4.144 | 25.061 | 0.398 |
| | 5 | 0.7 | 0.332 | 0.185 | 0.331 | 0.185 | 0.331 | 0.185 | 0.332 | 0.185 | 0.331 | 0.185 | 0.330 | 0.184 | 0.332 | 0.185 | 0.331 | 0.185 | 0.330 | 0.184 |
| | | 0.8 | 0.501 | 0.280 | 0.498 | 0.278 | 0.497 | 0.278 | 0.501 | 0.280 | 0.498 | 0.278 | 0.496 | 0.277 | 0.501 | 0.280 | 0.498 | 0.278 | 0.496 | 0.277 |
| | | 0.9 | 7.783 | 0.362 | 7.781 | 0.362 | 7.652 | 0.356 | 7.783 | 0.362 | 7.781 | 0.362 | 7.619 | 0.355 | 7.783 | 0.362 | 7.781 | 0.362 | 7.587 | 0.353 |
| | | 0.99 | 94.623 | 4.394 | 94.143 | 4.383 | 75.880 | 3.570 | 94.623 | 4.394 | 94.143 | 4.383 | 72.147 | 3.405 | 94.623 | 4.394 | 94.143 | 4.383 | 68.713 | 3.253 |
| | 10 | 0.7 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 | 0.039 | 0.020 |
| | | 0.8 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 | 0.059 | 0.032 |
| | | 0.9 | 4.049 | 0.362 | 4.049 | 0.362 | 4.032 | 0.360 | 4.049 | 0.362 | 4.049 | 0.362 | 4.028 | 0.360 | 7.783 | 0.362 | 7.781 | 0.362 | 7.587 | 0.353 |
| | | 0.99 | 49.779 | 4.394 | 49.716 | 4.391 | 46.971 | 4.159 | 49.779 | 4.394 | 49.716 | 4.391 | 46.310 | 4.103 | 94.623 | 4.394 | 94.143 | 4.383 | 68.713 | 3.253 |

Table 6: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 50$ and $p = 3$ when there are two outliers.

| $k$ | Sigma | Rho | $d = 0.2$ | | | | | | $d = 0.5$ | | | | | | $d = 0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{\mathrm{MRT}}$ | $\hat{\alpha}_{\mathrm{RTMME}}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{\mathrm{MRT}}$ | $\hat{\alpha}_{\mathrm{RTMME}}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{\mathrm{MRT}}$ | $\hat{\alpha}_{\mathrm{RTMME}}$ |
| 0.3 | 1 | 0.7 | 5.896 | 3.408 | 5.700 | 3.298 | 5.662 | 3.277 | 5.896 | 3.408 | 5.700 | 3.298 | 5.606 | 3.246 | 5.896 | 3.408 | 5.700 | 3.298 | 5.551 | 3.215 |
| | | 0.8 | 8.479 | 5.127 | 8.049 | 4.873 | 7.968 | 4.825 | 8.479 | 5.127 | 8.049 | 4.873 | 7.848 | 4.754 | 8.479 | 5.127 | 8.049 | 4.873 | 7.732 | 4.685 |
| | | 0.9 | 14.066 | 0.399 | 13.883 | 0.396 | 13.240 | 0.350 | 14.066 | 0.399 | 13.883 | 0.396 | 13.047 | 0.340 | 14.066 | 0.399 | 13.883 | 0.396 | 12.860 | 0.330 |
| | | 0.99 | 136.974 | 4.789 | 118.997 | 4.264 | 78.216 | 1.486 | 136.974 | 4.789 | 118.997 | 4.264 | 69.921 | 1.249 | 136.974 | 4.789 | 118.997 | 4.264 | 62.956 | 1.075 |
| | 5 | 0.7 | 0.355 | 0.201 | 0.355 | 0.201 | 0.354 | 0.201 | 0.355 | 0.201 | 0.355 | 0.201 | 0.354 | 0.201 | 0.355 | 0.201 | 0.355 | 0.201 | 0.354 | 0.201 |
| | | 0.8 | 0.520 | 0.300 | 0.519 | 0.300 | 0.519 | 0.299 | 0.520 | 0.300 | 0.519 | 0.300 | 0.519 | 0.299 | 0.520 | 0.300 | 0.519 | 0.300 | 0.518 | 0.299 |
| | | 0.9 | 3.085 | 0.399 | 3.083 | 0.399 | 3.077 | 0.397 | 3.085 | 0.399 | 3.083 | 0.399 | 3.076 | 0.396 | 3.085 | 0.399 | 3.083 | 0.399 | 3.074 | 0.396 |
| | | 0.99 | 30.921 | 4.789 | 30.710 | 4.766 | 29.955 | 4.449 | 30.921 | 4.789 | 30.710 | 4.766 | 29.724 | 4.370 | 30.921 | 4.789 | 30.710 | 4.766 | 29.498 | 4.294 |
| | 10 | 0.7 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 |
| | | 0.8 | 0.076 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 |
| | | 0.9 | 1.712 | 0.399 | 1.712 | 0.399 | 1.711 | 0.398 | 1.712 | 0.399 | 1.712 | 0.399 | 1.711 | 0.398 | 1.712 | 0.399 | 1.712 | 0.399 | 1.710 | 0.398 |
| | | 0.99 | 17.583 | 4.789 | 17.549 | 4.783 | 17.423 | 4.700 | 17.583 | 4.789 | 17.549 | 4.783 | 17.384 | 4.678 | 17.583 | 4.789 | 17.549 | 4.783 | 17.345 | 4.656 |
| 0.7 | 1 | 0.7 | 5.896 | 3.408 | 5.456 | 3.162 | 5.375 | 3.117 | 5.896 | 3.408 | 5.456 | 3.162 | 5.257 | 3.051 | 5.896 | 3.408 | 5.456 | 3.162 | 5.144 | 2.988 |
| | | 0.8 | 8.479 | 5.127 | 7.533 | 4.567 | 7.366 | 4.468 | 8.479 | 5.127 | 7.533 | 4.567 | 7.127 | 4.327 | 8.479 | 5.127 | 7.533 | 4.567 | 6.902 | 4.193 |
| | | 0.9 | 14.066 | 0.399 | 13.883 | 0.396 | 12.269 | 0.301 | 14.066 | 0.399 | 13.883 | 0.396 | 11.884 | 0.284 | 14.066 | 0.399 | 13.883 | 0.396 | 11.521 | 0.269 |
| | | 0.99 | 136.974 | 4.789 | 118.997 | 4.264 | 46.238 | 0.735 | 136.974 | 4.789 | 118.997 | 4.264 | 38.368 | 0.606 | 136.974 | 4.789 | 118.997 | 4.264 | 32.440 | 0.521 |
| | 5 | 0.7 | 0.355 | 0.201 | 0.354 | 0.201 | 0.354 | 0.201 | 0.355 | 0.201 | 0.354 | 0.201 | 0.354 | 0.201 | 0.355 | 0.201 | 0.354 | 0.201 | 0.353 | 0.201 |
| | | 0.8 | 0.520 | 0.300 | 0.518 | 0.299 | 0.517 | 0.298 | 0.520 | 0.300 | 0.518 | 0.299 | 0.517 | 0.298 | 0.520 | 0.300 | 0.518 | 0.299 | 0.516 | 0.298 |
| | | 0.9 | 3.085 | 0.399 | 3.083 | 0.399 | 3.068 | 0.394 | 3.085 | 0.399 | 3.083 | 0.399 | 3.064 | 0.393 | 3.085 | 0.399 | 3.083 | 0.399 | 3.060 | 0.391 |
| | | 0.99 | 30.921 | 4.789 | 30.710 | 4.766 | 28.770 | 4.056 | 30.921 | 4.789 | 30.710 | 4.766 | 28.284 | 3.902 | 30.921 | 4.789 | 30.710 | 4.766 | 27.816 | 3.758 |
| | 10 | 0.7 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 |
| | | 0.8 | 0.076 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.075 | 0.038 |
| | | 0.9 | 1.712 | 0.399 | 1.712 | 0.399 | 1.709 | 0.398 | 1.712 | 0.399 | 1.712 | 0.399 | 1.709 | 0.397 | 1.712 | 0.399 | 1.712 | 0.399 | 1.708 | 0.397 |
| | | 0.99 | 17.583 | 4.789 | 17.549 | 4.783 | 17.216 | 4.585 | 17.583 | 4.789 | 17.549 | 4.783 | 17.128 | 4.537 | 17.583 | 4.789 | 17.549 | 4.783 | 17.040 | 4.489 |
| 0.9 | 1 | 0.7 | 5.896 | 3.408 | 5.341 | 3.098 | 5.241 | 3.042 | 5.896 | 3.408 | 5.341 | 3.098 | 5.097 | 2.962 | 5.896 | 3.408 | 5.341 | 3.098 | 4.960 | 2.885 |
| | | 0.8 | 8.479 | 5.127 | 7.296 | 4.427 | 7.094 | 4.307 | 8.479 | 5.127 | 7.296 | 4.427 | 6.809 | 4.139 | 8.479 | 5.127 | 7.296 | 4.427 | 6.544 | 3.981 |
| | | 0.9 | 14.066 | 0.399 | 13.883 | 0.396 | 11.831 | 0.282 | 14.066 | 0.399 | 13.883 | 0.396 | 11.371 | 0.263 | 14.066 | 0.399 | 13.883 | 0.396 | 10.943 | 0.247 |
| | | 0.99 | 136.974 | 4.789 | 118.997 | 4.264 | 37.419 | 0.592 | 136.974 | 4.789 | 118.997 | 4.264 | 30.337 | 0.493 | 136.974 | 4.789 | 118.997 | 4.264 | 25.185 | 0.429 |
| | 5 | 0.7 | 0.355 | 0.201 | 0.354 | 0.201 | 0.354 | 0.201 | 0.355 | 0.201 | 0.354 | 0.201 | 0.353 | 0.200 | 0.355 | 0.201 | 0.354 | 0.201 | 0.353 | 0.200 |
| | | 0.8 | 0.520 | 0.300 | 0.517 | 0.298 | 0.517 | 0.298 | 0.520 | 0.300 | 0.517 | 0.298 | 0.516 | 0.297 | 0.520 | 0.300 | 0.517 | 0.298 | 0.515 | 0.297 |
| | | 0.9 | 3.085 | 0.399 | 3.083 | 0.399 | 3.063 | 0.393 | 3.085 | 0.399 | 3.083 | 0.399 | 3.058 | 0.391 | 3.085 | 0.399 | 3.083 | 0.399 | 3.053 | 0.389 |
| | | 0.99 | 30.921 | 4.789 | 30.710 | 4.766 | 28.216 | 3.881 | 30.921 | 4.789 | 30.710 | 4.766 | 27.620 | 3.699 | 30.921 | 4.789 | 30.710 | 4.766 | 27.051 | 3.532 |
| | 10 | 0.7 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 | 0.051 | 0.025 |
| | | 0.8 | 0.076 | 0.038 | 0.076 | 0.038 | 0.075 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.075 | 0.038 | 0.076 | 0.038 | 0.076 | 0.038 | 0.075 | 0.038 |
| | | 0.9 | 1.712 | 0.399 | 1.712 | 0.399 | 1.709 | 0.397 | 1.712 | 0.399 | 1.712 | 0.399 | 1.708 | 0.397 | 3.085 | 0.399 | 3.083 | 0.399 | 3.053 | 0.389 |
| | | 0.99 | 17.583 | 4.789 | 17.549 | 4.783 | 17.115 | 4.530 | 17.583 | 4.789 | 17.549 | 4.783 | 17.003 | 4.469 | 30.921 | 4.789 | 30.710 | 4.766 | 27.051 | 3.532 |

Table 7: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 100$ and $p = 3$ when there is no outlier.

| $k$ | Sigma | Rho | $d = 0.2$ | | | | | | $d = 0.5$ | | | | | | $d = 0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\widehat{\alpha}$ | $\widehat{\alpha}_M$ | $\widehat{\alpha}(k)$ | $\widehat{\alpha}_M(k)$ | $\widehat{\alpha}_{\text{MRT}}$ | $\widehat{\alpha}_{\text{RTMME}}$ | $\widehat{\alpha}$ | $\widehat{\alpha}_M$ | $\widehat{\alpha}(k)$ | $\widehat{\alpha}_M(k)$ | $\widehat{\alpha}_{\text{MRT}}$ | $\widehat{\alpha}_{\text{RTMME}}$ | $\widehat{\alpha}$ | $\widehat{\alpha}_M$ | $\widehat{\alpha}(k)$ | $\widehat{\alpha}_M(k)$ | $\widehat{\alpha}_{\text{MRT}}$ | $\widehat{\alpha}_{\text{RTMME}}$ |
| 0.3 | 1 | 0.7 | 3.350 | 2.231 | 3.289 | 2.188 | 3.277 | 2.180 | 3.350 | 2.231 | 3.289 | 2.188 | 3.259 | 2.168 | 3.350 | 2.231 | 3.289 | 2.188 | 3.241 | 2.155 |
| | | 0.8 | 4.756 | 3.200 | 4.621 | 3.106 | 4.594 | 3.088 | 4.756 | 3.200 | 4.621 | 3.106 | 4.555 | 3.061 | 4.756 | 3.200 | 4.621 | 3.106 | 4.517 | 3.035 |
| | | 0.9 | 0.189 | 0.200 | 0.188 | 0.199 | 0.176 | 0.186 | 0.189 | 0.200 | 0.188 | 0.199 | 0.173 | 0.183 | 0.189 | 0.200 | 0.188 | 0.199 | 0.170 | 0.180 |
| | | 0.99 | 2.079 | 2.210 | 2.005 | 2.137 | 1.034 | 1.100 | 2.079 | 2.210 | 2.005 | 2.137 | 0.908 | 0.966 | 2.079 | 2.210 | 2.005 | 2.137 | 0.807 | 0.858 |
| | 5 | 0.7 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 |
| | | 0.8 | 0.134 | 0.065 | 0.133 | 0.065 | 0.133 | 0.065 | 0.134 | 0.065 | 0.133 | 0.065 | 0.133 | 0.065 | 0.134 | 0.065 | 0.133 | 0.065 | 0.133 | 0.065 |
| | | 0.9 | 0.189 | 0.200 | 0.189 | 0.200 | 0.188 | 0.199 | 0.189 | 0.200 | 0.189 | 0.200 | 0.188 | 0.199 | 0.189 | 0.200 | 0.189 | 0.200 | 0.188 | 0.199 |
| | | 0.99 | 2.079 | 2.210 | 2.076 | 2.207 | 2.006 | 2.132 | 2.079 | 2.210 | 2.076 | 2.207 | 1.989 | 2.113 | 2.079 | 2.210 | 2.076 | 2.207 | 1.972 | 2.095 |
| | 10 | 0.7 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 |
| | | 0.8 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 |
| | | 0.9 | 0.189 | 0.200 | 0.189 | 0.200 | 0.189 | 0.199 | 0.189 | 0.200 | 0.189 | 0.200 | 0.189 | 0.199 | 0.189 | 0.200 | 0.189 | 0.200 | 0.189 | 0.199 |
| | | 0.99 | 2.079 | 2.210 | 2.079 | 2.209 | 2.061 | 2.190 | 2.079 | 2.210 | 2.079 | 2.209 | 2.056 | 2.185 | 2.079 | 2.210 | 2.079 | 2.209 | 2.051 | 2.180 |
| 0.7 | 1 | 0.7 | 3.350 | 2.231 | 3.210 | 2.134 | 3.183 | 2.115 | 3.350 | 2.231 | 3.210 | 2.134 | 3.143 | 2.088 | 3.350 | 2.231 | 3.289 | 2.188 | 3.241 | 2.155 |
| | | 0.8 | 4.756 | 3.200 | 4.450 | 2.988 | 4.392 | 2.948 | 4.756 | 3.200 | 4.450 | 2.988 | 4.308 | 2.891 | 4.756 | 3.200 | 4.621 | 3.106 | 4.517 | 3.035 |
| | | 0.9 | 0.189 | 0.200 | 0.188 | 0.199 | 0.161 | 0.170 | 0.189 | 0.200 | 0.188 | 0.199 | 0.156 | 0.164 | 0.189 | 0.200 | 0.188 | 0.199 | 0.150 | 0.159 |
| | | 0.99 | 2.079 | 2.210 | 2.005 | 2.137 | 0.583 | 0.620 | 2.079 | 2.210 | 2.005 | 2.137 | 0.487 | 0.517 | 2.079 | 2.210 | 2.005 | 2.137 | 0.419 | 0.445 |
| | 5 | 0.7 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 | 0.094 | 0.046 |
| | | 0.8 | 0.134 | 0.065 | 0.133 | 0.064 | 0.133 | 0.064 | 0.134 | 0.065 | 0.133 | 0.064 | 0.133 | 0.064 | 0.134 | 0.065 | 0.133 | 0.065 | 0.133 | 0.065 |
| | | 0.9 | 0.189 | 0.200 | 0.189 | 0.200 | 0.188 | 0.198 | 0.189 | 0.200 | 0.189 | 0.200 | 0.187 | 0.198 | 0.189 | 0.200 | 0.189 | 0.200 | 0.187 | 0.198 |
| | | 0.99 | 2.079 | 2.210 | 2.076 | 2.207 | 1.916 | 2.036 | 2.079 | 2.210 | 2.076 | 2.207 | 1.878 | 1.996 | 2.079 | 2.210 | 2.076 | 2.207 | 1.842 | 1.957 |
| | 10 | 0.7 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 | 0.022 | 0.012 |
| | | 0.8 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 | 0.032 | 0.018 |
| | | 0.9 | 0.189 | 0.200 | 0.189 | 0.200 | 0.189 | 0.199 | 0.189 | 0.200 | 0.189 | 0.200 | 0.188 | 0.199 | 0.189 | 0.200 | 0.189 | 0.200 | 0.188 | 0.199 |
| | | 0.99 | 2.079 | 2.210 | 2.079 | 2.209 | 2.036 | 2.164 | 2.079 | 2.210 | 2.079 | 2.209 | 2.026 | 2.153 | 2.079 | 2.210 | 2.079 | 2.209 | 2.015 | 2.142 |
| 0.9 | 1 | 0.7 | 0.175 | 0.185 | 0.165 | 0.175 | 0.163 | 0.173 | 0.175 | 0.185 | 0.165 | 0.175 | 0.161 | 0.170 | 0.175 | 0.185 | 0.165 | 0.175 | 0.159 | 0.168 |
| | | 0.8 | 0.276 | 0.293 | 0.251 | 0.266 | 0.247 | 0.262 | 0.276 | 0.293 | 0.251 | 0.266 | 0.241 | 0.255 | 0.276 | 0.293 | 0.251 | 0.266 | 0.236 | 0.250 |
| | | 0.9 | 0.189 | 0.200 | 0.188 | 0.199 | 0.155 | 0.164 | 0.189 | 0.200 | 0.188 | 0.199 | 0.148 | 0.156 | 0.189 | 0.200 | 0.188 | 0.199 | 0.142 | 0.150 |
| | | 0.99 | 2.079 | 2.210 | 2.005 | 2.137 | 0.476 | 0.505 | 2.079 | 2.210 | 2.005 | 2.137 | 0.396 | 0.420 | 2.079 | 2.210 | 2.005 | 2.137 | 0.342 | 0.361 |
| | 5 | 0.7 | 0.150 | 0.158 | 0.149 | 0.158 | 0.149 | 0.158 | 0.150 | 0.158 | 0.149 | 0.158 | 0.149 | 0.158 | 0.150 | 0.158 | 0.149 | 0.158 | 0.149 | 0.157 |
| | | 0.8 | 0.229 | 0.242 | 0.228 | 0.241 | 0.228 | 0.241 | 0.229 | 0.242 | 0.228 | 0.241 | 0.228 | 0.240 | 0.229 | 0.242 | 0.228 | 0.241 | 0.227 | 0.240 |
| | | 0.9 | 0.189 | 0.200 | 0.189 | 0.200 | 0.187 | 0.198 | 0.189 | 0.200 | 0.189 | 0.200 | 0.187 | 0.197 | 0.189 | 0.200 | 0.189 | 0.200 | 0.186 | 0.197 |
| | | 0.99 | 2.079 | 2.210 | 2.076 | 2.207 | 1.873 | 1.990 | 2.079 | 2.210 | 2.076 | 2.207 | 1.827 | 1.941 | 2.079 | 2.210 | 2.076 | 2.207 | 1.782 | 1.894 |
| | 10 | 0.7 | 0.156 | 0.164 | 0.156 | 0.164 | 0.155 | 0.163 | 0.156 | 0.164 | 0.156 | 0.164 | 0.155 | 0.163 | 0.156 | 0.164 | 0.156 | 0.164 | 0.155 | 0.163 |
| | | 0.8 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 |
| | | 0.9 | 0.189 | 0.200 | 0.189 | 0.200 | 0.188 | 0.199 | 0.189 | 0.200 | 0.189 | 0.200 | 0.188 | 0.199 | 0.189 | 0.200 | 0.189 | 0.200 | 0.188 | 0.199 |
| | | 0.99 | 2.079 | 2.210 | 2.079 | 2.209 | 2.024 | 2.151 | 2.079 | 2.210 | 2.079 | 2.209 | 2.011 | 2.137 | 2.079 | 2.210 | 2.079 | 2.209 | 1.998 | 2.123 |

Table 8: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 100$ and $p = 3$ when there is one outlier.

| $k$ | Sigma | Rho | $d = 0.2$ | | | | | | $d = 0.5$ | | | | | | $d = 0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ |
| 0.3 | 1 | 0.7 | 3.404 | 2.252 | 3.343 | 2.210 | 3.331 | 2.201 | 3.404 | 2.252 | 3.343 | 2.210 | 3.313 | 2.189 | 3.404 | 2.252 | 3.343 | 2.210 | 3.296 | 2.177 |
| | | 0.8 | 4.821 | 3.230 | 4.686 | 3.136 | 4.660 | 3.118 | 4.821 | 3.230 | 4.686 | 3.136 | 4.621 | 3.091 | 4.821 | 3.230 | 4.686 | 3.136 | 4.583 | 3.064 |
| | | 0.9 | 5.532 | 0.201 | 5.516 | 0.200 | 5.287 | 0.187 | 5.532 | 0.201 | 5.516 | 0.200 | 5.229 | 0.184 | 5.532 | 0.201 | 5.516 | 0.200 | 5.173 | 0.181 |
| | | 0.99 | 69.936 | 2.224 | 66.795 | 2.161 | 39.834 | 1.114 | 69.936 | 2.224 | 66.795 | 2.161 | 35.815 | 0.980 | 69.936 | 2.224 | 66.795 | 2.161 | 32.482 | 0.873 |
| | 5 | 0.7 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 |
| | | 0.8 | 0.144 | 0.068 | 0.144 | 0.067 | 0.144 | 0.067 | 0.144 | 0.068 | 0.144 | 0.067 | 0.144 | 0.067 | 0.144 | 0.068 | 0.144 | 0.067 | 0.144 | 0.067 |
| | | 0.9 | 1.261 | 0.201 | 1.261 | 0.201 | 1.259 | 0.200 | 1.261 | 0.201 | 1.261 | 0.201 | 1.258 | 0.200 | 1.261 | 0.201 | 1.261 | 0.201 | 1.258 | 0.200 |
| | | 0.99 | 15.725 | 2.223 | 15.695 | 2.221 | 15.269 | 2.146 | 15.725 | 2.223 | 15.695 | 2.221 | 15.158 | 2.127 | 15.725 | 2.223 | 15.695 | 2.221 | 15.050 | 2.109 |
| | 10 | 0.7 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 |
| | | 0.8 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 |
| | | 0.9 | 0.726 | 0.201 | 0.726 | 0.201 | 0.726 | 0.201 | 0.726 | 0.201 | 0.726 | 0.201 | 0.726 | 0.200 | 0.726 | 0.201 | 0.726 | 0.201 | 0.726 | 0.200 |
| | | 0.99 | 8.930 | 2.223 | 8.926 | 2.223 | 8.862 | 2.204 | 8.930 | 2.223 | 8.926 | 2.223 | 8.846 | 2.199 | 8.930 | 2.223 | 8.926 | 2.223 | 8.829 | 2.194 |
| 0.7 | 1 | 0.7 | 3.404 | 2.252 | 3.265 | 2.155 | 3.238 | 2.137 | 3.404 | 2.252 | 3.265 | 2.155 | 3.199 | 2.110 | 3.404 | 2.252 | 3.343 | 2.210 | 3.296 | 2.177 |
| | | 0.8 | 4.821 | 3.230 | 4.516 | 3.018 | 4.459 | 2.978 | 4.821 | 3.230 | 4.516 | 3.018 | 4.376 | 2.920 | 4.821 | 3.230 | 4.686 | 3.136 | 4.583 | 3.064 |
| | | 0.9 | 5.532 | 0.201 | 5.516 | 0.200 | 4.994 | 0.172 | 5.532 | 0.201 | 5.516 | 0.200 | 4.877 | 0.167 | 5.532 | 0.201 | 5.516 | 0.200 | 4.765 | 0.161 |
| | | 0.99 | 69.936 | 2.224 | 66.795 | 2.161 | 24.567 | 0.637 | 69.936 | 2.224 | 66.795 | 2.161 | 20.838 | 0.535 | 69.936 | 2.224 | 66.795 | 2.161 | 18.001 | 0.462 |
| | 5 | 0.7 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 | 0.100 | 0.047 |
| | | 0.8 | 0.144 | 0.068 | 0.143 | 0.067 | 0.143 | 0.067 | 0.144 | 0.068 | 0.143 | 0.067 | 0.143 | 0.067 | 0.144 | 0.068 | 0.144 | 0.067 | 0.144 | 0.067 |
| | | 0.9 | 1.261 | 0.201 | 1.261 | 0.201 | 1.256 | 0.199 | 1.261 | 0.201 | 1.261 | 0.201 | 1.255 | 0.199 | 1.261 | 0.201 | 1.261 | 0.201 | 1.253 | 0.199 |
| | | 0.99 | 15.725 | 2.223 | 15.695 | 2.221 | 14.698 | 2.049 | 15.725 | 2.223 | 15.695 | 2.221 | 14.461 | 2.009 | 15.725 | 2.223 | 15.695 | 2.221 | 14.232 | 1.971 |
| | 10 | 0.7 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 | 0.030 | 0.014 |
| | | 0.8 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 | 0.045 | 0.021 |
| | | 0.9 | 0.726 | 0.201 | 0.726 | 0.201 | 0.726 | 0.200 | 0.726 | 0.201 | 0.726 | 0.201 | 0.725 | 0.200 | 0.726 | 0.201 | 0.726 | 0.201 | 0.725 | 0.200 |
| | | 0.99 | 8.930 | 2.223 | 8.926 | 2.223 | 8.774 | 2.178 | 8.930 | 2.223 | 8.926 | 2.223 | 8.735 | 2.166 | 8.930 | 2.223 | 8.926 | 2.223 | 8.697 | 2.155 |
| 0.9 | 1 | 0.7 | 0.414 | 0.187 | 0.395 | 0.177 | 0.391 | 0.175 | 0.414 | 0.187 | 0.395 | 0.177 | 0.387 | 0.172 | 0.414 | 0.187 | 0.395 | 0.177 | 0.382 | 0.170 |
| | | 0.8 | 0.606 | 0.295 | 0.564 | 0.268 | 0.556 | 0.264 | 0.606 | 0.295 | 0.564 | 0.268 | 0.546 | 0.258 | 0.606 | 0.295 | 0.564 | 0.268 | 0.536 | 0.252 |
| | | 0.9 | 5.532 | 0.201 | 5.516 | 0.200 | 4.860 | 0.166 | 5.532 | 0.201 | 5.516 | 0.200 | 4.719 | 0.159 | 5.532 | 0.201 | 5.516 | 0.200 | 4.586 | 0.153 |
| | | 0.99 | 69.936 | 2.224 | 66.795 | 2.161 | 20.386 | 0.523 | 5.532 | 2.224 | 66.795 | 2.161 | 16.984 | 0.438 | 69.936 | 2.224 | 66.795 | 2.161 | 14.463 | 0.380 |
| | 5 | 0.7 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 5.532 | 0.160 | 0.211 | 0.160 | 0.210 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.210 | 0.160 |
| | | 0.8 | 0.316 | 0.246 | 0.315 | 0.245 | 0.315 | 0.245 | 5.532 | 0.246 | 0.315 | 0.245 | 0.314 | 0.244 | 0.316 | 0.246 | 0.315 | 0.245 | 0.314 | 0.244 |
| | | 0.9 | 1.261 | 0.201 | 1.261 | 0.201 | 1.255 | 0.199 | 5.532 | 0.201 | 1.261 | 0.201 | 1.253 | 0.199 | 1.261 | 0.201 | 1.261 | 0.201 | 1.251 | 0.198 |
| | | 0.99 | 15.725 | 2.223 | 15.695 | 2.221 | 14.428 | 2.004 | 5.532 | 2.223 | 15.695 | 2.221 | 14.136 | 1.955 | 15.725 | 2.223 | 15.695 | 2.221 | 13.854 | 1.907 |
| | 10 | 0.7 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 5.532 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 |
| | | 0.8 | 0.295 | 0.249 | 0.295 | 0.248 | 0.295 | 0.248 | 5.532 | 0.249 | 0.295 | 0.248 | 0.294 | 0.248 | 0.295 | 0.249 | 0.295 | 0.248 | 0.294 | 0.248 |
| | | 0.9 | 0.726 | 0.201 | 0.726 | 0.201 | 0.725 | 0.200 | 5.532 | 0.201 | 0.726 | 0.201 | 0.725 | 0.200 | 0.726 | 0.201 | 0.726 | 0.201 | 0.725 | 0.200 |
| | | 0.99 | 8.930 | 2.223 | 8.926 | 2.223 | 8.730 | 2.165 | 5.532 | 2.223 | 8.926 | 2.223 | 8.681 | 2.151 | 8.930 | 2.223 | 8.926 | 2.223 | 8.633 | 2.136 |

Table 9: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 100$ and $p = 3$ when there are two outliers.

| $k$ | Sigma | Rho | $d = 0.2$ | | | | | | $d = 0.5$ | | | | | | $d = 0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ |
| 0.3 | 1 | 0.7 | 3.529 | 2.360 | 3.463 | 2.316 | 3.450 | 2.307 | 3.529 | 2.360 | 3.463 | 2.316 | 3.431 | 2.294 | 3.529 | 2.360 | 3.463 | 2.316 | 3.412 | 2.281 |
| | | 0.8 | 5.046 | 3.388 | 4.901 | 3.289 | 4.873 | 3.269 | 5.046 | 3.388 | 4.901 | 3.289 | 4.831 | 3.241 | 5.046 | 3.388 | 4.901 | 3.289 | 4.790 | 3.212 |
| | | 0.9 | 2.737 | 0.207 | 2.706 | 0.206 | 2.596 | 0.194 | 2.737 | 0.207 | 2.706 | 0.206 | 2.563 | 0.191 | 2.737 | 0.207 | 2.706 | 0.206 | 2.531 | 0.188 |
| | | 0.99 | 33.739 | 2.273 | 28.776 | 2.151 | 18.246 | 1.161 | 33.739 | 2.273 | 28.776 | 2.151 | 16.274 | 1.026 | 33.739 | 2.273 | 28.776 | 2.151 | 14.661 | 0.918 |
| | 5 | 0.7 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 |
| | | 0.8 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 |
| | | 0.9 | 0.702 | 0.207 | 0.702 | 0.207 | 0.701 | 0.206 | 0.702 | 0.207 | 0.702 | 0.207 | 0.700 | 0.206 | 0.702 | 0.207 | 0.702 | 0.207 | 0.700 | 0.206 |
| | | 0.99 | 8.471 | 2.273 | 8.413 | 2.267 | 8.201 | 2.195 | 8.471 | 2.273 | 8.413 | 2.267 | 8.135 | 2.176 | 8.471 | 2.273 | 8.413 | 2.267 | 8.071 | 2.158 |
| | 10 | 0.7 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 |
| | | 0.8 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 |
| | | 0.9 | 0.447 | 0.207 | 0.447 | 0.207 | 0.447 | 0.207 | 0.447 | 0.207 | 0.447 | 0.207 | 0.447 | 0.207 | 0.447 | 0.207 | 0.447 | 0.207 | 0.447 | 0.207 |
| | | 0.99 | 5.298 | 2.273 | 5.289 | 2.271 | 5.254 | 2.253 | 5.298 | 2.273 | 5.289 | 2.271 | 5.243 | 2.248 | 5.298 | 2.273 | 5.289 | 2.271 | 5.232 | 2.243 |
| 0.7 | 1 | 0.7 | 3.529 | 2.360 | 3.378 | 2.258 | 3.349 | 2.238 | 3.529 | 2.360 | 3.378 | 2.258 | 3.307 | 2.209 | 3.529 | 2.360 | 3.463 | 2.316 | 3.412 | 2.281 |
| | | 0.8 | 5.046 | 3.388 | 4.718 | 3.163 | 4.656 | 3.121 | 5.046 | 3.388 | 4.718 | 3.163 | 4.566 | 3.059 | 5.046 | 3.388 | 4.901 | 3.289 | 4.790 | 3.212 |
| | | 0.9 | 2.737 | 0.207 | 2.706 | 0.206 | 2.431 | 0.179 | 2.737 | 0.207 | 2.706 | 0.206 | 2.365 | 0.173 | 2.737 | 0.207 | 2.706 | 0.206 | 2.304 | 0.168 |
| | | 0.99 | 33.739 | 2.273 | 28.776 | 2.151 | 10.921 | 0.678 | 33.739 | 2.273 | 28.776 | 2.151 | 9.207 | 0.574 | 33.739 | 2.273 | 28.776 | 2.151 | 7.924 | 0.499 |
| | 5 | 0.7 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 | 0.099 | 0.049 |
| | | 0.8 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 | 0.140 | 0.070 |
| | | 0.9 | 0.702 | 0.207 | 0.702 | 0.207 | 0.699 | 0.206 | 0.702 | 0.207 | 0.702 | 0.207 | 0.698 | 0.205 | 0.702 | 0.207 | 0.702 | 0.207 | 0.697 | 0.205 |
| | | 0.99 | 8.471 | 2.273 | 8.413 | 2.267 | 7.864 | 2.099 | 8.471 | 2.273 | 8.413 | 2.267 | 7.724 | 2.059 | 8.471 | 2.273 | 8.413 | 2.267 | 7.589 | 2.020 |
| | 10 | 0.7 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 | 0.024 | 0.015 |
| | | 0.8 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 | 0.034 | 0.021 |
| | | 0.9 | 0.447 | 0.207 | 0.447 | 0.207 | 0.446 | 0.207 | 0.447 | 0.207 | 0.447 | 0.207 | 0.446 | 0.207 | 0.447 | 0.207 | 0.447 | 0.207 | 0.446 | 0.206 |
| | | 0.99 | 5.298 | 2.273 | 5.289 | 2.271 | 5.196 | 2.227 | 5.298 | 2.273 | 5.289 | 2.271 | 5.171 | 2.216 | 5.298 | 2.273 | 5.289 | 2.271 | 5.146 | 2.204 |
| 0.9 | 1 | 0.7 | 0.835 | 0.192 | 0.811 | 0.182 | 0.807 | 0.180 | 0.835 | 0.192 | 0.811 | 0.182 | 0.800 | 0.178 | 0.835 | 0.192 | 0.811 | 0.182 | 0.794 | 0.176 |
| | | 0.8 | 1.220 | 0.302 | 1.163 | 0.276 | 1.153 | 0.271 | 1.220 | 0.302 | 1.163 | 0.276 | 1.138 | 0.265 | 1.220 | 0.302 | 1.163 | 0.276 | 1.123 | 0.260 |
| | | 0.9 | 2.737 | 0.207 | 2.706 | 0.206 | 2.356 | 0.172 | 2.737 | 0.207 | 2.706 | 0.206 | 2.278 | 0.166 | 2.737 | 0.207 | 2.706 | 0.206 | 2.205 | 0.159 |
| | | 0.99 | 33.739 | 2.273 | 28.776 | 2.151 | 9.001 | 0.561 | 33.739 | 2.273 | 28.776 | 2.151 | 7.469 | 0.473 | 33.739 | 2.273 | 28.776 | 2.151 | 6.351 | 0.413 |
| | 5 | 0.7 | 0.264 | 0.163 | 0.264 | 0.162 | 0.263 | 0.162 | 0.264 | 0.163 | 0.264 | 0.162 | 0.263 | 0.162 | 0.264 | 0.163 | 0.264 | 0.162 | 0.263 | 0.162 |
| | | 0.8 | 0.401 | 0.251 | 0.399 | 0.249 | 0.398 | 0.249 | 0.401 | 0.251 | 0.399 | 0.249 | 0.398 | 0.249 | 0.401 | 0.251 | 0.399 | 0.249 | 0.397 | 0.249 |
| | | 0.9 | 0.702 | 0.207 | 0.702 | 0.207 | 0.698 | 0.205 | 0.702 | 0.207 | 0.702 | 0.207 | 0.697 | 0.205 | 0.702 | 0.207 | 0.702 | 0.207 | 0.696 | 0.205 |
| | | 0.99 | 8.471 | 2.273 | 8.413 | 2.267 | 7.704 | 2.053 | 8.471 | 2.273 | 8.413 | 2.267 | 7.532 | 2.004 | 8.471 | 2.273 | 8.413 | 2.267 | 7.367 | 1.957 |
| | 10 | 0.7 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 |
| | | 0.8 | 0.463 | 0.265 | 0.462 | 0.265 | 0.462 | 0.265 | 0.463 | 0.265 | 0.462 | 0.265 | 0.462 | 0.265 | 0.463 | 0.265 | 0.462 | 0.265 | 0.462 | 0.265 |
| | | 0.9 | 0.447 | 0.207 | 0.447 | 0.207 | 0.446 | 0.207 | 0.447 | 0.207 | 0.447 | 0.207 | 0.446 | 0.206 | 0.447 | 0.207 | 0.447 | 0.207 | 0.446 | 0.206 |
| | | 0.99 | 5.298 | 2.273 | 5.289 | 2.271 | 5.168 | 2.214 | 5.298 | 2.273 | 5.289 | 2.271 | 5.136 | 2.200 | 5.298 | 2.273 | 5.289 | 2.271 | 5.105 | 2.186 |

Table 10: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 20$ and $p = 7$ when there is no outlier.

| $k$ | Sigma | Rho | $d=0.2$ | | | | | | $d=0.5$ | | | | | | $d=0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ |
| 0.3 | 1 | 0.7 | 1.430 | 1.543 | 1.140 | 1.228 | 1.098 | 1.182 | 1.430 | 1.543 | 1.140 | 1.228 | 1.041 | 1.120 | 1.430 | 1.543 | 1.140 | 1.228 | 0.991 | 1.066 |
| | | 0.8 | 2.160 | 2.320 | 1.529 | 1.641 | 1.449 | 1.555 | 2.160 | 2.320 | 1.529 | 1.641 | 1.345 | 1.443 | 2.160 | 2.320 | 1.529 | 1.641 | 1.258 | 1.349 |
| | | 0.9 | 4.469 | 4.885 | 2.335 | 2.534 | 2.145 | 2.326 | 4.469 | 4.885 | 2.335 | 2.534 | 1.918 | 2.076 | 4.469 | 4.885 | 2.335 | 2.534 | 1.739 | 1.880 |
| | | 0.99 | 56.157 | 59.856 | 4.310 | 4.575 | 3.580 | 3.798 | 56.157 | 59.856 | 4.310 | 4.575 | 2.829 | 2.998 | 56.157 | 59.856 | 4.310 | 4.575 | 2.323 | 2.458 |
| | 5 | 0.7 | 1.389 | 1.492 | 1.374 | 1.476 | 1.372 | 1.473 | 1.389 | 1.492 | 1.374 | 1.476 | 1.367 | 1.468 | 1.389 | 1.492 | 1.374 | 1.476 | 1.363 | 1.464 |
| | | 0.8 | 2.093 | 2.267 | 2.059 | 2.229 | 2.052 | 2.222 | 2.093 | 2.267 | 2.059 | 2.229 | 2.042 | 2.211 | 2.093 | 2.267 | 2.059 | 2.229 | 2.032 | 2.200 |
| | | 0.9 | 4.316 | 4.637 | 4.157 | 4.466 | 4.127 | 4.434 | 4.316 | 4.637 | 4.157 | 4.466 | 4.082 | 4.386 | 4.316 | 4.637 | 4.157 | 4.466 | 4.038 | 4.339 |
| | | 0.99 | 48.687 | 53.009 | 31.919 | 34.605 | 29.918 | 32.414 | 48.687 | 53.009 | 31.919 | 34.605 | 27.380 | 29.636 | 48.687 | 53.009 | 31.919 | 34.605 | 25.271 | 27.331 |
| | 10 | 0.7 | 1.034 | 1.107 | 1.032 | 1.105 | 1.032 | 1.105 | 1.034 | 1.107 | 1.032 | 1.105 | 1.031 | 1.104 | 1.034 | 1.107 | 1.032 | 1.105 | 1.031 | 1.104 |
| | | 0.8 | 1.561 | 1.679 | 1.558 | 1.675 | 1.557 | 1.674 | 1.561 | 1.679 | 1.558 | 1.675 | 1.556 | 1.672 | 1.561 | 1.679 | 1.558 | 1.675 | 1.555 | 1.671 |
| | | 0.9 | 3.151 | 3.404 | 3.134 | 3.385 | 3.130 | 3.381 | 3.151 | 3.404 | 3.134 | 3.385 | 3.125 | 3.375 | 3.151 | 3.404 | 3.134 | 3.385 | 3.120 | 3.370 |
| | | 0.99 | 34.931 | 37.511 | 32.402 | 34.797 | 31.942 | 34.304 | 34.931 | 37.511 | 32.402 | 34.797 | 31.279 | 33.592 | 34.931 | 37.511 | 32.402 | 34.797 | 30.644 | 32.911 |
| 0.7 | 1 | 0.7 | 1.430 | 1.543 | 0.915 | 0.983 | 0.860 | 0.923 | 1.430 | 1.543 | 0.915 | 0.983 | 0.792 | 0.849 | 1.430 | 1.543 | 0.915 | 0.983 | 0.738 | 0.790 |
| | | 0.8 | 2.160 | 2.320 | 1.132 | 1.213 | 1.044 | 1.118 | 2.160 | 2.320 | 1.132 | 1.213 | 0.940 | 1.005 | 2.160 | 2.320 | 1.132 | 1.213 | 0.859 | 0.917 |
| | | 0.9 | 4.469 | 4.885 | 1.499 | 1.617 | 1.342 | 1.445 | 4.469 | 4.885 | 1.499 | 1.617 | 1.166 | 1.252 | 4.469 | 4.885 | 1.499 | 1.617 | 1.036 | 1.110 |
| | | 0.99 | 56.157 | 59.856 | 1.751 | 1.848 | 1.439 | 1.516 | 56.157 | 59.856 | 1.751 | 1.848 | 1.145 | 1.201 | 56.157 | 59.856 | 1.751 | 1.848 | 0.963 | 1.006 |
| | 5 | 0.7 | 1.389 | 1.492 | 1.356 | 1.456 | 1.349 | 1.449 | 1.389 | 1.492 | 1.356 | 1.456 | 1.340 | 1.439 | 1.389 | 1.492 | 1.356 | 1.456 | 1.330 | 1.429 |
| | | 0.8 | 2.093 | 2.267 | 2.014 | 2.181 | 1.999 | 2.164 | 2.093 | 2.267 | 2.014 | 2.181 | 1.977 | 2.140 | 2.093 | 2.267 | 2.014 | 2.181 | 1.955 | 2.116 |
| | | 0.9 | 4.316 | 4.637 | 3.963 | 4.258 | 3.899 | 4.190 | 4.316 | 4.637 | 3.963 | 4.258 | 3.808 | 4.092 | 4.316 | 4.637 | 3.963 | 4.258 | 3.721 | 3.998 |
| | | 0.99 | 48.687 | 53.009 | 22.287 | 24.073 | 20.245 | 21.847 | 48.687 | 53.009 | 22.287 | 24.073 | 17.850 | 19.242 | 48.687 | 53.009 | 22.287 | 24.073 | 16.002 | 17.234 |
| | 10 | 0.7 | 1.034 | 1.107 | 1.030 | 1.103 | 1.029 | 1.102 | 1.034 | 1.107 | 1.030 | 1.103 | 1.028 | 1.101 | 1.034 | 1.107 | 1.030 | 1.103 | 1.027 | 1.100 |
| | | 0.8 | 1.561 | 1.679 | 1.553 | 1.669 | 1.551 | 1.667 | 1.561 | 1.679 | 1.553 | 1.669 | 1.548 | 1.664 | 1.561 | 1.679 | 1.553 | 1.669 | 1.546 | 1.662 |
| | | 0.9 | 3.151 | 3.404 | 3.111 | 3.360 | 3.103 | 3.351 | 3.151 | 3.404 | 3.111 | 3.360 | 3.091 | 3.338 | 3.151 | 3.404 | 3.111 | 3.360 | 3.079 | 3.325 |
| | | 0.99 | 34.931 | 37.511 | 29.583 | 31.771 | 28.717 | 30.842 | 34.931 | 37.511 | 29.583 | 31.771 | 27.518 | 29.555 | 34.931 | 37.511 | 29.583 | 31.771 | 26.424 | 28.380 |
| 0.9 | 1 | 0.7 | 1.430 | 1.543 | 0.839 | 0.901 | 0.784 | 0.840 | 1.430 | 1.543 | 0.839 | 0.901 | 0.717 | 0.767 | 1.430 | 1.543 | 0.839 | 0.901 | 0.665 | 0.710 |
| | | 0.8 | 2.160 | 2.320 | 1.011 | 1.082 | 0.927 | 0.991 | 2.160 | 2.320 | 1.011 | 1.082 | 0.829 | 0.885 | 2.160 | 2.320 | 1.011 | 1.082 | 0.755 | 0.804 |
| | | 0.9 | 4.469 | 4.885 | 1.286 | 1.384 | 1.145 | 1.229 | 4.469 | 4.885 | 1.286 | 1.384 | 0.990 | 1.059 | 4.469 | 4.885 | 1.286 | 1.384 | 0.877 | 0.936 |
| | | 0.99 | 56.157 | 59.856 | 1.339 | 1.409 | 1.113 | 1.167 | 56.157 | 59.856 | 1.339 | 1.409 | 0.906 | 0.945 | 56.157 | 59.856 | 1.339 | 1.409 | 0.782 | 0.811 |
| | 5 | 0.7 | 1.389 | 1.492 | 1.347 | 1.446 | 1.338 | 1.437 | 1.389 | 1.492 | 1.347 | 1.446 | 1.326 | 1.424 | 1.389 | 1.492 | 1.347 | 1.446 | 1.315 | 1.412 |
| | | 0.8 | 2.093 | 2.267 | 1.993 | 2.157 | 1.974 | 2.136 | 2.093 | 2.267 | 1.993 | 2.157 | 1.946 | 2.106 | 2.093 | 2.267 | 1.993 | 2.157 | 1.919 | 2.077 |
| | | 0.9 | 4.316 | 4.637 | 3.873 | 4.161 | 3.795 | 4.078 | 4.316 | 4.637 | 3.873 | 4.161 | 3.684 | 3.959 | 4.316 | 4.637 | 3.873 | 4.161 | 3.581 | 3.848 |
| | | 0.99 | 48.687 | 53.009 | 19.490 | 21.026 | 17.558 | 18.924 | 48.687 | 53.009 | 19.490 | 21.026 | 15.332 | 16.507 | 48.687 | 53.009 | 19.490 | 21.026 | 13.640 | 14.673 |
| | 10 | 0.7 | 1.034 | 1.107 | 1.029 | 1.102 | 1.028 | 1.101 | 1.034 | 1.107 | 1.029 | 1.102 | 1.027 | 1.099 | 1.034 | 1.107 | 1.029 | 1.102 | 1.025 | 1.098 |
| | | 0.8 | 1.561 | 1.679 | 1.550 | 1.666 | 1.548 | 1.664 | 1.561 | 1.679 | 1.550 | 1.666 | 1.545 | 1.660 | 1.561 | 1.679 | 1.550 | 1.666 | 1.541 | 1.657 |
| | | 0.9 | 3.151 | 3.404 | 3.099 | 3.347 | 3.089 | 3.336 | 3.151 | 3.404 | 3.099 | 3.347 | 3.074 | 3.320 | 3.151 | 3.404 | 3.099 | 3.347 | 3.059 | 3.304 |
| | | 0.99 | 34.931 | 37.511 | 28.363 | 30.462 | 27.356 | 29.380 | 34.931 | 37.511 | 28.363 | 30.462 | 25.983 | 27.907 | 34.931 | 37.511 | 28.363 | 30.462 | 24.754 | 26.587 |

Table 11: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 20$ and $p = 7$ when there is one outlier.

| $k$ | Sigma | Rho | $d = 0.2$ | | | | | | $d = 0.5$ | | | | | | $d = 0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ |
| 0.3 | 1 | 0.7 | 43.418 | 1.975 | 30.622 | 1.590 | 28.798 | 1.532 | 43.418 | 1.975 | 30.622 | 1.590 | 26.378 | 1.454 | 43.418 | 1.975 | 30.622 | 1.590 | 24.277 | 1.386 |
| | | 0.8 | 66.970 | 2.995 | 39.537 | 2.122 | 36.231 | 2.010 | 66.970 | 2.995 | 39.537 | 2.122 | 32.058 | 1.864 | 66.970 | 2.995 | 39.537 | 2.122 | 28.623 | 1.740 |
| | | 0.9 | 143.790 | 6.312 | 52.339 | 3.194 | 45.398 | 2.921 | 143.790 | 6.312 | 52.339 | 3.194 | 37.510 | 2.594 | 143.790 | 6.312 | 52.339 | 3.194 | 31.684 | 2.338 |
| | | 0.99 | 1790.873 | 76.051 | 36.821 | 5.131 | 29.299 | 4.271 | 1790.873 | 76.051 | 36.821 | 5.131 | 22.191 | 3.390 | 1790.873 | 76.051 | 36.821 | 5.131 | 17.718 | 2.798 |
| | 5 | 0.7 | 6.679 | 1.924 | 6.620 | 1.905 | 6.608 | 1.901 | 6.679 | 1.924 | 6.620 | 1.905 | 6.591 | 1.896 | 6.679 | 1.924 | 6.620 | 1.905 | 6.573 | 1.890 |
| | | 0.8 | 9.507 | 2.838 | 9.383 | 2.793 | 9.358 | 2.784 | 9.507 | 2.838 | 9.383 | 2.793 | 9.322 | 2.771 | 9.507 | 2.838 | 9.383 | 2.793 | 9.285 | 2.758 |
| | | 0.9 | 17.837 | 5.643 | 17.377 | 5.450 | 17.288 | 5.413 | 17.837 | 5.643 | 17.377 | 5.450 | 17.157 | 5.359 | 17.837 | 5.643 | 17.377 | 5.450 | 17.027 | 5.306 |
| | | 0.99 | 177.299 | 63.383 | 138.858 | 43.305 | 133.290 | 40.828 | 177.299 | 63.383 | 138.858 | 43.305 | 125.773 | 37.648 | 177.299 | 63.383 | 138.858 | 43.305 | 119.087 | 34.970 |
| | 10 | 0.7 | 2.837 | 1.405 | 2.832 | 1.403 | 2.831 | 1.403 | 2.837 | 1.405 | 2.832 | 1.403 | 2.830 | 1.402 | 2.837 | 1.405 | 2.832 | 1.403 | 2.828 | 1.401 |
| | | 0.8 | 4.370 | 2.150 | 4.358 | 2.144 | 4.355 | 2.143 | 4.370 | 2.150 | 4.358 | 2.144 | 4.352 | 2.141 | 4.370 | 2.150 | 4.358 | 2.144 | 4.348 | 2.139 |
| | | 0.9 | 8.934 | 4.360 | 8.882 | 4.334 | 8.871 | 4.329 | 8.934 | 4.360 | 8.882 | 4.334 | 8.855 | 4.322 | 8.934 | 4.360 | 8.882 | 4.334 | 8.840 | 4.314 |
| | | 0.99 | 103.740 | 48.970 | 96.088 | 45.343 | 94.699 | 44.685 | 103.740 | 48.970 | 96.088 | 45.343 | 92.695 | 43.735 | 103.740 | 48.970 | 96.088 | 45.343 | 90.779 | 42.827 |
| 0.7 | 1 | 0.7 | 43.418 | 1.975 | 21.166 | 1.281 | 18.955 | 1.204 | 43.418 | 1.975 | 21.166 | 1.281 | 16.297 | 1.108 | 43.418 | 1.975 | 21.166 | 1.281 | 14.218 | 1.029 |
| | | 0.8 | 66.970 | 2.995 | 23.850 | 1.561 | 20.674 | 1.436 | 66.970 | 2.995 | 23.850 | 1.561 | 17.086 | 1.287 | 66.970 | 2.995 | 23.850 | 1.561 | 14.451 | 1.171 |
| | | 0.9 | 143.790 | 6.312 | 24.476 | 1.998 | 20.200 | 1.778 | 143.790 | 6.312 | 24.476 | 1.998 | 15.822 | 1.534 | 143.790 | 6.312 | 24.476 | 1.998 | 12.891 | 1.355 |
| | | 0.99 | 1790.873 | 76.051 | 12.903 | 2.127 | 10.354 | 1.760 | 1790.873 | 76.051 | 12.903 | 2.127 | 7.935 | 1.409 | 1790.873 | 76.051 | 12.903 | 2.127 | 6.406 | 1.189 |
| | 5 | 0.7 | 6.679 | 1.924 | 6.542 | 1.880 | 6.515 | 1.872 | 6.679 | 1.924 | 6.542 | 1.880 | 6.475 | 1.859 | 6.679 | 1.924 | 6.542 | 1.880 | 6.436 | 1.846 |
| | | 0.8 | 9.507 | 2.838 | 9.221 | 2.736 | 9.166 | 2.716 | 9.507 | 2.838 | 9.221 | 2.736 | 9.085 | 2.687 | 9.507 | 2.838 | 9.221 | 2.736 | 9.005 | 2.659 |
| | | 0.9 | 17.837 | 5.643 | 16.801 | 5.214 | 16.609 | 5.136 | 17.837 | 5.643 | 16.801 | 5.214 | 16.329 | 5.024 | 17.837 | 5.643 | 16.801 | 5.214 | 16.058 | 4.916 |
| | | 0.99 | 177.299 | 63.383 | 108.816 | 31.115 | 101.158 | 28.426 | 177.299 | 63.383 | 108.816 | 31.115 | 91.433 | 25.213 | 177.299 | 63.383 | 108.816 | 31.115 | 83.324 | 22.686 |
| | 10 | 0.7 | 2.837 | 1.405 | 2.825 | 1.400 | 2.823 | 1.399 | 2.837 | 1.405 | 2.825 | 1.400 | 2.820 | 1.397 | 2.837 | 1.405 | 2.825 | 1.400 | 2.816 | 1.395 |
| | | 0.8 | 4.370 | 2.150 | 4.342 | 2.136 | 4.336 | 2.134 | 4.370 | 2.150 | 4.342 | 2.136 | 4.328 | 2.130 | 4.370 | 2.150 | 4.342 | 2.136 | 4.320 | 2.126 |
| | | 0.9 | 8.934 | 4.360 | 8.812 | 4.300 | 8.788 | 4.289 | 8.934 | 4.360 | 8.812 | 4.300 | 8.752 | 4.271 | 8.934 | 4.360 | 8.812 | 4.300 | 8.716 | 4.254 |
| | | 0.99 | 103.740 | 48.970 | 87.575 | 41.308 | 84.965 | 40.070 | 103.740 | 48.970 | 87.575 | 41.308 | 81.351 | 38.357 | 103.740 | 48.970 | 87.575 | 41.308 | 78.056 | 36.795 |
| 0.9 | 1 | 0.7 | 2.837 | 1.405 | 2.822 | 1.398 | 2.819 | 1.397 | 43.418 | 1.975 | 18.124 | 1.174 | 13.463 | 1.000 | 43.418 | 1.975 | 18.124 | 1.174 | 11.571 | 0.924 |
| | | 0.8 | 4.370 | 2.150 | 4.334 | 2.132 | 4.327 | 2.129 | 66.970 | 2.995 | 19.526 | 1.390 | 13.530 | 1.129 | 66.970 | 2.995 | 19.526 | 1.390 | 11.305 | 1.022 |
| | | 0.9 | 8.934 | 4.360 | 8.778 | 4.284 | 8.747 | 4.269 | 143.790 | 6.312 | 18.748 | 1.700 | 11.919 | 1.292 | 143.790 | 6.312 | 18.748 | 1.700 | 9.681 | 1.139 |
| | | 0.99 | 103.740 | 48.970 | 83.897 | 39.564 | 80.861 | 38.125 | 1790.873 | 76.051 | 9.532 | 1.641 | 5.913 | 1.119 | 1790.873 | 76.051 | 9.532 | 1.641 | 4.801 | 0.964 |
| | 5 | 0.7 | 6.679 | 1.924 | 6.504 | 1.868 | 6.470 | 1.857 | 6.679 | 1.924 | 6.504 | 1.868 | 6.419 | 1.841 | 6.679 | 1.924 | 6.504 | 1.868 | 6.369 | 1.825 |
| | | 0.8 | 9.507 | 2.838 | 9.143 | 2.708 | 9.073 | 2.683 | 9.507 | 2.838 | 9.143 | 2.708 | 8.971 | 2.647 | 9.507 | 2.838 | 9.143 | 2.708 | 8.870 | 2.612 |
| | | 0.9 | 17.837 | 5.643 | 16.528 | 5.103 | 16.289 | 5.008 | 17.837 | 5.643 | 16.528 | 5.103 | 15.944 | 4.872 | 17.837 | 5.643 | 16.528 | 5.103 | 15.613 | 4.743 |
| | | 0.99 | 177.299 | 63.383 | 98.186 | 27.421 | 90.186 | 24.816 | 177.299 | 63.383 | 98.186 | 27.421 | 80.244 | 21.759 | 177.299 | 63.383 | 98.186 | 27.421 | 72.146 | 19.393 |
| | 10 | 0.7 | 2.837 | 1.405 | 2.822 | 1.398 | 2.819 | 1.397 | 2.837 | 1.405 | 2.822 | 1.398 | 2.815 | 1.395 | 2.837 | 1.405 | 2.822 | 1.398 | 2.810 | 1.393 |
| | | 0.8 | 4.370 | 2.150 | 4.334 | 2.132 | 4.327 | 2.129 | 4.370 | 2.150 | 4.334 | 2.132 | 4.316 | 2.124 | 4.370 | 2.150 | 4.334 | 2.132 | 4.306 | 2.119 |
| | | 0.9 | 8.934 | 4.360 | 8.778 | 4.284 | 8.747 | 4.269 | 8.934 | 4.360 | 8.778 | 4.284 | 8.701 | 4.246 | 8.934 | 4.360 | 8.778 | 4.284 | 8.656 | 4.224 |
| | | 0.99 | 103.740 | 48.970 | 83.897 | 39.564 | 80.861 | 38.125 | 103.740 | 48.970 | 83.897 | 39.564 | 76.731 | 36.167 | 103.740 | 48.970 | 83.897 | 39.564 | 73.036 | 34.415 |

Table 12: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 20$ and $p = 7$ when there are two outliers.

| $k$ | Sigma | Rho | $d=0.2$ | | | | | | $d=0.5$ | | | | | | $d=0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ |
| 0.3 | 1 | 0.7 | 105.945 | 3.884 | 78.646 | 3.162 | 74.687 | 3.052 | 105.945 | 3.884 | 78.646 | 3.162 | 69.399 | 2.902 | 105.945 | 3.884 | 78.646 | 3.162 | 64.768 | 2.768 |
| | | 0.8 | 156.521 | 5.766 | 100.774 | 4.205 | 93.860 | 3.998 | 156.521 | 5.766 | 100.774 | 4.205 | 85.027 | 3.727 | 156.521 | 5.766 | 100.774 | 4.205 | 77.650 | 3.494 |
| | | 0.9 | 315.660 | 11.802 | 140.734 | 6.437 | 126.344 | 5.937 | 315.660 | 11.802 | 140.734 | 6.437 | 109.484 | 5.329 | 315.660 | 11.802 | 140.734 | 6.437 | 96.547 | 4.843 |
| | | 0.99 | 3440.840 | 126.654 | 194.473 | 11.626 | 157.663 | 9.599 | 3440.840 | 126.654 | 194.473 | 11.626 | 120.114 | 7.484 | 3440.840 | 126.654 | 194.473 | 11.626 | 94.981 | 6.041 |
| | 5 | 0.7 | 12.325 | 2.265 | 12.218 | 2.243 | 12.197 | 2.238 | 12.325 | 2.265 | 12.218 | 2.243 | 12.166 | 2.232 | 12.325 | 2.265 | 12.218 | 2.243 | 12.134 | 2.225 |
| | | 0.8 | 17.632 | 3.304 | 17.412 | 3.254 | 17.368 | 3.244 | 17.632 | 3.304 | 17.412 | 3.254 | 17.303 | 3.230 | 17.632 | 3.304 | 17.412 | 3.254 | 17.239 | 3.215 |
| | | 0.9 | 33.218 | 6.515 | 32.413 | 6.304 | 32.257 | 6.264 | 33.218 | 6.515 | 32.413 | 6.304 | 32.025 | 6.204 | 33.218 | 6.515 | 32.413 | 6.304 | 31.797 | 6.145 |
| | | 0.99 | 327.375 | 71.474 | 261.240 | 50.147 | 251.257 | 47.452 | 327.375 | 71.474 | 261.240 | 50.147 | 237.627 | 43.963 | 327.375 | 71.474 | 261.240 | 50.147 | 225.359 | 40.997 |
| | 10 | 0.7 | 7.152 | 3.480 | 7.139 | 3.474 | 7.137 | 3.473 | 7.152 | 3.480 | 7.139 | 3.474 | 7.133 | 3.471 | 7.152 | 3.480 | 7.139 | 3.474 | 7.129 | 3.470 |
| | | 0.8 | 10.655 | 5.157 | 10.626 | 5.144 | 10.621 | 5.141 | 10.655 | 5.157 | 10.626 | 5.144 | 10.612 | 5.137 | 10.655 | 5.157 | 10.626 | 5.144 | 10.603 | 5.133 |
| | | 0.9 | 21.143 | 10.222 | 21.022 | 10.165 | 20.998 | 10.154 | 21.143 | 10.222 | 21.022 | 10.165 | 20.962 | 10.137 | 21.143 | 10.222 | 21.022 | 10.165 | 20.926 | 10.121 |
| | | 0.99 | 222.411 | 106.264 | 207.282 | 99.298 | 204.530 | 98.029 | 222.411 | 106.264 | 207.282 | 99.298 | 200.553 | 96.195 | 222.411 | 106.264 | 207.282 | 99.298 | 196.749 | 94.439 |
| 0.7 | 1 | 0.7 | 105.945 | 3.884 | 57.827 | 2.562 | 52.814 | 2.408 | 105.945 | 3.884 | 57.827 | 2.562 | 46.673 | 2.212 | 105.945 | 3.884 | 57.827 | 2.562 | 41.757 | 2.049 |
| | | 0.8 | 156.521 | 5.766 | 67.178 | 3.150 | 60.021 | 2.906 | 156.521 | 5.766 | 67.178 | 3.150 | 51.678 | 2.608 | 156.521 | 5.766 | 67.178 | 3.150 | 45.309 | 2.369 |
| | | 0.9 | 315.660 | 11.802 | 79.699 | 4.177 | 69.053 | 3.734 | 315.660 | 11.802 | 79.699 | 4.177 | 57.385 | 3.224 | 315.660 | 11.802 | 79.699 | 4.177 | 48.936 | 2.837 |
| | | 0.99 | 3440.840 | 126.654 | 66.720 | 4.388 | 51.399 | 3.478 | 3440.840 | 126.654 | 66.720 | 4.388 | 36.861 | 2.605 | 3440.840 | 126.654 | 66.720 | 4.388 | 27.825 | 2.059 |
| | 5 | 0.7 | 12.325 | 2.265 | 12.079 | 2.214 | 12.031 | 2.204 | 12.325 | 2.265 | 12.079 | 2.214 | 11.959 | 2.190 | 12.325 | 2.265 | 12.079 | 2.214 | 11.889 | 2.175 |
| | | 0.8 | 17.632 | 3.304 | 17.126 | 3.190 | 17.028 | 3.168 | 17.632 | 3.304 | 17.126 | 3.190 | 16.883 | 3.136 | 17.632 | 3.304 | 17.126 | 3.190 | 16.740 | 3.104 |
| | | 0.9 | 33.218 | 6.515 | 31.399 | 6.044 | 31.058 | 5.958 | 33.218 | 6.515 | 31.399 | 6.044 | 30.560 | 5.834 | 33.218 | 6.515 | 31.399 | 6.044 | 30.078 | 5.715 |
| | | 0.99 | 327.375 | 71.474 | 206.265 | 36.676 | 191.848 | 33.624 | 327.375 | 71.474 | 206.265 | 36.676 | 173.336 | 29.931 | 327.375 | 71.474 | 206.265 | 36.676 | 157.747 | 26.990 |
| | 10 | 0.7 | 7.152 | 3.480 | 7.122 | 3.466 | 7.117 | 3.464 | 7.152 | 3.480 | 7.122 | 3.466 | 7.108 | 3.460 | 7.152 | 3.480 | 7.122 | 3.466 | 7.099 | 3.455 |
| | | 0.8 | 10.655 | 5.157 | 10.588 | 5.126 | 10.575 | 5.119 | 10.655 | 5.157 | 10.588 | 5.126 | 10.555 | 5.110 | 10.655 | 5.157 | 10.588 | 5.126 | 10.535 | 5.101 |
| | | 0.9 | 21.143 | 10.222 | 20.863 | 10.091 | 20.808 | 10.065 | 21.143 | 10.222 | 20.863 | 10.091 | 20.726 | 10.027 | 21.143 | 10.222 | 20.863 | 10.091 | 20.644 | 9.989 |
| | | 0.99 | 222.411 | 106.264 | 190.376 | 91.494 | 185.175 | 89.089 | 222.411 | 106.264 | 190.376 | 91.494 | 177.958 | 85.746 | 222.411 | 106.264 | 190.376 | 91.494 | 171.360 | 82.685 |
| 0.9 | 1 | 0.7 | 7.152 | 3.480 | 7.114 | 3.463 | 7.107 | 3.459 | 105.945 | 3.884 | 50.909 | 2.348 | 39.940 | 1.987 | 105.945 | 3.884 | 50.909 | 2.348 | 35.293 | 1.824 |
| | | 0.8 | 10.655 | 5.157 | 10.569 | 5.117 | 10.552 | 5.109 | 156.521 | 5.766 | 57.386 | 2.813 | 43.019 | 2.281 | 156.521 | 5.766 | 57.386 | 2.813 | 37.308 | 2.054 |
| | | 0.9 | 21.143 | 10.222 | 20.784 | 10.054 | 20.714 | 10.022 | 315.660 | 11.802 | 65.286 | 3.572 | 45.984 | 2.698 | 315.660 | 11.802 | 65.286 | 3.572 | 38.802 | 2.352 |
| | | 0.99 | 222.411 | 106.264 | 183.045 | 88.102 | 176.979 | 85.292 | 3440.840 | 126.654 | 46.447 | 3.182 | 24.966 | 1.885 | 3440.840 | 126.654 | 46.447 | 3.182 | 18.663 | 1.503 |
| | 5 | 0.7 | 12.325 | 2.265 | 12.010 | 2.200 | 11.949 | 2.188 | 12.325 | 2.265 | 12.010 | 2.200 | 11.858 | 2.169 | 12.325 | 2.265 | 12.010 | 2.200 | 11.769 | 2.151 |
| | | 0.8 | 17.632 | 3.304 | 16.986 | 3.159 | 16.862 | 3.131 | 17.632 | 3.304 | 16.986 | 3.159 | 16.679 | 3.091 | 17.632 | 3.304 | 16.986 | 3.159 | 16.500 | 3.051 |
| | | 0.9 | 33.218 | 6.515 | 30.914 | 5.922 | 30.491 | 5.816 | 33.218 | 6.515 | 30.914 | 5.922 | 29.876 | 5.665 | 33.218 | 6.515 | 30.914 | 5.922 | 29.284 | 5.522 |
| | | 0.99 | 327.375 | 71.474 | 186.213 | 32.474 | 170.948 | 29.471 | 327.375 | 71.474 | 186.213 | 32.474 | 151.794 | 25.902 | 327.375 | 71.474 | 186.213 | 32.474 | 136.061 | 23.107 |
| | 10 | 0.7 | 7.152 | 3.480 | 7.114 | 3.463 | 7.107 | 3.459 | 7.152 | 3.480 | 7.114 | 3.463 | 7.095 | 3.454 | 7.152 | 3.480 | 7.114 | 3.463 | 7.084 | 3.448 |
| | | 0.8 | 10.655 | 5.157 | 10.569 | 5.117 | 10.552 | 5.109 | 10.655 | 5.157 | 10.569 | 5.117 | 10.526 | 5.097 | 10.655 | 5.157 | 10.569 | 5.117 | 10.501 | 5.085 |
| | | 0.9 | 21.143 | 10.222 | 20.784 | 10.054 | 20.714 | 10.022 | 21.143 | 10.222 | 20.784 | 10.054 | 20.610 | 9.973 | 21.143 | 10.222 | 20.784 | 10.054 | 20.506 | 9.925 |
| | | 0.99 | 222.411 | 106.264 | 183.045 | 88.102 | 176.979 | 85.292 | 222.411 | 106.264 | 183.045 | 88.102 | 168.702 | 81.451 | 222.411 | 106.264 | 183.045 | 88.102 | 161.274 | 77.996 |

Table 13: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 50$ and $p = 7$ when there is no outlier.

| $k$ | Sigma | Rho | $d = 0.2$ | | | | | | $d = 0.5$ | | | | | | $d = 0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ |
| 0.3 | 1 | 0.7 | 0.355 | 0.379 | 0.337 | 0.360 | 0.334 | 0.356 | 0.355 | 0.379 | 0.337 | 0.360 | 0.329 | 0.351 | 0.355 | 0.379 | 0.337 | 0.360 | 0.325 | 0.346 |
| | | 0.8 | 0.556 | 0.593 | 0.513 | 0.546 | 0.506 | 0.538 | 0.556 | 0.593 | 0.513 | 0.546 | 0.495 | 0.527 | 0.556 | 0.593 | 0.513 | 0.546 | 0.485 | 0.516 |
| | | 0.9 | 1.188 | 1.272 | 0.980 | 1.049 | 0.948 | 1.015 | 1.188 | 1.272 | 0.980 | 1.049 | 0.905 | 0.968 | 1.188 | 1.272 | 0.980 | 1.049 | 0.866 | 0.927 |
| | | 0.99 | 14.115 | 14.999 | 3.778 | 3.995 | 3.350 | 3.542 | 14.115 | 14.999 | 3.778 | 3.995 | 2.865 | 3.026 | 14.115 | 14.999 | 3.778 | 3.995 | 2.500 | 2.639 |
| | 5 | 0.7 | 0.365 | 0.383 | 0.364 | 0.382 | 0.364 | 0.382 | 0.365 | 0.383 | 0.364 | 0.382 | 0.364 | 0.382 | 0.365 | 0.383 | 0.364 | 0.382 | 0.364 | 0.381 |
| | | 0.8 | 0.542 | 0.574 | 0.540 | 0.572 | 0.539 | 0.571 | 0.542 | 0.574 | 0.540 | 0.572 | 0.539 | 0.571 | 0.542 | 0.574 | 0.540 | 0.572 | 0.538 | 0.570 |
| | | 0.9 | 1.112 | 1.186 | 1.103 | 1.176 | 1.101 | 1.174 | 1.112 | 1.186 | 1.103 | 1.176 | 1.098 | 1.171 | 1.112 | 1.186 | 1.103 | 1.176 | 1.095 | 1.168 |
| | | 0.99 | 13.448 | 14.327 | 11.941 | 12.718 | 11.682 | 12.441 | 13.448 | 14.327 | 11.941 | 12.718 | 11.315 | 12.049 | 13.448 | 14.327 | 11.941 | 12.718 | 10.973 | 11.683 |
| | 10 | 0.7 | 0.320 | 0.339 | 0.320 | 0.339 | 0.320 | 0.339 | 0.320 | 0.339 | 0.320 | 0.339 | 0.320 | 0.339 | 0.320 | 0.339 | 0.320 | 0.339 | 0.320 | 0.339 |
| | | 0.8 | 0.490 | 0.515 | 0.489 | 0.515 | 0.489 | 0.515 | 0.490 | 0.515 | 0.489 | 0.515 | 0.489 | 0.515 | 0.490 | 0.515 | 0.489 | 0.515 | 0.489 | 0.514 |
| | | 0.9 | 1.022 | 1.083 | 1.020 | 1.081 | 1.020 | 1.081 | 1.022 | 1.083 | 1.020 | 1.081 | 1.020 | 1.080 | 1.022 | 1.083 | 1.020 | 1.081 | 1.019 | 1.080 |
| | | 0.99 | 12.424 | 13.112 | 12.100 | 12.773 | 12.038 | 12.707 | 12.424 | 13.112 | 12.100 | 12.773 | 11.945 | 12.610 | 12.424 | 13.112 | 12.100 | 12.773 | 11.854 | 12.514 |
| 0.7 | 1 | 0.7 | 0.355 | 0.379 | 0.318 | 0.338 | 0.311 | 0.332 | 0.355 | 0.379 | 0.318 | 0.338 | 0.303 | 0.323 | 0.355 | 0.379 | 0.318 | 0.338 | 0.296 | 0.315 |
| | | 0.8 | 0.556 | 0.593 | 0.468 | 0.498 | 0.455 | 0.484 | 0.556 | 0.593 | 0.468 | 0.498 | 0.437 | 0.465 | 0.556 | 0.593 | 0.468 | 0.498 | 0.422 | 0.448 |
| | | 0.9 | 1.188 | 1.272 | 0.807 | 0.863 | 0.764 | 0.816 | 1.188 | 1.272 | 0.807 | 0.863 | 0.709 | 0.757 | 1.188 | 1.272 | 0.807 | 0.863 | 0.664 | 0.708 |
| | | 0.99 | 14.115 | 14.999 | 2.033 | 2.144 | 1.744 | 1.837 | 14.115 | 14.999 | 2.033 | 2.144 | 1.437 | 1.511 | 14.115 | 14.999 | 2.033 | 2.144 | 1.224 | 1.284 |
| | 5 | 0.7 | 0.365 | 0.383 | 0.363 | 0.381 | 0.363 | 0.381 | 0.365 | 0.383 | 0.363 | 0.381 | 0.362 | 0.380 | 0.365 | 0.383 | 0.363 | 0.381 | 0.362 | 0.379 |
| | | 0.8 | 0.542 | 0.574 | 0.537 | 0.569 | 0.536 | 0.568 | 0.542 | 0.574 | 0.537 | 0.569 | 0.535 | 0.567 | 0.542 | 0.574 | 0.537 | 0.569 | 0.534 | 0.565 |
| | | 0.9 | 1.112 | 1.186 | 1.091 | 1.163 | 1.086 | 1.159 | 1.112 | 1.186 | 1.091 | 1.163 | 1.080 | 1.152 | 1.112 | 1.186 | 1.091 | 1.163 | 1.074 | 1.146 |
| | | 0.99 | 13.448 | 14.327 | 10.416 | 11.089 | 9.978 | 10.621 | 13.448 | 14.327 | 10.416 | 11.089 | 9.393 | 9.995 | 13.448 | 14.327 | 10.416 | 11.089 | 8.880 | 9.447 |
| | 10 | 0.7 | 0.320 | 0.339 | 0.320 | 0.339 | 0.320 | 0.338 | 0.320 | 0.339 | 0.320 | 0.339 | 0.320 | 0.338 | 0.320 | 0.339 | 0.320 | 0.339 | 0.320 | 0.338 |
| | | 0.8 | 0.490 | 0.515 | 0.489 | 0.514 | 0.489 | 0.514 | 0.490 | 0.515 | 0.489 | 0.514 | 0.488 | 0.514 | 0.490 | 0.515 | 0.489 | 0.514 | 0.488 | 0.514 |
| | | 0.9 | 1.022 | 1.083 | 1.018 | 1.079 | 1.017 | 1.078 | 1.022 | 1.083 | 1.018 | 1.079 | 1.016 | 1.077 | 1.022 | 1.083 | 1.018 | 1.079 | 1.015 | 1.075 |
| | | 0.99 | 12.424 | 13.112 | 11.695 | 12.348 | 11.560 | 12.206 | 12.424 | 13.112 | 11.695 | 12.348 | 11.363 | 12.000 | 12.424 | 13.112 | 11.695 | 12.348 | 11.173 | 11.801 |
| 0.9 | 1 | 0.7 | 0.355 | 0.379 | 0.309 | 0.329 | 0.302 | 0.322 | 0.355 | 0.379 | 0.309 | 0.329 | 0.293 | 0.311 | 0.355 | 0.379 | 0.309 | 0.329 | 0.284 | 0.302 |
| | | 0.8 | 0.556 | 0.593 | 0.450 | 0.478 | 0.435 | 0.462 | 0.556 | 0.593 | 0.450 | 0.478 | 0.416 | 0.441 | 0.556 | 0.593 | 0.450 | 0.478 | 0.399 | 0.423 |
| | | 0.9 | 1.188 | 1.272 | 0.747 | 0.798 | 0.702 | 0.749 | 1.188 | 1.272 | 0.747 | 0.798 | 0.647 | 0.689 | 1.188 | 1.272 | 0.747 | 0.798 | 0.602 | 0.642 |
| | | 0.99 | 14.115 | 14.999 | 1.644 | 1.731 | 1.402 | 1.473 | 14.115 | 14.999 | 1.644 | 1.731 | 1.152 | 1.208 | 14.115 | 14.999 | 1.644 | 1.731 | 0.986 | 1.030 |
| | 5 | 0.7 | 0.365 | 0.383 | 0.363 | 0.380 | 0.362 | 0.380 | 0.365 | 0.383 | 0.363 | 0.380 | 0.362 | 0.379 | 0.365 | 0.383 | 0.363 | 0.380 | 0.361 | 0.378 |
| | | 0.8 | 0.542 | 0.574 | 0.536 | 0.568 | 0.535 | 0.566 | 0.542 | 0.574 | 0.536 | 0.568 | 0.533 | 0.565 | 0.542 | 0.574 | 0.536 | 0.568 | 0.532 | 0.563 |
| | | 0.9 | 1.112 | 1.186 | 1.085 | 1.157 | 1.079 | 1.151 | 1.112 | 1.186 | 1.085 | 1.157 | 1.072 | 1.143 | 1.112 | 1.186 | 1.085 | 1.157 | 1.064 | 1.135 |
| | | 0.99 | 13.448 | 14.327 | 9.803 | 10.433 | 9.316 | 9.913 | 13.448 | 14.327 | 9.803 | 10.433 | 8.679 | 9.233 | 13.448 | 14.327 | 9.803 | 10.433 | 8.134 | 8.651 |
| | 10 | 0.7 | 0.320 | 0.339 | 0.320 | 0.338 | 0.320 | 0.338 | 0.320 | 0.339 | 0.320 | 0.338 | 0.320 | 0.338 | 0.320 | 0.339 | 0.320 | 0.338 | 0.319 | 0.338 |
| | | 0.8 | 0.490 | 0.515 | 0.488 | 0.514 | 0.488 | 0.514 | 0.490 | 0.515 | 0.488 | 0.514 | 0.488 | 0.513 | 0.490 | 0.515 | 0.488 | 0.514 | 0.488 | 0.513 |
| | | 0.9 | 1.022 | 1.083 | 1.017 | 1.078 | 1.016 | 1.077 | 1.022 | 1.083 | 1.017 | 1.078 | 1.014 | 1.075 | 1.022 | 1.083 | 1.017 | 1.078 | 1.013 | 1.073 |
| | | 0.99 | 12.424 | 13.112 | 11.503 | 12.147 | 11.336 | 11.971 | 12.424 | 13.112 | 11.503 | 12.147 | 11.094 | 11.718 | 12.424 | 13.112 | 11.503 | 12.147 | 10.863 | 11.476 |

Table 14: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 50$ and $p = 7$ when there is one outlier.

| $k$ | Sigma | Rho | $d = 0.2$ $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $d = 0.5$ $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $d = 0.8$ $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 1 | 0.7 | 2.368 | 0.394 | 2.266 | 0.377 | 2.246 | 0.374 | 2.368 | 0.394 | 2.266 | 0.377 | 2.218 | 0.370 | 2.368 | 0.394 | 2.266 | 0.377 | 2.191 | 0.365 |
| | | 0.8 | 3.716 | 0.626 | 3.435 | 0.582 | 3.384 | 0.574 | 3.716 | 0.626 | 3.435 | 0.582 | 3.311 | 0.563 | 3.716 | 0.626 | 3.435 | 0.582 | 3.241 | 0.552 |
| | | 0.9 | 7.504 | 1.319 | 6.208 | 1.102 | 6.004 | 1.068 | 7.504 | 1.319 | 6.208 | 1.102 | 5.725 | 1.023 | 7.504 | 1.319 | 6.208 | 1.102 | 5.472 | 0.982 |
| | | 0.99 | 88.326 | 15.690 | 21.517 | 4.214 | 18.804 | 3.735 | 88.326 | 15.690 | 21.517 | 4.214 | 15.750 | 3.191 | 88.326 | 15.690 | 21.517 | 4.214 | 13.476 | 2.781 |
| | 5 | 0.7 | 0.841 | 0.410 | 0.840 | 0.410 | 0.840 | 0.409 | 0.841 | 0.410 | 0.840 | 0.410 | 0.840 | 0.409 | 0.841 | 0.410 | 0.840 | 0.410 | 0.839 | 0.409 |
| | | 0.8 | 1.296 | 0.623 | 1.293 | 0.621 | 1.293 | 0.621 | 1.296 | 0.623 | 1.293 | 0.621 | 1.292 | 0.620 | 1.296 | 0.623 | 1.293 | 0.621 | 1.291 | 0.620 |
| | | 0.9 | 2.784 | 1.281 | 2.768 | 1.271 | 2.764 | 1.269 | 2.784 | 1.281 | 2.768 | 1.271 | 2.760 | 1.267 | 2.784 | 1.281 | 2.768 | 1.271 | 2.755 | 1.264 |
| | | 0.99 | 38.788 | 15.976 | 34.689 | 14.224 | 33.980 | 13.922 | 38.788 | 15.976 | 34.689 | 14.224 | 32.973 | 13.494 | 38.788 | 15.976 | 34.689 | 14.224 | 32.029 | 13.094 |
| | 10 | 0.7 | 0.428 | 0.347 | 0.428 | 0.347 | 0.428 | 0.347 | 0.428 | 0.347 | 0.428 | 0.347 | 0.428 | 0.346 | 0.428 | 0.347 | 0.428 | 0.347 | 0.428 | 0.346 |
| | | 0.8 | 0.669 | 0.529 | 0.669 | 0.529 | 0.669 | 0.529 | 0.669 | 0.529 | 0.669 | 0.529 | 0.669 | 0.529 | 0.669 | 0.529 | 0.669 | 0.529 | 0.668 | 0.529 |
| | | 0.9 | 1.399 | 1.111 | 1.397 | 1.109 | 1.397 | 1.109 | 1.399 | 1.111 | 1.397 | 1.109 | 1.396 | 1.108 | 1.399 | 1.111 | 1.397 | 1.109 | 1.395 | 1.108 |
| | | 0.99 | 17.302 | 13.500 | 16.828 | 13.151 | 16.737 | 13.084 | 17.302 | 13.500 | 16.828 | 13.151 | 16.601 | 12.984 | 17.302 | 13.500 | 16.828 | 13.151 | 16.468 | 12.886 |
| 0.7 | 1 | 0.7 | 2.368 | 0.394 | 2.144 | 0.358 | 2.104 | 0.352 | 2.368 | 0.394 | 2.144 | 0.358 | 2.048 | 0.344 | 2.368 | 0.394 | 2.144 | 0.358 | 1.995 | 0.336 |
| | | 0.8 | 3.716 | 0.626 | 3.125 | 0.535 | 3.030 | 0.521 | 3.716 | 0.626 | 3.125 | 0.535 | 2.900 | 0.503 | 3.716 | 0.626 | 3.125 | 0.535 | 2.781 | 0.486 |
| | | 0.9 | 7.504 | 1.319 | 5.078 | 0.918 | 4.780 | 0.871 | 7.504 | 1.319 | 5.078 | 0.918 | 4.397 | 0.810 | 7.504 | 1.319 | 5.078 | 0.918 | 4.075 | 0.761 |
| | | 0.99 | 88.326 | 15.690 | 10.595 | 2.257 | 8.826 | 1.933 | 88.326 | 15.690 | 10.595 | 2.257 | 6.951 | 1.587 | 88.326 | 15.690 | 10.595 | 2.257 | 5.654 | 1.347 |
| | 5 | 0.7 | 0.841 | 0.410 | 0.839 | 0.409 | 0.838 | 0.408 | 0.841 | 0.410 | 0.839 | 0.409 | 0.838 | 0.408 | 0.841 | 0.410 | 0.839 | 0.409 | 0.837 | 0.407 |
| | | 0.8 | 1.296 | 0.623 | 1.290 | 0.619 | 1.288 | 0.618 | 1.296 | 0.623 | 1.290 | 0.619 | 1.286 | 0.617 | 1.296 | 0.623 | 1.290 | 0.619 | 1.284 | 0.615 |
| | | 0.9 | 2.784 | 1.281 | 2.746 | 1.259 | 2.739 | 1.254 | 2.784 | 1.281 | 2.746 | 1.259 | 2.727 | 1.248 | 2.784 | 1.281 | 2.746 | 1.259 | 2.716 | 1.242 |
| | | 0.99 | 38.788 | 15.976 | 30.492 | 12.443 | 29.275 | 11.929 | 38.788 | 15.976 | 30.492 | 12.443 | 27.640 | 11.241 | 38.788 | 15.976 | 30.492 | 12.443 | 26.197 | 10.636 |
| | 10 | 0.7 | 0.428 | 0.347 | 0.428 | 0.346 | 0.427 | 0.346 | 0.428 | 0.347 | 0.428 | 0.346 | 0.427 | 0.346 | 0.428 | 0.347 | 0.428 | 0.346 | 0.427 | 0.346 |
| | | 0.8 | 0.669 | 0.529 | 0.668 | 0.529 | 0.668 | 0.528 | 0.669 | 0.529 | 0.668 | 0.529 | 0.668 | 0.528 | 0.669 | 0.529 | 0.668 | 0.529 | 0.668 | 0.528 |
| | | 0.9 | 1.399 | 1.111 | 1.394 | 1.107 | 1.393 | 1.106 | 1.399 | 1.111 | 1.394 | 1.107 | 1.392 | 1.105 | 1.399 | 1.111 | 1.394 | 1.107 | 1.390 | 1.103 |
| | | 0.99 | 17.302 | 13.500 | 16.236 | 12.715 | 16.039 | 12.569 | 17.302 | 13.500 | 16.236 | 12.715 | 15.751 | 12.357 | 17.302 | 13.500 | 16.236 | 12.715 | 15.474 | 12.153 |
| 0.9 | 1 | 0.7 | 2.368 | 0.394 | 2.088 | 0.350 | 2.040 | 0.343 | 2.368 | 0.394 | 2.088 | 0.350 | 1.974 | 0.333 | 2.368 | 0.394 | 2.088 | 0.350 | 1.911 | 0.325 |
| | | 0.8 | 3.716 | 0.626 | 2.992 | 0.516 | 2.882 | 0.500 | 3.716 | 0.626 | 2.992 | 0.516 | 2.733 | 0.479 | 3.716 | 0.626 | 2.992 | 0.516 | 2.599 | 0.461 |
| | | 0.9 | 7.504 | 1.319 | 4.663 | 0.852 | 4.348 | 0.803 | 7.504 | 1.319 | 4.663 | 0.852 | 3.952 | 0.742 | 7.504 | 1.319 | 4.663 | 0.852 | 3.625 | 0.692 |
| | | 0.99 | 88.326 | 15.690 | 8.212 | 1.820 | 6.737 | 1.548 | 88.326 | 15.690 | 8.212 | 1.820 | 5.216 | 1.266 | 88.326 | 15.690 | 8.212 | 1.820 | 4.192 | 1.078 |
| | 5 | 0.7 | 0.841 | 0.410 | 0.838 | 0.408 | 0.838 | 0.408 | 0.841 | 0.410 | 0.838 | 0.408 | 0.837 | 0.407 | 0.841 | 0.410 | 0.838 | 0.408 | 0.836 | 0.406 |
| | | 0.8 | 1.296 | 0.623 | 1.288 | 0.618 | 1.286 | 0.617 | 1.296 | 0.623 | 1.288 | 0.618 | 1.284 | 0.615 | 1.296 | 0.623 | 1.288 | 0.618 | 1.281 | 0.613 |
| | | 0.9 | 2.784 | 1.281 | 2.735 | 1.253 | 2.726 | 1.247 | 2.784 | 1.281 | 2.735 | 1.253 | 2.712 | 1.239 | 2.784 | 1.281 | 2.735 | 1.253 | 2.698 | 1.231 |
| | | 0.99 | 38.788 | 15.976 | 28.786 | 11.723 | 27.423 | 11.150 | 38.788 | 15.976 | 28.786 | 11.723 | 25.630 | 10.398 | 38.788 | 15.976 | 28.786 | 11.723 | 24.081 | 9.753 |
| | 10 | 0.7 | 0.428 | 0.347 | 0.427 | 0.346 | 0.427 | 0.346 | 0.428 | 0.347 | 0.427 | 0.346 | 0.427 | 0.346 | 0.428 | 0.347 | 0.427 | 0.346 | 0.427 | 0.346 |
| | | 0.8 | 0.669 | 0.529 | 0.668 | 0.528 | 0.668 | 0.528 | 0.669 | 0.529 | 0.668 | 0.528 | 0.667 | 0.528 | 0.669 | 0.529 | 0.668 | 0.528 | 0.667 | 0.528 |
| | | 0.9 | 1.399 | 1.111 | 1.393 | 1.106 | 1.392 | 1.104 | 1.399 | 1.111 | 1.393 | 1.106 | 1.390 | 1.103 | 1.399 | 1.111 | 1.393 | 1.106 | 1.388 | 1.101 |
| | | 0.99 | 17.302 | 13.500 | 15.955 | 12.508 | 15.711 | 12.327 | 17.302 | 13.500 | 15.955 | 12.508 | 15.359 | 12.067 | 17.302 | 13.500 | 15.955 | 12.508 | 15.022 | 11.818 |

Table 15: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 50$ and $p = 7$ when there are two outliers.

| $k$ | Sigma | Rho | $d = 0.2$ | | | | | | $d = 0.5$ | | | | | | $d = 0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{\mathrm{MRT}}$ | $\hat{\alpha}_{\mathrm{RTMME}}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{\mathrm{MRT}}$ | $\hat{\alpha}_{\mathrm{RTMME}}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{\mathrm{MRT}}$ | $\hat{\alpha}_{\mathrm{RTMME}}$ |
| 0.3 | 1 | 0.7 | 3.708 | 0.423 | 3.546 | 0.399 | 3.515 | 0.394 | 3.708 | 0.423 | 3.546 | 0.399 | 3.470 | 0.388 | 3.708 | 0.423 | 3.546 | 0.399 | 3.426 | 0.382 |
| | | 0.8 | 5.409 | 0.657 | 5.051 | 0.600 | 4.986 | 0.591 | 5.409 | 0.657 | 5.051 | 0.600 | 4.891 | 0.576 | 5.409 | 0.657 | 5.051 | 0.600 | 4.800 | 0.563 |
| | | 0.9 | 10.810 | 1.399 | 9.329 | 1.140 | 9.089 | 1.101 | 10.810 | 1.399 | 9.329 | 1.140 | 8.754 | 1.047 | 10.810 | 1.399 | 9.329 | 1.140 | 8.446 | 0.999 |
| | | 0.99 | 113.434 | 16.400 | 42.585 | 4.289 | 37.921 | 3.788 | 113.434 | 16.400 | 42.585 | 4.289 | 32.335 | 3.219 | 113.434 | 16.400 | 42.585 | 4.289 | 27.948 | 2.791 |
| | 5 | 0.7 | 1.359 | 0.435 | 1.357 | 0.434 | 1.357 | 0.434 | 1.359 | 0.435 | 1.357 | 0.434 | 1.356 | 0.434 | 1.359 | 0.435 | 1.357 | 0.434 | 1.356 | 0.434 |
| | | 0.8 | 2.109 | 0.665 | 2.104 | 0.663 | 2.103 | 0.663 | 2.109 | 0.665 | 2.104 | 0.663 | 2.102 | 0.662 | 2.109 | 0.665 | 2.104 | 0.663 | 2.101 | 0.662 |
| | | 0.9 | 4.488 | 1.361 | 4.462 | 1.351 | 4.457 | 1.349 | 4.488 | 1.361 | 4.462 | 1.351 | 4.449 | 1.346 | 4.488 | 1.361 | 4.462 | 1.351 | 4.441 | 1.343 |
| | | 0.99 | 59.623 | 17.056 | 53.650 | 15.213 | 52.613 | 14.894 | 59.623 | 17.056 | 53.650 | 15.213 | 51.140 | 14.443 | 59.623 | 17.056 | 53.650 | 15.213 | 49.758 | 14.020 |
| | 10 | 0.7 | 0.630 | 0.372 | 0.630 | 0.372 | 0.630 | 0.372 | 0.630 | 0.372 | 0.630 | 0.372 | 0.629 | 0.372 | 0.630 | 0.372 | 0.630 | 0.372 | 0.629 | 0.372 |
| | | 0.8 | 0.975 | 0.567 | 0.975 | 0.567 | 0.975 | 0.567 | 0.975 | 0.567 | 0.975 | 0.567 | 0.974 | 0.567 | 0.975 | 0.567 | 0.975 | 0.567 | 0.974 | 0.567 |
| | | 0.9 | 2.016 | 1.184 | 2.014 | 1.182 | 2.013 | 1.182 | 2.016 | 1.184 | 2.014 | 1.182 | 2.013 | 1.181 | 2.016 | 1.184 | 2.014 | 1.182 | 2.012 | 1.181 |
| | | 0.99 | 24.543 | 14.279 | 23.987 | 13.924 | 23.880 | 13.856 | 24.543 | 14.279 | 23.987 | 13.924 | 23.721 | 13.754 | 24.543 | 14.279 | 23.987 | 13.924 | 23.564 | 13.654 |
| 0.7 | 1 | 0.7 | 3.708 | 0.423 | 3.351 | 0.371 | 3.288 | 0.363 | 3.708 | 0.423 | 3.351 | 0.371 | 3.197 | 0.351 | 3.708 | 0.423 | 3.351 | 0.371 | 3.112 | 0.340 |
| | | 0.8 | 5.409 | 0.657 | 4.647 | 0.541 | 4.522 | 0.524 | 5.409 | 0.657 | 4.647 | 0.541 | 4.346 | 0.500 | 5.409 | 0.657 | 4.647 | 0.541 | 4.184 | 0.479 |
| | | 0.9 | 10.810 | 1.399 | 7.955 | 0.926 | 7.575 | 0.871 | 10.810 | 1.399 | 7.955 | 0.926 | 7.071 | 0.803 | 10.810 | 1.399 | 7.955 | 0.926 | 6.631 | 0.746 |
| | | 0.99 | 113.434 | 16.400 | 22.122 | 2.244 | 18.411 | 1.906 | 113.434 | 16.400 | 22.122 | 2.244 | 14.378 | 1.547 | 113.434 | 16.400 | 22.122 | 2.244 | 11.534 | 1.298 |
| | 5 | 0.7 | 1.359 | 0.435 | 1.355 | 0.433 | 1.354 | 0.433 | 1.359 | 0.435 | 1.355 | 0.433 | 1.353 | 0.433 | 1.359 | 0.435 | 1.355 | 0.433 | 1.352 | 0.432 |
| | | 0.8 | 2.109 | 0.665 | 2.098 | 0.661 | 2.096 | 0.660 | 2.109 | 0.665 | 2.098 | 0.661 | 2.093 | 0.658 | 2.109 | 0.665 | 2.098 | 0.661 | 2.090 | 0.657 |
| | | 0.9 | 4.488 | 1.361 | 4.428 | 1.338 | 4.416 | 1.333 | 4.488 | 1.361 | 4.428 | 1.338 | 4.398 | 1.327 | 4.488 | 1.361 | 4.428 | 1.338 | 4.381 | 1.320 |
| | | 0.99 | 59.623 | 17.056 | 47.501 | 13.333 | 45.711 | 12.790 | 59.623 | 17.056 | 47.501 | 13.333 | 43.298 | 12.061 | 59.623 | 17.056 | 47.501 | 13.333 | 41.162 | 11.420 |
| | 10 | 0.7 | 0.630 | 0.372 | 0.629 | 0.372 | 0.629 | 0.372 | 0.630 | 0.372 | 0.629 | 0.372 | 0.629 | 0.372 | 0.630 | 0.372 | 0.629 | 0.372 | 0.629 | 0.371 |
| | | 0.8 | 0.975 | 0.567 | 0.974 | 0.567 | 0.974 | 0.566 | 0.975 | 0.567 | 0.974 | 0.567 | 0.974 | 0.566 | 0.975 | 0.567 | 0.974 | 0.567 | 0.973 | 0.566 |
| | | 0.9 | 2.016 | 1.184 | 2.011 | 1.180 | 2.010 | 1.179 | 2.016 | 1.184 | 2.011 | 1.180 | 2.008 | 1.178 | 2.016 | 1.184 | 2.011 | 1.180 | 2.006 | 1.176 |
| | | 0.99 | 24.543 | 14.279 | 23.291 | 13.480 | 23.058 | 13.332 | 24.543 | 14.279 | 23.291 | 13.480 | 22.718 | 13.116 | 24.543 | 14.279 | 23.291 | 13.480 | 22.390 | 12.907 |
| 0.9 | 1 | 0.7 | 3.708 | 0.423 | 3.261 | 0.359 | 3.185 | 0.349 | 3.708 | 0.423 | 3.261 | 0.359 | 3.076 | 0.336 | 3.708 | 0.423 | 3.261 | 0.359 | 2.975 | 0.324 |
| | | 0.8 | 5.409 | 0.657 | 4.470 | 0.517 | 4.322 | 0.497 | 5.409 | 0.657 | 4.470 | 0.517 | 4.119 | 0.471 | 5.409 | 0.657 | 4.470 | 0.517 | 3.933 | 0.449 |
| | | 0.9 | 10.810 | 1.399 | 7.423 | 0.850 | 7.004 | 0.794 | 10.810 | 1.399 | 7.423 | 0.850 | 6.459 | 0.725 | 10.810 | 1.399 | 7.423 | 0.850 | 5.991 | 0.670 |
| | | 0.99 | 113.434 | 16.400 | 17.101 | 1.788 | 13.911 | 1.506 | 113.434 | 16.400 | 17.101 | 1.788 | 10.566 | 1.215 | 113.434 | 16.400 | 17.101 | 1.788 | 8.292 | 1.022 |
| | 5 | 0.7 | 1.359 | 0.435 | 1.354 | 0.433 | 1.353 | 0.432 | 1.359 | 0.435 | 1.354 | 0.433 | 1.351 | 0.432 | 1.359 | 0.435 | 1.354 | 0.433 | 1.350 | 0.431 |
| | | 0.8 | 2.109 | 0.665 | 2.095 | 0.659 | 2.093 | 0.658 | 2.109 | 0.665 | 2.095 | 0.659 | 2.089 | 0.657 | 2.109 | 0.665 | 2.095 | 0.659 | 2.085 | 0.655 |
| | | 0.9 | 4.488 | 1.361 | 4.411 | 1.331 | 4.396 | 1.326 | 4.488 | 1.361 | 4.411 | 1.331 | 4.373 | 1.317 | 4.488 | 1.361 | 4.411 | 1.331 | 4.351 | 1.309 |
| | | 0.99 | 59.623 | 17.056 | 44.990 | 12.572 | 42.977 | 11.965 | 59.623 | 17.056 | 44.990 | 12.572 | 40.319 | 11.168 | 59.623 | 17.056 | 44.990 | 12.572 | 38.014 | 10.483 |
| | 10 | 0.7 | 0.630 | 0.372 | 0.629 | 0.372 | 0.629 | 0.372 | 0.630 | 0.372 | 0.629 | 0.372 | 0.629 | 0.371 | 0.630 | 0.372 | 0.629 | 0.372 | 0.629 | 0.371 |
| | | 0.8 | 0.975 | 0.567 | 0.974 | 0.566 | 0.974 | 0.566 | 0.975 | 0.567 | 0.974 | 0.566 | 0.973 | 0.566 | 0.975 | 0.567 | 0.974 | 0.566 | 0.973 | 0.566 |
| | | 0.9 | 2.016 | 1.184 | 2.009 | 1.178 | 2.008 | 1.177 | 2.016 | 1.184 | 2.009 | 1.178 | 2.006 | 1.176 | 2.016 | 1.184 | 2.009 | 1.178 | 2.003 | 1.174 |
| | | 0.99 | 24.543 | 14.279 | 22.960 | 13.269 | 22.671 | 13.085 | 24.543 | 14.279 | 22.960 | 13.269 | 22.253 | 12.820 | 24.543 | 14.279 | 22.960 | 13.269 | 21.853 | 12.567 |

Table 16: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 100$ and $p = 7$ when there is no outlier.

| $k$ | Sigma | Rho | $d = 0.2$ | | | | | | $d = 0.5$ | | | | | | $d = 0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\widehat{\alpha}$ | $\widehat{\alpha}_M$ | $\widehat{\alpha}(k)$ | $\widehat{\alpha}_M(k)$ | $\widehat{\alpha}_{MRT}$ | $\widehat{\alpha}_{RTMME}$ | $\widehat{\alpha}$ | $\widehat{\alpha}_M$ | $\widehat{\alpha}(k)$ | $\widehat{\alpha}_M(k)$ | $\widehat{\alpha}_{MRT}$ | $\widehat{\alpha}_{RTMME}$ | $\widehat{\alpha}$ | $\widehat{\alpha}_M$ | $\widehat{\alpha}(k)$ | $\widehat{\alpha}_M(k)$ | $\widehat{\alpha}_{MRT}$ | $\widehat{\alpha}_{RTMME}$ |
| | | 0.7 | 0.175 | 0.185 | 0.171 | 0.182 | 0.171 | 0.181 | 0.175 | 0.185 | 0.171 | 0.182 | 0.170 | 0.180 | 0.175 | 0.185 | 0.171 | 0.182 | 0.169 | 0.179 |
| | 1 | 0.8 | 0.276 | 0.293 | 0.267 | 0.283 | 0.265 | 0.281 | 0.276 | 0.293 | 0.267 | 0.283 | 0.263 | 0.278 | 0.276 | 0.293 | 0.267 | 0.283 | 0.260 | 0.276 |
| | | 0.9 | 0.543 | 0.573 | 0.501 | 0.529 | 0.494 | 0.521 | 0.543 | 0.573 | 0.501 | 0.529 | 0.483 | 0.510 | 0.543 | 0.573 | 0.501 | 0.529 | 0.473 | 0.499 |
| | | 0.99 | 6.136 | 6.416 | 2.909 | 3.055 | 2.653 | 2.787 | 6.136 | 6.416 | 2.909 | 3.055 | 2.348 | 2.468 | 6.136 | 6.416 | 2.909 | 3.055 | 2.110 | 2.218 |
| | | 0.7 | 0.150 | 0.158 | 0.150 | 0.158 | 0.150 | 0.158 | 0.150 | 0.158 | 0.150 | 0.158 | 0.150 | 0.158 | 0.150 | 0.158 | 0.150 | 0.158 | 0.150 | 0.158 |
| 0.3 | 5 | 0.8 | 0.229 | 0.242 | 0.229 | 0.241 | 0.229 | 0.241 | 0.229 | 0.242 | 0.229 | 0.241 | 0.229 | 0.241 | 0.229 | 0.242 | 0.229 | 0.241 | 0.229 | 0.241 |
| | | 0.9 | 0.495 | 0.524 | 0.494 | 0.523 | 0.493 | 0.522 | 0.495 | 0.524 | 0.494 | 0.523 | 0.493 | 0.522 | 0.495 | 0.524 | 0.494 | 0.523 | 0.492 | 0.521 |
| | | 0.99 | 5.646 | 5.955 | 5.376 | 5.669 | 5.325 | 5.615 | 5.646 | 5.955 | 5.376 | 5.669 | 5.250 | 5.537 | 5.646 | 5.955 | 5.376 | 5.669 | 5.178 | 5.461 |
| | | 0.7 | 0.156 | 0.164 | 0.156 | 0.164 | 0.156 | 0.164 | 0.156 | 0.164 | 0.156 | 0.164 | 0.156 | 0.164 | 0.156 | 0.164 | 0.156 | 0.164 | 0.156 | 0.164 |
| | 10 | 0.8 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 |
| | | 0.9 | 0.457 | 0.480 | 0.456 | 0.480 | 0.456 | 0.480 | 0.457 | 0.480 | 0.456 | 0.480 | 0.456 | 0.480 | 0.457 | 0.480 | 0.456 | 0.480 | 0.456 | 0.479 |
| | | 0.99 | 5.366 | 5.735 | 5.308 | 5.672 | 5.296 | 5.660 | 5.366 | 5.735 | 5.308 | 5.672 | 5.279 | 5.641 | 5.366 | 5.735 | 5.308 | 5.672 | 5.262 | 5.623 |
| | | 0.7 | 0.175 | 0.185 | 0.167 | 0.177 | 0.165 | 0.175 | 0.175 | 0.185 | 0.167 | 0.177 | 0.163 | 0.173 | 0.175 | 0.185 | 0.167 | 0.177 | 0.162 | 0.171 |
| | 1 | 0.8 | 0.276 | 0.293 | 0.256 | 0.271 | 0.253 | 0.268 | 0.276 | 0.293 | 0.256 | 0.271 | 0.248 | 0.262 | 0.276 | 0.293 | 0.256 | 0.271 | 0.243 | 0.257 |
| | | 0.9 | 0.543 | 0.573 | 0.456 | 0.482 | 0.443 | 0.467 | 0.543 | 0.573 | 0.456 | 0.482 | 0.425 | 0.448 | 0.543 | 0.573 | 0.456 | 0.482 | 0.409 | 0.431 |
| | | 0.99 | 6.136 | 6.416 | 1.793 | 1.884 | 1.587 | 1.667 | 6.136 | 6.416 | 1.793 | 1.884 | 1.356 | 1.424 | 6.136 | 6.416 | 1.793 | 1.884 | 1.188 | 1.246 |
| | | 0.7 | 0.150 | 0.158 | 0.149 | 0.158 | 0.149 | 0.158 | 0.150 | 0.158 | 0.149 | 0.158 | 0.149 | 0.158 | 0.150 | 0.158 | 0.149 | 0.158 | 0.149 | 0.158 |
| 0.7 | 5 | 0.8 | 0.229 | 0.242 | 0.228 | 0.241 | 0.228 | 0.241 | 0.229 | 0.242 | 0.228 | 0.241 | 0.228 | 0.241 | 0.229 | 0.242 | 0.228 | 0.241 | 0.228 | 0.240 |
| | | 0.9 | 0.495 | 0.524 | 0.491 | 0.520 | 0.491 | 0.519 | 0.495 | 0.524 | 0.491 | 0.520 | 0.490 | 0.518 | 0.495 | 0.524 | 0.491 | 0.520 | 0.488 | 0.517 |
| | | 0.99 | 5.646 | 5.955 | 5.056 | 5.331 | 4.953 | 5.223 | 5.646 | 5.955 | 5.056 | 5.331 | 4.808 | 5.070 | 5.646 | 5.955 | 5.056 | 5.331 | 4.672 | 4.927 |
| | | 0.7 | 0.156 | 0.164 | 0.156 | 0.164 | 0.156 | 0.164 | 0.156 | 0.164 | 0.156 | 0.164 | 0.155 | 0.163 | 0.156 | 0.164 | 0.156 | 0.164 | 0.155 | 0.163 |
| | 10 | 0.8 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 |
| | | 0.9 | 0.457 | 0.480 | 0.456 | 0.479 | 0.456 | 0.479 | 0.457 | 0.480 | 0.456 | 0.479 | 0.455 | 0.479 | 0.457 | 0.480 | 0.456 | 0.479 | 0.455 | 0.479 |
| | | 0.99 | 5.366 | 5.735 | 5.232 | 5.591 | 5.206 | 5.563 | 5.366 | 5.735 | 5.232 | 5.591 | 5.167 | 5.521 | 5.366 | 5.735 | 5.232 | 5.591 | 5.129 | 5.481 |
| | | 0.7 | 0.175 | 0.185 | 0.165 | 0.175 | 0.163 | 0.173 | 0.175 | 0.185 | 0.165 | 0.175 | 0.161 | 0.170 | 0.175 | 0.185 | 0.165 | 0.175 | 0.159 | 0.168 |
| | 1 | 0.8 | 0.276 | 0.293 | 0.251 | 0.266 | 0.247 | 0.262 | 0.276 | 0.293 | 0.251 | 0.266 | 0.241 | 0.255 | 0.276 | 0.293 | 0.251 | 0.266 | 0.236 | 0.250 |
| | | 0.9 | 0.543 | 0.573 | 0.437 | 0.462 | 0.422 | 0.446 | 0.543 | 0.573 | 0.437 | 0.462 | 0.402 | 0.424 | 0.543 | 0.573 | 0.437 | 0.462 | 0.385 | 0.406 |
| | | 0.99 | 6.136 | 6.416 | 1.513 | 1.589 | 1.329 | 1.395 | 6.136 | 6.416 | 1.513 | 1.589 | 1.129 | 1.183 | 6.136 | 6.416 | 1.513 | 1.589 | 0.986 | 1.032 |
| | | 0.7 | 0.150 | 0.158 | 0.149 | 0.158 | 0.149 | 0.158 | 0.150 | 0.158 | 0.149 | 0.158 | 0.149 | 0.158 | 0.150 | 0.158 | 0.149 | 0.158 | 0.149 | 0.157 |
| 0.9 | 5 | 0.8 | 0.229 | 0.242 | 0.228 | 0.241 | 0.228 | 0.241 | 0.229 | 0.242 | 0.228 | 0.241 | 0.228 | 0.240 | 0.229 | 0.242 | 0.228 | 0.241 | 0.227 | 0.240 |
| | | 0.9 | 0.495 | 0.524 | 0.490 | 0.519 | 0.489 | 0.518 | 0.495 | 0.524 | 0.490 | 0.519 | 0.488 | 0.517 | 0.495 | 0.524 | 0.490 | 0.519 | 0.486 | 0.515 |
| | | 0.99 | 5.646 | 5.955 | 4.911 | 5.178 | 4.788 | 5.049 | 5.646 | 5.955 | 4.911 | 5.178 | 4.617 | 4.868 | 5.646 | 5.955 | 4.911 | 5.178 | 4.459 | 4.701 |
| | | 0.7 | 0.156 | 0.164 | 0.156 | 0.164 | 0.155 | 0.163 | 0.156 | 0.164 | 0.156 | 0.164 | 0.155 | 0.163 | 0.156 | 0.164 | 0.156 | 0.164 | 0.155 | 0.163 |
| | 10 | 0.8 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 | 0.229 | 0.243 |
| | | 0.9 | 0.457 | 0.480 | 0.456 | 0.479 | 0.455 | 0.479 | 0.457 | 0.480 | 0.456 | 0.479 | 0.455 | 0.478 | 0.457 | 0.480 | 0.456 | 0.479 | 0.455 | 0.478 |
| | | 0.99 | 5.366 | 5.735 | 5.195 | 5.551 | 5.162 | 5.516 | 5.366 | 5.735 | 5.195 | 5.551 | 5.113 | 5.464 | 5.366 | 5.735 | 5.195 | 5.551 | 5.066 | 5.413 |

TABLE 17: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 100$ and $p = 7$ when there is one outlier.

| $k$ | Sigma | Rho | $d=0.2$ | | | | | | $d=0.5$ | | | | | | $d=0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ |
| 0.3 | 1 | 0.7 | 0.414 | 0.187 | 0.407 | 0.184 | 0.406 | 0.183 | 0.414 | 0.187 | 0.407 | 0.184 | 0.404 | 0.182 | 0.414 | 0.187 | 0.407 | 0.184 | 0.402 | 0.181 |
| | | 0.8 | 0.606 | 0.295 | 0.591 | 0.285 | 0.588 | 0.284 | 0.606 | 0.295 | 0.591 | 0.285 | 0.583 | 0.281 | 0.606 | 0.295 | 0.591 | 0.285 | 0.579 | 0.278 |
| | | 0.9 | 1.155 | 0.578 | 1.092 | 0.534 | 1.081 | 0.526 | 1.155 | 0.578 | 1.092 | 0.534 | 1.065 | 0.515 | 1.155 | 0.578 | 1.092 | 0.534 | 1.049 | 0.504 |
| | | 0.99 | 12.070 | 6.477 | 7.416 | 3.111 | 6.944 | 2.842 | 12.070 | 6.477 | 7.416 | 3.111 | 6.345 | 2.521 | 12.070 | 6.477 | 7.416 | 3.111 | 5.843 | 2.270 |
| | 5 | 0.7 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 |
| | | 0.8 | 0.316 | 0.246 | 0.316 | 0.245 | 0.315 | 0.245 | 0.316 | 0.246 | 0.316 | 0.245 | 0.315 | 0.245 | 0.316 | 0.246 | 0.316 | 0.245 | 0.315 | 0.245 |
| | | 0.9 | 0.662 | 0.531 | 0.661 | 0.529 | 0.660 | 0.529 | 0.662 | 0.531 | 0.661 | 0.529 | 0.660 | 0.529 | 0.662 | 0.531 | 0.661 | 0.529 | 0.659 | 0.528 |
| | | 0.99 | 7.190 | 6.018 | 6.898 | 5.731 | 6.843 | 5.678 | 7.190 | 6.018 | 6.898 | 5.731 | 6.763 | 5.599 | 7.190 | 6.018 | 6.898 | 5.731 | 6.685 | 5.523 |
| | 10 | 0.7 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 |
| | | 0.8 | 0.295 | 0.249 | 0.295 | 0.249 | 0.295 | 0.249 | 0.295 | 0.249 | 0.295 | 0.249 | 0.295 | 0.249 | 0.295 | 0.249 | 0.295 | 0.249 | 0.295 | 0.249 |
| | | 0.9 | 0.603 | 0.499 | 0.603 | 0.498 | 0.602 | 0.498 | 0.603 | 0.499 | 0.603 | 0.498 | 0.602 | 0.498 | 0.603 | 0.499 | 0.603 | 0.498 | 0.602 | 0.498 |
| | | 0.99 | 6.738 | 5.870 | 6.666 | 5.806 | 6.651 | 5.794 | 6.738 | 5.870 | 6.666 | 5.806 | 6.630 | 5.775 | 6.738 | 5.870 | 6.666 | 5.806 | 6.609 | 5.757 |
| 0.7 | 1 | 0.7 | 0.414 | 0.187 | 0.399 | 0.179 | 0.396 | 0.177 | 0.414 | 0.187 | 0.399 | 0.179 | 0.392 | 0.175 | 0.414 | 0.187 | 0.399 | 0.179 | 0.388 | 0.173 |
| | | 0.8 | 0.606 | 0.295 | 0.572 | 0.274 | 0.566 | 0.270 | 0.606 | 0.295 | 0.572 | 0.274 | 0.557 | 0.265 | 0.606 | 0.295 | 0.572 | 0.274 | 0.549 | 0.260 |
| | | 0.9 | 1.155 | 0.578 | 1.023 | 0.487 | 1.002 | 0.473 | 1.155 | 0.578 | 1.023 | 0.487 | 0.973 | 0.453 | 1.155 | 0.578 | 1.023 | 0.487 | 0.946 | 0.436 |
| | | 0.99 | 12.070 | 6.477 | 5.119 | 1.933 | 4.612 | 1.713 | 12.070 | 6.477 | 5.119 | 1.933 | 4.004 | 1.467 | 12.070 | 6.477 | 5.119 | 1.933 | 3.528 | 1.286 |
| | 5 | 0.7 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.210 | 0.160 |
| | | 0.8 | 0.316 | 0.246 | 0.315 | 0.245 | 0.315 | 0.245 | 0.316 | 0.246 | 0.315 | 0.245 | 0.315 | 0.245 | 0.316 | 0.246 | 0.315 | 0.245 | 0.314 | 0.244 |
| | | 0.9 | 0.662 | 0.531 | 0.658 | 0.527 | 0.657 | 0.526 | 0.662 | 0.531 | 0.658 | 0.527 | 0.656 | 0.525 | 0.662 | 0.531 | 0.658 | 0.527 | 0.655 | 0.524 |
| | | 0.99 | 7.190 | 6.018 | 6.551 | 5.393 | 6.439 | 5.285 | 7.190 | 6.018 | 6.551 | 5.393 | 6.280 | 5.132 | 7.190 | 6.018 | 6.551 | 5.393 | 6.131 | 4.988 |
| | 10 | 0.7 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 |
| | | 0.8 | 0.295 | 0.249 | 0.295 | 0.249 | 0.295 | 0.248 | 0.295 | 0.249 | 0.295 | 0.249 | 0.295 | 0.248 | 0.295 | 0.249 | 0.295 | 0.249 | 0.295 | 0.248 |
| | | 0.9 | 0.603 | 0.499 | 0.602 | 0.498 | 0.602 | 0.498 | 0.603 | 0.499 | 0.602 | 0.498 | 0.602 | 0.497 | 0.603 | 0.499 | 0.602 | 0.498 | 0.601 | 0.497 |
| | | 0.99 | 6.738 | 5.870 | 6.572 | 5.724 | 6.539 | 5.696 | 6.738 | 5.870 | 6.572 | 5.724 | 6.492 | 5.654 | 6.738 | 5.870 | 6.572 | 5.724 | 6.445 | 5.613 |
| 0.9 | 1 | 0.7 | 0.163 | 0.173 | 0.414 | 0.187 | 0.395 | 0.177 | 0.414 | 0.187 | 0.395 | 0.177 | 0.387 | 0.172 | 0.414 | 0.187 | 0.395 | 0.177 | 0.382 | 0.170 |
| | | 0.8 | 0.247 | 0.262 | 0.606 | 0.295 | 0.564 | 0.268 | 0.606 | 0.295 | 0.564 | 0.268 | 0.546 | 0.258 | 0.606 | 0.295 | 0.564 | 0.268 | 0.536 | 0.252 |
| | | 0.9 | 0.422 | 0.446 | 1.155 | 0.578 | 0.993 | 0.467 | 1.155 | 0.578 | 0.993 | 0.467 | 0.935 | 0.429 | 1.155 | 0.578 | 0.993 | 0.467 | 0.905 | 0.411 |
| | | 0.99 | 1.329 | 1.395 | 12.070 | 6.477 | 4.422 | 1.634 | 12.070 | 6.477 | 4.422 | 1.634 | 3.354 | 1.222 | 12.070 | 6.477 | 4.422 | 1.634 | 2.915 | 1.068 |
| | 5 | 0.7 | 0.149 | 0.158 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.210 | 0.160 | 0.211 | 0.160 | 0.211 | 0.160 | 0.210 | 0.160 |
| | | 0.8 | 0.228 | 0.241 | 0.316 | 0.246 | 0.315 | 0.245 | 0.316 | 0.246 | 0.315 | 0.245 | 0.314 | 0.244 | 0.316 | 0.246 | 0.315 | 0.245 | 0.314 | 0.244 |
| | | 0.9 | 0.489 | 0.518 | 0.662 | 0.531 | 0.657 | 0.526 | 0.662 | 0.531 | 0.657 | 0.526 | 0.654 | 0.523 | 0.662 | 0.531 | 0.657 | 0.526 | 0.652 | 0.522 |
| | | 0.99 | 4.788 | 5.049 | 7.190 | 6.018 | 6.393 | 5.241 | 7.190 | 6.018 | 6.393 | 5.241 | 6.069 | 4.930 | 7.190 | 6.018 | 6.393 | 5.241 | 5.894 | 4.762 |
| | 10 | 0.7 | 0.155 | 0.163 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 | 0.199 | 0.168 |
| | | 0.8 | 0.229 | 0.243 | 0.295 | 0.249 | 0.295 | 0.248 | 0.295 | 0.249 | 0.295 | 0.248 | 0.294 | 0.248 | 0.295 | 0.249 | 0.295 | 0.248 | 0.294 | 0.248 |
| | | 0.9 | 0.455 | 0.479 | 0.603 | 0.499 | 0.602 | 0.498 | 0.603 | 0.499 | 0.602 | 0.498 | 0.601 | 0.497 | 0.603 | 0.499 | 0.602 | 0.498 | 0.601 | 0.497 |
| | | 0.99 | 5.162 | 5.516 | 6.738 | 5.870 | 6.526 | 5.684 | 6.738 | 5.870 | 6.526 | 5.684 | 6.425 | 5.596 | 6.738 | 5.870 | 6.526 | 5.684 | 6.366 | 5.544 |

Table 18: Estimated MSEs of OLS, MRE, MRE, MRT, and RTMME at $n = 100$ and $p = 7$ when there are two outliers.

| $k$ | Sigma | Rho | $d = 0.2$ | | | | | | $d = 0.5$ | | | | | | $d = 0.8$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ | $\hat{\alpha}$ | $\hat{\alpha}_M$ | $\hat{\alpha}(k)$ | $\hat{\alpha}_M(k)$ | $\hat{\alpha}_{MRT}$ | $\hat{\alpha}_{RTMME}$ |
| 0.3 | 1 | 0.7 | 0.835 | 0.192 | 0.827 | 0.188 | 0.825 | 0.188 | 0.835 | 0.192 | 0.827 | 0.188 | 0.823 | 0.187 | 0.835 | 0.192 | 0.827 | 0.188 | 0.820 | 0.186 |
| | | 0.8 | 1.220 | 0.302 | 1.200 | 0.292 | 1.196 | 0.290 | 1.220 | 0.302 | 1.200 | 0.292 | 1.191 | 0.288 | 1.220 | 0.302 | 1.200 | 0.292 | 1.185 | 0.285 |
| | | 0.9 | 2.304 | 0.589 | 2.220 | 0.546 | 2.204 | 0.538 | 2.304 | 0.589 | 2.220 | 0.546 | 2.181 | 0.527 | 2.304 | 0.589 | 2.220 | 0.546 | 2.158 | 0.516 |
| | | 0.99 | 23.201 | 6.580 | 15.780 | 3.202 | 14.892 | 2.929 | 23.201 | 6.580 | 15.780 | 3.202 | 13.730 | 2.604 | 23.201 | 6.580 | 15.780 | 3.202 | 12.725 | 2.348 |
| | 5 | 0.7 | 0.264 | 0.163 | 0.264 | 0.163 | 0.264 | 0.163 | 0.264 | 0.163 | 0.264 | 0.163 | 0.264 | 0.163 | 0.264 | 0.163 | 0.264 | 0.163 | 0.264 | 0.163 |
| | | 0.8 | 0.401 | 0.251 | 0.400 | 0.250 | 0.400 | 0.250 | 0.401 | 0.251 | 0.400 | 0.250 | 0.400 | 0.250 | 0.401 | 0.251 | 0.400 | 0.250 | 0.399 | 0.250 |
| | | 0.9 | 0.830 | 0.540 | 0.827 | 0.538 | 0.826 | 0.538 | 0.830 | 0.540 | 0.827 | 0.538 | 0.825 | 0.537 | 0.830 | 0.540 | 0.827 | 0.538 | 0.824 | 0.537 |
| | | 0.99 | 8.931 | 6.102 | 8.546 | 5.810 | 8.474 | 5.755 | 8.931 | 6.102 | 8.546 | 5.810 | 8.369 | 5.675 | 8.931 | 6.102 | 8.546 | 5.810 | 8.266 | 5.598 |
| | 10 | 0.7 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 |
| | | 0.8 | 0.463 | 0.265 | 0.463 | 0.265 | 0.463 | 0.265 | 0.463 | 0.265 | 0.463 | 0.265 | 0.463 | 0.265 | 0.463 | 0.265 | 0.463 | 0.265 | 0.463 | 0.265 |
| | | 0.9 | 0.949 | 0.537 | 0.948 | 0.537 | 0.948 | 0.537 | 0.949 | 0.537 | 0.948 | 0.537 | 0.948 | 0.537 | 0.949 | 0.537 | 0.948 | 0.537 | 0.948 | 0.537 |
| | | 0.99 | 10.099 | 6.192 | 10.000 | 6.126 | 9.980 | 6.113 | 10.099 | 6.192 | 10.000 | 6.126 | 9.951 | 6.094 | 10.099 | 6.192 | 10.000 | 6.126 | 9.921 | 6.075 |
| 0.7 | 1 | 0.7 | 0.835 | 0.192 | 0.816 | 0.184 | 0.813 | 0.182 | 0.835 | 0.192 | 0.816 | 0.184 | 0.807 | 0.180 | 0.835 | 0.192 | 0.816 | 0.184 | 0.802 | 0.179 |
| | | 0.8 | 1.220 | 0.302 | 1.175 | 0.281 | 1.167 | 0.277 | 1.220 | 0.302 | 1.175 | 0.281 | 1.155 | 0.272 | 1.220 | 0.302 | 1.175 | 0.281 | 1.143 | 0.267 |
| | | 0.9 | 2.304 | 0.589 | 2.120 | 0.499 | 2.088 | 0.485 | 2.304 | 0.589 | 2.120 | 0.499 | 2.042 | 0.466 | 2.304 | 0.589 | 2.120 | 0.499 | 1.998 | 0.450 |
| | | 0.99 | 23.201 | 6.580 | 11.231 | 2.004 | 10.155 | 1.779 | 23.201 | 6.580 | 11.231 | 2.004 | 8.836 | 1.526 | 23.201 | 6.580 | 11.231 | 2.004 | 7.779 | 1.339 |
| | 5 | 0.7 | 0.264 | 0.163 | 0.264 | 0.163 | 0.264 | 0.162 | 0.264 | 0.163 | 0.264 | 0.163 | 0.263 | 0.162 | 0.264 | 0.163 | 0.264 | 0.163 | 0.263 | 0.162 |
| | | 0.8 | 0.401 | 0.251 | 0.399 | 0.250 | 0.399 | 0.250 | 0.401 | 0.251 | 0.399 | 0.250 | 0.398 | 0.249 | 0.401 | 0.251 | 0.399 | 0.250 | 0.398 | 0.249 |
| | | 0.9 | 0.830 | 0.540 | 0.823 | 0.536 | 0.822 | 0.535 | 0.830 | 0.540 | 0.823 | 0.536 | 0.820 | 0.533 | 0.830 | 0.540 | 0.823 | 0.536 | 0.818 | 0.532 |
| | | 0.99 | 8.931 | 6.102 | 8.092 | 5.466 | 7.947 | 5.356 | 8.931 | 6.102 | 8.092 | 5.466 | 7.741 | 5.200 | 8.931 | 6.102 | 8.092 | 5.466 | 7.548 | 5.053 |
| | 10 | 0.7 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 |
| | | 0.8 | 0.463 | 0.265 | 0.462 | 0.265 | 0.462 | 0.265 | 0.463 | 0.265 | 0.462 | 0.265 | 0.462 | 0.265 | 0.463 | 0.265 | 0.462 | 0.265 | 0.462 | 0.265 |
| | | 0.9 | 0.949 | 0.537 | 0.948 | 0.536 | 0.947 | 0.536 | 0.949 | 0.537 | 0.948 | 0.536 | 0.947 | 0.536 | 0.949 | 0.537 | 0.948 | 0.536 | 0.946 | 0.536 |
| | | 0.99 | 10.099 | 6.192 | 9.870 | 6.041 | 9.826 | 6.012 | 10.099 | 6.192 | 9.870 | 6.041 | 9.760 | 5.969 | 10.099 | 6.192 | 9.870 | 6.041 | 9.695 | 5.926 |
| 0.9 | 1 | 0.7 | 0.835 | 0.192 | 0.811 | 0.182 | 0.807 | 0.180 | 0.835 | 0.192 | 0.811 | 0.182 | 0.800 | 0.178 | 0.835 | 0.192 | 0.811 | 0.182 | 0.794 | 0.176 |
| | | 0.8 | 1.220 | 0.302 | 1.163 | 0.276 | 1.153 | 0.271 | 1.220 | 0.302 | 1.163 | 0.276 | 1.138 | 0.265 | 1.220 | 0.302 | 1.163 | 0.276 | 1.123 | 0.260 |
| | | 0.9 | 2.304 | 0.589 | 2.074 | 0.480 | 2.035 | 0.464 | 2.304 | 0.589 | 2.074 | 0.480 | 1.980 | 0.443 | 2.304 | 0.589 | 2.074 | 0.480 | 1.929 | 0.424 |
| | | 0.99 | 23.201 | 6.580 | 9.746 | 1.698 | 8.670 | 1.496 | 23.201 | 6.580 | 9.746 | 1.698 | 7.389 | 1.273 | 23.201 | 6.580 | 9.746 | 1.698 | 6.394 | 1.113 |
| | 5 | 0.7 | 0.264 | 0.163 | 0.264 | 0.162 | 0.263 | 0.162 | 0.264 | 0.163 | 0.264 | 0.162 | 0.263 | 0.162 | 0.264 | 0.163 | 0.264 | 0.162 | 0.263 | 0.162 |
| | | 0.8 | 0.401 | 0.251 | 0.399 | 0.249 | 0.398 | 0.249 | 0.401 | 0.251 | 0.399 | 0.249 | 0.398 | 0.249 | 0.401 | 0.251 | 0.399 | 0.249 | 0.397 | 0.249 |
| | | 0.9 | 0.830 | 0.540 | 0.821 | 0.534 | 0.819 | 0.533 | 0.830 | 0.540 | 0.821 | 0.534 | 0.817 | 0.531 | 0.830 | 0.540 | 0.821 | 0.534 | 0.814 | 0.530 |
| | | 0.99 | 8.931 | 6.102 | 7.886 | 5.310 | 7.712 | 5.178 | 8.931 | 6.102 | 7.886 | 5.310 | 7.469 | 4.994 | 8.931 | 6.102 | 7.886 | 5.310 | 7.244 | 4.824 |
| | 10 | 0.7 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 | 0.311 | 0.178 |
| | | 0.8 | 0.463 | 0.265 | 0.462 | 0.265 | 0.462 | 0.265 | 0.463 | 0.265 | 0.462 | 0.265 | 0.462 | 0.265 | 0.463 | 0.265 | 0.462 | 0.265 | 0.462 | 0.265 |
| | | 0.9 | 0.949 | 0.537 | 0.947 | 0.536 | 0.947 | 0.536 | 0.949 | 0.537 | 0.947 | 0.536 | 0.946 | 0.536 | 0.949 | 0.537 | 0.947 | 0.536 | 0.946 | 0.535 |
| | | 0.99 | 10.099 | 6.192 | 9.807 | 6.000 | 9.750 | 5.963 | 10.099 | 6.192 | 9.807 | 6.000 | 9.667 | 5.908 | 10.099 | 6.192 | 9.807 | 6.000 | 9.585 | 5.855 |

Table 19: Estimated MSE values of the OLSE, ME, RE, RME, MRT, and RTMME in Hussein data set.

| Coefficients | $\alpha$ | $\widehat{\alpha}_M$ | $\widehat{\alpha}(k)$ | $\widehat{\alpha}_M(k_{\text{HM}})$ | $\widehat{\alpha}_{\text{MRT}}(k_{\text{HMR}}, d)$ | $\widehat{\alpha}_{\text{RTMME}}((\widetilde{k}_{\text{HMR}}, d)$ |
|---|---|---|---|---|---|---|
| $\beta_0$ | 208.874 | 173.331 | 178.6184 | 155.4684 | 178.6184 | 155.4684 |
| $\beta_1$ | −2.0171 | −1.8846 | −2.0167 | −1.8844 | −2.0167 | −1.8844 |
| $\beta_2$ | 1.51533 | 1.6340 | 1.5153 | 1.6339 | 1.5153 | 1.6339 |
| $\beta_3$ | −1.3143 | −1.3527 | −1.3143 | −1.3527 | −1.3143 | −1.3527 |
| MSE | 20875.62 | 9751.86 | 2269.33 | 1952.60 | 2269.33 | 629.58 |

$k = 1.9136$, $k_{\text{HM}} = 1.2980$, $k_{\text{HMR}} = 0.9813$, $k_{\text{HMR}} = 0.6656$, and $d = 0.95$.

## 6. Numerical Example

Hussein data were originally adopted by Hussein and Zari [18] and later by Lukman et al. [19, 20]. The data contain 31 observations and three explanatory variables. The variance inflation factor for the three explanatory variables is greater than 10 (VIF> 10), which indicates multicollinearity. Lukman et al. [20] identified the following observations as outliers in the $y$-direction: 12, 14, 15, 16, 30, and 31. Hence, this data set suffers both problems considered in this study. Table 19 shows that the estimated MSE value of the new estimator, RTMME, is smaller compared to the ridge, M-ridge, and MRT estimator.

The theoretical results in this study are validated through the Hussein data as follows:

(i) $\sum_{i=1}^{p} \Omega_{ii} = 883.55 < \sum_{i=1}^{p} \sigma^2 \lambda_i^{-1} = 1850.482$; thus, $\text{MSE}(\widehat{\alpha}_{\text{RTMME}}(k, d)) < \text{MSE}(\widehat{\alpha}_{\text{MRT}}(k, d))$ for every $k > 0$. By Theorem 1, this implies that the RTMME outperforms the MRT estimator.

(ii) From Theorem 2, $k_{1i}$ is calculated as [0.2470, 0.0520, 0.0519, 0.0002]. For $k > 1.2980$ $(k > k_{1i})$, $\text{MSE}(\widehat{\alpha}_{\text{RTMME}}(k, d)) < \text{MSE}(\widehat{\alpha}_M(k))$.

(iii) The necessary condition from Theorem 3 is also proven from $\sum_{i=1}^{p} \Omega_{ii} < \sum_{i=1}^{p} \sum_{i=1}^{p} (k(1+d)\alpha_i^2)/(2\lambda_i + k(1+d)) = 3026.59$ $\Rightarrow \text{MSE}(\widehat{\alpha}_{\text{RTMME}}(k, d)) < \text{MSE}(\widehat{\alpha}_M)$.

## 7. Conclusion

We proposed a two-parameter ridge-type modified M-estimator to jointly handle the problem of multicollinearity and outliers in a linear regression model. Theoretically, the new estimator outperforms the existing estimators under certain conditions. The results of the simulation study and numerical example agree with the theoretical findings. A right choice of $k$ and $d$ also produces better estimates using the proposed estimator. Thus, in the presence of multicollinearity and outliers, this estimator can effectively replace the following estimators: the ordinary least squares estimator, the M-estimator, the ridge estimator, the M-ridge estimator, and the modified ridge-type estimator.

## Data Availability

The data used to support the findings of this study are available in page 7 of [19].

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## References


[1] D. N. Gujarati, *Basic Econometrics*, McGraw-Hill, New York, NY, USA, 1995.

[2] A. F. Lukman, *Classification-Based Ridge*, Lambert Academic Publishing, Beau-Bassin, Mauritius, 2018.

[3] A. F. Lukman and K. Ayinde, "Review and Classifications of the ridge parameter estimation techniques," *Hacetteppe Journal of Mathematics and Statistic*, vol. 46, no. 5, pp. 953–967, 2017.

[4] A. E. Hoerl and R. W. Kennard, "Ridge regression: biased estimation for nonorthogonal problems," *Technometrics*, vol. 12, no. 1, pp. 55–67, 1970.

[5] K. Liu, "A new class of biased estimate in linear regression," *Communication in Statistics-Theory and Methods*, vol. 22, pp. 393–402, 1993.

[6] C. Stein, "Inadmissibility of the usual estimator for mean of multivariate normal distribution," in *Proceedings of the Third Berkley Symposium on Mathematical and Statistics Probability*, J. Neyman, Ed., vol. 1, pp. 197–206, University of California Press, Berkeley, CA, USA, 1956.

[7] W. F. Massy, "Principal components regression in exploratory statistical research," *Journal of the American Statistical Association*, vol. 60, no. 309, pp. 234–256, 1965.

[8] B. F. Swindel, "Good ridge estimators based on prior information," *Communications in Statistics - Theory and Methods*, vol. 5, no. 11, pp. 1065–1075, 1976.

[9] P. J. Huber, *Robust Statistics*, Wiley, New York, NY, USA, 1981.

[10] M. J. Silvapulle, "Robust ridge regression based on an M-estimator," *Australian Journal of Statistics*, vol. 33, no. 3, pp. 319–333, 1991.

[11] K. Ayinde, A. F. Lukman, and O. T. Arowolo, "Robust regression diagnostics of influential observations in linear regression model," *Open Journal of Statistics*, vol. 5, pp. 1–11, 2015.

[12] A. V. Dorugade, "Adjusted ridge estimator and comparison with Kibria's method in linear regression," *Journal of the Association of Arab Universities for Basic and Applied Sciences*, vol. 21, no. 1, pp. 96–102, 2016.

[13] A. F. Lukman, K. Ayinde, S. Binuomote, and O. A. Clement, "Modified ridge-type estimator to combat multicollinearity: applications to chemical data," *Journal of Chemometrics*, vol. 33, no. 5, p. e3125, 2019.

[14] F. R. Hampel, E. M. Ronchetti, P. J. Rousseeuw, and W. A. Stahel, *Robust Statistics. The Approach Based on Influence Function*, Wiley, New York, NY, USA, 1986.

[15] B. M. G. Kibria, "Performance of some new ridge regression estimators," *Communication in Statistics-Simulation and Computation*, vol. 32, no. 1, pp. 419–435, 2003.

[16] G. C. McDonald and D. I. Galarneau, "A Monte Carlo evaluation of some ridge-type estimators," *Journal of the American Statistical Association*, vol. 70, no. 350, pp. 407–416, 1975.

[17] A. F. Lukman, K. Ayinde, and A. S. Ajiboye, "Monte Carlo study of some classification-based ridge parameter estimators," *Journal of Modern Applied Statistical Methods*, vol. 16, no. 1, pp. 428–451, 2017.

[18] Y. Hussein and M. Zari, "Generalized two stage ridge regression estimator TR for multicollinearity and autocorrelated errors," *Canadian Journal on Science and Engineering Mathematics*, vol. 3, no. 3, pp. 79–85, 2012.

[19] A. F. Lukman, K. Ayinde, S. K. Sek, and E. Adewuyi, "A modified new two-parameter estimator in a linear regression model," *Modelling and Simulation in Engineering*, vol. 2019, Article ID 6342702, 10 pages, 2019.

[20] A. F. Lukman, O. I. Osowole, and K. Ayinde, "Two stage robust ridge method in a linear regression model," *Journal of Modern Applied Statistical Methods*, vol. 14, no. 2, pp. 53–67, 2015.