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Research Article

Closed-Form Expression for the Bit Error Probability of the M-QAM for a Channel Subjected to Impulsive Noise and Nakagami Fading

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In this paper, new exact expressions for evaluating the bit error probability (BEP) of the M-ary quadrature amplitude modulation scheme (M-QAM) for a channel model characterized by double gated additive white Gaussian noise (G^2AWGN) and Nakagami fading are presented. The derivation of the BEP is performed considering a method in which the multiplicative fading is transformed in an additive noise R obtained by dividing the received signal by the estimated fading envelope. In addition, the probability density function (PDF) and cumulative density function (CDF) of the random variable R that represents the aforementioned noise are obtained for the G^2AWGN. The BEP curves as a function of the signal to permanent noise ratio for several values of the signal to impulsive noise ratio, modulation order, and fading parameters are also presented.

1. Introduction

Modern factories use wireless sensor networks (WSNs) to monitor and control industrial processes [1, 2]. This technology has advantages over wired networks that include greater flexibility, low cost, and facility of installation and maintenance, avoiding installation tests and cable replacement needs [3]. However, a WSN presents several challenges, such as noise and interference in the range of spectrum considered for wireless communication. In addition, industrial environments include several objects and machines, usually made of very reflective metal materials [4, 5]. Therefore, the fading phenomenon caused by the reflection, refraction, diffraction, and scattering, which affects the received signal, has the potential to worsen the performance of the digital communications link in such environment [6, 7].

In industrial environments, signal sources for 2.4 GHz industrial applications are numerous, such as welding equipment, microwave ovens and wireless communication networks, such as WiFi and Bluetooth [6, 8]. Other sources of interference usually found in such environments are electric motors and frequency converters. In addition, many industries also present features that make the wireless channel nonstationary for long periods of time, causing abrupt changes in their characteristics over time [9, 10]. For short periods of time, the wireless channel can be modeled as Wide Sense Stationary (WSS), despite the movements of the transmitter and/or receiver [11].

Another challenge found in industrial environments is the presence of impulsive noise [8], whose effects can affect the performance of the communication links [12]. In addition, its nonstationary behavior makes it more difficult to be analyzed as compared to white Gaussian noise [13]. Studies on impulsive noise began with Middleton [14] in 1951, and since then works have been developed, such as the studies involving double gated additive white Gaussian noise (G^2AWGN) binary [15, 16]. The noise model G^2AWGN mathematically characterizes the occurrence of noisy bursts and pulses. For other noise models described in the literature, such as the one of Ghosh [17] and symmetric alpha stable [18], for example, the corresponding probability density function (PDF) is a Gaussian mixture, which characterizes noise from multiple sources. In addition, papers have also emphasized the relevance of mathematical analysis and characterization of impulsive noise in 5G systems and wireless sensor networks. Some works on 5G systems are concerned with the effects of this type of noise on millimeter communication systems. In [19], for example, it is shown that the performance of those systems can be considerably affected by the impulsive noise characterized by Gaussian mixtures, such as the type of noise considered in the present paper.

In this article, novel exact expressions for evaluating the bit error probability (BEP), P_e, of M-QAM signals in a channel model composed of G^2AWGN noise and Nakagami fading are presented. Several papers use the Nakagami-m distribution to characterize the effects of fading in industrial environments [11, 20, 21]. That distribution is used to characterize mathematically strong or weak fading conditions and include, as special cases, the Rayleigh, Rice, and Hoyt distributions. In addition, the Nakagami-m distribution is simple, easy to manipulate, and capable of modeling different environments subjected to the effects of fading. In this way, the channel model adopted in the present work, composed of impulsive noise and Nakagami-m fading, may be appropriate to model channels in scenarios such as [1, 22, 23] domestic environments, industrial parks, agricultural fields, laboratories, oil platforms, shopping malls, energy substations, and outside environments of large cities. In addition, this channel model can also be used to characterize wireless communications channels in which there is traffic of unmanned aerial vehicles (UAVs), such as drones, which has increasingly attracted the attention of various researchers due to its wide range of applications, including communications, photography, agriculture, surveillance, and numerous public services [24]. In that channel model, the noise is considered to be AWGN, coming from mobile telephone networks and satellite signals, for example. By its turn, in that scenario, fading is modeled mathematically by the Rice distribution, since, usually, the aircraft has a line of sight with the land stations with which they communicate [25]. The phenomenon of fading occurs in that channel model because the environment in which aircraft fly over is composed of several obstacles, both natural and artificial, that are randomly located in the ground, such as rivers, mountains, and buildings [25]. It is worth mentioning that the AWGN noise model and the Rice fading model are encompassed, respectively, by the G^2AWGN and Nakagami-m models, addressed in our manuscript.

For determining the new BEP expressions, the method presented in [26] is considered for facilitating the mathematical development. In this method, the division of the received signal by the fading envelope estimate, in a signaling interval T_s, is performed, transforming the communication channel model subjected to multiplicative fading in a channel model subjected to an additive noise represented by the random variable $R(t)$, characterized by the ratio of two random variables. In the present work, one random variable is characterized by the G^2AWGN noise distribution and the other characterized by the Nakagami-m distribution. The method shown in [26] can also be applied to reduce the computational complexity of signal detection in the presence of fading.

In [26], the ratio of two random variables was considered to determine exact expressions for the BEP of quadrature phase shift keying (QPSK) signals subjected to multiplicative effect of Rayleigh fading and additive white Gaussian noise (AWGN). Other articles have considered this alternative method in the BEP calculation. In [27], the same approach was taken into account to determine exact expressions for BEP of R-QAM signals subjected to Rayleigh fading as well as AWGN. In [28], expressions for BEP of M-PAM and $I \times J$-QAM signals subjected to Nakagami fading and AWGN were determined from this alternative method.

The remaining of this article is organized as follows. In Section 2, works related to the calculation of the BEP with the communication channel subject to noise and fading are presented. In Section 3, impulsive noise models described in the literature are presented. Sections 4 and 5 are devoted to the mathematical model of the received signal, as well as the G^2AWGN noise model and its PDF. In Section 6, expressions for PDF of the additive noise $R(t)$ are presented, obtained by the ratio of the variables that characterize the impulsive noise and the Nakagami fading envelope. Section 7 provides the BEP for M-QAM scheme in the channel model in which the G^2AWGN noise and Nakagami fading act together. In Section 8, BEP curves as a function of the signal to permanent noise ratio for different parameters are presented and Section 9 presents the conclusions.

2. Related Works

In the literature, recent works related to the computation of the BEP with the communications channel under the effects of noise and fading are presented. In [29], for example, the performance of wireless networks with multihop routing, with the channel subject to the generalized fading α-μ and the additive white generalized Gaussian noise (AWGGN), is evaluated in terms of the average BEP, considering the modulation scheme M-QAM. The expression presented for the average BEP is exact, new, closed-form, written in terms of the H Fox function. Some numerical results, with the purpose of analyzing the influence of the parameters of the mathematical models of fading and noise adopted in the characterization of the communications channel, are also presented in [29]. In addition, closed-form expressions for the average BEP of the M-QAM scheme, considering the communications channel subject to the effects of the Nakagami-m fading or Rayleigh fading and AWGN noise, are also obtained as special cases. All results shown in [29] are validated by simulations performed with the Monte Carlo method.

In [30], a mathematical analysis of the combined effects of the Nakagami-m fading and the gated additive white Gaussian noise or double gated additive white Gaussian noise in a communication system is presented. In addition, new and exact analytical expressions, corroborated by simulations, for the average BEP of the modulation scheme M-QAM, written in terms of the Gauss hypergeometric function, are also presented. Average BEP curves as a function of the signal to permanent noise ratio for different values of the modulation order M, fading intensity, and signal to impulsive noise ratio are also shown in [30].

Mathur *et al.* [31] consider a cooperative communication system in which the data are initially transmitted by a wireless communication link and then transmitted via communication over an electric network. In the modeling of the communication link, fading is characterized mathematically by the Nakagami-m distribution and the noise considered is the AWGN. In the power line communication (PLC) link, the fading is modeled by the Log-normal distribution and the noise by a Gaussian mixture. A performance analysis of the M-ary frequency shift keying modulation (M-FSK), in terms of BEP, is performed in [31]. From this analysis, a new and closed-form approximated expression is obtained for the average BEP of the cooperative communication system considered. BEP curves as a function of the signal-to-noise ratio (SNR), theoretical as well as obtained by simulation with the Monte Carlo method, in order to corroborate the theoretical results obtained, are also presented in [31].

In the papers of Badarneh and Almehmadi [29], Queiroz *et al.* [30], and Mathur *et al.* [31], average BEP expressions are calculated from a classical approach, which consists of weighing the bit error probability conditioned to fading by the probability density function of fading. Differently from those articles, the present work is based on the results presented by Lopes and Alencar in [26], in which the BEP is determined from an alternative method, which presents greater mathematical simplicity as compared to the classical approach.

3. Impulsive Noise Models

In the literature, several models of impulsive noise are described, among them those of Middleton [14], symmetric alpha stable [18], Ghosh [17], and the gated and double gated [15, 16, 30], for example.

Middleton, in [32–34], presents physical-statistical models of electromagnetic interference to represent impulsive noise. These models are classified taking into account the noise bandwidth compared to the operating bandwidth of the receiver and are defined in classes such as A, B, and C. Class A noise is characterized by a narrower spectrum than the operational bandwidth of the receiver, while Class B noise refers to impulsive noise characterized by a broader one. Although it is possible to accurately model broadband impulse noise, its practical applications are limited because of the complex shape of its PDF, which has five parameters and an inflection point determined empirically. In turn, Class C noise corresponds to the linear combination of noise Classes A and B.

Another model of impulsive noise is the symmetric alpha stable, denoted by $S\alpha S$. It is mainly used for mathematical characterization of impulsive noise in wireless communications systems. If η is a random variable with symmetric alpha stable distribution, its characteristic function is given by [18]

$$\varphi_\eta(\omega) = e^{j\delta\omega - |b\omega|^\alpha (1 - jc\,\mathrm{sgn}(\omega)\Phi(\omega,\alpha))}, \quad (1)$$

in which

$$\Phi(\omega,\alpha) = \begin{cases} \tan\left(\pi\dfrac{\alpha}{2}\right), & \alpha \neq 1 \\ -\dfrac{2}{\pi}\log|\omega|, & \alpha = 1, \end{cases} \quad (2)$$

$\mathrm{sgn}(x) = 2\mathrm{u}(x) - 1$, and $\mathrm{u}(x)$ represents the unit step function.

The distribution of the symmetric alpha stable model is characterized by four parameters. The exponent α belongs to the range $(0; 2]$ and determines the format of the PDF, the parameter c determines the asymmetry of the PDF, δ is a location parameter, and b^α is the dispersion. This distribution is said to be symmetric if c and δ are equal to zero.

There is no exact mathematical expression for the PDF of the symmetric alpha stable distribution, except in cases where $\alpha = 2$ (corresponding to the Gaussian distribution) and $\alpha = 1$ (corresponding to the Cauchy distribution). The PDF expression of η can be written, from (1), as [18]

$$f_\eta(x) = \frac{1}{\pi}\mathrm{Re}\left[\int_0^\infty e^{-j\omega(x-\delta)} e^{-(b\omega)^\alpha(1-jc\Phi(\omega,\alpha))} d\omega\right] \quad (3)$$

and in the case where δ and c are null, it can be written as in [18], for the modeling of $S\alpha S$ impulsive noise in a communication link under Rayleigh fading, so that

$$f_\eta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-(b\omega)^\alpha} e^{-j\omega x} d\omega. \quad (4)$$

Another model of impulsive noise is that of Ghosh [17]. In his paper, Ghosh demonstrated the equivalence between the Bernoulli-Gauss model in the discrete time domain and the continuous time Poisson model. He assumes that the noise in the communications systems, in the k-th discrete instant of time, can be written as

$$\eta(k) = \eta_g(k) + b(k)\eta_i(k), \quad (5)$$

where $\eta(k)$ represents the total noise, and the terms $\eta_g(k)$ and $\eta_i(k)$ are characterized, respectively, by random AWGN processes of variances σ_g^2 and σ_i^2, indexed by the discrete time k. In that model, the impulsive noise component $b(k)\eta_i(k)$ is seen as a product of a Bernoulli process $b(k)$ of probability p and a complex Gaussian process.

In [17], a characteristic function is also presented for the process $\eta(k)$ in (5), written as

$$\varphi_{\eta(k)}(\omega_1,\omega_2) = e^{-(\sigma_g^2/2)(\omega_1^2+\omega_2^2)}\left[(1-p) + pe^{-(\sigma_i^2/2)(\omega_1^2+\omega_2^2)}\right], \quad (6)$$

as well as its PDF,

$$f_{\eta(k)}(\eta) = (1-p)\, f_{\eta_g(k)}(\eta) + p f_{\eta_g(k)}(\eta) * f_{\eta_i(k)}(\eta), \quad (7)$$

where $*$ represents the convolution operator and $f_{\eta_g(k)}(\eta)$ and $f_{\eta_i(k)}(\eta)$ are the PDFs of noise $\eta_g(k)$ and noise $\eta_i(k)$, respectively.

Another model of impulsive noise is the gated or double gated, whose studies are described in [15, 16]. In [16], a general model of binary and multilevel double gated impulsive noise is presented, denoted by G^2AWGN, able to characterize random occurrences of noisy bursts and pulses. This model includes, as special cases, other types of simpler noise that, depending on the application, can be used. In addition, the G^2AWGN model is able to characterize mathematically impulsive noise in indoor and outdoor environments and its PDF is given by a Gaussian mixture, which characterizes noise composed of multiple noisy sources [30, 35]. The PDF of this model is presented in (13).

4. Mathematical Model for the Communication System

Consider that a received signal model can be written as a transmitted signal component affected by fading added to a random variable representing noise. Mathematically, one has

$$Y(t) = \gamma X(t) + \eta(t), \quad (8)$$

in which $X(t)$ represents the transmitted signal, $\eta(t)$ is the G^2AWGN noise, $Y(t)$ is the received signal, and γ represents the Nakagami fading [36]. The fading is considered slow and nonselective in frequency, implying that the multiplicative parameter γ can be considered constant during a signaling interval.

The received signal $Y(t)$ can be seen as a component $X(t)$ subjected to an additive noise $R(t)$, by dividing the received signal $Y(t)$ by γ. Mathematically,

$$\frac{Y(t)}{\gamma} = X(t) + \frac{\eta(t)}{\gamma} = X(t) + R(t). \quad (9)$$

The additive noise $R(t)$ in (9) can be written, in instant t, as a random variable defined as the ratio of the random variable $\eta(t)$ and the random variable of fading γ [26, 37].

5. Mathematical Model for the Noise

The mathematical model of the noise G^2AWGN, denoted by $\eta(t)$, can be written as [15]

$$\eta(t) = \eta_g(t) + C(t)\,\eta_i(t), \quad (10)$$

in which $\eta_i(t)$ represents a white Gaussian random process with variance σ_i^2, $C(t)$ is a signal that models the occurrence of noise $\eta_i(t)$, represented by a continuous time and discrete Bernoulli random process, the term $\eta_g(t)$ is the background Gaussian noise that acts permanently on the system and has zero mean and variance σ_g^2, and the product $C(t)\eta_i(t)$ characterizes the noise $\eta_i(t)$ gated by the $C(t)$ process.

In the double gated impulsive noise model, the signal $C(t)$ is written as $C(t) = C_1(t)C_2(t)$, in which the auxiliary continuous time random processes $C_1(t)$ and $C_2(t)$ take values into a discrete set $\{0,1\}$ and determine the occurrence of bursts and pulses, influencing the duration and instants of occurrence of bursts and pulses, respectively. The process $C_1(t)$ is represented by

$$C_1(t) = \sum_{k=-\infty}^{\infty} m_k P_{R_1}(t - kT_1), \quad (11)$$

in which m_k is the k-th bit of the alphabet $\{0,1\}$ with probability distribution $p(m_k = 1) = p_1$ and $p(m_k = 0) = 1 - p_1$. The pulse $P_{R_1}(t)$ assumes unit amplitude at $0 \le t \le \beta T_1$, with β taking values in $[0,1]$. The signal $C_2(t)$ also assumes values zero and one in a random manner and can be written as

$$C_2(t) = \sum_{l=-\infty}^{\infty} m_l P_{R_2}(t - lT_2), \quad (12)$$

in which m_l is the l-th bit of the alphabet $\{0,1\}$ with probability distribution of $p(m_l = 1) = p_2$ and $p(m_l = 0) = 1 - p_2$. The pulse $P_{R_2}(t)$ assumes unit amplitude at $0 \le t \le \alpha_p T_2$, with α_p taking values in $[0,1]$. The PDF of the noise for this model, $f_{\eta(t)}(\eta)$, can be written as [16]

$$f_{\eta(t)}(\eta) = \frac{\alpha_p \beta p_1 p_2}{\sqrt{2\pi(\sigma_g^2 + \sigma_i^2)}} e^{-\eta^2/2(\sigma_g^2+\sigma_i^2)} + \frac{(1 - \alpha_p \beta p_1 p_2)}{\sqrt{2\pi\sigma_g^2}} e^{-\eta^2/2\sigma_g^2}. \quad (13)$$

6. Distribution of Noise Random Variable $R(t)$

The additive noise $R(t)$, expressed by

$$R(t) = \frac{\eta(t)}{\gamma}, \quad (14)$$

in which $\eta(t)$ denotes the noise G^2AWGN and γ the fading envelope, is modeled as the ratio of two random variables, one characterized by a Gaussian mixture and the other characterized by the Nakagami distribution. The PDF of the Nakagami distribution is given by [38]

$$f_\gamma(\gamma) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m \gamma^{2m-1} e^{-m\gamma^2/\Omega} \mathrm{u}(\gamma), \quad (15)$$

in which m is the shape factor of the Nakagami distribution. Thus, signals with weaker fading can be fitted to the Nakagami model for which the value of m is greater than 1.0, which corresponds to Rayleigh fading. The parameter Ω is the average power of the transmitted signal, $\Gamma(\cdot)$ is the Gamma function, and $\mathrm{u}(\cdot)$ represents the unit step function.

The PDF of the additive noise $R(t)$, defined as the ratio of two random variables, is given by [39]

$$f_{R(t)}(r) = \int_{-\infty}^{\infty} |\gamma| \, f_{\eta(t),\gamma}(r\gamma, \gamma)\, d\gamma, \quad (16)$$

in which $f_{\eta(t),\gamma}(r\gamma,\gamma)$ is the joint PDF of $\eta(t)$ and γ. Considering that $\eta(t)$ and γ are independent random variables, one has

$$f_{\eta(t),\gamma}(r\gamma,\gamma) = f_{\eta(t)}(r\gamma)\, f_\gamma(\gamma) = \frac{2\alpha_p \beta p_1 p_2}{\Gamma(m)\sqrt{2\pi(\sigma_g^2+\sigma_i^2)}} \left(\frac{m}{\Omega}\right)^m \cdot \gamma^{2m-1} e^{-r^2\gamma^2/2(\sigma_g^2+\sigma_i^2) - m\gamma^2/\Omega} \mathrm{u}(\gamma)$$

$$+\frac{2\left(1-\alpha_p\beta p_1 p_2\right)}{\Gamma(m)\sqrt{2\pi\sigma_g^2}}\left(\frac{m}{\Omega}\right)^m \cdot \gamma^{2m-1}e^{-r^2\gamma^2/2\sigma_g^2-m\gamma^2/\Omega}\mathrm{u}(\gamma). \tag{17}$$

Thus, the PDF of R can be rewritten as

$$\begin{aligned} f_{R(t)}(r) &= \frac{\alpha_p\beta p_1 p_2 m^m}{\sqrt{\pi\left(N_{\eta_g}+N_{\eta_i}\right)}\Omega^m} \cdot \frac{\Gamma(m+1/2)}{\Gamma(m)\left[r^2/\left(N_{\eta_g}+N_{\eta_i}\right)+m/\Omega\right]^{m+1/2}} \\ &+ \frac{\left(1-\alpha_p\beta p_1 p_2\right)m^m}{\sqrt{\pi N_{\eta_g}}\Omega^m} \cdot \frac{\Gamma(m+1/2)}{\Gamma(m)\left[r^2/N_{\eta_g}+m/\Omega\right]^{m+1/2}}, \end{aligned} \tag{18}$$

where $N_{\eta_g}=2\sigma_g^2$ and $N_{\eta_i}=2\sigma_i^2$.

The CDF of the noise $R(t)$, $F_{R(t)}(r)$, is obtained by calculating

$$\begin{aligned} F_{R(t)}(r) &= \int_{-\infty}^{r} f_{R(t)}(r)\,dr = \frac{\alpha_p\beta p_1 p_2 m^m}{\sqrt{\pi}\Omega^m} \cdot \frac{\Gamma(m+1/2)}{\Gamma(m)}\int_{-\infty}^{r/\sqrt{N_{\eta_g}+N_{\eta_i}}}\left(x^2+\frac{m}{\Omega}\right)^{-(m+1/2)}dx \\ &+ \frac{\left(1-\alpha_p\beta p_1 p_2\right)m^m}{\sqrt{\pi}\Omega^m}\frac{\Gamma(m+1/2)}{\Gamma(m)} \cdot \int_{-\infty}^{r/\sqrt{N_{\eta_g}}}\left(x^2+\frac{m}{\Omega}\right)^{-(m+1/2)}dx. \end{aligned} \tag{19}$$

Since

$$\int\left(x^2+a\right)^{-(2m+1)/2}dx = x\cdot {}_2\mathrm{F}_1\left(\frac{1}{2},m+\frac{1}{2};\frac{3}{2};-\frac{x^2}{a}\right)a^{-(m+1/2)}, \tag{20}$$

in which ${}_2\mathrm{F}_1(a,b;c;x)$ is the Gaussian hypergeometric function, it follows that

$$\int\left(x^2+\frac{m}{\Omega}\right)^{-(2m+1)/2}dx = \frac{x\cdot {}_2\mathrm{F}_1\left(1/2,m+1/2;3/2;-\Omega x^2/m\right)}{(m/\Omega)^{m+1/2}}. \tag{21}$$

Evaluating the limit of (21) when x tends to minus infinity, one has

$$\lim_{x\to-\infty}\frac{x\cdot {}_2\mathrm{F}_1\left(1/2,m+1/2;3/2;-\Omega x^2/m\right)}{(m/\Omega)^{m+1/2}} = -\frac{\Omega^m\Gamma(3/2)\Gamma(m)}{m^m\Gamma(m+1/2)}. \tag{22}$$

Thus, $F_R(t)(r)$ can be written as

$$\begin{aligned} F_{R(t)}(r) &= \alpha_p\beta p_1 p_2\left\{\frac{\Gamma(m+1/2)}{\sqrt{\pi}\Gamma(m)}\frac{r}{\sqrt{N_{\eta_g}+N_{\eta_i}}}\left(\frac{m}{\Omega}\right)^{-1/2} \times {}_2\mathrm{F}_1\left(\frac{1}{2},m+\frac{1}{2};\frac{3}{2};-\frac{\Omega}{m}\frac{r^2}{\left(N_{\eta_g}+N_{\eta_i}\right)}\right)+\frac{1}{2}\right\} \\ &+ \left(1-\alpha_p\beta p_1 p_2\right)\left\{\frac{\Gamma(m+1/2)}{\sqrt{\pi}\Gamma(m)}\frac{r}{\sqrt{N_{\eta_g}}}\left(\frac{m}{\Omega}\right)^{-1/2} \times {}_2\mathrm{F}_1\left(\frac{1}{2},m+\frac{1}{2};\frac{3}{2};-\frac{\Omega x^2}{m}\frac{r^2}{N_{\eta_g}}\right)+\frac{1}{2}\right\}. \end{aligned} \tag{23}$$

7. BEP of M-QAM Scheme on a Channel with G^2AWGN Noise and Nakagami Fading

In this section, the expressions presented by Cho and Yoon [40] are used to obtain new exact expressions for the BEP of a M-QAM scheme in a channel subjected to G^2AWGN noise and Nakagami fading. Cho and Yoon proposed an exact expression for computing the BEP, P_e, of the QAM scheme with arbitrary constellation size, considering a channel with AWGN noise. This expression is given by [40]

$$P_e = \frac{1}{\log_2\sqrt{M}}\sum_{k=1}^{\log_2\sqrt{M}}P_e(k), \tag{24}$$

with $P_e(k)$ given by

$$P_e(k) = \frac{1}{\sqrt{M}}\sum_{i=0}^{(1-2^{-k})\sqrt{M}-1}\left\{w(i,k,M) \times \operatorname{erfc}\left((2i+1)\sqrt{\frac{3\log_2 M\cdot E_b}{2(M-1)N_0}}\right)\right\}, \tag{25}$$

and weights $w(i,k,M)$ defined as

$$w(i,k,M) = (-1)^{\lfloor (i\cdot 2^{k-1})/\sqrt{M}\rfloor}\cdot\left(2^{k-1}-\left\lfloor\frac{i\cdot 2^{k-1}}{\sqrt{M}}+\frac{1}{2}\right\rfloor\right), \tag{26}$$

in which E_b/N_0 denotes the signal-to-noise ratio per bit, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x, and erfc(·) denotes the complementary error function. Note that (25) is written in terms of a weighted sum of functions erfc(·). The term erfc(·) corresponds to twice the probability that the additive noise exceeds $(2i+1)\sqrt{3\log_2 M \cdot E_b/2(M-1)}$ and the weights $w(i,k,M)$ include the effect, in P_e, of the position of the k-th bit in a symbol with $\log_2 M$ bits.

Under the channel model subjected to noise G^2AWGN and Nakagami fading, twice the probability of the additive noise exceeding $(2i+1)\sqrt{3\log_2 M \cdot E_b/(M-1)}$ can be written as

$$\operatorname{erfc}\left(\sqrt{\frac{a(i,M)E_b}{2}}\right) = 2P\left(r \geq \sqrt{a(i,M)E_b}\right) = 2 - 2F_R\left(\sqrt{a(i,M)E_b}\right), \quad (27)$$

in which

$$a(i,M) = \frac{3(2i+1)^2 \log_2 M}{M-1}. \quad (28)$$

Using the result obtained for the CDF presented in (23), we obtain the expression for the BEP P_e of the M-QAM scheme subjected to noise G^2AWGN and the Nakagami fading, as shown in (24), with $P_e(k)$ written as

$$\begin{aligned} P_e(k) = {} & \frac{1}{\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} w(i,k,M) \times \Bigg\{ 1 \\ & - \frac{2\Gamma(m+1/2)\sqrt{\Omega}}{\sqrt{m\pi}\Gamma(m)} \\ & \cdot \alpha_p \beta p_1 p_2 \left(a(i,M)\frac{\delta_g \delta_i}{\delta_g + \delta_i}\right)^{1/2} \\ & \times {}_2F_1\left(\frac{1}{2}, m+\frac{1}{2}; \frac{3}{2}; -\frac{\Omega a(i,M)\delta_g\delta_i}{m(\delta_g+\delta_i)}\right) \\ & - \frac{2\Gamma(m+1/2)\sqrt{\Omega}}{\sqrt{m\pi}\Gamma(m)}\left(1-\alpha_p\beta p_1 p_2\right) \\ & \cdot \left(a(i,M)\delta_g\right)^{1/2} \\ & \times {}_2F_1\left(\frac{1}{2}, m+\frac{1}{2}; \frac{3}{2}; -\frac{\Omega a(i,M)}{m}\delta_g\right)\Bigg\}, \end{aligned} \quad (29)$$

in which the signal to permanent noise ratio, $\delta_g = E_b/N_{\eta_g}$, is defined as the ratio of the signal power to the power of the background Gaussian noise that is always present in the system. By its turn, the signal to impulsive noise ratio, $\delta_i = E_b/N_{\eta_i}$, is defined as the ratio between the power of the signal and the power of the impulsive noise that acts in the system.

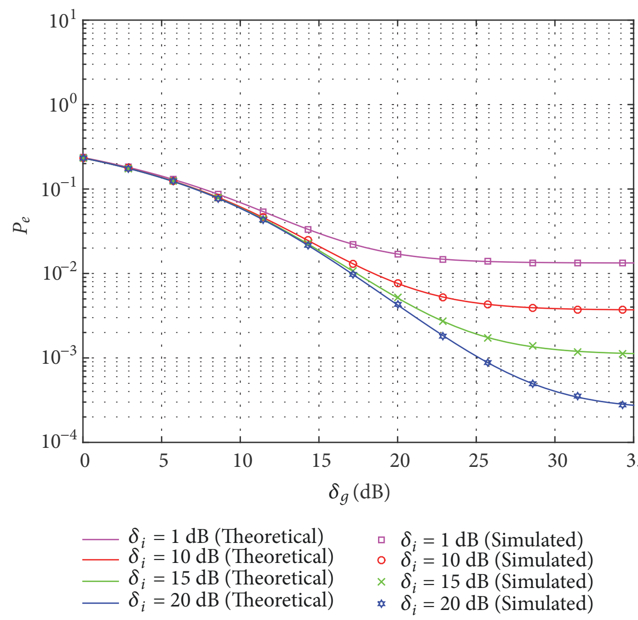

FIGURE 1: BEP of the 64-QAM modulation scheme under the effect of noise G^2AWGN and Nakagami fading, for four distinct values of the signal to impulsive noise ratio.

8. Results

In the numerical evaluation of the mathematical expressions obtained in this article, as well as in the simulations, the values considered for the impulsive noise parameters and order M of the M-QAM modulation are based on [15, 35]. In this article, simulations were performed using the Monte Carlo method, considering 5×10^6 transmitted bits.

Figure 1 shows the BEP curves of the 64-QAM modulation scheme under G^2AWGN noise and Nakagami fading, as a function of δ_g, for different values of δ_i, with $m = 1.5$, $\alpha_p = 0.5$, $p_2 = 0.5$, $\beta = 0.5$, and $p_1 = 0.5$. For $\delta_i = 1$ dB, we notice that the BEP, P_e, is practically constant as δ_g increases, for values of δ_g comprised between 25 and 35 dB. For high values of δ_i, such as 15 and 20 dB, the term $\delta_g\delta_i/(\delta_g+\delta_i)$ in (29) can be approximated by δ_g and thus, the error probability is a function of only the parameter δ_g, being more influenced by the permanent background noise. It can also be seen that, for values of δ_g smaller than 8 dB, the four values of δ_i under consideration lead to practically the same values of BEP. For $\delta_g = 35$ dB, a BEP difference of one order of magnitude is observed in the curves corresponding to $\delta_i = 1$ dB and $\delta_i = 15$ dB.

In Figure 2, BEP curves of 64-QAM under the effect of the G^2AWGN noise and Nakagami fading are presented, with $\delta_i = 20$ dB, $\alpha_p = 0.5$, $p_2 = 0.5$, $\beta = 0.5$, and $p_1 = 0.5$, for different values of the parameter m. As m decreases, the value of the BEP for fixed values of δ_g is greater because more severe conditions of fading are present in the channel. It can also be seen that the BEP equal to 10^{-3} is obtained with $\delta_g \approx 20$ dB for $m = 2.5$, and it is obtained with $\delta_g \approx 35$ dB for $m = 1.0$. When $\delta_g = 20$ dB, a BEP difference in terms of one

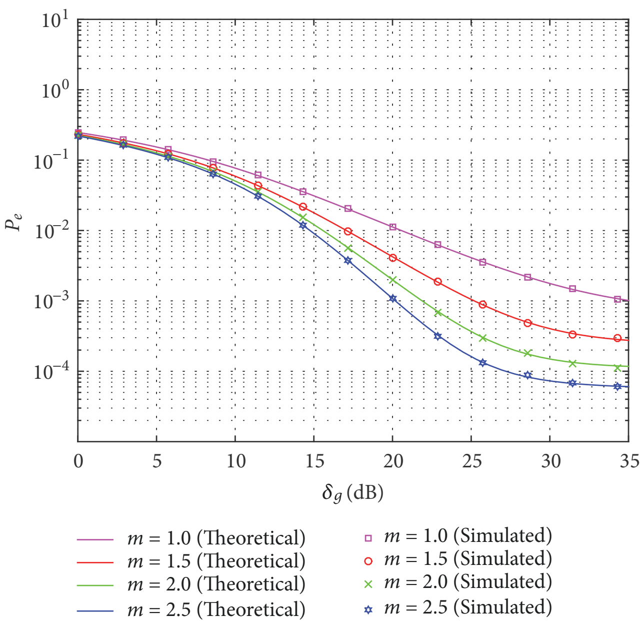

Figure 2: BEP of the 64-QAM modulation scheme under the effect of the noise G^2AWGN and Nakagami fading for different values of the parameter m.

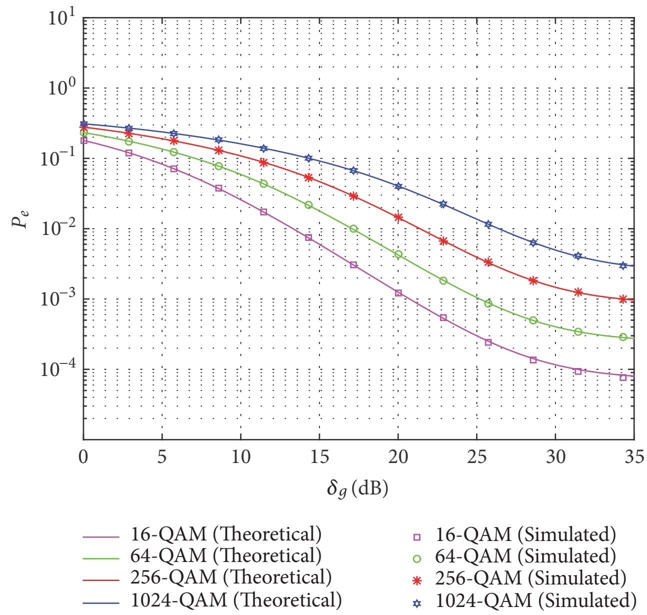

Figure 3: BEP for different values of constellation order M under the effect of noise G^2AWGN and Nakagami fading.

order of magnitude is observed in the curves corresponding to $m = 1.0$ and $m = 2.5$.

In Figure 3, BEP curves for G^2AWGN noise and Nakagami fading for four different values of M, considering $\delta_i = 20$ dB, $m = 1.5$, $\alpha_p = 0.5$, $p_2 = 0.5$, $\beta = 0.5$, and $p_1 = 0.5$, are shown. The less dense the constellation, the smaller the BEP obtained, since the symbols are more spaced and therefore less susceptible to the effects of noise. It is observed that a BEP equal to 10^{-3} is obtained with $\delta_g \approx 20.8$ dB for $M = 16$ while it is obtained with $\delta_g \approx 35$ dB for $M = 256$. For $\delta_g = 20$ dB, a P_e decrease of one order of magnitude is obtained when replacing 1024-QAM by 64-QAM.

9. Conclusions

In this paper, exact expressions were presented for computing the bit error probability (BEP) of the M-ary quadrature amplitude modulation (M-QAM) scheme for a channel model subjected to double gated additive white Gaussian noise (G^2AWGN) and Nakagami fading. The derivation of the expression for the BEP was performed from a method which consists in dividing the received signal by the fading envelope, transforming the multiplicative effect of the fading on an additive noise. Furthermore, BEP curves were presented, as a function of the signal to permanent noise ratio (δ_g) under the effect of G^2AWGN noise and Nakagami fading, for different values for the modulation order M and fading intensity.

As future works, new exact expressions for the bit probability of the modulation scheme M-QAM considering the channel under the combined effects of the noise G^2AWGN and fading α-μ [41] are intended to be determined.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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