

# Research Article SW and GB (N) ARQ Protocols under Markovian Interruptions

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This paper discusses packet data multiplexing using stop-and-wait (SW) and go-back-N (GBN) automatic repeat request (ARQ) protocols under Markovian interruption. The Markov process shows the output channel by examining the Markovian interruption using inactive and active states. We assume that whenever the voice signal is active the output link is used and will be blocked for the data packet, and data traffic input is exponentially distributed in increments via the Poisson process, with each data packet transmitted within an individual time slot. Active and inactive periods of the original voice signal are geometrically distributed with their unique parameters. The study introduces the concept of average service time and average queueing delay to simplify the analysis and shows that data multiplexers using SW and GBN ARQ schemes exhibit queueing behaviour when the interruption signal follows a Markov process. Moreover, we derived the effective capacity that features the average arrival rate at the transmitter queue under the quality of service (QoS) constraints. Also from the results system stability depends on the error probability and Markovian interruptions occurrence. Simulation results verify the theoretical analysis.

# 1. Introduction

This study assumes that updates are generated comparative results of average service time and average delay time for SW [1] and GBN [2] ARQ protocols interrupted by Markovian interruptions. We derived the average arrival rate and packet error probability at the transmitter queue under QoS constraints [3]. Reference [2] presented an overview of QoS paradigms for heterogeneous networks, focusing on those based on deterministic and probabilistic QoS. Reference [4] classified QoS routing protocols into two categories, which are probabilistic and deterministic, which in turn include soft real time and hard real time QoS routing protocol. Also error control techniques are used to provide reliable communication over noisy channels. Commonly used techniques include forward error correction (FEC) [5], automatic repeat request (ARQ) [5], or a combination, called hybrid ARQ [6]. FEC method is many more bits are needed for error correction. Instead, it adopts an ARQ error correction method that retransmits in error correction. ARQ data is protected by error detecting codes. If the receiver detects errors, the corresponding frame is retransmitted. ARQ protocols for error control are divided into 3 schemes: stop-and-wait (SW), go-back-N (GBN), and selective repeat (SR).

We choose voice traffic as the Markovian interruption signal over the data packet. Markov chain is commonly used for the performance and reliability evaluation of computer and communication systems [7]. Reference [8] analysed a Markovian model scheme, which takes account of the complex interaction of voice and data traffic sharing the optimal call admission control parameters. In this type of integrated system, data and voice traffic may be transmitted alternately through a single transmission link. Several models have been used to study SW and GBN ARQ protocols. Reference [9] described the system performance for service time, waiting time, Poisson arrivals, and server interruptions through the on-off Markov process. Reference [10] analyzed some queueing models in the integrated voice and data network. Reference [9] analyzed a discrete time single queue with arrivals where services were subject to interruptions, with particular attention to the probability generating function of the steady-state performance, and customer delay time of the vacation model. Besides [11] is presented analysis of the MMBP/Geo/1 queue with correspondence between positive and negative customer arrivals for radio link layer systems. Reference [6] considered Poisson arrivals and general service time and referred to two different HARQ system as the



FIGURE 1: Transmission diagram for SW ARQ protocol (round trip time r = 4 slots).

M/G/1/1 queue. A recent study by [1] analyzed the throughput of average packet delay and end-to-end packet delay of the cognitive SW hybrid ARQ system. Their analytical approach was probability and discrete time Markov chain based. Reference [3] investigated queueing of packet delay within transmitted data blocks and buffer occupancy.

Convergence of different types of wired, wireless, mobile, and cellular networks is crucial for success of next generation networks. Reference [5] analysed comparison of transmission efficiency for GBN and SR ARQ with forward error correction over channel bit error probability. Also [2] compared to IEEE 802.11e wireless networks; SW and GBN schemes showed that burst acknowledgement, utilized for GBN, performed better for medium sized networks with large window size and low frame error rate.

The inspiration for the current study is from our previous studies [12–14] investigating the achievable average buffer occupancy and corresponding average GBN and SW ARQ waiting times in voice integrated networks. Data and voice traffic are transmitted alternately in a single transmission link, and we assumed that retransmission interval time is equal to time slots. The output link channel changed by available (A) and blocked (B) states. Then the channel enters a blocking state generated by the Markovian voice interruption. Practical applications of this type of Markovian interruption [12–14] with data packets include voice-data integrated packet networks.

In particular, we compare SW and GBN ARQ schemes in single transmission links. Generally, the sender, i.e., transmitter, waits for acknowledgement before starting the next transmission. During this wait time, the transmitter is not allowed to retransmit a packet. However, a simulation study [12–14] considered the transmitter continuously transmitting N packets in every time slot without waiting for acknowledgement. The current paper combines the system states and identifies the service and delay time z-transformation, hence obtaining average service time and corresponding average delay time.

The remainder of the paper is organized as follows. Section 2 describes the proposed system model and assumptions,

and Section 3 derives average service time and queueing delay for SW and GBN ARQ schemes under a single Markovian interruption signal. Section 4 provides performance results and graphical illustrations, and Section 5 summarizes and concludes the paper.

#### 2. Proposed Model and Assumptions

We propose a model where data is transmitted using SW and GBN ARQ schemes over a link with cross traffic, such as voice data, and derive comparative analytical expressions. The proposed model is then compared with a conventional priority queue model. The models are equivalent if high priority traffic characteristics in the conventional priority queue are Markovian. However, when interruption traffic is heavy, it becomes more difficult for data traffic to be transmitted and delays become longer. On the hand, when interruption traffic is light, it is easier for data traffic to be transmitted and delays shorten [9].

Traffic pattern for a single voice can be modeled as a two state Markov signal. If there is voice traffic then data traffic should be blocked since a voice signal has higher priority than data traffic. Therefore, the voice signal constitutes an interruption signal for data packets.

We do not consider voice signal retransmission in this paper. Voice activity usually follows exponentially distributed ON/OFF patterns. Once voice and data integration can be successfully analyzed for the single voice case, then multiple voice case(s) can be considered.

2.1. SW ARQ Scheme. Figure 1 shows a typical SW ARQ protocol, where the transmitter checks the acknowledgement (ACK) feedback signal before sending the next packet after the next available round-trip delay time. However, in the case of transmission error, the transmitter receives the NACK signal from the receiver and retransmits the current packet at the next round-trip delay time after one slot.

2.2. GBN ARQ Scheme. Figure 2 shows some practical operation examples. The time axis consists of consecutive



FIGURE 2: Transmission diagram for GBN ARQ protocol (round trip time r = 4 slots).

time slots, and a time slot can hold only one packet at a time. We assume retransmission interval time is 4 time slots. After packets P(1) to P(4) are successfully transmitted, packet P(5) is affected by an error. The corresponding NACK(5) appears at the end of the 1st available period. The channel then enters a blocking state generated due to the Markovian voice interruption. When the channel changes available state again, retransmission of packets P(5) to P(7) begins at the second available period. If P(5) transmission fails again, it will finally be successfully transmitted at the 3rd transmission, and the corresponding ACK(6) signal produced from the receiver side.

Figures 1 and 2 shows the output link channel status. The channel can change between available (A) and blocked (B) states. When a NACK response is received, channel state changes from  $A_s$  to  $A_r$  if the output channel is available, or to  $B_r$  if the output channel is blocked by a Markovian interruption, where  $A_s$  and  $A_r$  mean the channel is available for a slot or round trip based period, respectively; and  $B_s$  and  $B_r$  mean that the channel is blocked for a slot or round trip based period.

To simplify the analysis, we do not consider transmission errors on the ACK (or NACK) signals in the reverse channel or timed out events; i.e., the transmitter receives either ACK or NACK as soon as the round trip delay has passed.

The queueing problem is modeled in the time axis, separated into a sequence of time slots, the packets have fixed size, and each packet can be forwarded into one slot time. There is no transmission error at the feedback channel. Round-trip delay, *r*, is defined as the time delay from the end of data packet transmission time to the corresponding ACK (or NACK) packet's instantaneous reception time (in slots).

Data packets are assumed to arrive at the transmitter at the beginning of a time slot, and buffer size is assumed to be infinite. The arrival process is assumed to be Poisson with arrival probability  $\lambda$  in a time slot,

$$P(\mathbf{k}) = \frac{e^{-\lambda}\lambda^k}{\mathbf{k}!}.$$
 (1)

Thus, the z-transformation for the number of arrivals per slot is

$$\Gamma(z) = \sum_{k=0}^{\infty} P(j) z^{k} = e^{\lambda(z-1)},$$
(2)

and the *z*-transformation of the number of arrivals R(z) during the retransmission time interval is

$$R(z) = \{T(z)\}^{r}$$
. (3)

#### 3. SW and GBN ARQ Scheme Behavior

Figure 3 shows the data multiplexer four state transition diagram for SW and GBN ARQ schemes under Markovian interruption. System state changes from  $A_s$  to  $B_s$  if the channel is changed to blocked state *B* because of the interruption signal. In  $A_s$  state the channel is available and one data packet will be transmitted if the buffer is not empty. If the transmission is successful, the transmitter is eligible to transmit at the next time slot.

State transition probabilities  $\alpha_s$ ,  $\alpha_r$ , and  $\beta_s$  are defined as follows:

 $\alpha_s$ : probability that the system in state  $A_s$  was in state  $A_s$  at the preceding time slot.

 $\beta_s$ : probability that the system in state  $B_s$  was in state  $B_s$  at the preceding time slot.

 $\alpha_r$ : probability the system changes from state  $A_s$  to state  $A_r$  during the round-trip delay time.

*SW Scheme.* The system state at the next time slot will be  $A_s$  with probability  $\alpha_s$ , or  $B_s$  with probability  $(1 - \alpha_s)$ . If transmission is successful,  $A_r$  occurs when ACK is received and the channel is available. State  $B_r$  occurs when NACK is received and the channel is blocked. State  $B_s$  can also be entered during a slot time from  $A_r$  or  $B_r$  if the output link is blocked during the time slot.

*GBN Scheme.* If the transmission is successful,  $A_s$  occurs when ACK is received and the channel is available. State  $A_r$  occurs when NACK is received and the channel is available. State  $B_r$  occurs when NACK is received and the channel is blocked. State  $B_s$  can also be entered from  $A_r$  or  $B_r$  if the output link is blocked during the time slot.



FIGURE 3: Four state transition diagram for SW and GBN ARQ protocol.

The probability that no Markovian interruptions exist is [13]

$$P(A) = \frac{1 - \beta_s}{2 - \alpha_s - \beta_s}.$$
(4)

The periods are random following the geometric probability density function with discrete time output channels (time slots),

$$P_A(m) = (1 - \alpha) \alpha^{m-1},$$
 (5)

and

$$P_B(m) = (1 - \beta) \beta^{m-1}.$$
 (6)

Hence the average periods are

$$\overline{A} = \frac{1}{(1-\alpha)} \tag{7}$$

and

$$\overline{B} = \frac{1}{(1-\beta)},\tag{8}$$

and the arrival rate is

$$\lambda \le \frac{1 - \beta_s}{2 - \alpha_s - \beta_s} \cdot \frac{1 - p}{1 - p + rp}.$$
(9)

3.1. Average Buffer Occupancy. The state transition equations for z-transformation of buffer occupancy  $A_s(z)$ ,  $A_r(z) B_s(z)$  and  $B_r(z)$  can be derived for each schemes. The distribution of buffer occupancy is

$$N(z) = A_{s}(z) + B_{s}(z) + \sum_{m=0}^{r} \frac{A_{r}(z) + B_{r}(z)}{T^{m}(z)}, \quad (10)$$

and average buffer occupancy can be obtained by differentiating (10) and substituting z=1,

$$N'(1) = A_{s}'(1) + B_{s}'(1) + r \left\{ A_{r}'(1) + B_{r}'(1) \right\} - \frac{r(r-1)}{2} \left\{ A_{r}(1) + B_{r}(1) \right\} T'(1).$$
(11)

3.1.1. SW ARQ Scheme Behavior. In the SW ARQ protocol, the sender waits until the receiving ACK/NACK before starting its next transmission. Hence the minimum time interval for consecutive transmission includes the round-trip delay time, and the systems can be separated into 4 states based on the following observations.

(1) If an error occurs during transmission, the packet is retransmitted after the round-trip delay r-1 slots; whereas if packet transmission is successful, buffer occupancy is reduced by one after the round-trip delay time. Therefore, the time epochs of interest are divided into slot and round-trip delay based time epochs.

(2) If a voice signal occurs, it will be transmitted first since the voice signal is real time traffic with higher priority. Data traffic is blocked during voice transmission. The probability of a packet error during transmission is p and no packet error is q = 1-p.

The z-transformations of buffer occupancy can be expressed as [14]

$$SA_{s}(z) = \alpha_{s} \{SA_{s}(0) + SA_{r}(0)\} T(z) + (1 - \beta_{s}) \cdot \{SB_{s}(z) + SB_{r}(z)\} T(z),$$
(12)

$$SB_{s}(z) = (1 - \alpha_{s}) \{SA_{s}(0) + SA_{r}(0)\} T(z) + \beta_{s} \{SB_{s}(z) + SB_{r}(z)\} T(z),$$
(13)

$$SA_{r}(z) = (qz^{-1} + p)$$

$$\cdot \alpha_{r} \{SA_{r}(z) - SA_{r}(0) + SA_{s}(z) - SA_{s}(0)\} R(z),$$
(14)

and

$$SB_{r}(z) = (qz^{-1} + p)(1 - \alpha_{r})$$
  
 
$$\cdot \{SA_{r}(z) + SA_{s}(z) - SA_{r}(0) - SA_{s}(0)\}$$
(15)  
 
$$\cdot R(z),$$

where

$$\frac{1 - \beta_S}{2 - \alpha_S - \beta_S} - \frac{\lambda}{1 - p} = SA_s(0) + SA_r(0).$$
(16)

Since

$$SA_{s}(1) = \lim_{z \to 1} SA_{s}(z) = SA(0) \lim_{z \to 1} \frac{SN_{AS}(z)}{SD_{AS}(z)}$$
 (17)

and the numerator  $(SN_{AS}(z), SN_{AR}(z), SN_{BS}(z) \text{ or } SN_{BR}(z))$ , and the denominator  $(SD_{AS}(z), SD_{AR}(z), SD_{BS}(z) \text{ or } D_{BR}(z))$ go to zero as z goes to 1; L'Hospital's rule [15] can be used to simplify (12)–(15)

$$SA_{s}(1) = SA(0) \lim_{z \to 1} \frac{SN_{AS}'(z)}{SD_{AS}'(z)},$$
 (18)

.

$$SB_{s}(1) = SA(0) \lim_{z \to 1} \frac{SN_{BS}'(z)}{SD_{BS}'(z)},$$
(19)

$$SA_r(1) = SA(0) \lim_{z \to 1} \frac{SN_{AR}'(z)}{SD_{AR}'(z)},$$
 (20)

and

$$SB_r(1) = SA(0) \lim_{z \to 1} \frac{SN_{BR}'(z)}{SD_{BR}'(z)}.$$
 (21)

Thus,

$$SA_{s}'(1) = \left\{ \frac{SN_{AS}''(1)}{2SD_{AS}'(1)} - \frac{SN_{AS}'(1)SD_{AS}''(1)}{2\left\{SD_{AS}'(1)\right\}^{2}} \right\} SA(0), \qquad (22)$$
$$SB_{s}'(1)$$

$$= \left\{ \frac{SN_{BS}''(1)}{2SD_{BS}'(1)} - \frac{SN_{BS}'(1)SD_{BS}''(1)}{2\left\{SD_{BS}'(1)\right\}^2} \right\} SA(0),$$
(23)

$$= \left\{ \frac{SN_{AR}''(1)}{2SD_{AR}'(1)} - \frac{SN_{AR}'(1)SD_{AR}''(1)}{2\left\{SD_{AR}'(1)\right\}^2} \right\} SA(0),$$
(24)

and

 $SA_{r}'(1)$ 

$$SB_{r}'(1) = \left\{ \frac{SN_{BR}''(1)}{2SD_{BR}'(1)} - \frac{SN_{BR}'(1)SD_{BR}''(1)}{2\left\{SD_{BR}'(1)\right\}^{2}} \right\} SA(0),$$
(25)

where  $SN_{AS}''(1)$ ,  $SD_{AS}''(1)$ ,  $SN_{AR}''(1)$ ,  $SD_{AR}''(1)$ ,  $SN_{BS}''(1)$ ,  $SD_{BS}''(1)$ ,  $SN_{BR}''(1)$ ,  $SD_{BR}''(1)$ ,  $SX_1''(1)$ , and  $SX_2''(1)$  are shown in the Appendix ((A.13)–(A.20), respectively).

*3.1.2. GBN ARQ Scheme Behavior.* Similar to the SW case, GBN ARQ scheme state and buffer occupancy are

$$GA_{s}(z) = q\alpha_{s} \{GA_{s}(z) - GA_{s}(0) + GA_{r}(z) - GA_{r}(0)\}$$

$$\cdot z^{-1}T(z) + \alpha_{s} \{GA_{s}(0) + GA_{r}(0)\}T(z)$$

$$+ (1 - \beta_{s}) \{GB_{s}(z) + GB_{r}(z)\}T(z),$$
(26)

$$GB_{s}(z) = q(1 - \alpha_{s})$$

$$\cdot \{GA_{s}(z) - GA_{s}(0) + GA_{r}(z) - GA_{r}(0)\}$$

$$\cdot z^{-1}T(z) + \beta_{s}\{GB_{s}(z) + GB_{r}(z)\}T(z)$$

$$+ (1 - \alpha_{s})\{GA_{s}(0) + GA_{r}(0)\}T(z),$$

$$GA_{r}(z)$$
(27)

$$= p\alpha_{r} \{GA_{s}(z) - GA_{s}(0) + GA_{r}(z) - GA_{r}(0)\}$$
(28)  
  $\cdot R(z),$ 

$$GB_{r}(z) = p(1 - \alpha_{r})$$

$$\cdot \{GA_{s}(z) - GA_{s}(0) + GA_{r}(z) - GA_{r}(0)\} R(z),$$
(29)

where

$$GA_{s}(0) = \left[\frac{p(1-\alpha_{r})(1-\alpha_{s}-\beta_{s})}{(1-\beta_{s})(1-p\alpha_{r})} - \frac{pr}{1-p\alpha_{r}} + \frac{GN'(1)}{GD'(1)}\left\{1 + \frac{pr}{1-p\alpha_{r}} + \frac{q(1-\alpha_{s}) + p\beta_{s}(1-\alpha_{r})}{(1-\beta_{s})(1-p\alpha_{r})}\right\}\right]^{-1}.$$
(30)

Using L'Hospital's rule [15], the steady probabilities that the system is in state  $GA_s(z)$ ,  $GA_r(z)$ ,  $GB_s(z)$ , or  $GB_r(z)$  are

$$GA_{s}(1) = \lim_{z \to 1} \frac{GN'(z)}{GD'(z)} GA_{s}(0) = \frac{GN'(1)}{GD'(1)} GA_{s}(0), \quad (31)$$

 $GB_{s}(1)$ 

$$= \frac{GA_s(0)}{(1-\beta_s)(1-\alpha_r p)} \left[ (1-\alpha_r)(1-\alpha_s-\beta_s) p + \{(1-\alpha_s)q + (1-\alpha_r)\beta_s p\} \frac{GN'(1)}{GD'(1)} \right],$$
(32)

$$GA_{r}(1) = \frac{\alpha_{r}p}{1 - \alpha_{r}p} \{GA_{s}(1) - GA_{s}(0)\}$$

$$= \frac{\alpha_{r}p}{1 - \alpha_{r}p} \left\{\frac{GN'(1)}{GD'(1)} - 1\right\} GA_{s}(0),$$
(33)

and

$$GB_{r}(1) = \frac{(1 - \alpha_{r})p}{1 - \alpha_{r}p} \{GA_{s}(1) - GA_{s}(0)\}$$

$$= \frac{(1 - \alpha_{r})p}{1 - \alpha_{r}p} \left\{\frac{GN'(1)}{GD'(1)} - 1\right\} GA_{s}(0),$$
(34)

respectively; and hence

$$GA_{s}'(1) = \left\{ \frac{GN''(1) GD'(1) - GN'(1) GD''(1)}{\left\{ 2GD'(1) \right\}^{2}} + \frac{GN'(1)}{GD'(1)}T'(1) \right\} GA_{s}(0),$$
(35)

$$GA_{r}'(1) = \frac{\alpha_{r}pR'(1)}{(1-\alpha_{r}p)^{2}} \{GA_{s}(1) - GA_{s}(0)\} + \frac{\alpha_{r}p}{1-\alpha_{r}p}$$
(36)  
$$\cdot GA_{s}'(1),$$

$$GB_{s}'(1) = \frac{(1 - \alpha_{r}) \beta_{s} pR'(1) + (1 - \alpha_{s}) q \{\alpha_{r} p + \alpha_{r} pR'(1) - 1\}}{(1 - \alpha_{r} p)^{2}} \cdot \frac{1}{1 - \beta_{s}} \cdot \{GA_{s}(1) - GA_{s}(0)\} + \frac{T'(1)}{(1 - \beta_{s})^{2}} \left[ \frac{(1 - \alpha_{s}) q + (1 - \alpha_{r}) \beta_{s} p}{1 - \alpha_{r} p} (GA_{s}(1) - GA_{s}(0)) + (1 - \alpha_{s}) GA_{s}(0) \right] + \frac{1}{1 - \beta_{s}} \cdot \frac{(1 - \alpha_{s}) q + (1 - \alpha_{r}) \beta_{s} p}{1 - \alpha_{r} p} GA_{s}'(1)$$
(37)

and

$$GB_{r}'(1) = \frac{(1 - \alpha_{r}) p R''(1)}{(1 - \alpha_{r} p)^{2}} \{GA_{s}(1) - GA_{s}(0)\} + \frac{(1 - \alpha_{r}) p}{1 - \alpha_{r} p} GA_{s}'(1),$$
(38)

where GN'(1), GD'(1), GN''(1), and GD''(1) are shown in the Appendix as (A.23)–(A.26), respectively.

3.2. Average Waiting Time. Let E(W) denote the average system waiting time in the system. Then, from Little's formula [11]

$$E(W) = \frac{N'(1)}{\lambda}.$$
(39)

We use (11) and (39) as throughput efficiency for both schemes, and since r,  $\lambda$ , and T'(1) are the same for both schemes, the analysis is simplified to determine  $A_s(z)$ ,  $B_s(z)$ ,  $A_r(z)$ , and  $B_r(z)$  depending on the type of each SW and GBN ARQ transmission protocol in Sections 3.1.1 and 3.1.2.

3.3. Average Service Time. We need to know the average service time packet to calculate the average delay time. If many errors have occurred in a packet then the service time is the slot based transmission service time and round-trip based transmission service time. Therefore, the service time for any state is divided into pure transmission time and blocking time by the Markovian interruption. Pure service time consists of transmission times and NACK period, whereas blocking time includes only the Markovian interruption service time.

Thus, we need to calculate the slot based transmission time and its z-transform. If a Markovian interruption occurs while transmitting a packet, transmission is stopped and the transmitter waits until the Markovian interruption ends. Therefore, the probability of density function (PDF) for the first transmission time is

$$s_{s}(n) = \begin{cases} \alpha_{s}, & \text{if } n = 1\\ (1 - \alpha_{s}) \beta_{s} (1 - \beta_{s})^{n-2}, & \text{if } n > 1, \end{cases}$$
(40)

and z-transformations of slot-based transmission time are as follows:

$$S_{s}(z) = \sum_{n=0}^{\infty} z^{n} s_{s}(n) = \alpha_{s} z + \frac{(1-\alpha_{s}) \beta_{s} z^{2}}{1-(1-\beta_{s}) z}.$$
 (41)

As discussed above, Markovian interruption occurs with probability  $\alpha_r$ , hence retransmission, such as round-trip delay based transmission service time PDF, is

$$s_{r}(n) = \begin{cases} \alpha_{r}, & \text{if } n = 1 + R\\ (1 - \alpha_{r}) \beta_{s} (1 - \beta_{s})^{n - 2 - R}, & \text{if } n > 1 + R, \end{cases}$$
(42)

and z-transformations of Markovian interruptions are as follows:

$$S_r(z) = \sum_{n=0}^{\infty} z^n s_r(n) = \alpha_r z^{R+1} + \frac{(1-\alpha_r)\beta_s z^{R+2}}{1-(1-\beta_s)z}.$$
 (43)

If we know how many errors occur in transmitting a packet, the *z*-transform PDF of the total service time is [7, 11]

$$S(z \mid i) = S_s(z) S_r^i(z),$$
 (44)

where i is number of times errors occurred. The PDF for how many times the transmitter retransmits a packet until successful receiver is given by the geometric distribution,

$$l(n) = (1 - p) p^{n},$$
 (45)

where n = 0, 1, 2, 3, ...

Thus, the z-transformation of service time is

$$S(z) = \sum_{i=0}^{\infty} l(n) S(z \mid i) = (1-p) S_s(z) \frac{1}{1-pS_r(z)}, \quad (46)$$

and average service time is

$$E(S) = \frac{dS(z)}{dz}\Big|_{z=1} = S'_{s}(1) + p\frac{S'_{r}(1)}{1-p}$$

$$= 1 + \frac{1-\alpha_{s}}{\beta_{s}} + \frac{p}{1-p}\left[R + 1 + \frac{1-\alpha_{r}}{\beta_{s}}\right].$$
(47)

For the proposed 4 state Markov chain model,

$$S_{s}(z) = A_{s}(z) + B_{s}(z),$$

$$S_{s}'(z) = A_{s}'(z) + B_{s}'(z)$$

$$S_{r}(z) = A_{r}(z) + B_{r}(z),$$

$$S_{r}'(z) = A_{r}'(z) + B_{r}'(z)$$
(49)

Let E(S) be the average service time in system. It is similar to (47); we have

$$E(S) = \frac{dS(z)}{dz}\Big|_{z=1}$$

$$= \frac{(1-p)\left\{S'_{s}(1) - p\left(S'_{s}(1)S_{r}(1) + S_{s}(1)S'_{r}(1)\right)\right\}}{(1-pS_{r}(1))^{2}}.$$
(50)

3.4. Average Delay Time. The average system delay time is then

$$E(D) = E(W) + E(S),$$
 (51)

where E(W) and E(S) are from (39) and (50), respectively.

### 4. Performance Results

We verified the achieved theoretical analysis results using extensive simulation, based on a discrete event simulator [5] coded in C++ and Java languages in Windows 7. Default parameters were chosen following [12]: average available period  $\overline{A}$ =1800ms, average blocked period  $\overline{B}$ =1300ms, time slot T=5ms, retransmission interval time r = 4 time slots, and the average available and blocked periods were derived from (8). Hence, we calculated parameters  $\alpha_s = 0.9972$ ,  $\beta_s = 0.9962$ , and  $\alpha_r = 0.9889$ .

*Simulation Method.* In the simulation, we considered a commonly used the PDF to check whether data is available or not. If data is available, the packet is transmitted to another state occurs based on the proposed model in the paper. This process can be performed in a single-threaded environment, and the proposed model does not require multiple machine systems. This will complete the test by comparing the analytical and simulation results of our proposed model. Simulations were performed under the following conditions to partially verify the above analysis validity.

(1) Buffer length was infinite.

(2) Number of arrivals in a time slot was given by *r*: for a generated random number 0 < RN < 1, r packets arrive if  $CDF[r-1] \le RN \le CDF[r]$ .

(3) Simulations were run for 1,000,000 timeslots.

Figure 4 shows simulation and theoretical results for buffer occupancy as a function of arrival rate with packet error probabilities ( $\alpha_s$  and  $\beta_s$ ) for the proposed SW and GBN ARQ retransmission schemes. The GBN ARQ protocol parameters are always higher than the SW ARQ parameters for good (p=0.000001) or bad (p=0.5) channel conditions. As expected buffer occupancy increases very abruptly after threshold as traffic intensity ( $\lambda$ ) increases. When *p*=0.000001, simulation and theoretical average buffer occupancy equal for both SW and GBN ARQ schemes. Figure 5 shows average system service time with Markovian interruptions for different round-trip delay time from Eq. (50).

Figures 6 and 7 show average service and delay times for four state Markov chains under GBN and SW ARQ protocols. Delay time is non zero even under very low traffic load. If a packet arrives during state *B*, the next packet must wait until



FIGURE 4: Average buffer occupancy for GBN and SW ARQ protocols as a function of arrival rate ( $\lambda$ ) with packet error probabilities (p) as a parameter  $\alpha_s$  and  $\beta_s$ .



FIGURE 5: Average service time for the system with Markovian interruptions as a function of packet error probability (p) for different round-trip delay time (r).

*B* ends; hence the average delay time under very low traffic load is  $1/(1 - \beta)$ . Average data queue delay time for *r*=10 is larger than for *r*=4 due to difference service times.

Figures 8 and 9 show average service and delay times for four state Markov chain under GBN and SW ARQ protocols as a function of arrival rate ( $\lambda$ ) with packet error probability (p). GBN ARQ protocol service and delay time is always higher than the SW ARQ protocol for good (*p*=0.000001) and bad (*p*=0.5) channel conditions. As expected service time and delay time increase very abruptly after some threshold as



FIGURE 6: Average service time for 4 state Markov chain under GBN and SW ARQ protocols as a function of packet error probability (p) for different round-trip delay time (r).



FIGURE 7: Average delay time for 4 state Markov chain under GBN and SW ARQ protocols as a function of packet error probability (p) for different round-trip delay time (r).

traffic intensity ( $\lambda$ ) increases. For example, when p=0.5 the curve slop abrupt steepens after  $\lambda$ =0.06 and has no stationary probability behavior when the traffic load is over 0.1 for GBN ARQ scheme. When p=0.000001, simulation and theoretical buffer occupancy and service and delay times are equal for both ARQ schemes. Thus, system stability depends on the error probability and Markovian interruption occurrence.

From the analytical and simulation results, the roundtrip delay time (r) has to be carefully picked out under the given packet error probability to prevent from over service time and delay time. The closed form solution for delay time is important when queuing theory is applied to data communications.



FIGURE 8: Average service time for 4 state Markov chain under GBN and SW ARQ protocols as a function of arrival rate ( $\lambda$ ) for different packet error probability (p).



FIGURE 9: Average delay time for 4 state Markov chain under GBN and SW ARQ protocols as a function of arrival rate ( $\lambda$ ) for different packet error probability (p).

#### 5. Conclusions

This paper analyzed a packet voice/data multiplexer with fully reliable SW and GBN ARQ schemes. Different retransmission techniques with ARQ protocols were studied, comparing average service and delay times. Better throughput was obtained comparing in a time slotted packet multiplexer with Markovian interruptions of service time at the transmitter. The system was modeled as a two state Markov chain, and buffer behavior and service time for SW and GBN ARQ protocols were found to depend on voice signal (inactive/active) activity. Available and blocked states were divided into round trip and time slot time based states, depending on whether or not transmission error occurred, providing 4 system states. Available and blocked periods for the original voice signal were geometrically distributed with unique parameters. Simulation results were consistent with the theoretical analysis and verified that system stability depends on the error probability and Markovian interruption occurrence.

The limitation of this study has the retransmission of voice signals will not be considered in this paper. Rather, only the retransmission of the data packet discussed. To simplify the analysis, the transmission errors of ACK (NACK) in the reverse channel are not considered. Furthermore our analysis can be extended in analysis of Selective Repeat ARQ scheme under Markovian interruptions. Will compare with the presented results of this paper and extensive study of a fully adaptive ARQ scheme.

#### Appendix

Following [14] and Appendix (A.13)–(A.20) we differentiate (18)–(21) from Section 3 substitute z = 1:

$$SN_{AS}'(1) = (1 - X_1(1)) T'(1) (2 - \alpha_s - 2\beta_s) - SX_2'(1) - (1 - \beta_s) SX_1'(1),$$
(A.1)

$$SD_{AS}'(1) = -(1 - \beta_s) SX_1'(1) - \beta_s T'(1) (1 - SX_1(1)) - SX_2'(1),$$
(A.2)

$$SN_{AR}'(1) = SX_1(1) \left( SN_{AS}'(1) - SD_{AS}'(1) \right),$$
 (A.3)

$$SD_{AR}'(1) = (1 - SX_1(1))SD_{AS}'(1),$$
 (A.4)

$$SN_{BS}'(1) = (1 - \alpha_s) (1 - SX_1(1)) SD_{AS}'(1) + (1 - \alpha_r) \beta_s \{SN_{AS}'(1) - SD_{AS}'(1)\},$$
(A.5)

$$SD_{BS}'(1) = (1 - \beta_s) (1 - SX_1(1)) SD_{AS}'(1),$$
 (A.6)

$$SN_{BR}'(1) = (1 - \alpha_r) \left\{ SN_{AS}'(1) - SD_{AS}'(1) \right\}$$
(A.7)

and

$$SD_{BR}'(1) = (1 - SX_1(1)) SD_{AS}'(1),$$
 (A.8)

where

$$SX_1(1) = \alpha_r, \tag{A.9}$$

$$SX_2(1) = (1 - \alpha_r)(1 - \beta_s),$$
 (A.10)

$$SX_{1}'(1) = \alpha_r \left\{ R'(1) - q \right\},$$
 (A.11)

$$SX_{2}'(1) = (1 - \alpha_{r})(1 - \beta_{s}) \{R'(1) + T'(1) - q\}.$$
 (A.12)

Differentiating (A.1)–(A.8) again and substituting 
$$z = 1$$

$$SN_{AS}''(1) = (1 - SX_1(1)) (4 - 3\alpha_s - 4\beta_s) T''(1)$$
  
-  $(4 - 2\alpha_s - 4\beta_s) T'(1) SX_1'(1) - (1 - \beta_s)$  (A.13)  
 $\cdot SX_1''(1) - SX_2''(1)$ 

$$SD_{AS}''(1) = -\beta_s (1 - SX_1(1)) T''(1) - SX_2''(1) + 2\beta_s SX_1'(1) T'(1) - (1 - \beta_s) SX_1''(1),$$
(A.14)

$$SN_{AR}''(1) = 2SX_1'(1) (SN_{AS}'(1) - SD_{AS}'(1)) + SX_1(1) (SN_{AS}''(1) - SD_{AS}''(1)),$$
(A.15)

$$SD_{AR}''(1) = (1 - SX_1(1)) SD_{AS}''(1) - 2SX_1'(1)$$
  
  $\cdot SD_{AS}'(1),$  (A.16)

$$SN_{BS}''(1) = (1 - \alpha_{s})$$

$$\cdot \left[ (2T'(1) SD_{AS}'(1) + SD_{AS}''(1)) (1 - SX_{1}(1)) - 2SX_{1}'(1) SD_{AS}'(1) \right] + (1 - \alpha_{r}) \beta_{s} \qquad (A.17)$$

$$\cdot \left\{ SN_{AS}''(1) - SD_{AS}''(1) \right\} + (1 - \alpha_{r}) \beta_{s} 2 (T'(1) + R'(1) - q) \left\{ SN_{AS}'(1) - SD_{AS}'(1) \right\},$$

$$SD_{BS}''(1) = (1 - \beta_{s}) (1 - SX_{1}(1)) SD_{AS}''(1) - (1 - \beta_{s}) 2SX_{1}'(1) SD_{AS}'(1) - 2\beta_{s} (1 - SX_{1}(1)) \qquad (A.18)$$

$$\cdot T'(1) SD_{AS}'(1),$$

$$SN_{BR}''(1) = 2(1 - \alpha_r) \{SN_{AS}'(1) - SD_{AS}'(1)\}$$
  

$$\cdot (R'(1) - q) + (1 - \alpha_r) \{SN_{AS}''(1)$$
(A.19)  

$$- SD_{AS}''(1)\},$$

and

$$SD_{BR}^{\ \prime\prime}(1) = (1 - SX_1(1))SD_{AS}^{\ \prime\prime}(1)$$
  
 $- 2SX_1^{\ \prime}(1)SD_{AS}^{\ \prime\prime}(1).$  (A.20)

Hence from (22) to (25),

$$SX_{1}^{\prime\prime}(1) = \alpha_{r} \left\{ 2q - 2qR^{\prime}(1) + R^{\prime\prime}(1) \right\}$$
(A.21)

and

$$SX_{2}''(1) = (1 - \alpha_{r})(1 - \beta_{s})$$

$$\cdot \{2q(1 - R'(1) - T'(1)) + R''(1) \quad (A.22)$$

$$+ 2T'(1)R'(1) + T''(1)\}$$

where R(z) is given in (3) and  $R''(1) = \lambda^2 r^2$ . Finally, [13]

$$GD'(1) = (1 - \beta_s) q - T'(1) [\beta_s + 2(1 - \beta_s) q + (1 - \alpha_r - \beta_s) p - \alpha_s q] - (1 - \beta_s) pR'(1),$$
(A.23)

$$GN'(1) = T'(1)(1 - \alpha_s - \beta_s)(1 - \alpha_r p - q) + (1$$

$$-\beta_s)R'(1) + (1 - \beta_s)q$$

$$GN''(1) = (1 - \alpha_s - \beta_s)[T''(1)(1 - q - \alpha_r p)$$

$$+ 2T'(1)\{q - \alpha_r pR'(1)\}] - (1 - \beta_s)\{pR''(1) \quad (A.25)$$

$$+ 2q\},$$

$$GD''(1) = 2\alpha_s qT'(1) - (1 - \beta_s)\{2q - pR''(1)$$

$$+ T''(1)(q+2) - T''(1)(1 - 2\alpha_r p)$$

$$+ \alpha_s (1 - 2q)).$$
(A.26)

#### **Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

#### **Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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