

## Research Article

# Energy Efficiency Maximized Resource Allocation for Opportunistic Relay-Aided OFDMA Downlink with Subcarrier Pairing

Tao Wang , Chao Ma, Yanzan Sun, Shunqing Zhang, and Yating Wu

*Shanghai Institute for Advanced Communication and Data Science, Key laboratory of Specialty Fiber Optics and Optical Access Networks, Joint International Research Laboratory of Specialty Fiber Optics and Advanced Communication, Shanghai University, Shanghai, China*

Correspondence should be addressed to Tao Wang; [twang@shu.edu.cn](mailto:twang@shu.edu.cn)

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This paper studies the energy efficiency (EE) maximization for an orthogonal frequency division multiple access (OFDMA) downlink network aided by a relay station (RS) with subcarrier pairing. A highly flexible transmission protocol is considered, where each transmission is executed in two time slots. Every subcarrier in each slot can either be used in direct mode or be paired with a subcarrier in another slot to operate in relay mode. The resource allocation (RA) in such a network is highly complicated, because it has to determine the operation mode of subcarriers, the assignment of subcarriers to users, and the power allocation of the base station and RS. We first propose a mathematical description of the RA strategy. Then, a RA algorithm is derived to find the globally optimum RA to maximize the EE. Finally, we present extensive numerical results to show the impact of minimum required rate of the network, the user number, and the relay position on the maximum EE of the network.

## 1. Introduction

With rapid growth of multimedia services, requirements for high-speed wireless communications are growing fast. To meet these requirements, telecom operators arrange a large number of base stations, which lead to a high amount of energy consumption [1]. To address this issue, many scholars have proposed energy-saving methods to minimize total energy consumption for a variety of wireless communication systems, such as Device-to-Device (D2D) communications, wireless sensor networks (WSNs), and cellular networks [2, 3]. Recently, energy efficiency- (EE-) based optimization design, which aims to maximize the EE defined as the number of transmitted bits per Joule total energy consumption, has attracted much interest from academia and industry [4–10].

Orthogonal frequency division multiple access (OFDMA) has been widely recognized as one of the dominant wireless technologies for high-data-rate wireless multimedia services. One of the main reasons behind this fact is that performance of OFDMA systems can be significantly improved by proper

resource allocation (RA) when transmitter channel state information (CSI) is available [11–16]. Lately, relay-aided cooperation schemes have been widely used in combination with OFDMA networks to improve spectral efficiency. Under the constraint of guaranteeing users' communication rate, some works designed RA algorithms to minimize the total transmission power of networks [17–24]. In [17], to minimize the maximum value between transmission power of the BS and transmission power of all the RSs, Muller et al. designed a RA algorithm for decode-and-forward (DF) relay-aided OFDMA networks. In [21], knowing the relay selection, Huang et al. proposed an optimization algorithm of subcarriers and power allocation to minimize the total power of BS and all the RSs. Chen et al. designed a strategy of user assignment for subcarriers, RS's choice, and modulation scheme to minimize the total transmission power of networks for amplify-and-forward (AF) relay-aided downlink OFDMA networks [23]. The above works ignore the influence of circuit power of the BS and RSs, so these algorithms cannot ensure high EE of networks.

It is interesting to further study how to improve EE of relay-aided OFDMA networks. When users lie outside the BS's radio coverage, EE maximized RA problems for OFDMA networks using subcarrier-pair-based DF protocols have been addressed in [25–29]. In these works, every subcarrier in the first time slot is paired with a subcarrier in the second time slot for the relay-aided transmission. In most cases, BS can also transmit messages to users directly; designing EE maximized RA algorithm for flexible transmission protocols is more meaningful. In [30–34], the authors adopted more flexible transmission protocols and proposed EE maximized RA algorithms for downlink OFDMA networks when the total transmission power is constrained to be smaller than a prescribed value. In this case, when the network reaches the maximum EE, the total communication rate might be too small to meet the needs of users.

In this paper, we focus on the optimum energy-efficient RA for downlink OFDMA with a RS using subcarrier-pair-based DF relaying, when the sum rate is constrained above a prescribed value. An opportunistic relay-aided transmission protocol is considered. User message bits are transmitted during two consecutive equal-duration time slots. In the first slot, the BS broadcasts OFDM symbols to RSs and users. In the second slot, subcarriers in direct mode can transmit to users directly; other subcarriers can be paired with subcarriers in first slot to transmit messages with the help of the RS. To be more specific, our contributions are summarized as follows:

- (i) An EE maximized RA problem is formulated, and a polynomial-complexity algorithm is designed to find the optimum RA to maximize the EE based on the Dinkelbach method as well as the dual method to solve a subproblem.
- (ii) Extensive numerical results are shown to exhibit the impact of system parameters (including minimum communication rate required by the network, RS deployment, and user number) on the network EE.

The rest of this paper is organized as follows. In the next section, the transmission protocol of the network is described. After that, the EE maximized RA algorithm is developed in Section 3. Numerical experiments are shown in Section 4. Finally, Section 5 concludes the paper.

*Notations.*  $\mathcal{E}(x) = \log_2(1 + x)$ .

## 2. Network Model and Transmission Protocol

*2.1. General Introduction of the Network and Protocol.* We consider an OFDMA downlink network as shown in Figure 1. The network under consideration consists of a BS, a RS, and multiple users. Both the BS and the RS adopt OFDMA scheme using the same frequency band of bandwidth  $B$  Hz and with  $K$  subcarriers, which means that each OFDM symbol has a duration of  $K/B$  seconds.

For illustration purpose, the channel coefficient and noise-power normalized channel gain at any subcarrier  $k$  ( $k \in \{1, \dots, K\}$ ) from BS to RS, from RS to any user  $u$ ,

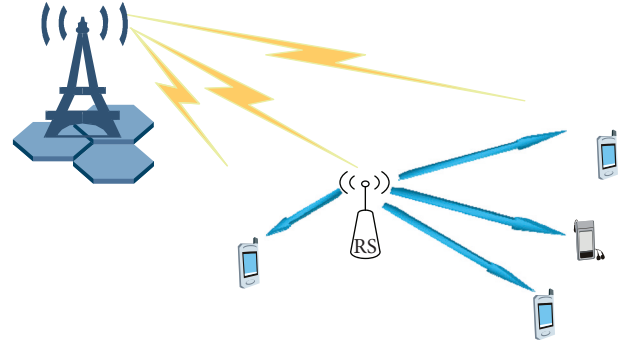


FIGURE 1: The relay-aided OFDMA downlink network under consideration.

TABLE 1: Channel coefficient and gain for subcarrier  $k \in \{1, \dots, K\}$ .

Channel for subcarrier $k$	Coefficient	Gain
BS to RS	$h_{sr}^k$	$G_{sr}^k = \frac{ h_{sr}^k ^2}{\sigma^2}$
BS to user $u$	$h_{su}^k$	$G_{su}^k = \frac{ h_{su}^k ^2}{\sigma^2}$
RS to user $u$	$h_{ru}^k$	$G_{ru}^k = \frac{ h_{ru}^k ^2}{\sigma^2}$

and from BS to any user  $u$  are defined in Table 1, where  $\sigma^2$  is assumed as the power of additive white Gaussian noise at each subcarrier.

The transmission protocol under consideration is carried out as follows. Each transmission needs two time slots denoted by slot-1 and slot-2, respectively. Each subcarrier in every slot can either operate in direct mode (it is used by the BS for transmission to a user directly) or be paired with a subcarrier in another slot to operate in relay mode (the relay helps the transmission as explained in Section 2.2). The protocol is illustrated by Figure 2. In the following subsections, the transmission procedures for the direct and relay mode are explained in detail.

*2.2. Transmission for a Subcarrier Pair in Relay Mode.* Suppose that a subcarrier  $k$  in slot-1 is paired with a subcarrier  $l$  in slot-2 to operate in relay mode, and this relay-link is assigned to user  $u$ . Denote this subcarrier pair as  $(k, l, u)$ , which is shown in Figure 3. Over this link, the transmission procedure is carried out as follows.

In slot-1, the BS broadcasts a symbol  $x$  to both the relay and the user with power  $P_1$ . The received signal at the relay is expressed as

$$y_r = \sqrt{P_1} h_{sr}^k x + n_r, \quad (1)$$

and that at the user is expressed as

$$y_{u,1} = \sqrt{P_1} h_{su}^k x + n_{u,1}. \quad (2)$$

At the end of slot-1, the relay decodes  $y_r$  and recovers  $x$ . To enable the relay to successfully decode the message bits, the

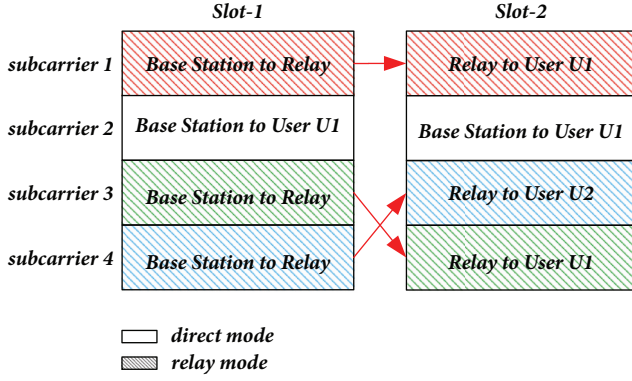


FIGURE 2: The transmission procedure over the subcarrier pair  $(k, l, u)$ .

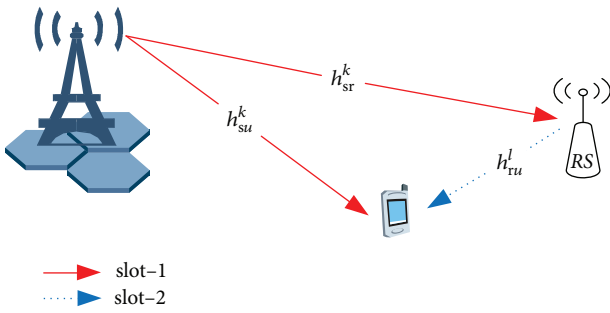


FIGURE 3: The transmission procedure over the subcarrier pair  $(k, l, u)$ .

maximum transmission rate is no greater than  $\log_2(1 + P_1 G_{sr}^k)$  bits/symbol.

In slot-2, the relay simply emits  $x$  at subcarrier  $l$  to the user with power  $P_2$ . The received signal at the user is expressed as

$$y_{u,2} = \sqrt{P_2} h_{ru}^l x + n_{u,2}. \quad (3)$$

At the end of slot-2, the source combines  $y_{u,1}$  and  $y_{u,2}$  with maximum-ratio combining (MRC) to maximize the received SNR. The final signal used for decoding at the user can be expressed as

$$z = \alpha y_{u,1} + \beta y_{u,2}. \quad (4)$$

It can be shown that when  $\alpha = \sqrt{P_1} h_{su}^k$  and  $\beta = \sqrt{P_2} h_{ru}^l$ , the MRC is achieved [35]. The maximum received SNR is

$$\text{SNR}_{k,l,u} = G_{su}^k P_1 + G_{ru}^l P_2. \quad (5)$$

As a result, the maximum transmission rate over this relay link should be the minimum between the source-relay rate and the source-relay-user rate. It can be evaluated as

$$C_{klu} = \min \left\{ \mathcal{C}(\text{SNR}_{klu}), \mathcal{C}(G_{sr}^k P_1) \right\}, \quad (6)$$

in the unit of bits/symbol.

Suppose that  $P'$  is the sum power of  $P_1$  and  $P_2$ ; the optimums  $P_1$  and  $P_2$  for maximizing the rate are the optimum solution for

$$\begin{aligned} \max_{P_1, P_2} \quad & \min \{ P_1 G_{sr}^k, P_1 G_{su}^k + P_2 G_{ru}^l \}, \\ \text{s.t.} \quad & P_1 + P_2 = P', \quad P_1 \geq 0, \quad P_2 \geq 0. \end{aligned} \quad (7)$$

Using the same method as in [36], it can easily be shown that the optimums  $P_1$  and  $P_2$  are

$$\begin{aligned} P_1 &= \begin{cases} \frac{G_{ru}^l}{\Delta_{u,k} + G_{ru}^l} P' & \text{if } \min \{ G_{sr}^k, G_{ru}^l \} > G_{su}^k, \\ P' & \text{if } \min \{ G_{sr}^k, G_{ru}^l \} \leq G_{su}^k, \end{cases} \\ P_2 &= \begin{cases} \frac{G_{sr}^k - G_{su}^k}{\Delta_{u,k} + G_{ru}^l} P' & \text{if } \min \{ G_{sr}^k, G_{ru}^l \} > G_{su}^k, \\ 0 & \text{if } \min \{ G_{sr}^k, G_{ru}^l \} \leq G_{su}^k, \end{cases} \end{aligned} \quad (8)$$

where  $\Delta_{u,k} = G_{sr}^k - G_{su}^k$ .

The maximum rate associated with the above solution is equal to

$$\begin{aligned} C_{klu} &= \frac{B}{2K} \mathcal{C}(G_{klu} P') \quad (\text{bits/second}), \\ G_{klu} &= \begin{cases} \frac{G_{sr}^k G_{ru}^l}{\Delta_{u,k} + G_{ru}^l} & \text{if } \min \{ G_{sr}^k, G_{ru}^l \} > G_{su}^k, \\ \min \{ G_{sr}^k, G_{su}^k \} & \text{if } \min \{ G_{sr}^k, G_{ru}^l \} \leq G_{su}^k. \end{cases} \end{aligned} \quad (9)$$

**2.3. Transmission for a Subcarrier in Direct Mode.** Every subcarrier in either slot-1 or slot-2 can be assigned to operate in direct mode. In this mode, a direct link from the BS to a certain user is formed at this subcarrier. Suppose that a subcarrier  $k$  in either slot-1 or slot-2 is assigned to user  $u$  and operates in the direct mode, and BS uses power  $P$  for this subcarrier. Therefore, the average transmission rate over this direct link can be evaluated as

$$R(P) = \frac{B}{2K} \mathcal{C}(G_{su}^k P) \quad (\text{bits/second}). \quad (10)$$

### 3. Energy-Efficient RA Algorithm Design

Before data transmission, we assume that the BS controller knows all CSI, that is,  $\{G_{sr}^k, G_{su}^k, G_{ru}^k \mid \forall k, \forall u\}$ . Based on the a priori knowledge, the BS runs an algorithm to find the optimum RA strategy to maximize the network EE.

To be more specific, the RA algorithm for the transmission protocol needs to be optimized:

- (i) *Subcarrier operation mode*: how to decide whether each subcarrier should operate in direct or relay mode
- (ii) *Subcarrier pairing for relay mode*: how to pair subcarriers in relay mode
- (iii) *Subcarrier assignment to users*: how to allocate subcarriers to users

- (1) evaluate  $G_{klu}, \forall k, l, u$ ;
- (2)  $\theta_{\min} = 0$ ;  $\theta_{\max}$  is set by a large value;  $\theta = \theta_{\max}$ ;  $\delta = 10^{-3}$ ;
- (3) **while**  $|F(\theta)| \leq \delta$  **do**
- (4) solve (P2) for  $\mathbf{S}(\theta)$  using Algorithm 2;
- (5) update  $\theta$  by  $\min\{\theta_{\max}, \max\{R(\mathbf{S}(\theta))/P(\mathbf{S}(\theta)), \theta_{\min}\}\}$ ;
- (6) **end while**
- (7) output  $\theta$  as  $\eta^*$  and  $\mathbf{S}(\theta)$  as the optimum RA strategy.

ALGORITHM 1: The algorithm to solve (P1).

- (iv) *Power allocation*: how to allocate BS's and RS's transmission power for each subcarrier

To design the RA algorithm, we proceed as follows. First, a mathematical description of the RA strategy and network EE is proposed in Section 3.1. Then, a Dinkelbach-method-based RA algorithm, namely, Algorithm 1, is designed in Section 3.2. Moreover, algorithms called by Algorithm 1 are designed in Sections 3.3 and 3.4.

*3.1. Description of the RA Strategy and Network EE.* We first define the following variables to describe the RA strategy:

- (i)  $t_{klu} \in \{0, 1\}, \forall k, l, u$ : indicating subcarrier  $k$  in slot-1 is paired with subcarrier  $l$  in slot-2 when  $t_{klu} = 1$ .
- (ii)  $p_{klu} \geq 0, \forall k, l, u$ : indicating the total transmission power (i.e.,  $P_1 + P_2$  as mentioned in Section 2.2) for the subcarrier pair  $(k, l, u)$ .
- (iii)  $t_{klab} \in \{0, 1\}, \forall k, l, a, b$ : indicating that subcarrier  $k$  in slot-1 and subcarrier  $l$  in slot-2 are, respectively, allocated to user  $a$  and user  $b$  when  $t_{klab} = 1$ .
- (iv)  $p_{klab}^1 \geq 0$  and  $p_{klab}^2 \geq 0$ : respectively indicating the BS's transmission power for subcarrier  $k$  in slot-1 and  $l$  in slot-2 for the direct mode.

Let us define a RA strategy as  $\mathbf{S} = \{\mathbf{I}, \mathbf{P}\}$ , where  $\mathbf{I}$  collects all indicator variables and  $\mathbf{P}$  collects all power variables. A feasible  $\mathbf{S}$  must satisfy

$$\begin{aligned} t_{klu} &\in \{0, 1\}, \\ t_{klab} &\in \{0, 1\}, \\ &\forall k, l, u, a, b, \end{aligned} \quad (11)$$

$$\sum_l \left( \sum_u t_{klu} + \sum_{ab} t_{klab} \right) \leq 1, \quad \forall k, \quad (12)$$

$$\sum_k \left( \sum_u t_{klu} + \sum_{ab} t_{klab} \right) \leq 1, \quad \forall l, \quad (13)$$

$$\begin{aligned} p_{klu} &\geq 0, \\ p_{klab}^1 &\geq 0, \\ p_{klab}^2 &\geq 0, \\ &\forall k, l, u, a, b, \end{aligned} \quad (14)$$

where constraints (12) and (13) guarantee that each subcarrier in either slot-1 or slot-2 must operate in a single mode and be assigned to a single user.

For given  $\mathbf{S}$ , the network EE is formulated as

$$\eta(\mathbf{S}) = \frac{R(\mathbf{S})}{P(\mathbf{S})} \quad (\text{bits/Joule}), \quad (15)$$

where  $R(\mathbf{S})$  represents the sum rate for the network:

$$\begin{aligned} R(\mathbf{S}) &= \frac{B}{2K} \left( \sum_{klu} t_{klu} \mathcal{C}(p_{klu} G_{klu}) \right. \\ &\quad \left. + \sum_{klab} t_{klab} \left( \mathcal{C}(p_{klab}^1 G_{su}^k) + \mathcal{C}(p_{klab}^2 G_{su}^l) \right) \right), \end{aligned} \quad (16)$$

and  $P(\mathbf{S})$  is the sum power consumption for the network:

$$\begin{aligned} P(\mathbf{S}) &= \frac{1 + \alpha}{2} \left( \sum_{klu} t_{klu} p_{klu} + \sum_{klab} t_{klab} (p_{klab}^1 + p_{klab}^2) \right) \\ &\quad + P_{\text{cir}}, \end{aligned} \quad (17)$$

with  $\alpha$  being the loss factor of the power amplifiers (PA) used by the BS and RS and  $P_{\text{cir}}$  representing the total power consumption by the BS's and RS's circuit devices.

*3.2. Dinkelbach-Method-Based RA Algorithm Design.* We consider the EE maximization problem when the sum rate must be greater than a prescribed value  $R_{\text{req}}$ . This problem can be expressed as (P1):

$$\begin{aligned} \max_{\mathbf{S}} \quad & \eta(\mathbf{S}) \\ \text{s.t.} \quad & (11), (12), (13), (14) \\ & R(\mathbf{S}) \geq R_{\text{req}}. \end{aligned} \quad (18)$$

It can be seen that solving problem (P1) is highly challenging due to the following reasons:

- (i)  $\eta(\mathbf{S})$  has a fractional structure, which is highly non-linear.
- (ii) (P1) is a mixed-integer problem containing both binary and continuous variables.

To solve (P1) for the optimum RA (denoted by  $\mathbf{S}^*$ ) and the maximum EE (denoted by  $\eta^*$ ), we make use of the Dinkelbach method. To be more specific, we define a parameter  $\theta$  and a function  $F(\theta)$  as the optimum objective value for the following problem (P2):

$$\begin{aligned} F(\theta) &= \max_{\mathbf{S}} R(\mathbf{S}) - \theta P(\mathbf{S}) \\ \text{s.t.} \quad & (11), (12), (13), (14), (18), \end{aligned} \quad (19)$$

whose optimum solution is  $\mathbf{S}(\theta)$ .

Based on Dinkelbach method,  $\eta^*$  must satisfy  $F(\eta^*) = 0$ , and  $\mathbf{S}(\eta^*)$  is the optimum solution for problem (P1). Moreover,  $\eta^*$  can be found by iterative procedures as elaborated in [6].

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(1)  $\mu = 0$ ;
(2) solve (P3) for  $\mathbf{S}_\theta(0)$  using Algorithm 3;
(3) if  $\mathbf{S}_\theta(0)$  is feasible for (P2) then
(4)   output  $\mathbf{S}_\theta(0)$  as  $\mathbf{S}(\theta)$ 
(5) else
(6)    $\mu_{\min} = 0$ ;  $\mu_{\max}$  is set as a sufficiently large value;
(7)   while  $\mu_{\max} - \mu_{\min} > \delta$  do
(8)      $\mu = \frac{\mu_{\max} + \mu_{\min}}{2}$ ;
(9)     solve (P3) for  $\mathbf{S}_\theta(\mu)$  using Algorithm 3;
(10)    if  $\gamma_\theta(\mu) > 0$  then
(11)       $\mu_{\max} = \mu$ ;
(12)    else if  $\gamma_\theta(\mu) < 0$  then
(13)       $\mu_{\min} = \mu$ ;
(14)    else if  $\gamma_\theta(\mu) = 0$  then
(15)      break;
(16)    end if
(17)  end while
(18) end if
(19) Output  $\mathbf{S}_\theta(\mu)$  as  $\mathbf{S}(\theta)$ .

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ALGORITHM 2: The algorithm to solve (P2).

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(1) compute  $p_{klu}, p_{klab}^1, p_{klab}^2, \forall k, l, a, b, u$  with (25);
(2) compute  $A_{klu}, B_{klab}, \forall k, l, u, a, b$ ; and  $C_{kl}, \forall k, l$ ;
(3) solve (P5) for its optimum solution  $\{t_{kl}^* \mid \forall k, l\}$  with
    the Hungarian algorithm;
(4) construct the optimum  $\mathbf{I}$  for (P4) by assigning for every
    combination of  $k$  and  $l$ , all entries in  $\{t_{klu}, t_{klab} \mid \forall k, l\}$  to
    zero, except for the one with the metric equal to  $C_{kl}$  to  $t_{kl}^*$ .
(5)  $\mathbf{S}_1 = \{\mathbf{I}, \mathbf{P}_1\}$  is output as  $\mathbf{S}_\theta(\mu)$ .

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ALGORITHM 3: The algorithm to find  $\mathbf{S}_\theta(\mu)$ .

Motivated by the above principle, the RA algorithm to solve (P1) for the EE maximized RA strategy is summarized in Algorithm 1. The iterative update of  $\theta$  has a superlinear convergence rate [37]. We will elaborate on the design of Algorithm 2 to solve (P2) in Section 3.3.

**3.3. Design of Algorithm 2 to Solve (P2).** Using the same arguments as those in [38], we can show that (P2)'s duality gap is zero; therefore the dual method can be used to solve (P2). To this end, define

(i)  $\mu$  as Lagrange multiplier for the constraint (18);

(ii) Lagrangian function

$$L_\theta(\mu, \mathbf{S}) = (R(\mathbf{S}) - \theta P(\mathbf{S})) + \mu (R(\mathbf{S}) - R_{\text{req}}); \quad (20)$$

(iii) Lagrange relaxation problem for (P2) as (P3):

$$\begin{aligned} \max_{\mathbf{S}} \quad & L_\theta(\mu, \mathbf{S}) \\ \text{s.t.} \quad & (11), (12), (13), (14) \end{aligned} \quad (21)$$

For (P3), we define its optimum solution as  $\mathbf{S}_\theta(\mu)$  and its dual function as  $d_\theta(\mu) = L_\theta(\mu, \mathbf{S}_\theta(\mu))$ . It can be shown that the dual function is convex of  $\mu \geq 0$ , and

$$\gamma_\theta(\mu) = R(\mathbf{S}_\theta(\mu)) - R_{\text{req}} \quad (22)$$

is a subgradient of  $d_\theta(\mu)$  satisfying

$$\forall \mu', d_\theta(\mu') \geq d_\theta(\mu) + (\mu' - \mu) \gamma_\theta(\mu). \quad (23)$$

The key for the dual method is to find the dual optimum:

$$\mu_\theta^* = \arg \min_{\mu \geq 0} d_\theta(\mu), \quad (24)$$

and then  $\mathbf{S}_\theta(\mu_\theta^*)$  is the optimum solution for (P2).

According to the dual method,  $\mu_\theta^*$  is the  $\mu$  satisfying the following two conditions: (1)  $\mu \gamma_\theta(\mu) = 0$  (i.e., the complementary slackness condition) and (2)  $\gamma_\theta(\mu) \geq 0$  (i.e.,  $\mathbf{S}_\theta(\mu)$  is feasible for (P2)). To find  $\mu_\theta^*$ , we use the following method:

(i) First, we compute  $\mathbf{S}_\theta(0)$ . If  $\gamma_\theta(0) \geq 0$ ,  $\mu = 0$  and  $\mathbf{S}_\theta(0)$  satisfy the above conditions. Therefore,  $\mathbf{S}_\theta(0)$  is the optimum for (P2).

- (ii) Otherwise,  $\gamma_\theta(0) < 0$  and  $\mu_\theta^* > 0$  must hold. Note that  $\gamma_\theta(\mu)$  is increasing of  $\mu$ . Once  $\mu > 0$  satisfying  $\gamma_\theta(\mu) = 0$  is found,  $\mu$  and  $\mathbf{S}_\theta(\mu)$  satisfy the above two conditions and hence can be taken as  $\mu_\theta^*$  and  $\mathbf{S}(\theta)$ , respectively. We will find  $\mu > 0$  satisfying  $\gamma_\theta(\mu) = 0$  with the bisection method.

To complete this subsection, the above procedures to solve (P2) are summarized in Algorithm 2 as follows. Algorithm 2 has a polynomial complexity with respect to  $K$ . We will elaborate on the design of Algorithm 3 to solve (P3) in Section 3.4.

*3.4. Design of Algorithm 3 to Solve (P3).* We show how to find  $\mathbf{S}_\theta(\mu)$  as follows:

- (i) First, the optimum  $\mathbf{P}$  for (P3) with fixed  $\mathbf{I}$  is found and denoted by  $\mathbf{P}_\mathbf{I}$ .
- (ii) Second, define  $\mathbf{S}_\mathbf{I} = \{\mathbf{I}, \mathbf{P}_\mathbf{I}\}$ . Then we find the optimum  $\mathbf{I}$  to maximize  $L_\theta(\mu, \mathbf{S}_\mathbf{I})$  subject to the constraints on  $\mathbf{I}$  in (P3).
- (iii) Finally,  $\mathbf{S}_\mathbf{I}$  corresponding to this optimum  $\mathbf{I}$  can be taken as  $\mathbf{S}_\theta(\mu)$ .

As for the first step, the elements in the optimum  $\mathbf{P}_\mathbf{I}$  can be computed according to KKT conditions as follows [39]:

$$\begin{aligned} p_{klu} &= \Lambda(\theta, \mu, G_{klu}), \quad \forall k, l, u \\ p_{klab}^1 &= \Lambda(\theta, \mu, G_{su}^k), \quad \forall k, l, a, b \\ p_{klab}^2 &= \Lambda(\theta, \mu, G_{su}^l), \quad \forall k, l, a, b, \end{aligned} \quad (25)$$

where

$$\Lambda(\theta, \mu, G) = \left[ \frac{(\mu + 1)B \cdot \log_2(e)}{(1 + \alpha)\theta \cdot K} - \frac{1}{G} \right]^+ \quad (26)$$

As for the second step, it can readily be shown that

$$L_\theta(\mu, \mathbf{S}_\mathbf{I}) = -\mu R_{\text{req}} + \sum_{klu} t_{klu} A_{klu} + \sum_{klab} t_{klab} B_{klab}, \quad (27)$$

where

$$\begin{aligned} A_{klu} &= \frac{(\mu + 1)B}{2K} \mathcal{E}(G_{klu} \Lambda(\theta, \mu, G_{klu})) \\ &\quad - \frac{\theta(1 + \alpha)}{2} \Lambda(\theta, \mu, G_{klu}) \\ B_{klab} &= \frac{(\mu + 1)B}{2K} \mathcal{E}(G_{su}^k \Lambda(\theta, \mu, G_{su}^k)) \\ &\quad - \frac{\theta(1 + \alpha)}{2} \Lambda(\theta, \mu, G_{su}^k) \\ &\quad + \frac{(\mu + 1)B}{2K} \mathcal{E}(G_{su}^l \Lambda(\theta, \mu, G_{su}^l)) \\ &\quad - \frac{\theta(1 + \alpha)}{2} \Lambda(\theta, \mu, G_{su}^l) \end{aligned} \quad (28)$$

Finally, we find the optimum  $\mathbf{I}$  for maximizing  $L_\theta(\mu, \mathbf{S}_\mathbf{I})$ . This problem is equivalent to solving (P4):

$$\begin{aligned} \max_{\mathbf{I}, \{t_{kl} | \forall k, l\}} \quad & \sum_{kl} \sum_{uab} (t_{klu} A_{klu} + t_{klab} B_{klab}) \\ \text{s.t.} \quad & t_{klu} \geq 0, \\ & t_{klab} \geq 0, \end{aligned} \quad (29)$$

$\forall k, l, u, a, b,$

$$\sum_l t_{kl} = 1, \quad \forall k, \quad (30)$$

$$\sum_k t_{kl} = 1, \quad \forall l, \quad (31)$$

$$t_{kl} = \sum_u t_{klu} + \sum_{ab} t_{klab}, \quad \forall k, l \quad (32)$$

Note that

$$\sum_{uab} (t_{klu} A_{klu} + t_{klab} B_{klab}) \leq t_{kl} C_{kl} \quad (33)$$

holds, where  $C_{kl} = \max\{\max_u A_{klu}, \max_{a,b} B_{klab}\}$ . Call  $A_{klu}$  the metric for  $t_{klu}$  and  $B_{klab}$  the metric for  $t_{klab}$ ; the inequality is tightened when all entries of  $\{t_{klu}, t_{klab} \mid \forall u, a, b\}$  are assigned to zero, except that the one with the metric equal to  $C_{kl}$  is assigned to  $t_{kl}^*$ .

Therefore, after problem (P5)

$$\begin{aligned} \max_{\{t_{kl} | \forall k, l\}} \quad & \sum_{kl} t_{kl} C_{kl} \\ \text{s.t.} \quad & (30), (31), \quad t_{kl} \in \{0, 1\}, \quad \forall k, l \end{aligned} \quad (34)$$

is solved for its optimum solution  $\{t_{kl}^* \mid \forall k, l\}$ , an optimum (32) can be constructed by assigning for every combination of  $k$  and  $l$  all entries in  $\{t_{klu}, t_{klab} \mid \forall k, l\}$  to zero, except for the one with the metric equal to  $C_{kl}$  to  $t_{kl}^*$ .

Most interestingly, (P5) is a standard assignment problem; hence every entry in  $\{t_{kl}^* \mid \forall k, l\}$  is either 0 or 1 and  $\{t_{kl}^* \mid \forall k, l\}$  can be found efficiently by the Hungarian algorithm [40]. After knowing Hungarian algorithm, the optimum  $\mathbf{I}$  can be constructed according to the way mentioned earlier.

Motivated by the above principle, the method to solve (P2) is summarized in Algorithm 3 as follows. The complexity of computing  $A_{klu}$  and  $B_{klab}$  is  $O(K^2(U + U^2))$ . We use the Hungarian algorithm to solve (P5); the complexity is  $O(K^3)$ . As a result, the complexity of Algorithm 3 is  $O(K^2(U + U^2 + K))$ .

#### 4. Numerical Experiments and Discussions

We will first introduce system setup for numerical experiments, as well as two benchmark protocols for comparison purpose. Then, results and discussions are presented to show the impact of different parameters on the network EE.

TABLE 2: Network Parameters.

Meanings	Parameters	Values
Bandwidth of the network	$B$	20 MHz
Subcarriers' number	$K$	32
Loss factor of the PA	$\alpha$	0.3
Circuit power of BS and RS	$P_{\text{cir}}$	500 w
Radius of UR	$R$	0.2 km
Distance between BS and the UR center	$D$	1.5 km

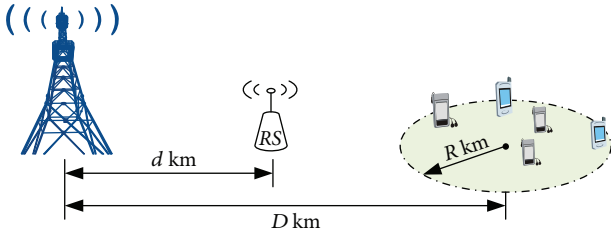


FIGURE 4: The downlink OFDMA system considered in numerical experiments.

**4.1. System Setup and Benchmark Protocols.** In numerical experiments, we consider the downlink OFDMA network with a RS exhibit in Figure 4.  $U$  represents the number of users in service and they are randomly distributed in a circular region of radius  $R$  km. The RS is located between the BS and the user-region (UR) center, and the BS-RS distance is  $d$  km. The distance between the BS and the user-region center is  $D$  km. The bandwidth of the system is  $B$  Hz, and the OFDMA uses  $K$  subcarriers. The network parameters are listed in Table 2.

The channels are independent of each other and are generated in the same way as in [7]. For every user  $u$ , the impulse response of the source-to- $u$  channel is modeled as a tapped delay line with  $L = 6$  taps, which are independently generated from circularly symmetric complex Gaussian distributions with zero mean and variance equal to  $(1/L)(d_{su}/d_{\text{ref}})^{-4}$ , where  $d_{\text{ref}} = 1$  km and  $d_{su}$  represents the source-to- $u$  distance. The source-to-relay and relay-to- $u$  channels are generated in the same way, with each tap having variance as  $(1/L)(d_{sr}/d_{\text{ref}})^{-4}$  and  $(1/L)(d_{ru}/d_{\text{ref}})^{-4}$ , respectively, where  $d_{ru}$  represents the relay-to- $u$  distance. The CSI  $\{h_{sr}^k | \forall k\}$ ,  $\{h_{su}^k | \forall k, u\}$ , and  $\{h_{ru}^k | \forall k, u\}$  are computed by making  $K$ -point FFT over the impulse response of the associated channels.

In order to illustrate the benefit of optimized subcarrier pairing and opportunistic relaying, we also consider two other benchmark protocols, namely, BP-1 and BP-2. BP-1 is similar to the considered protocol, except that subcarrier  $k$  in slot-1 can only be paired with subcarrier  $k$  in slot-2 in relay mode. BP-2 is a simplified version of the considered protocol, and the simplification lies in the fact that each subcarrier in every slot should be allocated to users in direct mode. The RA algorithms for both BP-1 and BP-2 can be derived in the same way as that for the considered protocol, and therefore the derivation is omitted for the sake of clarity.

TABLE 3: Complexity comparison.

Algorithm	Complexity
Proposed algorithm	$O(K^2(U + U^2 + K))$
BP-1	$O(K(U + U^2))$
BP-2	$O(KU)$

The complexity of the three algorithms is shown in Table 3. It can be seen that the proposed algorithm has the highest complexity, while the BP-2 algorithm has the lowest complexity.

**4.2. Impact of  $R_{\text{req}}$  on the Optimum EE and Corresponding Sum Rate.** To show the influence of  $R_{\text{req}}$  on the EE, we choose  $U = 10$  and  $d = 0.6$  km and then evaluate the average optimum EE for every protocol over 1000 random channel realizations, when  $R_{\text{req}}$  increases from 0 to 40 Mbits/s. The results are shown in Figure 5.

Compared with BP-1 and BP-2, we can see that the proposed protocol and algorithm always correspond to a higher average EE as shown in Figure 5(a). Since BP-2 does not utilize opportunistic relaying as the proposed protocol and BP-1, it is reasonable that BP-2 corresponds to much lower average EE. The proposed protocol can achieve higher EE than BP-1, because a subcarrier in every slot can be paired with the other slot's subcarrier freely.

Figure 5(a) also shows that the average EE of these methods decreases with the increase of  $R_{\text{req}}$ . This is because the feasible set of the problem shrinks with the increase of  $R_{\text{req}}$ . From Figures 5(a), 5(b), and 5(c), when  $R_{\text{req}} < 20$  Mbits/s and the average EE reaches the optimum value, the average communication rate is larger than  $R_{\text{req}}$ , the average total power remains stable, and the average EE of the network maintains high value. When  $R_{\text{req}} \geq 20$  Mbits/s and the average EE reaches the optimum value, the average communication rate is equal to  $R_{\text{req}}$ , the average total power increases rapidly, and the average EE of the network decreases. The above phenomenon indicates that the restricted condition  $R(\mathbf{S}) \geq R_{\text{req}}$  influences the choice of the optimum solution.

**4.3. Impact of Relay Position on the Optimum EE and Corresponding Sum Rate.** To show the impact of relay position on the EE, we choose  $U = 10$  and  $R_{\text{req}} = 20$  Mbits/s and then evaluate the average optimum EE for every protocol over 1000 random channel realizations, when  $d$  increases from 0.2 to 1.2 km. The results are shown in Figure 6.

It is shown that the proposed protocol leads to a higher average EE than the BP-1 and BP-2 for every relay position. Moreover, the average EE improves as the RS moves towards the middle region between the BS and the users. This can be interpreted as follows. In theory, the optimum EE enhances if  $\forall k, l, u, G_{klu}$  is more likely to take a high value. Note that  $G_{klu}$  takes a high value only if both  $G_{sr}^k$  and  $G_{ru}^l$  are much higher than  $G_{su}^k$ . When RS lies in the middle between the BS and the users' region, it is more likely to have  $G_{sr}^k$  and  $G_{ru}^l$ , both much greater than  $G_{su}^k$ , and thus  $G_{klu}$  is more likely to take a high value. Moreover, BP-2 is a direct transmission protocol; the

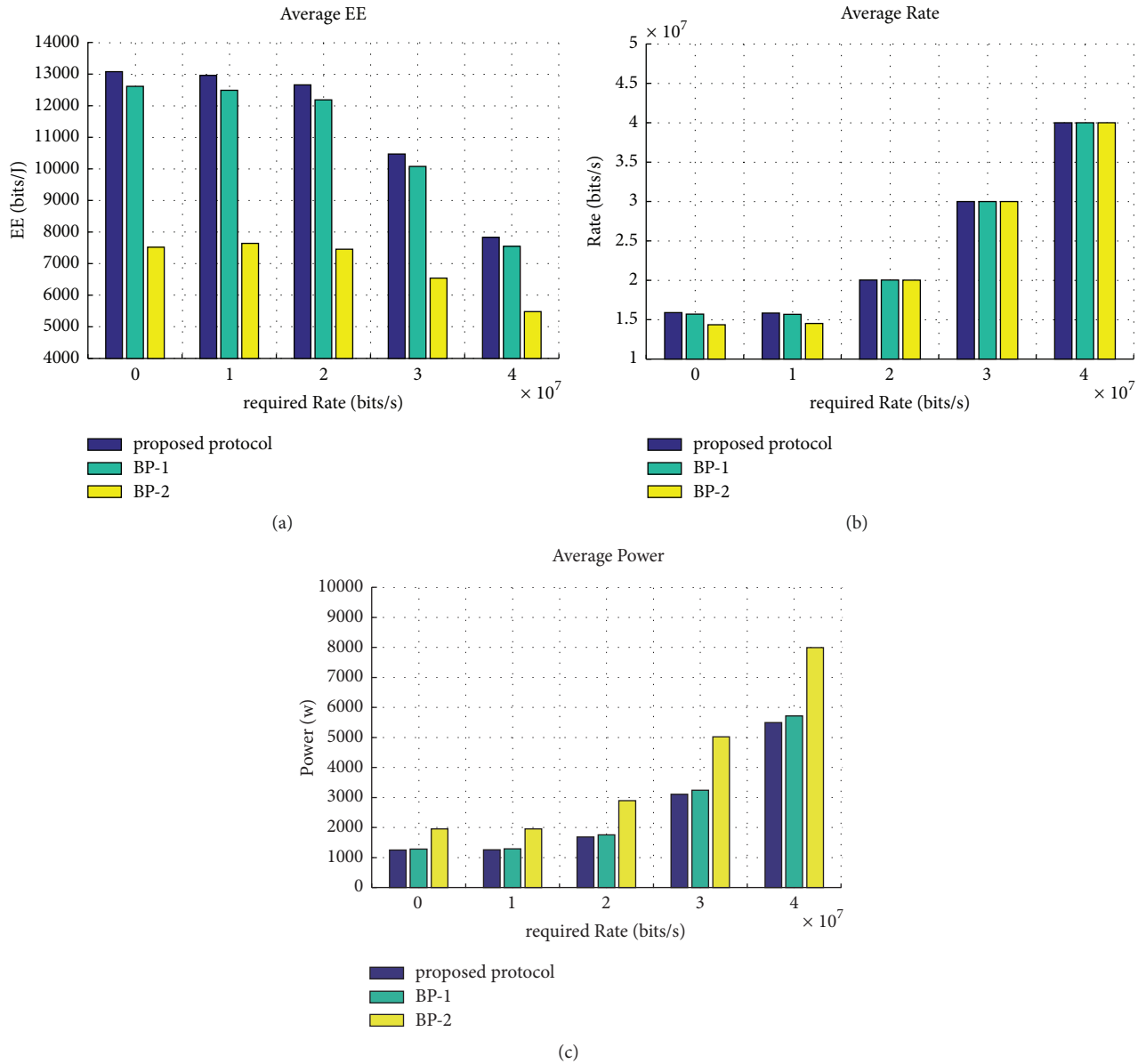


FIGURE 5: The average EE as the minimum rate changes.

RS does not help to transmit signals. It is reasonable that the average EE remains steady when the relay position changes.

**4.4. Impact of User Number on the Optimum EE and Corresponding Sum Rate.** To show the impact of user number on the EE, we choose  $d = 0.6$  km and  $R_{\text{req}} = 20$  Mbits/s and then evaluate the average optimum EE for every protocol over 1000 random channel realizations, when  $U$  increases from 5 to 30. The results are shown in Figure 7.

From Figure 7, we see that the average EEs of the three methods increase with the increase of user number. This is because when the number of users in the network increases, the subcarrier assignment has more flexibility. The numbers of  $A_{klu}$  and  $B_{klab}$  increase with the increase of user number,

which can improve the probability of  $C_{kl}$  taking a larger value. In this way, the average EE of the network will be improved.

## 5. Conclusions

We have addressed an EE maximized RA problem for cooperative OFDM transmission using the improved DF protocol with optimized subcarrier pairing when the network's communication rate is larger than a required value. The subcarrier-pair-based opportunistic DF relay-aided protocol has two operation modes: direct mode and relay mode. This scheme improves the flexibility of the communication network. Subcarriers can choose the mode that can improve the network's EE to send messages. Based on the above protocol,



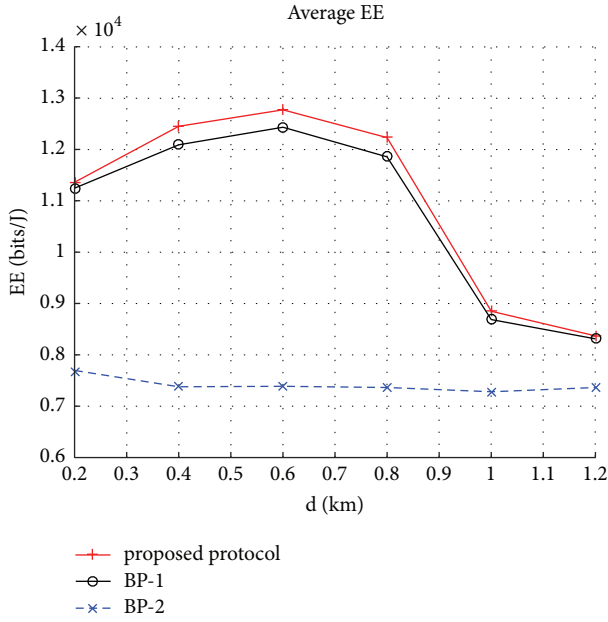


FIGURE 6: The average EE as the relay position changes.

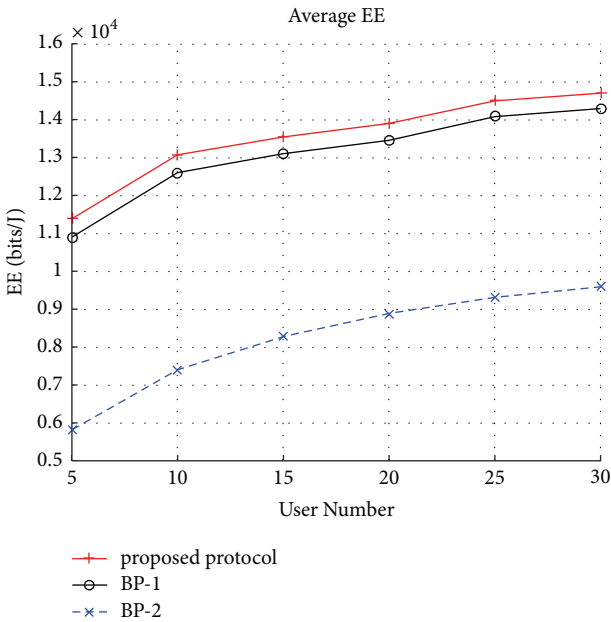


FIGURE 7: The average EE as the user number changes.

we formulate the optimization problem of maximizing the network's EE.

The problem is polynomial complexity, so we solve it with the following three steps. In the first step, we eliminate the fractional structure with the help of Dinkelbach method and transfer problem (P1) into problem (P2). In the second step, we get the Lagrangian function by using the dual method. In the third step, we use KKT conditions and Hungarian algorithm to solve the Lagrangian function. Then we can get the RA algorithm of maximizing the network's EE. Numerical experiments show that the proposed RA algorithm can

improve the EE of the downlink OFDMA networks. And the experiments also illustrated the impact of minimum required communication rate, relay position, and the user number. Theoretical analysis has been presented to interpret what is observed in numerical experiments.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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