

Research Article

Robust Destination Jamming Aided Secrecy Precoding for an AF MIMO Untrusted Relay System

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We consider a secrecy dual-hop amplify-and-forward (AF) untrusted relay network, where the relay is willing to forward the signal to the destination while acting as a potential eavesdropper. Assuming that all these nodes are equipped with multiple antennas, we propose a joint destination aided cooperative jamming and precoding at both the source and the relay scheme, with the objective to maximize the worst case secrecy rate. The formulated problem is highly non-convex due to the maximization of the difference of several logarithmic determinant (log-det) functions in the CSI uncertainty region. To handle this challenge, we propose to linearize these log-det terms. After linearization, we tackle the CSI uncertainty based on epigraph reformulation and the sign-definiteness lemma. Finally, an alternating optimization (AO) algorithm is proposed to solve the reformulated problem. Numerical results are provided to demonstrate the performance of the proposed scheme.

1. Introduction

Due to the openness of wireless transmission medium, wireless information is susceptible to eavesdropping. Thus, the secure communication is a critical issue for wireless systems [1]. Physical layer security (PLS) is a newly emerging secrecy communication technique [2]. Relay is considered as a popular approach to extend network coverage and provide spatial degrees of freedom (DoF) [3]. PLS method in relay networks has raised great attention [4].

In practice, relay is not always helpful for secure communication. A misbehaving relay may try to eavesdrop the confidential information from the source node, which is termed as untrusted relay [5]. The untrusted multipleinput multiple-output (MIMO) relay was first studied in [6], in which a joint beamforming design was proposed. Furthermore, the untrusted two-way MIMO relay systems are studied in [7, 8], while, in [9], the authors investigated a novel joint destination-aided cooperative jamming (CJ) and precoding scheme at both the source and the untrusted relay node, which was further extended to untrusted energy harvested relay network in [10]. Recently, in [11], the authors proposed a joint beamforming alignment method with suboptimal power allocation for a untrusted two-way relay. In [12], the authors investigated the secrecy performance of untrusted relay systems with a full-duplex jamming destination, while, in [13], the authors investigated the energy-efficient secure transmission design for the internet of things (IoT) with an untrusted relay. In [14], the authors investigated the secure transmission in untrusted wireless-powered two-way relay networks.

All these above literatures consider perfect channel state information (CSI) case. However, in practice, perfect CSI is hard to obtain due the channel estimation and quantization errors. To overcome this obstacle, in [15], the authors investigated the untrusted amplify-and-forward (AF) relay design with imperfect CSI, while, in [16], the authors proposed an artificial noise (AN) aided secure precoding for untrusted two-way MIMO relay systems with imperfect CSI.

Based on these observations, in this paper, we investigated the robust secrecy design in a untrusted AF MIMO relay network. Specifically, assuming that all the nodes are equipped with multiple antennas and only imperfect CSI



can be obtained, we propose a robust destination-aided CJ scheme to maximize the worst-case secrecy rate. However, the formulated problem is highly non-convex and challenging to solve. To handle this challenge, we propose to linearize the difference of multiple logarithmic determinant (log-det) functions. After linearization, we tackle the channel uncertainty based on epigraph reformulation and the sign-definiteness lemma. Finally, an alternating optimization (AO) method is proposed to solve the reformulated problem. Simulation results demonstrate the performance of our proposed design.

The rest of this paper is organized as follows. The system model and problem formulation are described in Section 2. The AO based method is discussed in Section 3. Numerical results are provided in Section 4, and Section 5 concludes the paper.

Notations. Throughout this paper, boldface lowercase and uppercase letters denote vectors and matrices, respectively. The transpose, conjugate transpose, and trace of matrix **A** are denoted as \mathbf{A}^T , \mathbf{A}^H , and $\operatorname{Tr}(\mathbf{A})$, respectively. $\mathbf{a} = \operatorname{vec}(\mathbf{A})$ denotes stacking the columns of matrix **A** into a vector **a**. $\mathbf{A} \geq \mathbf{0}$ indicates that **A** is a positive semidefinite matrix. $\|\cdot\|$ and \otimes represent the Frobenius norm and the Kronecker product, respectively. $\operatorname{Re}\{a\}$ denotes the real part of a complex variable a. $[x]^+$ and $\mathbb{E}[.]$ indicates $\max(0, x)$ and the statistical expectation, respectively. $\mathbf{0}_n$ denotes n by n zeros matrix. $\mathbf{0}_{m \times n}$ denotes m by n zeros matrix. \mathbf{I}_n denotes n by n identity matrix. $\mathbf{I}_{m \times n}$ denotes m by n identity matrix. $\mathcal{CN}(\mathbf{0}, \mathbf{I})$ denotes a circularly symmetric complex Gaussian random vector with mean **0** and covariance **I**.

2. System Model and Problem Statement

2.1. System Model. Let us consider a secure MIMO relay system, which consists of one source node (S), one relay (R), and one destination (D), as shown in Figure 1. It is assumed that S, R, and D are equipped with N_s , N_r , and N_d antennas, respectively. The channel coefficients between S to R, D to R, and R to D are denoted as $\mathbf{F} \in \mathbb{C}^{N_r \times N_s}$, $\mathbf{H} \in \mathbb{C}^{N_r \times N_d}$, and $\mathbf{G} \in \mathbb{C}^{N_d \times N_r}$, respectively.

In this paper, R is assumed to forward the signal to D and, at the same time, acts as an eavesdropper (Eve) which interprets the signal from S. Besides, we assume that only imperfect S, D, and R's CSIs can be obtained. Based on the bounded CSI model in [15, 16], the imperfect CSIs are modeled as

$$\mathscr{F} = \left\{ \mathbf{F} \mid \mathbf{F} = \overline{\mathbf{F}} + \Delta \mathbf{F}, \|\Delta \mathbf{F}\| \le \chi_F \right\}, \tag{1a}$$

$$\mathscr{H} = \left\{ \mathbf{H} \mid \mathbf{H} = \overline{\mathbf{H}} + \Delta \mathbf{H}, \|\Delta \mathbf{H}\| \le \chi_H \right\}, \qquad (1b)$$

$$\mathscr{G} = \left\{ \mathbf{G} \mid \mathbf{G} = \overline{\mathbf{G}} + \Delta \mathbf{G}, \|\Delta \mathbf{G}\| \le \chi_G \right\}, \qquad (1c)$$

where $\overline{\mathbf{F}}$, $\overline{\mathbf{H}}$, and $\overline{\mathbf{G}}$ denote the estimates of \mathbf{F} , \mathbf{H} , and \mathbf{G} , respectively; $\Delta \mathbf{F}$, $\Delta \mathbf{H}$, and $\Delta \mathbf{G}$ are their respective channel uncertainties; χ_F , χ_H , and χ_G denote the respective sizes of the bounded channel uncertainties region.

In the first phase, the signal vector $\mathbf{s} \in \mathbb{C}^{d \times 1}$ $(d \leq N_s)$ is precoded by the precoding matrix $\mathbf{W}_s \in \mathbb{C}^{N_s \times d}$, and the jamming vector $\mathbf{z} \in \mathbb{C}^{z \times 1}$ $(j \leq N_d)$ is emitted by D with precoding matrix $\mathbf{W}_d \in \mathbb{C}^{N_d \times z}$ to protect the confidential information. Both \mathbf{s} and \mathbf{z} satisfy $\mathscr{CN}(0, \mathbf{I})$. Thus, the received signal at R can be expressed as

$$\mathbf{y}_r = \mathbf{F}\mathbf{W}_s\mathbf{s} + \mathbf{H}\mathbf{W}_d\mathbf{z} + \mathbf{n}_r,\tag{2}$$

where $\mathbf{n}_r \in \mathbb{C}^{N_r \times 1}$ is the noise vector at the relay with $\mathbf{n}_r \sim \mathscr{CN}(\mathbf{0}, \sigma_r^2 \mathbf{I})$.

In the second phase, R utilizes precoding matrix W_r to forward the information; thus, the received signal at D can be expressed as

$$\mathbf{y}_{d} = \mathbf{GW}_{r} \left(\mathbf{FW}_{s} \mathbf{s} + \mathbf{HW}_{d} \mathbf{z} \right) + \mathbf{GW}_{r} \mathbf{n}_{r} + \mathbf{n}_{d}, \qquad (3)$$

where $\mathbf{n}_d \in \mathbb{C}^{N_r \times 1}$ is the noise vector at the destination with $\mathbf{n}_d \sim \mathscr{CN}(\mathbf{0}, \sigma_d^2 \mathbf{I})$.

Furthermore, \mathbf{y}_d can be expressed as

$$\mathbf{y}_{d} = \mathbf{G}\mathbf{W}_{r}\mathbf{F}\mathbf{W}_{s}\mathbf{s} + \underbrace{\mathbf{G}\mathbf{W}_{r}\mathbf{H}\mathbf{W}_{d}\mathbf{z}}_{self \ interference} + \Delta\mathbf{G}\mathbf{W}_{r}\mathbf{M}\mathbf{W}_{d}\mathbf{z} + \underbrace{\Delta\mathbf{G}\mathbf{W}_{r}\Delta\mathbf{H}\mathbf{W}_{d}\mathbf{z}}_{high \ order} + \mathbf{G}\mathbf{W}_{r}\mathbf{n}_{r} \qquad (4)$$

 $+ \mathbf{n}_d$.

Here, we assume that D can partially remove the self interference signal based on the estimated CSI, and the high-order term with respect to (w.r.t) the uncertain CSI is small enough compared with the channel strength. Thus, the received signal at D can be approximately given by

$$\overline{\mathbf{y}}_{d} = \mathbf{G}\mathbf{W}_{r}\mathbf{F}\mathbf{W}_{s}\mathbf{s} + \Delta\mathbf{G}\mathbf{W}_{r}\mathbf{H}\mathbf{W}_{d}\mathbf{z} + \mathbf{G}\mathbf{W}_{r}\Delta\mathbf{H}\mathbf{W}_{d}\mathbf{z} + \mathbf{G}\mathbf{W}_{r}\mathbf{n}_{r} + \mathbf{n}_{d},$$
(5)

Accordingly, the worst case secrecy rate can be expressed as

$$R_{s} = \min_{\forall \mathbf{F} \in \mathscr{F}, \forall \mathbf{H} \in \mathscr{H}, \forall \mathbf{G} \in \mathscr{G}} \left[R_{d} - R_{r} \right]^{+}, \tag{6}$$

where R_d and R_r denote the mutual information at R and D, respectively, and are given by

$$R_d = \frac{1}{2} \ln \left| \mathbf{I}_d + \mathbf{W}_s^H \mathbf{F}^H \mathbf{W}_r^H \mathbf{G}^H \mathbf{D}^{-1} \mathbf{G} \mathbf{W}_r \mathbf{F} \mathbf{W}_s \right|, \qquad (7a)$$

$$R_r = \frac{1}{2} \ln \left| \mathbf{I}_d + \mathbf{W}_s^H \mathbf{F}^H \mathbf{R}^{-1} \mathbf{F} \mathbf{W}_s \right|, \tag{7b}$$

with **D** and **R** being given as

$$\mathbf{D} = \Delta \mathbf{G} \mathbf{W}_{r} \overline{\mathbf{H}} \mathbf{W}_{d} \mathbf{W}_{d}^{H} \overline{\mathbf{H}}^{H} \mathbf{W}_{r}^{H} \Delta \mathbf{G}^{H}$$
$$+ \overline{\mathbf{G}} \mathbf{W}_{r} \Delta \mathbf{H} \mathbf{W}_{d} \mathbf{W}_{d}^{H} \Delta \mathbf{H}^{H} \mathbf{W}_{r}^{H} \overline{\mathbf{G}}^{H}$$
(8)

 $+ \sigma_r^2 \mathbf{G} \mathbf{W}_r \mathbf{W}_r^H \mathbf{G}^H + \sigma_d^2 \mathbf{I}_{N_d},$

and

$$\mathbf{R} = \sigma_r^2 \mathbf{I}_{N_r} + \mathbf{H} \mathbf{W}_d \mathbf{W}_d^H \mathbf{H}^H.$$
(9)

2.2. Problem Statement. In this paper, we investigated the joint precoding matrices design to maximize the worst case secrecy rate, under the power constraints at the respective nodes. Mathematically, the problem can be formulated as

$$\max_{W_s, W_r, W_d \forall \mathbf{F} \in \mathscr{F}, \forall \mathbf{H} \in \mathscr{H}, \forall \mathbf{G} \in \mathscr{G}} \min_{R_d - R_r,$$
(10a)

s.t.
$$\|\mathbf{W}_{s}\|^{2} \leq P_{s},$$
 (10b)
 $\|\mathbf{W}_{d}\|^{2} \leq P_{d},$
 $\|\mathbf{W}_{r}\mathbf{F}\mathbf{W}_{s}\|^{2} + \|\mathbf{W}_{r}\mathbf{H}\mathbf{W}_{d}\|^{2}$
 $+ \sigma_{r}^{2} \|\mathbf{W}_{r}\|^{2} \leq P_{r},$ (10c)

where P_s , P_d , and P_r are the maximum achievable powers for S, D, and R, respectively.

3. An Alternating Optimization (AO) Method

The formulated problem is highly non-convex due to the maximization of the difference of several log-det functions in the CSI uncertainty region. In this section, we will propose to linearize these log-det terms and tackle the CSI uncertainty based on epigraph reformulation.

Firstly, we introduce the following Lemma.

Lemma 1 (see [17]). Define m by m matrix function,

$$\mathbb{E} (\mathbf{U}, \mathbf{V}) \triangleq \mathbf{U}^{H} \mathbf{N} \mathbf{U} + \left(\mathbf{I}_{m} - \mathbf{U}^{H} \mathbf{M} \mathbf{V}\right) \left(\mathbf{I}_{m} - \mathbf{U}^{H} \mathbf{M} \mathbf{V}\right)^{H},$$
(11)

where N is any positive definite matrix. Then, the following three equations hold true.

Equation 1. For any positive definite matrix $\mathbf{S} \in \mathbb{C}^{m \times m}$, we have

$$\mathbf{S}^{-1} = \underset{\mathbf{T} \succ \mathbf{0}}{\operatorname{arg max} \ln |\mathbf{T}|} - \operatorname{Tr}(\mathbf{TS}), \qquad (12)$$

and

$$-\ln |\mathbf{S}| = \max_{\mathbf{T} > \mathbf{0}} \ln |\mathbf{T}| - \mathrm{Tr} (\mathbf{TS}) + m.$$
(13)

Equation 2. For any positive definite matrix T, we have

$$\widetilde{\mathbf{U}} \triangleq \arg\min_{\mathbf{U}} \operatorname{Tr} \left(\mathbf{T} \mathbb{E} \left(\mathbf{U}, \mathbf{V}\right)\right)$$

$$= \left(\mathbf{N} + \mathbf{M} \mathbf{V} \mathbf{V}^{H} \mathbf{M}^{H}\right)^{-1} \mathbf{M} \mathbf{V},$$
(14)

and

$$\mathbb{E}\left(\widetilde{\mathbf{U}},\mathbf{V}\right) = \mathbf{I}_m - \widetilde{\mathbf{U}}^H \mathbf{M} \mathbf{V} = \left(\mathbf{I}_m + \mathbf{V}^H \mathbf{M}^H \mathbf{N}^{-1} \mathbf{M} \mathbf{V}\right)^{-1}.$$
 (15)

Equation 3. We have

$$\ln \left| \mathbf{I}_{m} + \mathbf{M} \mathbf{V} \mathbf{V}^{H} \mathbf{M}^{H} \mathbf{N}^{-1} \right|$$

= arg max ln |T| - Tr (TE) + m. (16)
T>0,U

Equations 1 and 2 can be proven by the first-order optimality condition, while Equation 3 directly follows from Equations 1 and 2 and the identity $\ln |\mathbf{I}_m + \mathbf{AB}| = \ln |\mathbf{I}_n + \mathbf{BA}|$.

Equations (10a), (10b), and (10c) are hard to handle due to the coupled variables and non-convex objective and constraints. In the following, we will decouple (10a), (10b), and (10c) based on these above equations.

To utilize the above equations, we rewritten R_s as

$$= \frac{1}{2} \underbrace{\ln \left| \mathbf{I}_{d} + \mathbf{GW}_{r} \mathbf{FW}_{s} \left(\mathbf{GW}_{r} \mathbf{FW}_{s} \right)^{H} \mathbf{D}^{-1} \right|}_{\triangleq f_{1}} + \frac{1}{2} \underbrace{\ln \left| \mathbf{I}_{d} + \sigma_{r}^{-2} \mathbf{HW}_{d} \mathbf{W}_{d}^{H} \mathbf{H}^{H} \right|}_{\triangleq f_{2}} - \frac{1}{2} \underbrace{\ln \left| \mathbf{I}_{d} + \sigma_{r}^{-2} \mathbf{FW}_{s} \mathbf{W}_{s}^{H} \mathbf{F}^{H} + \sigma_{r}^{-2} \mathbf{HW}_{d} \mathbf{W}_{d}^{H} \mathbf{H}^{H} \right|}_{\triangleq f_{3}},$$

$$(17)$$

where

 R_s

$$f_{1} = \max_{\mathbf{V}_{1} \succ \mathbf{0}, \mathbf{U}_{1}} \ln |\mathbf{V}_{1}| - \operatorname{Tr} \left(\mathbf{V}_{1} \mathbb{E}_{1} \left(\mathbf{U}_{1}, \mathbf{W}_{s}, \mathbf{W}_{d}, \mathbf{W}_{r}\right)\right) + d,$$
(18a)

$$f_{2} = \max_{\mathbf{V}_{2} \succ \mathbf{0}, \mathbf{U}_{2}} \ln |\mathbf{V}_{2}| - \operatorname{Tr} \left(\mathbf{V}_{2} \mathbb{E}_{2} \left(\mathbf{U}_{2}, \mathbf{W}_{d}\right)\right) + N_{r}, \qquad (18b)$$

$$f_{3} = \max_{\mathbf{V}_{3} \succ \mathbf{0}} |\mathbf{V}_{3}| - \operatorname{Tr}\left(\mathbf{V}_{3}\left(\mathbf{I}_{N_{r}} + \sigma_{r}^{-2}\mathbf{F}\mathbf{W}_{s}\mathbf{W}_{s}^{H}\mathbf{F}^{H} + \sigma_{r}^{-2}\mathbf{H}\mathbf{W}_{d}\mathbf{W}_{d}^{H}\mathbf{H}^{H}\right)\right) + N_{e}.$$
(18c)

Furthermore, the matrix functions \mathbb{E}_1 and \mathbb{E}_2 are as follows:

$$\mathbb{E}_{1} \left(\mathbf{U}_{1}, \mathbf{W}_{s}, \mathbf{W}_{d}, \mathbf{W}_{r} \right) \triangleq \left(\mathbf{I}_{N_{d} \times d} - \mathbf{U}_{1}^{H} \mathbf{G} \mathbf{W}_{r} \mathbf{F} \mathbf{W}_{s} \right)$$

$$\cdot \left(\mathbf{I}_{N_{d} \times d} - \mathbf{U}_{1}^{H} \mathbf{G} \mathbf{W}_{r} \mathbf{F} \mathbf{W}_{s} \right)^{H} + \mathbf{U}_{1}^{H} \mathbf{D} \mathbf{U}_{1},$$

$$\mathbb{E}_{2} \left(\mathbf{U}_{2}, \mathbf{W}_{d} \right) \triangleq \left(\mathbf{I}_{N_{r} \times z} - \sigma_{r}^{-1} \mathbf{U}_{2}^{H} \mathbf{H} \mathbf{W}_{d} \right)$$

$$\cdot \left(\mathbf{I}_{N_{r} \times z} - \sigma_{r}^{-1} \mathbf{U}_{2}^{H} \mathbf{H} \mathbf{W}_{d} \right)^{H} + \mathbf{U}_{2}^{H} \mathbf{U}_{2}.$$
(19a)
(19b)

Based on the above relationships, (10a), (10b), and (10c) can be rewritten as

$$\max_{\mathbf{w}_{s},\mathbf{w}_{r},\mathbf{w}_{d},\{\mathbf{V}_{1},\mathbf{V}_{2},\mathbf{V}_{3}\}>\mathbf{0},\mathbf{U}_{1},\mathbf{U}_{2},\forall\mathsf{Fe}\mathscr{F},\forall\mathsf{He}\mathscr{F},\forall\mathsf{He}\mathscr{F},\forall\mathsf{Ge}\mathscr{G}} \min |\mathbf{V}_{1}|
+ \ln |\mathbf{V}_{2}| + \ln |\mathbf{V}_{3}| - \beta_{1} - \beta_{2} - \beta_{3},$$
(20a)

s.t.
$$\operatorname{Tr}\left(\mathbf{V}_{1}\mathbb{E}_{1}\left(\mathbf{U}_{1},\mathbf{W}_{s},\mathbf{W}_{d},\mathbf{W}_{r}\right)\right) \leq \beta_{1},$$
 (20b)

$$\operatorname{Tr}\left(\mathbf{V}_{2}\mathbb{E}_{2}\left(\mathbf{U}_{2},\mathbf{W}_{d}\right)\right) \leq \beta_{2},$$
(20c)

$$\operatorname{Tr}\left(\mathbf{V}_{3}\left(\mathbf{I}_{N_{r}}+\sigma_{r}^{-2}\mathbf{F}\mathbf{W}_{s}\mathbf{W}_{s}^{H}\mathbf{F}^{H}+\sigma_{r}^{-2}\mathbf{H}\mathbf{W}_{d}\mathbf{W}_{d}^{H}\mathbf{H}^{H}\right)\right)\leq\beta_{3},\qquad(20d)$$

Furthermore, by introducing auxiliary variables $\{\alpha_i\}_{i=0}^9$, $\{\beta_k\}_{k=1}^3$ and denoting $\{\mathbf{S}_1 \geq \mathbf{0}_{N_d}, \mathbf{S}_1 = \mathbf{V}_1^{1/2}\}$, $\{\mathbf{S}_i \geq \mathbf{0}_{N_r}, \mathbf{S}_i = \mathbf{V}_i^{1/2}\}_{i=2}^3$, we obtain the following relationships:

$$\operatorname{Tr}\left(\mathbf{S}_{1}\mathbf{S}_{1}^{H}\left(\mathbf{I}_{N_{d}\times d}-\mathbf{U}_{1}^{H}\mathbf{G}\mathbf{W}_{r}\mathbf{F}\mathbf{W}_{s}\right)\right.$$

$$\left.\left(\mathbf{I}_{N_{d}\times d}-\mathbf{U}_{1}^{H}\mathbf{G}\mathbf{W}_{r}\mathbf{F}\mathbf{W}_{s}\right)^{H}\right)\leq\alpha_{0},$$
(21a)

$$\operatorname{Tr}\left(\mathbf{S}_{1}\mathbf{S}_{1}^{H}\mathbf{U}_{1}^{H}\mathbf{U}_{1}\right) \leq \alpha_{1},$$
(21b)

$$\operatorname{Tr}\left(\mathbf{S}_{1}\mathbf{S}_{1}^{H}\mathbf{U}_{1}^{H}\Delta\mathbf{G}\mathbf{W}_{r}\overline{\mathbf{H}}\mathbf{W}_{d}\mathbf{W}_{d}^{H}\overline{\mathbf{H}}^{H}\mathbf{W}_{r}^{H}\Delta\mathbf{G}^{H}\mathbf{U}_{1}\right)$$
(21c)

$$\leq \alpha_2$$
,

$$\operatorname{Tr}\left(\mathbf{S}_{1}\mathbf{S}_{1}^{H}\mathbf{U}_{1}^{H}\overline{\mathbf{G}}\mathbf{W}_{r}\Delta\mathbf{H}\mathbf{W}_{d}\mathbf{W}_{d}^{H}\Delta\mathbf{H}^{H}\mathbf{W}_{r}^{H}\overline{\mathbf{G}}^{H}\mathbf{U}_{1}\right)$$
(21d)

$$\leq \alpha_3,$$

$$\operatorname{Tr}\left(\mathbf{S}_{1}\mathbf{S}_{1}^{H}\mathbf{U}_{1}^{H}\mathbf{G}\mathbf{W}_{r}\mathbf{W}_{r}^{H}\mathbf{G}^{H}\mathbf{U}_{1}\right) \leq \alpha_{4},$$
(21e)

$$\alpha_0 + \left(1 + \sigma_d^2\right)\alpha_1 + \alpha_2 + \alpha_3 + \sigma_r^2\alpha_4 \le \beta_1, \tag{21f}$$

$$\operatorname{Tr}\left(\mathbf{S}_{2}\mathbf{S}_{2}^{H}\left(\mathbf{I}_{N_{r}\times z}-\sigma_{r}^{-1}\mathbf{U}_{2}^{H}\mathbf{H}\mathbf{W}_{d}\right)$$
(21g)

$$\cdot \left(\mathbf{I}_{N_r \times z} - \sigma_r^{-1} \mathbf{U}_2^H \mathbf{H} \mathbf{W}_d \right)^H \right) \le \alpha_5,$$

$$\operatorname{Tr}\left(\mathbf{S}_{2}\mathbf{S}_{2}^{H}\mathbf{U}_{2}^{H}\mathbf{U}_{2}\right) \leq \alpha_{6},$$
(21h)

$$\alpha_5 + \alpha_6 \le \beta_2, \tag{21i}$$

$$\operatorname{Tr}\left(\mathbf{S}_{3}\mathbf{S}_{3}^{H}\right) \leq \alpha_{7},\tag{21j}$$

$$\operatorname{Tr}\left(\mathbf{S}_{3}\mathbf{S}_{3}^{H}\mathbf{F}\mathbf{W}_{s}\mathbf{W}_{s}^{H}\mathbf{F}^{H}\right) \leq \alpha_{8},$$
(21k)

$$\operatorname{Tr}\left(\mathbf{S}_{3}\mathbf{S}_{3}^{H}\mathbf{H}\mathbf{W}_{d}\mathbf{W}_{d}^{H}\mathbf{H}^{H}\right) \leq \alpha_{9},$$
(211)

$$\alpha_7 + \sigma_r^{-2} \alpha_8 + \sigma_r^{-2} \alpha_9 \le \beta_3, \tag{21m}$$

Equations (21a)–(21m) are still hard to handle due to the CSI uncertainty. Next, we will transform (21a)–(21m) into a more convenient reformulation and employ the following sign-definiteness lemma to handle the CSI uncertainty.

Lemma 2 (the sign-definiteness Lemma [18]). Given a Hermitian matrix **A** and arbitrary matrices $\{\mathbf{P}_i, \mathbf{Q}_i\}_{i=1}^N$, the semi-infinite linear matrix inequality (LMI)

$$\mathbf{A} \succeq \sum_{i=1}^{N} \left(\mathbf{P}_{i}^{H} \mathbf{X}_{i} \mathbf{Q}_{i} + \mathbf{Q}_{i}^{H} \mathbf{X}_{i}^{H} \mathbf{P}_{i} \right), \quad \forall \mathbf{X}_{i} : \left\| \mathbf{X}_{i} \right\| \leq \chi_{i}$$
(22)

holds if and only if there exist real numbers $\{\lambda_i \geq 0\}_{i=1}^N$ such that

$$\begin{bmatrix} \mathbf{A} - \sum_{i=1}^{N} \lambda_i \mathbf{Q}_i^H \mathbf{Q}_i & -\chi_1 \mathbf{P}_1^H & \dots & -\chi_N \mathbf{P}_N^H \\ -\chi_1 \mathbf{P}_1 & \lambda_1 \mathbf{I} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ -\chi_N \mathbf{P}_N & \mathbf{0} & \dots & \lambda_N \mathbf{I} \end{bmatrix} \succeq \mathbf{0} \quad (23)$$

Firstly, via the Schur complement [19], (21a) can be transformed into

$$\operatorname{Tr}\left(\mathbf{S}_{1}\mathbf{S}_{1}^{H}\left(\mathbf{I}_{N_{d}\times d}-\mathbf{U}_{1}^{H}\mathbf{G}\mathbf{W}_{r}\mathbf{F}\mathbf{W}_{s}\right)\left(\mathbf{I}_{N_{d}\times d}-\mathbf{U}_{1}^{H}\mathbf{G}\mathbf{W}_{r}\mathbf{F}\mathbf{W}_{s}\right)^{H}\right) \leq \alpha_{0} \Longrightarrow$$

$$\begin{bmatrix}\alpha_{0}\mathbf{I}_{N_{d}} \qquad \mathbf{S}_{1}^{H}\left(\mathbf{I}_{N_{d}\times d}-\mathbf{U}_{1}^{H}\left(\overline{\mathbf{G}}+\Delta\mathbf{G}\right)\mathbf{W}_{r}\left(\overline{\mathbf{F}}+\Delta\mathbf{F}\right)\mathbf{W}_{s}\right)\\\left(\mathbf{I}_{N_{d}\times d}-\mathbf{U}_{1}^{H}\left(\overline{\mathbf{G}}+\Delta\mathbf{G}\right)\mathbf{W}_{r}\left(\overline{\mathbf{F}}+\Delta\mathbf{F}\right)\mathbf{W}_{s}\right)^{H}\mathbf{S}_{1} \qquad \mathbf{I}_{d} \end{bmatrix} \geq \mathbf{0}_{N_{d}+d} \Longrightarrow$$

$$\begin{bmatrix}\alpha_{0}\mathbf{I}_{N_{d}} \qquad \mathbf{S}_{1}^{H}\left(\mathbf{I}_{N_{d}\times d}-\mathbf{U}_{1}^{H}\overline{\mathbf{G}}\mathbf{W}_{r}\overline{\mathbf{F}}\mathbf{W}_{s}\right)\\\left(\mathbf{I}_{N_{d}\times d}-\mathbf{U}_{1}^{H}\overline{\mathbf{G}}\mathbf{W}_{r}\overline{\mathbf{F}}\mathbf{W}_{s}\right)^{H}\mathbf{S}_{1} \qquad \mathbf{I}_{d} \end{bmatrix} \geq \begin{bmatrix}\mathbf{0}_{N_{d}} \qquad \mathbf{S}_{1}^{H}\mathbf{U}_{1}^{H}\overline{\mathbf{G}}\mathbf{W}_{r}\Delta\mathbf{F}\mathbf{W}_{s}\\\mathbf{W}_{s}^{H}\Delta\mathbf{F}^{H}\mathbf{W}_{r}^{H}\overline{\mathbf{G}}^{H}\mathbf{U}_{1}\mathbf{S}_{1} \qquad \mathbf{0}_{d} \end{bmatrix}$$

$$+\begin{bmatrix}\mathbf{0}_{N_{d}} \qquad \mathbf{S}_{1}^{H}\mathbf{U}_{1}^{H}\Delta\mathbf{G}\mathbf{W}_{r}\overline{\mathbf{F}}\mathbf{W}_{s}\\\mathbf{W}_{s}^{H}\overline{\mathbf{F}}^{H}\mathbf{W}_{r}^{H}\Delta\mathbf{G}^{H}\mathbf{U}_{1}\mathbf{S}_{1} \qquad \mathbf{0}_{d}\end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \mathbf{0}_{N_{d} \times N_{s}} \\ \mathbf{W}_{s}^{H} \end{bmatrix}}_{N_{2}} \Delta \mathbf{F}^{H} \begin{bmatrix} \mathbf{W}_{r}^{H} \overline{\mathbf{G}}^{H} \mathbf{U}_{1} \mathbf{S}_{1} & \mathbf{0}_{N_{r} \times d} \end{bmatrix} + \begin{bmatrix} \mathbf{S}_{1}^{H} \mathbf{U}_{1}^{H} \overline{\mathbf{G}} \mathbf{W}_{r} \\ \mathbf{0}_{d \times N_{r}} \end{bmatrix} \Delta \mathbf{F} \begin{bmatrix} \mathbf{0}_{N_{s} \times N_{d}} & \mathbf{W}_{s} \end{bmatrix}}_{N_{2}}$$
$$+ \underbrace{\begin{bmatrix} \mathbf{0}_{N_{d} \times N_{r}} \\ \mathbf{W}_{s}^{H} \overline{\mathbf{F}}^{H} \mathbf{W}_{r}^{H} \end{bmatrix}}_{N_{3}} \Delta \mathbf{G}^{H} \begin{bmatrix} \mathbf{U}_{1} \mathbf{S}_{1} & \mathbf{0}_{N_{d} \times d} \end{bmatrix} + \begin{bmatrix} \mathbf{S}_{1}^{H} \mathbf{U}_{1}^{H} \\ \mathbf{0}_{d \times N_{d}} \end{bmatrix}}_{N_{3}} \Delta \mathbf{G} \begin{bmatrix} \mathbf{0}_{N_{r} \times N_{d}} & \mathbf{W}_{r} \overline{\mathbf{F}} \mathbf{W}_{s} \end{bmatrix}}_{N_{3}}$$
(24)

To handle the uncertainties with respect to ΔF and $\Delta G,$ we denote

$$\mathbf{A} = \begin{bmatrix} \alpha_0 \mathbf{I}_{N_d} & \mathbf{S}_1^H \left(\mathbf{I}_{N_d \times d} - \mathbf{U}_1^H \overline{\mathbf{G}} \mathbf{W}_r \overline{\mathbf{F}} \mathbf{W}_s \right) \\ \left(\mathbf{I}_{d \times N_d} - \mathbf{W}_s^H \overline{\mathbf{F}}^H \mathbf{W}_r^H \overline{\mathbf{G}}^H \mathbf{U}_1 \right) \mathbf{S}_1 & \mathbf{I}_d \end{bmatrix}$$
(25)

and $\mathbf{P}_1 = [\mathbf{0}_{N_s \times N_d} \ \mathbf{W}_s], \ \mathbf{P}_2 = [\mathbf{0}_{N_r \times N_d} \ \mathbf{W}_r \overline{\mathbf{F}} \mathbf{W}_s], \ \mathbf{Q}_1 = [\mathbf{W}_r^H \overline{\mathbf{G}}^H \mathbf{U}_1 \mathbf{S}_1 \ \mathbf{0}_{N_r \times d}], \ \mathbf{Q}_2 = [\mathbf{U}_1 \mathbf{S}_1 \ \mathbf{0}_{N_d \times d}].$

Then, based on Lemma 2, $\mathbf{N}_1 \succeq \mathbf{N}_2 + \mathbf{N}_3$ can be reformulated as

$$\begin{bmatrix} \mathbf{A} - \lambda_1 \mathbf{Q}_1^H \mathbf{Q}_1 - \lambda_2 \mathbf{Q}_2^H \mathbf{Q}_2 & -\chi_F \mathbf{P}_1^H & -\chi_G \mathbf{P}_2^H \\ -\chi_F \mathbf{P}_1 & \lambda_1 \mathbf{I}_{N_s} & \mathbf{0}_{N_s \times N_r} \\ -\chi_G \mathbf{P}_2 & \mathbf{0}_{N_r \times N_s} & \lambda_2 \mathbf{I}_{N_r} \end{bmatrix}$$
(26)

$$\geq \mathbf{0}_{d+N_s+N_d+N_r}$$

where $\{\lambda_1 \geq 0, \lambda_2 \geq 0\}$ are introduced auxiliary variables.

$$\begin{bmatrix} \alpha_{0}\mathbf{I}_{N_{d}} - \lambda_{2}\mathbf{S}_{1}^{H}\mathbf{U}_{1}^{H}\mathbf{U}_{1}\mathbf{S}_{1} & \mathbf{S}_{1}^{H}\left(\mathbf{I}_{N_{d}\times d} - \mathbf{U}_{1}^{H}\overline{\mathbf{G}}\mathbf{W}_{r}\overline{\mathbf{F}}\mathbf{W}_{s}\right) & \mathbf{0}_{N_{d}\times N_{s}} & \mathbf{0}_{N_{d}\times N_{r}} \\ \begin{pmatrix} -\lambda_{1}\mathbf{S}_{1}^{H}\mathbf{U}_{1}^{H}\overline{\mathbf{G}}\mathbf{W}_{r}\mathbf{W}_{r}^{H}\overline{\mathbf{G}}^{H}\mathbf{U}_{1}\mathbf{S}_{1} & \mathbf{I}_{d} & -\chi_{F}\mathbf{W}_{s}^{H} - \chi_{G}\mathbf{W}_{s}^{H}\overline{\mathbf{F}}^{H}\mathbf{W}_{r}^{H} \\ \begin{pmatrix} \mathbf{0}_{N_{s}\times N_{d}} & -\chi_{F}\mathbf{W}_{s} & \lambda_{1}\mathbf{I}_{N_{s}} & \mathbf{0}_{N_{s}\times N_{r}} \\ \mathbf{0}_{N_{r}\times N_{d}} & -\chi_{G}\mathbf{W}_{r}\overline{\mathbf{F}}\mathbf{W}_{s} & \mathbf{0}_{N_{r}\times N_{s}} & \lambda_{2}\mathbf{I}_{N_{r}} \end{bmatrix} \geq \mathbf{0}_{d+N_{s}+N_{d}+N_{r}}$$
(27)

Similarly, the constraints (21b)–(21e), (21g), (21h), and (21j)–(21l) can be transformed into the following LMIs:

$$\begin{bmatrix} \alpha_1 \mathbf{I}_{N_d} & \mathbf{S}_1^H \mathbf{U}_1^H \\ \mathbf{U}_1 \mathbf{S}_1 & \mathbf{I}_{N_d} \end{bmatrix} \succeq \mathbf{0}_{2N_d}$$
(28)

$$\begin{bmatrix} \alpha_{2}\mathbf{I}_{z} & \mathbf{0}_{z \times N_{d}} & -\chi_{G}\mathbf{W}_{d}^{H}\overline{\mathbf{H}}^{H}\mathbf{W}_{r}^{H} \\ \mathbf{0}_{N_{d} \times z} & \mathbf{I}_{N_{d}} - \lambda_{3}\mathbf{S}_{1}^{H}\mathbf{U}_{1}^{H}\mathbf{U}_{1}\mathbf{S}_{1} & \mathbf{0}_{N_{d} \times N_{r}} \\ -\chi_{G}\mathbf{W}_{r}\overline{\mathbf{H}}\mathbf{W}_{d} & \mathbf{0}_{N_{r} \times N_{d}} & \lambda_{3}\mathbf{I}_{N_{r}} \end{bmatrix} \succeq \mathbf{0}_{N_{d} + N_{r} + z}$$
(29)

By substituting **A**, \mathbf{P}_1 , \mathbf{P}_2 , \mathbf{Q}_1 , and \mathbf{Q}_2 into (25), (26) can be recast as

$$\begin{bmatrix} \alpha_{3}\mathbf{I}_{N_{d}} & \mathbf{0}_{N_{d}\times z} & -\chi_{H}\mathbf{W}_{d}^{H} \\ \mathbf{0}_{z\times N_{d}} & \mathbf{I}_{z} - \lambda_{4}\mathbf{S}_{1}^{H}\mathbf{U}_{1}^{H}\overline{\mathbf{G}}\mathbf{W}_{r}\mathbf{W}_{r}^{H}\overline{\mathbf{G}}^{H}\mathbf{U}_{1}\mathbf{S}_{1} & \mathbf{0}_{N_{d}} \\ -\chi_{H}\mathbf{W}_{d} & \mathbf{0}_{N_{d}} & \lambda_{4}\mathbf{I}_{N_{d}} \end{bmatrix} \succeq \mathbf{0}_{2N_{d}+z}$$
(30)

$$\begin{bmatrix} \alpha_{4}\mathbf{I}_{N_{d}} - \lambda_{5}\mathbf{S}_{1}^{H}\mathbf{U}_{1}^{H}\mathbf{U}_{1}\mathbf{S}_{1} \quad \mathbf{S}_{1}^{H}\mathbf{U}_{1}^{H}\overline{\mathbf{G}}\mathbf{W}_{r} \quad \mathbf{0}_{N_{d}\times N_{r}} \\ \mathbf{W}_{r}^{H}\overline{\mathbf{G}}^{H}\mathbf{U}_{1}\mathbf{S}_{1} \qquad \mathbf{I}_{N_{r}} \quad \chi_{G}\mathbf{W}_{r}^{H} \\ \mathbf{0}_{N_{r}\times N_{d}} \qquad \chi_{G}\mathbf{W}_{r} \quad \lambda_{5}\mathbf{I}_{N_{r}} \end{bmatrix} \succeq \mathbf{0}_{N_{d}+2N_{r}}$$
(31)

$$\begin{bmatrix} \alpha_{5}\mathbf{I}_{N_{r}} - \lambda_{6}\mathbf{S}_{2}^{H}\mathbf{U}_{2}^{H}\mathbf{U}_{2}\mathbf{S}_{2} & \mathbf{S}_{2}^{H}\left(\mathbf{I}_{N_{r}\times z} - \sigma_{r}^{-1}\mathbf{U}_{2}^{H}\overline{\mathbf{H}}\mathbf{W}_{d}\right) & \mathbf{0}_{N_{r}\times N_{d}} \\ \left(\mathbf{I}_{z\times N_{r}} - \sigma_{r}^{-1}\mathbf{W}_{d}^{H}\overline{\mathbf{H}}^{H}\mathbf{U}_{2}\right)\mathbf{S}_{2} & \mathbf{I}_{z} & -\chi_{H}\mathbf{W}_{d}^{H} \\ \mathbf{0}_{N_{d}\times N_{r}} & -\chi_{H}\mathbf{W}_{d} & \lambda_{6}\mathbf{I}_{N_{d}} \end{bmatrix} \succeq \mathbf{0}_{N_{r}+z+N_{d}}$$
(32)

$$\begin{bmatrix} \alpha_{6} \mathbf{I}_{N_{r}} & \mathbf{S}_{2}^{H} \mathbf{U}_{2}^{H} \\ \mathbf{U}_{2} \mathbf{S}_{2} & \mathbf{I}_{N_{r}} \end{bmatrix} \succeq \mathbf{0}_{2N_{r}}$$
(33)

$$\begin{bmatrix} \alpha_{7} \mathbf{I}_{N_{r}} & \mathbf{S}_{3}^{H} \\ \mathbf{S}_{3} & \mathbf{I}_{N_{r}} \end{bmatrix} \succeq \mathbf{0}_{2N_{r}}$$
(34)

$$\begin{aligned} & \alpha_{8}\mathbf{I}_{N_{r}} - \lambda_{7}\mathbf{S}_{3}^{H}\mathbf{S}_{3} \quad \mathbf{S}_{3}^{H}\overline{\mathbf{F}}\mathbf{W}_{s} \quad \mathbf{0}_{N_{r} \times N_{s}} \\ & \mathbf{W}_{s}^{H}\overline{\mathbf{F}}^{H}\mathbf{S}_{3} \qquad \mathbf{I}_{d} \qquad \chi_{F}\mathbf{W}_{s}^{H} \\ & \mathbf{0}_{N_{s} \times N_{r}} \qquad \chi_{F}\mathbf{W}_{s} \quad \lambda_{7}\mathbf{I}_{N_{s}} \end{aligned} \right] \succeq \mathbf{0}_{N_{r}+d+N_{s}}$$
(35)

$$\alpha_{9}\mathbf{I}_{N_{r}} - \lambda_{8}\mathbf{S}_{3}^{H}\mathbf{S}_{3} \quad \mathbf{S}_{3}^{H}\overline{\mathbf{H}}\mathbf{W}_{d} \quad \mathbf{0}_{N_{r}\times d}$$

$$\mathbf{W}_{d}^{H}\overline{\mathbf{H}}^{H}\mathbf{S}_{3} \qquad \mathbf{I}_{z} \qquad \chi_{H}\mathbf{W}_{d}^{H}$$

$$\mathbf{0}_{d\times N_{r}} \qquad \chi_{H}\mathbf{W}_{d} \quad \lambda_{8}\mathbf{I}_{N_{d}}$$

$$(36)$$

where $\{\lambda_m \ge 0\}_{m=3}^8$ are introduced auxiliary variables.

Lastly, the robust relay power constraint (e.g., (10c)) can be turned into the following LMIs:

$$\begin{bmatrix} \alpha_{10}\mathbf{I}_{d} - \lambda_{9}\mathbf{W}_{s}^{H}\mathbf{W}_{s} & \mathbf{W}_{s}^{H}\overline{\mathbf{F}}^{H}\mathbf{W}_{r}^{H} & \mathbf{0}_{d \times N_{r}} \\ \mathbf{W}_{r}\overline{\mathbf{F}}\mathbf{W}_{s} & \mathbf{I}_{N_{r}} & \chi_{F}\mathbf{W}_{r} \\ \mathbf{0}_{N,\times d} & \chi_{F}\mathbf{W}_{r}^{H} & \lambda_{9}\mathbf{I}_{N_{r}} \end{bmatrix} \succeq \mathbf{0}_{2N_{r}+d} \quad (37)$$

$$\begin{bmatrix} \alpha_{11}\mathbf{I}_{z} - \lambda_{10}\mathbf{W}_{d}^{H}\mathbf{W}_{d} & \mathbf{W}_{d}^{H}\overline{\mathbf{F}}^{H}\mathbf{W}_{r}^{H} & \mathbf{0}_{z \times N_{r}} \\ \mathbf{W}_{r}\overline{\mathbf{F}}\mathbf{W}_{d} & \mathbf{I}_{N_{r}} & \chi_{H}\mathbf{W}_{r} \\ \mathbf{0}_{N_{r} \times z} & \chi_{H}\mathbf{W}_{r}^{H} & \lambda_{10}\mathbf{I}_{N_{r}} \end{bmatrix} \succeq \mathbf{0}_{2N_{r}+z} \quad (38)$$
$$\begin{bmatrix} \alpha_{12}\mathbf{I}_{N_{r}} & \mathbf{W}_{r} \\ \mathbf{W}_{r}^{H} & \mathbf{I}_{N_{r}} \end{bmatrix} \succeq \mathbf{0}_{2N_{r}} \quad (39)$$

where $\{\alpha_{10}, \alpha_{11}, \alpha_{12}\}$ and $\{\lambda_9 \ge 0, \lambda_{10} \ge 0\}$ are introduced auxiliary variables.

Bases on these operations, we obtain the following problem:

$$\max_{\mathbf{w}_{s},\mathbf{w}_{r},\mathbf{w}_{d},\{\mathbf{S}_{1},\mathbf{S}_{2},\mathbf{S}_{3}\}>\mathbf{0},\mathbf{U}_{1},\mathbf{U}_{2},\,\forall \mathbf{F}\in\mathscr{F},\forall \mathbf{H}\in\mathscr{H},\forall \mathbf{G}\in\mathscr{G}} 2\ln|\mathbf{S}_{1}|$$

$$+ 2\ln|\mathbf{S}_{2}| + 2\ln|\mathbf{S}_{3}| - \beta_{1} - \beta_{2} - \beta_{3},$$
(40a)

s.t.
$$\operatorname{Tr}\left(\mathbf{V}_{1}\mathbb{E}_{1}\left(\mathbf{U}_{1},\mathbf{W}_{s},\mathbf{W}_{d},\mathbf{W}_{r}\right)\right) \leq \beta_{1},$$
 (40b)

$$\operatorname{Tr}\left(\mathbf{V}_{2}\mathbb{E}_{2}\left(\mathbf{U}_{2},\mathbf{W}_{d}\right)\right) \leq \beta_{2},\tag{40c}$$

$$\operatorname{Tr}\left(\mathbf{V}_{3}\left(\mathbf{I}+\sigma_{r}^{-2}\mathbf{F}\mathbf{W}_{s}\mathbf{W}_{s}^{H}\mathbf{F}^{H}+\sigma_{r}^{-2}\mathbf{H}\mathbf{W}_{d}\mathbf{W}_{d}^{H}\mathbf{H}^{H}\right)\right)$$

$$\leq\beta_{3},$$
(40d)

$$(10b), (37), (38), (39),$$
 (40e)

To this end, we have turned (10a), (10b), and (10c) into an equivalent problem (40a)–(40e). Equations (40a)–(40e) are still non-convex with respect to all these optimization variables, but is convex with respect to given variables when other variables are fixed. Specifically, these variables can be divided into four groups, { W_s , W_d }, W_r , { S_1 , S_2 , S_3 }, { U_1 , U_2 }. Then, (40a)–(40e) can be decoupled into four subproblems with respect to these variables. Both these subproblems can be effectively solved by the convex optimization tool CVX [20], and the optimal local solution to (10a), (10b), and (10c) can be achieved in an alternating method.

4. Simulation Results

In this section, we evaluate the performance of our design through Monte Carlo simulations. The simulation settings are



FIGURE 2: The convergency of the proposed method.

assumed as follows: $P_s = 10$ dBW, $P_r = 10$ dBW, $P_d = 10$ dBW, $\sigma_d^2 = \sigma_r^2 = 10^{-6}$. Each entry of $\overline{\mathbf{F}}$, $\overline{\mathbf{H}}$, and $\overline{\mathbf{G}}$ is randomly generated by $\mathscr{CN}(0, 10^{-2})$ and the channel uncertainties are $\chi_F^2 = \chi_H^2 = \chi_G^2 = 2 \times 10^{-4}$. In addition, we compare our algorithm with the following methods: (1) the perfect CSI case, which can be treated as the upper bound of our robust design; (2) the no jamming method, e.g., setting $\mathbf{W}_d = \mathbf{0}$ with only optimize \mathbf{W}_s and \mathbf{W}_r . The three methods are legend as "the proposed design", "the perfect CSI case", and "the no jamming case", respectively.

Firstly, we investigated the convergency performance of our proposed AO method by comparing the worst case secrecy rate R_s with the iterative numbers. Figure 2 shows four examples of the convergence behavior with random channel realizations. From this figure, we can see that the proposed AO algorithm can always converge to the optimal solution in limited iterative numbers.

In Figure 3, we show the worst case secrecy rate R_s versus the source transmit power P_s . From this figure, we can see that R_s increase with the increase of P_s for our proposed design and the perfect CSI case. On the other hand, for the no jamming case, R_s firstly increase with the increase of P_s to some point, then decrease. This is a typical phenomenon in untrusted relay case.

In Figure 4, we illustrate the impact of the destination transmit power P_d on R_s . From this figure, we can see that R_s increase with the increase of P_d for all these methods. Since in this case, the more jamming signal used to interfere the untrusted relay, the high secrecy performance can be obtained.

In Figure 5, we show the worst case secrecy rate R_s versus the relay transmit power P_r . From this figure, we can see that R_s increase with the increase of P_r for our proposed design and the perfect CSI case. On the other hand, for the



FIGURE 3: The worst case secrecy rate versus the source transmit power.



FIGURE 4: The worst case secrecy rate versus the destination transmit power.

no jamming case, R_s firstly increase with the increase of P_r , then decrease at some point.

In Figure 6, we illustrate the impact of the relay antenna numbers N_r on the worst case secrecy rate R_s . It is an interesting result that as the number of antennas of the relay increases, R_s firstly increase to a certain point and then decrease. This is because that as the DoF of the relay increases, the untrusted relay has more ability to cancel the jamming signal and further intercepts the confidential message successfully.



FIGURE 5: The worst case secrecy rate versus the relay transmit power.



FIGURE 6: The worst case secrecy rate versus the relay antenna numbers.

5. Conclusion

In this paper, we investigated an AF MIMO relay system where the relay is untrusted. Assuming that only imperfect CSI can be obtained, we aim to maximize the worst case secrecy rate via designing the precoding matrices. To solve the formulated highly non-convex problem, we propose to linearize the difference of multiple log-det functions. After linearization, we handle the CSI uncertainty based on epigraph reformulation and the sign-definiteness lemma. Finally, an AO method was proposed to solve the reformulated problem. Simulation results demonstrated the performance of our proposed design.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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