

# Research Article **The Construction Method of BeiDou Satellite Navigation Measurement Error System**

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Based on the measurement error of pseudorange in BeiDou satellite navigation system, this paper analyzes the measurement principle of the system. Aiming at the difficulties in the system measurement error index system, an overall construction method of measurement error system based on empirical estimation method and error distribution model is proposed. Based on the Analytic Hierarchy Process, the correlation analysis model of the measurement error index is constructed, the relationship between the indicators is analyzed, and the system measurement error index hierarchy is constructed. Based on the empirical estimation method and the error distribution model, the index values are decomposed and assigned based on the final service performance of the system, and a clear representation of the complex relationship of index matching is achieved. Finally, by analyzing the principle of the positioning function in the ground transportation control mode, taking the satellite clock error as an example, the model is decomposed layer by layer and the relevant indicator items are established. The relationship between index terms was studied, and the value of the indicators was quantified. Compared with the actual operating conditions of the current system, the correctness of the method was verified, which provided a basis for the demonstration of the index values of satellite navigation systems. Based on the empirical estimation method and error distribution model as a new type of calculation method, the index system established under a certain set of conditions is reasonable, and it can be applied to the error control adjustment in satellite navigation system engineering construction.

#### **1. Introduction**

The Global Navigation Satellite System (GNSS) is satellitebased radio navigation and positioning system. It can provide precise three-dimensional position, three-dimensional speed, navigation data, accurate satellite time reference, and other information for all types of users. It shows more and more important uses in military and civilian applications and its application prospects far beyond people's imagination [1]. Based on the empirical estimation method and error distribution model as a new type of calculation method, the index system established under a certain set of conditions is reasonable, and it can be applied to the error control adjustment in satellite navigation system engineering construction [2–8].

The construction of the BeiDou satellite navigation system is a complex and huge system project. When the BeiDou

satellite navigation system is using for position or navigation, there are various errors in the observation and measurement, such as the BeiDou system's own error of the signal which includes satellite ephemeris error and satellite clock error; the propagation error of the BeiDou signal from the satellite to the user receiving antenna includes ionospheric delay correction error and tropospheric delay correction error, etc.; BeiDou user receiver generated signal measurement error, including observation noise error and antenna phase center error. This paper defines the measurement error system of the BeiDou satellite navigation system as an organic combination of error index elements in the measurement process of the BeiDou satellite navigation system [9–15]. It is the correlation and constraint relationship between the performance of a single measurement error indicator and multiple measurement error indicators, and an intrinsic part of the satellite navigation system as well as a crucial foundation for ensuring



FIGURE 1: Measurement principle of BeiDou satellite navigation system code pseudorange.

the stable operation of satellite navigation systems. It determines and restricts the functions and performance of satellite navigation systems.

The various measurement errors existing in the BeiDou system will have a certain influence on the measurement accuracy of the system, and the error is unavoidable. However, some techniques can be used to reduce the influence of the error on the measurement accuracy. In order to ensure that the system's final service accuracy meets the design requirements, in the system's demonstration design, various errors need to be repeatedly adjusted and simulated to calculate the system positioning accuracy according to the system's service accuracy requirements and finally be given under the condition of meeting the accuracy index in accordance with the current technical development level of the indicators of the distribution of errors, and the error control of each link in the process of engineering development is guided.

How to start from the system service performance, establish the index system, and clearly describe the complex relationship existing in the index system has become a hot topic in BeiDou satellite navigation system research. Based on the research of BeiDou satellite navigation system measurement error index, this thesis builds a hierarchy of system measurement error indicators guided by Analytic Hierarchy Process, and proposes an overall measurement error system based on empirical prediction method and error distribution model methods to study the relationship between the indicators of the index system and quantify the relevant index values.

#### 2. BeiDou Satellite Navigation System Positioning Principle and Error Analysis

2.1. System Positioning Principle. The principle of the positioning of the BeiDou satellite navigation system is mainly based on the relevant principles of spatial geometry and physical knowledge, using satellites in the spatial distribution and the distance between the satellite and the surface of the earth to calculate the specific location of the ground point. Unlike the ground-based optoelectronic distance measurement method, BeiDou satellite navigation system implements pseudorange measurement [16-19]. In principle, it needs two clocks, one is called a satellite clock and the other is called a user receiver clock (local clock). Because there is a clock error between the satellite clock and the user receiver clock, the distance measurement is called a pseudorange. In order to facilitate the analysis of the influence of various errors on the measurement accuracy, errors are usually attributed to the pseudorange parameter measurement of the satellite and can be regarded as the equivalent error of the pseudorange value. This paper reanalyzed the physical definition of the code pseudorange and the various error source parameters combed out [20-23]. Assume that the time is called the transmission time when the satellite transmits the signal, and the time is called the reception time when the user receives the signal. Figure 1 shows BeiDou satellite navigation system signal code pseudorange measuring principle.

Usually, the user clock is not synchronized with the BeiDou satellite clock. The BeiDou navigation system time (BDT) is actually equal to t. Then the user receiver time at t is  $t_u(t)$ , and the difference between the user receiver time and the BeiDou navigation system time is recorded  $\delta t_u(t)$ ; that is,

$$t_u(t) = t + \delta t_u(t) \tag{1}$$

Similarly,

$$t^{(s)}(t) = t + \delta t^{(s)}(t)$$
(2)

Pseudorange is defined as the distance between signal reception time and transmission time:

$$\rho(t) = c \left( t_u(t) - t^{(s)}(t - \tau) \right)$$
(3)

where  $\tau$  represents the actual propagation time of the signal from the satellite to the receiver.

Substitute (1) into the above equation; that is,

$$\rho(t) = c\tau + c\left(\delta t_u(t) - \delta t^{(s)}(t-\tau)\right) \tag{4}$$

The actual propagation time of the BeiDou satellite navigation signal consists of two parts: one is the time when the signal travels a geometric distance, and the other is the propagation delay caused by the atmosphere; that is,

$$\tau = \frac{r\left(t - \tau, t\right)}{c} + I\left(t\right) + T\left(t\right) \tag{5}$$

The atmospheric propagation delay can be decomposed into two parts: the ionosphere delay I(t) and the troposphere delay T(t). Substituting (5) into (4),

$$\rho(t) = r(t - \tau, t) + c\left(\delta t_u(t) - \delta t^{(s)}(t - \tau)\right) + cI(t)$$

$$+ cT(t) + \varepsilon_o(t)$$
(6)

In the formula,  $\rho(t)$  represents the pseudorange observation;  $r(t - \tau, t)$  is the real distance from the satellite to the receiver;  $\varepsilon_{\rho}(t)$  represents the noise error. Equation (6) is often referred to as the pseudorange observation equation, which is the basic equation for the user receiver to use a pseudorange to achieve a single point absolute positioning.

BeiDou satellite navigation system error positioning accuracy is

$$M_p = PDOP \times U_p \tag{7}$$

In the formula, PDOP is the spatial position geometric precision factor;  $U_p$  is the measuring error factor.

2.2. Measurement Error Analysis. There are many errors in the accuracy of the positioning accuracy produced by the BeiDou satellite navigation system, for example, the orbit of the satellites, the atmospheric refraction, and the BeiDou positioning receivers themselves. These errors have a great influence on the positioning accuracy of the BeiDou satellite navigation system. In the navigation and positioning system, the distance error caused by the measurement error can all be equivalent to the error due to the pseudorange measurement. These errors are collectively referred to as the user equivalent distance error (UERE).

In order to analyze the effects of various errors on the accuracy, it is assumed that the error sources affecting the positioning accuracy are all independent, and the satellite approximately is represented as a zero-mean Gaussian random variable whose variance is determined by the sum of the variance of each component. The measurement error factor is satellites *UERE*.

$$UERE = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2}$$
(8)

According to formula (6), BeiDou satellite navigation system positioning accuracy is closely related to measurement error. Measurement error is affected by many factors, such as satellite orbit error, satellite clock error, ionosphere error, tropospheric error, multipath, and thermal noise. According



FIGURE 2: Schematic diagrams of measurement error statistics.

to the different statistical characteristics of measurement error, these errors can be divided into accidental error, systematic error, and abnormal errors, as shown in Figure 2.

Systematic error refers to the error of a certain law, such as constant system error, periodic system error, etc., caused by the systematic influence of certain factors. The effects of systematic errors are cumulative. For example, dynamic model errors, satellite orbit errors, coordinate system errors, ionosphere delay errors, and tropospheric delay errors are system errors. Random error refers to errors caused by various random factors. Individuals of this type of error are random and irregular, but statistically obey specific statistical rules, such as normal distribution or heavy tail distribution (Huber distribution), for example, satellite ranging error, spatial signal jitter error, etc. Abnormal error refers to the error caused by equipment abnormalities and abnormal changes in observation conditions and is generally expressed in the form of abnormal values. Satellite ranging errors or carrier measurement errors caused by satellite orbit maneuvers, satellite equipment failures, receiver failures, and other extreme errors are abnormal errors.

## 3. Measurement Error Index Construction Model Based on Analytic Hierarchy Process

3.1. Systematical Measurement Error Level Decomposition Method. The principle of the AHP is to decompose the decision-making first, draw a number of important influencing factors, and classify it to construct a multilevel structural model. After starting from the lower level, analyze the relative importance of the underlying factors to the upper factors and rank according to the degree of importance. The characteristics of AHP are quantification and hierarchization. It decomposes complex problem into a number of relatively simple and small problems, then calculates, and analyzes in turn.

The specific application steps of the AHP are as follows: Determine the specific issues that need to be decided, and decompose the problem into target layer, criterion layer, and indicator layer. The decision-making problem is the target level, the final solution is the indicator level, and the criteria level includes the key criteria that need to be considered when making decisions, as shown in Figure 3.

According to the Analytic Hierarchy Process (AHP), the measurement error elements of the BeiDou satellite navigation system are decomposed into system objectives, criteria, and indicators. Based on this, qualitative and quantitative analysis is performed. The complex system measurement error problem is represented as an ordered hierarchical structure. The hierarchical analysis structure is shown in



FIGURE 4: AHP determines the measurement error indicator.

Figure 5. The energy efficiency status of navigation satellites, signal propagation environments, and ground-based receiving equipment directly reflect the source of the measurement error and the level of energy efficiency of the entire system. Well, therefore, navigation satellites, signal propagation environments, ground-based receiving equipment (including BeiDou user receiver and ground transport accused of BeiDou receiver), and the above three (navigation satellite, signal propagation environments, and ground-based receiving equipment) of different period of coupling are difficult to define clearly the error as the criterion layer 2 indicators.

At the same time, satellite delay error, satellite error, its ionosphere, troposphere, and BeiDou user's receiver noise error, multipath error, and many other factors have great impact on measurement error energy efficiency, subdividing them into the 3rd factor indicator level according to the rule, as shown in Figure 4.

3.2. System Measurement Error Indicator Correlation Analysis Model. There is a quantitative relationship between the Bei-Dou satellite navigation system measurement indicators in the physical sense, but the changes in the visual relationship between any two measurement and the linkage relationship between the measurement error indicator data and through one or a few indicators if it is possible to distinguish the other indicators of changes in laws or is a measure of whether changes depend on one or several indexes such as relationship remain to be further analyzed.

The quantitative relationship of the measurement error indicators of the BeiDou satellite navigation system can be divided into two types: one is a deterministic relationship, which is called a functional relationship; the other is an uncertainty relationship, called the correlation relationship. The functional relationship between the system measurement error indicators is determined by the physical characteristics of the BeiDou satellite navigation system operation and the definition of measurement error characteristic indicators. The correlation relationship reflects the relevant forms and correlation degree of the studied variables or reflects the regular when the variable changes, the other variable will follow the law of the corresponding change, and the value of this change is uncertain. Therefore, the initial search for such uncertainties in the BeiDou satellite navigation system measurement error index can be determined by doing correlation analysis for the measurement error characteristic indices and finding the correlation coefficient between the error characteristic index data.

The correlation analysis of measurement error index data is hoped to mine the inherent law hidden in statistical index data through data mining technology. In most cases, the correlation analysis we perform is performed between the two indicators. This requires the use of a binary variable correlation analysis. Different types of variable data should



FIGURE 5: Flow chart of measurement error indicator construction model.

use different correlation analysis methods. When the statistical indicators of measurement error are used for correlation analysis, the error indicators are generally numerical variables. Therefore, the Pearson correlation coefficient analysis method is used to determine the correlation coefficient between the load characteristics law.

Let two random variables be X and y, then the correlation coefficient of the two variables is

$$\rho = \frac{\operatorname{cov}(X, Y)}{\sqrt{\operatorname{var}(X)}\sqrt{\operatorname{var}(Y)}}$$
(9)

where cov(X, Y) is the covariance of two variables; var(X) var(Y) are the variance of X and Y; the overall correlation coefficient is a measure of the correlation coefficient between the two variables.

However, in fact, the overall correlation coefficient is generally unknown and needs to be estimated using the sample correlation coefficient. For measurement error characteristics, let  $X = (x_1, x_2, \dots, x_n)$ ,  $Y = (y_1, y_2, \dots, y_n)$  be the two time series from measurement error characteristic index X and index Y, respectively, then the correlation coefficient between indexes is r:

$$r = \frac{\sum_{i=1}^{n} \left( x_i - \overline{x} \right) \left( y_i - \overline{y} \right)}{\sqrt{\sum_{i=1}^{n} \left( x_i - \overline{x} \right)^2 \left( y_i - \overline{y} \right)^2}}$$
(10)

 $i = 1, 2, \dots, n, \overline{x}$  and  $\overline{y}$  represent the mean values of X and Y sequences, and the measurement error characteristic index data sample correlation coefficient  $\rho$  is a uniform estimator of the overall correlation coefficient of index.

3.3. System Measurement Error Distribution Model. The establishment of the measurement error indicator of the BeiDou satellite navigation system is actually the process of error allocation based on the system service design requirements. The allocation of errors should take into account the distribution of all error components. For the BeiDou satellite navigation system measurement error, according to the "Bei-Dou II" satellite navigation system engineering construction experience, and for a given system error, the impact can be removed first from the total error, and then the remaining random error and the undetermined systematic error distribution problems are analyzed. If the error allowable range of each index item is determined, other errors are assigned according to the determined error. If the error allowable range of the index item cannot be determined, it is distributed according to the equal action principle.

The principle of equal action is to first consider that the error of each part of the BeiDou satellite navigation system has equal influence on the overall error. That is to say, when allocating errors, the error factors are all random errors and are not related to each other, then the error transfer formula of

TABLE 1: Correlation analysis of factor level indicators.

	r11	r12	r13	r14	r15	r16	r17	r18
r11	1.00	0.15	0.08	0.72	057	0.59	0.41	-0.83
r 21	0.15	1.00	0.04	0.09	-0.11	0.06	03	0.19
r31	0.08	-0.04	1.00	0.05	0.12	0.07	0.01	-0.10
r41	0.72	0.09	-0.05	1.00	0.52	0.49	0.32	0.65
r51	0.57	0.11	0.12	0.52	1.00	0.02	0.07	0.81
r61	0.59	0.06	0.07	0.49	-0.02	1.00	0.08	0.85
r71	0.41	0.03	-0.01	0.32	0.07	-0.08	1.00	0.87
r81	0.83	0.19	0.10	0.65	-0.81	0.85	-0.87	1.00

the arbitrary function  $y = f(x_1, x_2, \dots, x_m)$  of the unrelated variable is

$$\sigma_y = \sqrt{\sum_{i=1}^m \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_i^2} = \sqrt{\sum_{i=1}^m \left(D_i\right)^2}$$
(11)

In the formula  $D_i = |\partial f/\partial x_i|\sigma_i$ ,  $\sigma_i$  is the fractional error of the direct measurement. According to the requirement of error distribution, when given  $\sigma_y$ , determine the value of  $D_i$  or  $\sigma_i$ , that needs to meet the formula  $\sqrt{D_1^2 + D_2^2 + \cdots + D_m^2} \leq \sigma_y$ , according to the principle of distribution, that requires

$$D_1 = D_2 = \dots = D_m = \sqrt{\frac{\sigma_y^2}{m}} = \left|\frac{\partial f}{\partial x_i}\right|\sigma_i$$
 (12)

The distribution of errors by the principle of equal action may appear unreasonable, because the calculated local errors are all equal. This point is easy to achieve for some measurement values to ensure that its measurement error is not beyond the allowable range, and some of these measurement values are difficult to meet the requirements; to meet its measurement accuracy, it is bound to use expensive highprecision instruments or to pay a larger labor. From (9), we can see that when the error of each direct quantity is fixed, the corresponding measurement error is inversely proportional to the error transfer coefficient. Therefore, each measurement error is not equal, and sometimes the phase difference may be large when the local errors are equal. Because of the above two kinds of situations, the errors allocated to the principle of equal action must be adjusted according to the specific circumstances. For error terms that are difficult to guarantee during measurement, the allowable error values should be appropriately expanded. For errors that are easy to guarantee in measurement, the allowable error values should be reduced as much as possible. After the error is adjusted, the total error should also be calculated according to the error distribution formula to see if it exceeds the allowable value of the given function error.

3.4. Measurement Error Index Construction Model. Selecting the appropriate process for accuracy analysis and error index allocation is the key to the design process. The process flow shown in Figure 5 has been used in the analysis of the actual model and achieved the effect of the engineering application.

## 4. Results and Discussion Analysis on Measurement Errors of BeiDou Satellite Navigation System

4.1. Relevance Decomposition of Systematic Measurement Error Factor Layer Indicators. In order to analyze the correlation between measurement indicators of the BeiDou satellite navigation system error indicators and determine whether it meets the principle established by the index system, we can use a method of calculating correlations to assess the correlation between index items.

Take BeiDou satellite navigation system data released as an example. The data used includes ① satellite clock error (II), ② satellite delay error (I2), ③ relativistic effect error (I3), ④ satellite ephemeris error (I4), ⑤ ionospheric refractive error (I5), ⑥ tropospheric refractive error (I6), ⑦ multipath or occlusion error (I7), and ⑧ BeiDou satellite navigation receiver measurement error (I8); in the ground control state, ground control section errors are contained within ephemeris errors and satellite clock errors. The correlation calculation results are shown in Table 1.

Calculate the correlation value r according to formula (10). The value of r is between -1 and 1 and describes the degree and direction of the linear correlation between the two measurement error characteristic indices: r > 0, there is a positive correlation between the two measurement error characteristic indicators; r < 0, there is a negative correlation between the two measurement error characteristic indices;  $r = \pm 1$ , there is a complete correlation between the two load characteristic indexes; r = 0, there is no linear correlation between the two measurement error characteristic indexes. According to experience, the degree of correlation is divided into the following situations: when |r| > 0.8, it can be regarded as a high degree of correlation between the load characteristics; when 0.5  $\leq$   $|r| \leq$  0.8, it can be regarded as a measurement error with a moderate degree of correlation; when  $0.3 \le |r| \le 0.5$ , it is considered that the measurement error characteristic index is low-degree related; when |r| <0.3, the measurement error characteristic index correlation is extremely weak; it may be regarded as irrelevant. The degree of correlation between the measurement error characteristic indicators is determined according to the size of the correlation coefficient, and sorting is performed to remove irrelevant indexes.



FIGURE 6: Satellite clock error decomposition indicators.

Table 1 has certain significance, but it does not fully represent the correlation of measurement error indicators. Here, only typical examples are provided for the construction of model methods. According to Table 1, there is a strong correlation between the satellite clock error and other error indicators. That is, the satellite clock error is used as an example for further decomposition until the disaggregated indicator items are independent of each other. The error sources related to satellite clock error errors mainly include the performance of the satellite clock, the accuracy of satellite-to-earth time comparison, and the update rate of the star clock parameters. The satellite clock errors are related to the decomposition index items, as shown in Figure 6.

## 4.2. Index Item Allocation Based on Empirical Estimation Method

4.2.1. Estimated Distribution of Experience for Each Indicator Item. The availability of the BeiDou satellite navigation system is related to the cut-off angle used by the receiver. Decreasing the cut-off angle can lead to better usability. However, lowering the cut-off angle to observe more satellites will introduce larger atmospheric errors, so a reasonable selection of cut-off angles should be made on the basis of reaching the availability index. According to statistics of system availability at different cut-off angles, it can be seen that when the cut-off angle is 5°, the availability is greater than 98%, which can meet the general demand for availability. Therefore, to ensure generality, when the PDOP value is calculated from the measurement error index value of this paper, the cut-off angle is 5°. At this time, when the PDOP is less than or equal to 2.5, the system is available and the system availability is good. The availability in China is 100%. The system has been able to better meet the positioning and navigation needs in China and its neighboring regions.

The error positioning accuracy of the BeiDou satellite navigation system is related to the geometric accuracy factor of the spatial position and the error factor for the measurement of  $U_p$ . In the ground operation control mode, the positioning accuracy of the system is better than 10m, and

TABLE 2: Relativity correction error caused by inaccurate.

$t - t_k$	$\sigma_{s}$	$\sigma_d$	$\sigma\left(t ight)$	Corresponding ranging error
4 hours	$1 \times 10^{-15}/s$	$1 \times 10^{-15}$ /day	0.17ps	5.1E-5m
1 day	$1 \times 10^{-15}/s$	$1 \times 10^{-15}$ /day	0.45ps	/

the position accuracy factor of the system is PDOP=2.5, then

$$UERE = 10m/2.5 = 4m$$
 (13)

From Figures 3 and 5, the square root of the sum of the squares of the measurement error index of the indicator layer is 4m; that is,

$$\sigma_{UERE} = \sqrt{\sigma_{\text{satellite delay}}^2 + \sigma_{\text{satellite clock}}^2 + \sigma_{\text{The theory of relativity}}^2 + \sigma_{\text{ephemeris}}^2 + \sigma_{\text{The ionosphere}}^2 + \sigma_{\text{troposphere}}^2 + \sigma_{\text{receiver}}^2 + \sigma_{\text{multipath}}^2$$
(14)

① Satellite Delay Error Indicator Allocation. Delay from the zero point of the satellite system to the output of the signal conversion circuit, the delay from the output of the signal conversion circuit to the output of the wave modulator, the delay from the output of the microwave modulator to the output of the power amplifier, the time delay from the output of the power amplifier to the phase center of the antenna, and the error between the signals, the phase deviation of the transmitting antenna phase, and the satellite-to-earth time ratio error on-board are the navigation satellite delay errors. At present, the impact of satellite delay error on positioning is expected to be within 0.2ns.

<sup>(2)</sup> *Distribution of Relativistic Effect Error Indicators.* According to the principle of relativity, clock oscillators at different speeds of motion will produce frequency offsets, and clock oscillators with different gravitational bits will generate gravitational shifts. During BeiDou satellite navigation and positioning surveys, due to the different statuses of the BeiDou satellite clock and the receiver clock, their movement speed and gravitational force are different.

The frequency stability expression of a satellite-borne atomic clock can be approximated by  $\sigma_y(\tau) = \sigma_s \tau^{-1/2} + \sigma_d$  (where  $\sigma_s$  is the second stability,  $\sigma_d$  is the day stability, and  $\tau$  is the measurement interval). The satellite clock time offset variance  $\sigma^2(t)$  caused by it is determined by the time interval and calibration method of satellite clock synchronization calibration (the ground clock is generally based on the hydrogen clock),  $\sigma^2(t) = \sigma_s^2 \times (t-t_k) + \sigma_d^2 \times (t-t_k)^2$  (where *t* is the current time and  $t_k$  is the calibration time). Table 2 shows the relationship between  $\sigma_s$ ,  $\sigma_d$ , synchronization calibration intervals  $t - t_k$  and  $\sigma(t)$ , and ranging error.

Therefore, the satellite has been revised, and the influence of relativity on the stability of the bell is below 1E-15. This item can be ignored.

③ Allocation of Satellite Ephemeris Error Indicators. Satellite ephemeris error is also called satellite orbit error. Estimating and processing satellite orbital errors is more difficult because satellites are subject to the combined effects of multiple perturbations in orbital operations, and it is difficult for ground monitoring systems to accurately grasp the changing laws of these forces. The BeiDou system satellites are equipped with laser reflectors. The accuracy of the satellite laser reflectors can reach 1 to 2cm, and the existing orbit determination technology and the perturbation model have been improved; therefore, the accuracy of precision orbit determination can theoretically reach the order of decimeters and even centimeters. However, since all three ground monitoring stations of the BeiDou satellite navigation system are located in China, there are few tracking arcs for the satellites, and the distribution is extremely uneven. Therefore, using the observation data from 3 monitoring stations to determine the accuracy of the track is difficult to be improved. At present, the influence of ephemeris errors on positioning is expected to be controlled within 1 m.

④ Ionospheric Refractive Error Index Allocation. Ionospheric refractive errors are errors in observations due to ionospheric effects. When the BeiDou satellite navigation signal passes through the ionosphere, the path of the navigation signal will be bent, the propagation speed will also change, the carrier propagation speed will be accelerated, and the code propagation speed will be reduced, so that the measured distance will be deviated. This effect is called for ionospheric refraction. Our country is in the midlatitudes of the northern hemisphere, some regions in the south are located in the anomalous areas of the equator, the difference in the elevation angle between the antenna and the satellite oscillates the ionospheric refraction error, and an average estimate of the ionospheric refraction error index needs to be performed. At this time, taking the ionospheric refraction at an angle of 300 as the average value, the ionospheric refractive error is about 20m, and the equivalent error distance is about 6m, as shown in Figure 7. And the division of ionospheric grids in China is shown in Figure 8.

Using the Klobuchar model to correct ionospheric time refraction, the average effective rate reaches over 70% in the midlatitudes of the northern hemisphere. Combined with an ionospheric error grid correction algorithm and using a grid model with an interval of  $5^{\circ} \times 5^{\circ}$ , the vertical ionospheric delay at the user station's longitude and latitude at the point of penetration was calculated. It is concluded that the residual error of atmospheric refraction after the ionospheric model



FIGURE 7: Ionospheric grid model.



FIGURE 8: Division of ionospheric grids in China.

parameter correction is expected to be controlled within 3m.

(5) *Tropospheric Refractive Error Index Allocation*. Because the atmospheric density is greater than the ionosphere and the state of the atmosphere is also more complicated, at the same time, the troposphere is in contact with the ground and receives radiant heat energy from the ground. Its temperature decreases as the altitude increases. Therefore, when the BeiDou satellite navigation signal passes through the troposphere, it will also cause the propagation path to be bent, thus causing a deviation in the measurement distance. This phenomenon is called tropospheric refraction. There is no better way to correct tropospheric delay errors. Usually, corrections are made using models such as Hopfield and Sastomonin. The residual error after this correction is expected to reach lm. (•) Allocation of Multipath or Occlusion Error Indicators. The error caused by the multipath effect is difficult to eliminate completely and its influence can only be weakened as much as possible. The common practices are to avoid strong reflection surfaces during positioning, use a receiving antenna with anti-multipath effects and use an extended observation time and averaging method. In short, the multipath error can be controlled within 2m using a suitable method.

⑦ BeiDou Satellite Navigation Receiver Measurement Error Indicator Allocation. The noise of BeiDou satellite navigation receivers has a wide range of meanings, including the receiver clock skew, code tracking errors caused by thermal noise, interference, etc., also known as pseudorange measurement errors. It also includes the heat of the antenna, amplifier and various electronic devices thermal noise, signal quantization error, cross-correlation between satellite signals, algorithm error in determining code phase and carrier phase, and various calculation errors in receiver software. Based on comprehensive statistics, the impact of the noise error of the former BeiDou satellite navigation receivers on positioning can be controlled within 1 ns.

(8) Distribution of Ground Motion Control Segment Error Indicators. The ground segment of the satellite navigation system is a complex, which is the control center of the entire navigation system. It is a typical mission-critical system and is responsible for the operation and management of the entire navigation system. It is responsible for satellite time synchronization, precision orbit determination, and ionospheric delay processing, system integrity monitoring and wide-area differential processing in key service areas, uplink injection of navigation message parameters, and management and maintenance of satellite constellations and payloads. The impact of systematic errors on positioning is expected to be less than 0.3 ns, within the ground station ranging receiver accuracy and ground transceiver channel calibration error.

4.2.2. Satellite Clock Indicator Item Allocation. According to satellite delay error, relativity error, tropospheric error, tropospheric error, receiver measurement error, and satellite ephemeris error, the satellite clock error indicator is quantitatively calculated:

 $\sigma_{\rm Satellite\ clock\ error}$ 

(15)

$$= \sqrt{\sigma_{UERE}^2 - \left(\sigma_{Satellite delay}^2 + \sigma_{Satellite clock}^2 + \sigma_{The theory of relativity}^2 + \sigma_{ephemeris}^2 + \sigma_{ionosphere}^2 + \sigma_{troposphere}^2 + \sigma_{receiver}^2 + \sigma_{multipath}^2\right)}$$

By substituting the above-mentioned qualitatively assigned indicators into the above equation, the equivalent distance error of the satellite clock error can be obtained:

$$\sigma_{\text{Satellite clock error}} = \sqrt{4^2 - (0.06^2 + 1 + 3^2 + 1 + 2^2 + 0.3^2)}$$
(16)  
$$\approx 0.952m$$

That is, in order to achieve a positioning service accuracy of 10 meters, the satellite clock bias forecast accuracy is better than 3.2 ns when the system is performing a satellite clock bias forecast (between the two forecasts, the precision error caused by the clock difference should be less than 0.952m).

4.3. Error Allocation Modeling of Satellite Clock Error Index Value. There are frequency and phase deviation and phase noise between the carrier, pseudocode signal, and nominal carrier and pseudorange signal by navigation satellites. The jitter error of the navigation signal at the receiver is mainly reflected in the ability to capture and distinguish the signal. The front-end of the receiver removes the interference signals of adjacent frequency bands through signal filtering. At the same time, in order to better capture weak signals, the signal power should be put in about 1010-1011 times. That is, if the carrier frequency error is less than 1E-11Hz, then the receiver cannot discriminate the carrier frequency error during signal acquisition and recovery and has no effect on system service accuracy. The second-order stability of the satellite-borne time-frequency reference 10.23M signal is generally 0.5 to 1 order of magnitude higher. According to the development of the satellite-borne atomic clock of the BeiDou satellite navigation system, the second-order stability of the satellite-borne time-frequency reference 10.23M signal is 5E-12, and the corresponding second-order atomic clock satellite stability is at least 5E-12.

According to Figure 7, the indicators related to satellite clock error indicators include satellite clock performance, satellite-to-ground bidirectional time comparison, and satellite clock parameter update rates. The update rate of the satellite clock parameter is a constraint indicator, but it plays a key role in the control of the satellite clock error. In general, the higher the satellite clock parameter update rate is, the higher the satellite clock accuracy is, and at the same time, it can compensate for the error caused by insufficient satellite clock performance. In this paper, the forecasting strategy of one-hour forecasting for one hour is used to decompose the indicator value to improve the update rate of the star clock parameter. 4.3.1. Distribution of Satellite Clock Performance Error Indicators. According to atomic clock noise characteristics, the satellite clock is stable at 5E-12 seconds. The frequency stability within the range of tens of seconds is related to the time interval  $\tau$ -1/2. Since the observation strategy is to observe 1 hour forecast for 1 hour, the 1-hour stability of the satellite atomic clock is

$$\sigma(3600s) = \sqrt{\left(\frac{1}{3600}\right)} \times \sigma(1s) = 8.3E - 14$$
(17)

The calculation results based on the error estimation formula  $\Delta t = \sigma(\tau) \times \tau$  represent the ultimate accuracy of the model prediction. The actual model prediction results will have a certain degree of precision attenuation. In order to ensure the validity of the actual accuracy, a margin of 30% must be reserved, and the performance of the satellite atomic clock can be obtained (1-hour stability) as

$$\sigma(3600s) = 8.3E - 14 \times (1 - 30\%) \approx 5.8E - 14$$
(18)

Similarly, the other stability of the satellite clock can be estimated as follows:

Second stability:

$$\sigma\left(1s\right) = 5E - 12\tag{19}$$

Ten seconds stability:

$$\sigma(10s) = \left(\frac{1}{10}\right)^{1/2} \times \sigma(1s) \times (1 - 30\%) \approx 1.1E - 12 \quad (20)$$

100 second stability:

$$\sigma (100s) = \left(\frac{1}{100}\right)^{1/2} \times \sigma (1s) \times (1 - 30\%)$$

$$= 3.5E - 13$$
(21)

Thousand seconds stability:

$$\sigma (1000s) = \left(\frac{1}{1000}\right)^{1/2} \times \sigma (1s) \times (1 - 30\%)$$
  
\$\approx 1.1E - 13\$ (22)

Million second's stability:

$$\sigma (10000s) = \left(\frac{1}{10000}\right)^{1/2} \times \sigma (1s) \times (1 - 30\%)$$
  
= 3.5E - 14 (23)

Day stability:

$$\sigma (86400s) = \left(\frac{1}{86400}\right)^{1/2} \times \sigma (1s) \times (1 - 30\%)$$

$$\approx 2E - 14$$
(24)

The effect of satellite atomic clock performance (1 hour stability) on satellite clock error is

$$5.8 \times 10^{-14} \times 3.0 \times 10^8 \times 3600 = 0.06m \tag{25}$$

4.3.2. Decomposition of the Two-Dimensional Time-to-Time Ratio Accuracy Index. According to the principle of error distribution, the two-way time accuracy error of satellites and satellites is

$$\sigma_{\text{The precision of the two-way}} = \sqrt{\sigma_{\text{Satellite clock error}}^2 - \sigma_{\text{Satellite clock performance}}^2}$$
(26)
$$= \sqrt{0.952^2 - 0.06^2} \approx 0.95m$$

The precision of the two-way time comparison between the star and the earth is divided into the accuracy of the clock error forecast model and the source accuracy of the clock error forecast model. Among them, the application of the clock error prediction model is using some linear models as commonly used models, including polynomial models, gray models and time series models. The error introduced by the characteristics of the clock error prediction model is 0.1ns to 0.01ns. If it exceeds 0.1ns, the forecast model has no use value. Taking the maximum error of 0.1ns and converting the equivalent distance error to 0.03 m, the data source accuracy of the clock error prediction model is

$$\sigma$$
 the precision of the data source of the bell difference prediction model

$$= \sqrt{\sigma_{\text{precision of bell difference}}^2 - \sigma_{\text{precision of bell difference}}^2} \qquad (27)$$
$$= \sqrt{0.95^2 - 0.03^2} \approx 0.94m$$

4.3.3. Clock Error Forecast Model Data Source Precision Index Item Quantization Decomposition. The error introduced by the data source accuracy of the clock bias forecast model is divided into systematic error and random error. The systematic error mainly refers to the systematic error introduced by the time comparison link in the clock difference data observation process. According to the principle of error allocation

$$\sigma_{\text{the precision random error of the data source}} \sigma_{\text{the bell difference prediction model}} = \sigma_{\text{error of accuracy system of data source}} \sigma_{\text{bell difference prediction model}}$$
(28)
$$= \sqrt{\frac{1}{2}\sigma_{\text{the precision of the data source of the}}^2} \approx 0.66m$$

 $\sqrt{2}$  bell difference prediction model ① Clock Error Forecast Model Data Source Accuracy Random Error. According to Figure 7, the random error of the data source of the clock error prediction model is mainly caused by the error caused by the time comparison between the satellite clock and the injection station and the error of the injection station clock compared with the main clock.

$$\sigma_{\text{satellite clock and injection station}} = \sigma_{\text{the injection station clock and the main}} = \sqrt{\frac{1}{2}\sigma_{\text{the precision of the data source of the}}^2} \approx 0.46m$$

$$= \sqrt{\frac{1}{2}\sigma_{\text{the precision of the data source of the}}^2} \approx 0.46m$$

<sup>(2)</sup> Comparison of the Quantification of Random Error Indicator Terms between Satellite Clocks and Monitoring Stations. According to the correlation decomposition, the random error between the satellite clock and the monitoring station clock can be divided into satellite-to-ground link random error, satellite clock time-frequency reference performance, and injection station clock time-frequency reference performance.

Since the hypothetical observation strategy is observation for 1 hour and forecast for 1 hour, the performance of the satellite-borne time-frequency reference is directly related to the 1-hour stability of the satellite clock. According to the above chapter, the performance of the satellite clock is decomposed, and the 1-hour stability of the satellite clock is  $\sigma(3600s) = 8.3E - 14 \times (1 - 30\%) \approx 5.8E - 14$ . The equivalent distance error is  $(5.8E - 14) \times 3E8m/s \times 3600s \approx 0.06m$ .

The performance of the injected station clock is slightly better than that of the satellite clock, but it is reflected in the user equivalent distance error, which is almost equal.

According to the principle of error allocation,

$$\sigma_{\text{the time of the star is compared with the random error of link}} = \sqrt{\sigma_{\text{injection station clock and satellite}}^2 - \sigma_{\text{satellite clock time-frequency}}^2 - \sigma_{\text{injection station clock time base}}^2} = \sqrt{0.46^2 - 0.06^2 - 0.06^2} \approx 0.45m$$
(30)

The equivalent distance error of the time-to-space error of the star-to-ground random error is 0.45m; that is, the

time-to-station random error of the satellite-to-ground time is 1.5ns. From the point of view of model analysis, the time-to-station comparisons between stations and satellites are equivalent to random errors.

④ *Star-Time Comparison of Random Error Indicator Terms.* According to Figure 7, using correlation decomposition, the link-to-link random error of satellites and satellites includes pseudorange measurement accuracy, equipment delay error, ionospheric delay error, and multipath error, etc., according to the principle of error allocation:

$$\sigma_{\text{accuracy of pseudo distance}} = \sigma_{\text{equipment delay}} = \sigma_{\text{ionospheric delay}} = \sigma_{\text{multipath}} = \sigma_{\text{ionospheric delay}} = \sigma_{\text{multipath}} = \sqrt{\frac{1}{4}\sigma_{\text{star time comparison}}^2} \approx 0.22m$$
(31)

According to formula (31), the user equivalent distance error of pseudorange measurement accuracy, device delay error, ionospheric delay error, and multipath error is 0.22m, which is 0.7ns.

(5) *The Main Clock Performance Index Quantitative Decomposition.* Since the observation strategy is observation for 1 hour and forecast for 1 hour, it can be first concluded that the 1-hour stability of the main bell is equal to the stability of the satellite clock for 1 hour; that is, the second stability is 5E-12. However, in practice, when the main control station selects the main clock, the selected main clock is at least half a second higher than the satellite clock, so the second-degree stability assigned to the main clock is 1E-12. According to the foregoing chapter's index decomposition theory of bell performance can estimate the stability indicator of the main bell:

Seconds stability:

$$\sigma(1s) \approx 1E - 12 \tag{32}$$

Ten seconds stability:

$$\sigma(10s) = \left(\frac{1}{10}\right)^{1/2} \times \sigma(1s) \times (1 - 30\%) \approx 2.2E - 13 \quad (33)$$

100 second stability:

$$\sigma(100s) = \left(\frac{1}{100}\right)^{1/2} \times \sigma(1s) \times (1 - 30\%) \approx 1E - 13 \quad (34)$$

Thousand seconds stability:

$$\sigma (1000s) = \left(\frac{1}{1000}\right)^{1/2} \times \sigma (1s) \times (1 - 30\%)$$

$$\approx 3E - 14$$
(35)

Million seconds stability:

$$\sigma (10000s) = \left(\frac{1}{10000}\right)^{1/2} \times \sigma (1s) \times (1 - 30\%)$$
  
\$\approx 1E - 14\$ (36)

Day stability:

$$\sigma (86400s) = \left(\frac{1}{86400}\right)^{1/2} \times \sigma (1s) \times (1 - 30\%)$$

$$\approx 3E - 15$$
(37)

There is a maximum requirement for the constant term of the clock difference prediction model in the navigation message broadcast by the BeiDou satellite navigation system, and the physical deviation between the satellite clock time and the system time should be less than 1 millisecond. If it is higher than 1 millisecond, the atomic clock needs to be physically adjusted, and this adjustment will affect the stability output indicator of the on-board atomic clock. It requires less adjustment, the general adjustment interval should be greater than 100 days, and it can be estimated that the frequency accuracy of the satellite clock is

$$1ms/100d = 1E - 10 \tag{38}$$

In order to ensure the reliability of practical applications, it is necessary to reserve a certain margin and take an index value of one order of magnitude higher; that is, the frequency accuracy of the satellite clock is about 1E-11. According to experience, the frequency accuracy of the main clock is 2 orders of magnitude higher than that of the satellite clock. Therefore, the frequency accuracy of the main clock can be set to 1E-13.

The frequency drift rate of a satellite clock can be approximated by

$$1ms/2(100d)^2 = 2.7E - 17/s = 2.3E - 12/d$$
(39)

In order to ensure the reliability of practical applications, it is necessary to reserve a certain margin and take an index value of one order of magnitude higher; that is, the frequency drift rate of the satellite clock is about 2E-13. After each comparison, the traceability of its accuracy needs to be increased by one order of magnitude. The frequency drift rate of the main clock is one order of magnitude higher than the frequency drift rate of the satellite clock. Therefore, the frequency drift rate of the main clock can be taken as 2E-14.

## 5. BeiDou Satellite Navigation System Measurement Error Index Quantitative Verification

Through the analysis of the magnitude and value relationship of the measurement error indicators of the BeiDou satellite navigation system, the magnitude of measurement error indicators of the BeiDou satellite navigation system in the ground control mode can be summed up. Comparing with the BeiDou satellite navigation system measurement error indicators currently completed can verify the correctness of the method, as shown in Table 3.

The accuracy of clock bias forecasting is an important indicator that affects the measurement accuracy of BeiDou satellite navigation system. It uses the indicator system under ground control mode as input, quantifies the satellite clock

BeiDou Satellite Navigation System Measurement Err	Equivalent distance error (m) quantified according to the model	Completion of BeiDou Satellite Navigation System Measurement Error Index (m)	
positioning accuracy		10m	10m
System position accuracy factor PDOP	2.5	2.5	
UERE		4	4
Satellite delay error		0.06	0.06 (Qualitative)
Relativity effect error		_	_
Satellite ephemeris error		1	1
Ionospheric refractive error		3	3
Tropospheric refractive error		1	1
Multipath or occlusion error		2	2
BeiDou satellite navigation receiver measurement error		0.3	0.3
Satellite clock difference		0.952	0.9
Spaceborne atomic clock performance indicators	Frequency stability	Second stability 5E-12, Ten-second stability 1.1E-12, 100-second stability 3.5E-14, Thousand-second stability 1.1E-14, Million-second stability 3.5E-14, Day stability 2E-14	Second stability 3E-12, Ten-second stability 1E-12,100-second stability 3E-13, Thousand-second stability 1E-13, Million-second stability 3E-14, Day stability 2E-14
	Frequency accuracy	1E-11	1E-11
	Frequency drift rate	2E-13/d	1E-13
Inject station clock performance index	Frequency stability	Second stability 5E-12, Ten-second stability 1.1E-12, 100-second stability 3.5E-14, Thousand-second stability 1.1E-14, Million-second stability 3.5E-14, Day stability 2E-14	Second stability 3E-12, Ten-second stability 1E-12, 100-second stability 3E-13, Thousand-second stability 1E-13, Million-second stability 3E-14, Day stability 2E-14
	Frequency accuracy	1E-11	1E-11
	Frequency drift rate	2E-13	1E-13
Main clock performance index	Frequency stability	Second stability 1E-12, Ten-second stability 2.2E-13,100-second stability 1E-13, Thousand-second stability 3E-14, Million-second stability 1E-14, Day stability 3E-15	Million-second stability: 1E-14, Day stability: 3E-15
	Frequency accuracy	1E-13	3E-14
	Frequency drift rate	2E-13/d	1E-14/d
	Pseudorange measurement accuracy	0.7ns	0.5~1ns
Star time comparison	Device delay error	0.7ns	0.5~1ns
	Ionospheric delay error	0.7ns	0.5~1ns
	Multipath error	0.7ns	0.3~1ns

#### TABLE 3: ERE index allocation table.

error indicator, and compares with the indicators of measurement error of the Compass satellite navigation system completed in the construction. The arguments deduced that the main clock, the satellite clock, and the injection clock are in the seconds, ten seconds, and the hundred-second stability index which is within the accuracy of the system's true value. More than 1000 seconds is away from the accuracy range of the system, which is consistent with the hopping characteristics of the chime clock after thousands of seconds. The accuracy of the ground pseudorange measurement, the on-board pseudorange measurement accuracy, the equipment delay error, and the ionospheric delay error are 0.7ns in accordance with the error allocation principle. Among them, the measurement accuracy of ground pseudorange and the precision of on-board pseudorange measurement are related to the ranging code used by the navigation signal. Therefore, 0.7 ns represents the orientation of the scope. Within the scope of the real value, prove the validity of the method. Equipment time delay error of the emitting and receiving equipment include time delay and instability of satellite repeater delay etc. 0.7ns is within the true value range, the verification method is correct, and the ionospheric delay error can be further reduced to 0.5ns by using multistation and dual-frequency monitoring observations, indicating that 0.7ns is within the verification range and the method is effective. The multipath error is related to the position and environment of the receiver, and the quantified value is within the true value of the system, which has a typical significance.

## 6. Conclusion

During the demonstration process of the BeiDou satellite navigation system, it is necessary to repeatedly adjust the errors and simulate the calculation system positioning accuracy according to the system's service accuracy requirement; finally, the indicator distributions that meet the current technological development level under the condition of satisfying the accuracy index are given, and the error control of each link in the engineering development process is guided. In this paper, based on the pseudorange measurement error in BeiDou satellite navigation system, an overall construction method of measurement error system based on empirical prediction method and quantitative decomposition modeling is proposed. A clear representation of the complex relationship of index matching was achieved and qualitatively combined with the numerical matching relationship and constraint relationship between the index values of the indicators related to system service performance and the measurement error index system of the BeiDou satellite navigation system which was established. From the verification results, this method is feasible and can be used to guide the error control in the system engineering construction.

#### **Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

#### **Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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