

Research Article

Consensus Learning Control for Leader-Following Nonlinear Multiagent Systems with Control Delay

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In this paper, the consensus tracking problem of leader-following nonlinear control time-delay multiagent systems with directed communication topology is addressed. An improved high-order iterative learning control scheme with time-delay is proposed, where the local information between agents is considered. The uniformly global Lipschitz condition is applied to deal with the nonlinear dynamics. Then, a sufficient condition is driven, which guarantees that all the following agents track the trajectory of leader. Also, the convergence of proposed control protocol is analyzed by the norm theory. Finally, two cases are provided to illustrate the validity of theoretical results.

1. Introduction

In recent years, cooperative control problems of multiagent systems have been paid considerable attention owing to their applications in multiple unmanned aerial vehicle systems, mobile robot systems, and sensor networks [1–3]. The key problem of cooperative control is consensus, which means that the states of a group of agents arrive at the agreement under a designed control law.

In the existing literature, the consensus problems of linear multiagent systems, such as the leader-following consensus tracking problem [4, 5], the random packet dropout problem [6], and the finite-time formation problem [7], have been widely considered. However, compared with the linear multiagent systems, the consensus problems of nonlinear multiagent systems are more popular for researchers. For example, the robust tracking problem of heterogeneous nonlinear multiagent systems with bounded disturbances was developed in [8]. In [9], the consensus problem of nonlinear multiagent systems with unknown control directions was addressed, and the quantized consensus protocol for secondorder nonlinear multiagent systems was studied in [10]. In addition, the formation control problems of nonlinear multiagent systems [11, 12] and the tracking problem of highorder nonlinear multiagent systems [13] were investigated as well. It should be pointed out that the consensus problems considered in the above papers do not take into consideration the case of time delay.

In practice, the time delay occurred due to the impact of physical factors of sensors. It is of great significance to study the control problem of time-delay systems. In [14], Lyapunov matrices for a type of time-delay systems were designed. In [15], the stabilization problem of time-delay switched control systems was analyzed, and a compensation approach for a time-delay system was proposed in [16]. By now, the consensus problems of multiagent systems with time delay have been investigated in some papers. In [17], an eventtriggered consensus protocol for the leader-following firstorder linear multiagent systems with time-varying delays was proposed. In [18], the consensus problem of second-order linear multiagent systems with communication delay was addressed. In [19, 20], the consensus problems of general linear multiagent systems with time-delay were studied, while the consensus problem of delayed linear multiagent systems was fully considered and studied in [21]. Moreover, the leader-following consensus problem of nonlinear multiagent systems with mixed delays and the formation control problem of multiagent systems with time-varying delays were addressed [22, 23]. Different from [17-20], the case of control delay was considered in [24]. Based on the above papers, it is not difficult to see that there are few efforts focusing on the consensus problems of multiagent systems with control delay. Hence, the first motivation of this work is to discuss the control delay problem of multiagent systems.

As we know, the iterative learning control is based upon the idea that the performance of a system that performs the same task repeatedly can be improved by learning from previous iterations [25]. Currently, the method has been used to achieve the consensus problem of multiagent systems. In [26], the optimal iterative learning control protocol for the consensus tracking of multiagent systems was presented, while the event-triggered iterative learning control protocol for the same problem was designed in [27]. In [28], the formation control of multiagent systems with iterative learning control was studied, and the finite-time consensus problem of multiagent systems with iterative learning control scheme was discussed in [29]. Furthermore, the distributed adaptive iterative learning control scheme [30], the sliding mode iterative learning control approach [31], and the highorder iterative learning control protocol [32] were also designed to deal with the consensus of multiagent systems. The results obtained in the above literature indicate that the iterative learning control in solving the consensus problem of multiagent systems is effective. Accordingly, the second motivation of this work is to consider the iterative learning control for the consensus problem.

Inspired by the above analysis, the consensus tracking problem of leader-following nonlinear multiagent systems with control delay is investigated in this paper. The iterative learning control approach is introduced to design the control protocol. The main contributions of this work are summarized as follows: (i) compared with [17-20], the consensus problem of nonlinear multiagent systems with control delay under directed topology is investigated. It is assumed that the delay of control input is fixed; (ii) different from [32], an improved high-order iterative learning control protocol with time delay is designed, where the local information between agents is considered in control protocol design. In addition, the uniformly global Lipschitz condition is applied to process the nonlinear terms; (iii) based on the norm theory, the convergence of the proposed control protocol is verified, and a sufficient condition is driven. The condition can guarantee all the following agents to track the trajectory of the leader.

The rest of this paper is organized as follows. In Section 2, the graph theory, some useful definitions, and lemmas are introduced. In Section 3, the problem formulation on the leader-following nonlinear multiagent systems with control delay is described. The control protocol design and convergence analysis are shown in Section 4, and simulation examples are provided in Section 5. Finally, some conclusions are briefly drawn in Section 6.

2. Preliminaries

2.1. Graph Theory. A multiagent system which consists of n agents can be defined as a graph \mathcal{C} . Let $\mathcal{V} = (v_1, \dots, v_n)$ be a node set and let $\mathcal{C} = \{(i, j), i, j \in \mathcal{V}, \text{ and } i \neq j\}$ be a directed edge set. The adjacency matrix is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$, where $a_{ij} > 0$ if and only if $(i, j) \in \mathcal{C}$ and $a_{ij} = 0$ otherwise. It is assumed that $a_{ii} = 0$. Let $\mathcal{N}_i = \{v_j : (v_j, v_i) \in \mathcal{C}\}$ represent the set of neighbors of agent *i*. The Laplacian matrix \mathcal{S} is

denoted by $\mathscr{L} = \mathscr{D} - \mathscr{A}$, where $\mathscr{D} = \text{diag}\{d_1, \dots, d_n\}$ with $d_i = \sum_{i=1}^{\mathcal{N}_i} a_{ii}$.

In this paper, an augmented graph $\overline{\mathscr{G}}$ consists of *n* following agents and one leader agent is considered. The communication between following agents and the leader is defined as $\mathscr{B} = \text{diag}\{b_1, \dots, b_n\}$. If the agent *i* can obtain the information of the leader, then $b_i > 0$ and $b_i = 0$ otherwise.

2.2. Definitions and Lemmas

Definition 1 (see [33]). For a function $h(t) : [0, T] \longrightarrow \mathbb{R}^n$, the λ norm is defined as

$$\|\boldsymbol{h}(t)\|_{\lambda} = \sup_{t \in [0,T]} e^{-\lambda t} \|\boldsymbol{h}(t)\|, \quad \lambda > 0$$
(1)

The following property for λ norm is held.

Property 2. For the functions $f(t), h(t) : [0, T] \longrightarrow \mathbb{R}^n$, if $h(t) = \int_0^t e^{a(t-s)} f(s) ds$, there exists

$$\|\boldsymbol{h}(t)\|_{\lambda} \le \frac{1 - e^{(a-\lambda)T}}{\lambda - a} \|\boldsymbol{f}(t)\|_{\lambda}$$
(2)

and if $h(t) = \int_0^t \int_0^\sigma e^{a(\sigma-s)} f(s) ds d\sigma$, then we have

$$\|\boldsymbol{h}(t)\|_{\lambda} \leq \frac{1 - e^{-\lambda T}}{\lambda} \frac{1 - e^{(a-\lambda)T}}{\lambda - a} \|\boldsymbol{f}(t)\|_{\lambda}, \quad (0 < a < \lambda) \quad (3)$$

Lemma 3 (Bellman-Gronwall lemma). Suppose that the functions x(t) and y(t) are real continuous functions on the interval [0, T]; if there exists

$$x(t) \le c + \int_{0}^{t} (ax(s) + by(s)) ds$$
 (4)

then we have

$$x(t) \le ce^{at} + \int_0^t by(s) e^{a(t-s)} ds$$
 (5)

where $a \ge 0$ and $b, c \in R$.

Lemma 4 (see [34]). For a real positive series $\{a_n\}(n \in \{1, \dots, \infty\})$, if it satisfies

$$a_n \le \beta_1 a_{n-1} + \beta_2 a_{n-2} + \dots + \beta_N a_{N-2} + \varphi$$

 $n = N + 1, N + 2, \dots$
(6)

where $\beta_m \ge 0$ ($m = 1, \dots, N$), $\varphi \ge 0$ and there exists $\beta = \sum_{i=1}^{N} \beta_m < 1$, then we have

$$\lim_{n \to \infty} a_n \le \frac{\varphi}{1 - \beta} \tag{7}$$

3. Problem Formulation

In this section, a type of leader-following nonlinear multiagent systems with control delay is considered. The dynamics of the ith following agent at the kth iteration are described as

$$\dot{x}_{i}^{k}(t) = v_{i}^{k}(t)$$

$$\dot{v}_{i}^{k}(t) = f\left(z_{i}^{k}(t), t\right) + B\left(z_{i}^{k}(t), t\right)u_{i}^{k}(t-\tau)$$

$$y_{i}^{k}(t) = \left[x_{i}^{k}(t), v_{i}^{k}(t)\right]^{\mathrm{T}} = g\left(z_{i}^{k}(t), t\right)$$
(8)

where $x_i^k(t) \in R$, $v_i^k(t) \in R$, and $u_i^k(t) \in R$ are the position, velocity, and control input of the *i*th following agent, respectively; $y_i^k(t) \in R^2$ is the specified output; $z_i^k(t) = [x_i^k(t), v_i^k(t)]^T$ and $t \in [0, T]$; $\tau < T$ is known time delay. The nonlinear functions $f(\cdot, \cdot) : R^2 \times [0, T] \longrightarrow R$ and $B(\cdot, \cdot) : R^2 \times [0, T] \longrightarrow R$ are piecewise continuous on the interval [0, T]; $g(\cdot, \cdot) : R^2 \times [0, T] \longrightarrow R^2$ is differentiable in zand t, with $g_z(\cdot, \cdot)$ and $g_t(\cdot, \cdot)$.

The vector form of (8) is written as

$$\dot{\boldsymbol{z}}^{k}(t) = \boldsymbol{f}\left(\boldsymbol{z}^{k}(t), t\right) + \boldsymbol{B}\left(\boldsymbol{z}^{k}(t), t\right)\boldsymbol{u}^{k}(t-\tau)$$

$$\boldsymbol{y}^{k}(t) = \boldsymbol{g}\left(\boldsymbol{z}^{k}(t), t\right)$$
(9)

where $\boldsymbol{z}^{k}(t) = [(z_{1}^{k}(t))^{\mathrm{T}}, \cdots, (z_{n}^{k}(t))^{\mathrm{T}}]^{\mathrm{T}}, \boldsymbol{y}^{k}(t) = [(y_{1}^{k}(t))^{\mathrm{T}}, \cdots, (y_{n}^{k}(t))^{\mathrm{T}}]^{\mathrm{T}}, \boldsymbol{u}^{k}(t) = [u_{1}^{k}(t-\tau), \cdots, u_{n}^{k}(t-\tau)]^{\mathrm{T}}, \boldsymbol{f}(\boldsymbol{z}^{k}(t), t), t) = [f_{1}^{\mathrm{T}}(z_{1}^{k}(t), t), \cdots, f_{n}^{\mathrm{T}}(z_{n}^{k}(t), t)]^{\mathrm{T}} \text{ with } f_{i}(z_{i}^{k}(t), t) = [z_{i}^{k}(t), t) = diag\{B_{1}(z_{1}^{k}(t), t), \cdots, B_{n}(z_{n}^{k}(t), t)\}$ with $B_{i}(z_{i}^{k}(t), t) = [0, B(z_{i}^{k}(t), t)]^{\mathrm{T}}, \text{ and } \boldsymbol{g}(\boldsymbol{z}^{k}(t), t) = [\boldsymbol{g}^{\mathrm{T}}(z_{1}^{k}(t), t), \cdots, \boldsymbol{g}^{\mathrm{T}}(z_{n}^{k}(t), t)]^{\mathrm{T}}.$

The dynamics of leader agent are given as

$$\dot{x}_{o}(t) = v_{o}(t)$$

$$\dot{v}_{o}(t) = f(z_{o}(t), t) + B(z_{o}(t), t) u_{o}(t - \tau)$$
(10)
$$y_{o}(t) = [x_{o}(t), v_{o}(t)]^{T} = g(z_{o}(t), t)$$

where $z_o(t) = [x_o(t), v_o(t)]^T$; $x_o(t) \in R$, $x_o(t) \in R$, $u_o(t) \in R$, and $y_o(t) \in R^2$ are the position, velocity, and control input and output of the leader agent, respectively. It is assumed that the input $u_o(t)$ is bounded; that is, there exists $||u_o(t)|| \le b_{uo}$

with b_{uo} being a positive constant. The following assumptions are provided for the system (9).

Assumption 5. The functions f, B, g, g_{z^k} , and g_{t^k} for z^k are uniformly globally Lipschitz on the interval [0, T]; that is, there exists constant h_{α} such that

$$\left\|\boldsymbol{\alpha}\left(\boldsymbol{z}_{1}^{k}\left(t\right),t\right)-\boldsymbol{\alpha}\left(\boldsymbol{z}_{2}^{k}\left(t\right),t\right)\right\| \leq h_{\boldsymbol{\alpha}}\left\|\boldsymbol{z}_{1}^{k}\left(t\right)-\boldsymbol{z}_{2}^{k}\left(t\right)\right\|$$
(11)

where $\boldsymbol{\alpha} \in \{\boldsymbol{f}, \boldsymbol{B}, \boldsymbol{g}, \boldsymbol{g}_{\boldsymbol{z}^k}, \boldsymbol{g}_{t^k}\}.$

Assumption 6. The functions g_z and B are bounded with bounds b_{qz} and b_B that are denoted by

$$b_{gz} = \max_{k \to \infty, t \in [-\tau, T-\tau]} \left\| \boldsymbol{g}_{\boldsymbol{z}^{k}} \left(\boldsymbol{z}^{k} \left(t + \tau \right), t + \tau \right) \right\|$$

$$b_{B} = \max_{k \to \infty, t \in [-\tau, T-\tau]} \left\| \boldsymbol{B} \left(\boldsymbol{z}^{k} \left(t + \tau \right), t + \tau \right) \right\|$$
(12)

Assumption 7. The resetting condition is considered for each iteration; that is,

$$\boldsymbol{z}^{k}(0) = \mathbf{1}_{n} \otimes \boldsymbol{z}_{o}(0), \quad (k = 0, 1, \cdots)$$
 (13)

where $z_o(0)$ is the initial state of leader, \otimes represents the Kronecker product, and $\mathbf{l}_n = [1, \dots, 1]^{\mathrm{T}}$.

Remark 8. In Assumption 5, the uniformly globally Lipschitz condition is satisfied for the nonlinear dynamics. However, the radial basis function neural network is applied to approximate the nonlinear dynamics in [30, 35]. Due to the fact that the number of hidden layers impacts the approximation accuracy of neural network, the method is not adopted in this paper.

In this paper, the objective is to find a control scheme $\boldsymbol{u}^{k}(t)$ such that the output of all the following agents can track the trajectory of leader as k tends to infinity; that is, $\lim_{k\to\infty} x_i^k(t) = x_o(t)$ and $\lim_{k\to\infty} v_i^k(t) = v_o(t)$ as $t \in [0, T]$ for $i = 1, \dots, n$.

4. Main Results

In this section, the consensus problem of leader-following nonlinear multiagent systems with control delay is discussed, and the control protocol design and convergence analysis will be described.

Consider the multiagent systems (8) and (10); the consensus tracking error of the following agent i at the kth iteration is defined as

$$\varepsilon_{i}^{k}(t) = \sum_{j \in \mathcal{N}_{i}} a_{ij} \left(y_{j}^{k}(t) - y_{i}^{k}(t) \right) + b_{i} \left(y_{o}(t) - y_{i}^{k}(t) \right)$$
(14)

and we have

$$\boldsymbol{\varepsilon}^{k}\left(t\right) = \overline{\boldsymbol{\mathscr{H}}}\boldsymbol{e}^{k}\left(t\right) \tag{15}$$

where $\overline{\mathcal{H}} = \mathcal{H} \otimes I_2$, $\mathcal{H} = \mathcal{L} + \mathcal{B}$, $e^k(t) = y_o(t) - y^k(t)$, and $y_o(t) = \mathbf{1}_n \otimes y_o(t)$; I_2 is the unit matrix with 2 dimensions and $t \in [0, T]$.

For the sake of the discussion simplification, some notations are provided as follows.

$$\begin{aligned} \boldsymbol{\alpha}^{k}\left(t\right) &= \boldsymbol{\alpha}\left(\boldsymbol{z}^{k}\left(t\right), t\right), \\ \boldsymbol{\alpha}_{o}\left(t\right) &= \boldsymbol{\alpha}\left(\boldsymbol{z}_{o}\left(t\right), t\right), \\ \delta \boldsymbol{\alpha}^{k}\left(t\right) &= \boldsymbol{\alpha}_{o}\left(t\right) - \boldsymbol{\alpha}^{k}\left(t\right) \\ \boldsymbol{\alpha}_{z^{k}}\left(t\right) &= \left.\frac{\partial \boldsymbol{\alpha}\left(\boldsymbol{x}\left(t\right), t\right)}{\partial \boldsymbol{x}\left(t\right)}\right|_{\boldsymbol{x}\left(t\right)=\boldsymbol{z}^{k}\left(t\right)}, \\ \boldsymbol{\alpha}_{t^{k}}\left(t\right) &= \left.\frac{\partial \boldsymbol{\alpha}\left(\boldsymbol{x}\left(t\right), t\right)}{\partial t}\right|_{\boldsymbol{x}\left(t\right)=\boldsymbol{z}^{k}\left(t\right)} \\ \boldsymbol{\alpha}_{z_{o}}\left(t\right) &= \left.\frac{\partial \boldsymbol{\alpha}\left(\boldsymbol{x}_{o}\left(t\right), t\right)}{\partial \boldsymbol{x}_{o}\left(t\right)}\right|_{\boldsymbol{x}_{o}\left(t\right)=\boldsymbol{z}_{o}\left(t\right)}, \end{aligned}$$
(16)
$$\boldsymbol{\alpha}_{t_{o}}\left(t\right) &= \left.\frac{\partial \boldsymbol{\alpha}\left(\boldsymbol{x}_{o}\left(t\right), t\right)}{\partial \boldsymbol{x}_{o}\left(t\right)}\right|_{\boldsymbol{x}_{o}\left(t\right)=\boldsymbol{z}_{o}\left(t\right)}, \end{aligned}$$

where α represents a function concerned and $z_o(t) = \mathbf{1}_n \otimes z_o(t)$.

According to the multiagent systems (8) and (10), an improved high-order iterative learning control protocol with time delay is presented as

$$\boldsymbol{u}^{k+1}(t) = \boldsymbol{u}^{l}(t) + \sum_{m=1}^{N} \boldsymbol{Q}_{m}(t) \boldsymbol{\varepsilon}^{l}(t+\tau)$$

+
$$\sum_{m=1}^{N} \boldsymbol{R}_{m}(t) \boldsymbol{\dot{\varepsilon}}^{l}(t+\tau)$$
(17)
+
$$\sum_{m=1}^{N} \boldsymbol{S}_{m}(t) \int_{-\tau}^{t} \boldsymbol{\varepsilon}^{l}(s+\tau) ds$$

where $\varepsilon^{l}(t + \tau) = \mathcal{\mathcal{H}} \varepsilon^{l}(t + \tau)$, $t \in [-\tau, T - \tau]$; l = k - m + 1, $N \ge 1$ is the order of the control protocol; $Q_m(t)$, $R_m(t)$, and $S_m(t)$ are learning matrices with appropriate dimensions. In addition, it is supposed that $Q_m(t)\mathcal{\mathcal{H}}$, $R_m(t)\mathcal{\mathcal{H}}$, and $S_m(t)\mathcal{\mathcal{H}}$ are bounded and their bounds are defined as b_Q , b_R , and b_S , respectively; that is,

$$b_{Q} = \max_{t \in [-\tau, T-\tau]} \left\| \boldsymbol{Q}_{m}(t) \, \overline{\boldsymbol{\mathcal{H}}} \right\|,$$

$$b_{R} = \max_{t \in [-\tau, T-\tau]} \left\| \boldsymbol{R}_{m}(t) \, \overline{\boldsymbol{\mathcal{H}}} \right\|,$$

$$b_{S} = \max_{t \in [-\tau, T-\tau]} \left\| \boldsymbol{S}_{m}(t) \, \overline{\boldsymbol{\mathcal{H}}} \right\|$$
(18)

Remark 9. The introduction of learning matrices $Q_m(t)$, $R_m(t)$, and $S_m(t)$ is to adjust the performance of control protocol (17). Through adjusting these learning matrices, the desired control effect can be obtained. However, as can be seen from the below analysis, the learning matrices $Q_m(t)$ and $S_m(t)$ do not affect the convergence result of control protocol (17). A sufficient condition with $R_m(t)$ will be shown in Theorem 10.

Based on the above analysis, we have Theorem 10.

Theorem 10. Consider the multiagent systems (9) and (10) under directed topology and suppose that Assumptions 5–7 are satisfied, and the control protocol (17) is applied. If there exists

$$\sum_{m=1}^{N} \boldsymbol{P}_{m}\left(t\right) = \boldsymbol{I}_{n} \tag{19}$$

and positive numbers β_m satisfy

$$\left\|\boldsymbol{P}_{m}\left(t\right) - \boldsymbol{R}_{m}\left(t\right)\overline{\boldsymbol{\mathcal{H}}}\boldsymbol{g}_{\boldsymbol{z}}\left(\boldsymbol{z}^{k}\left(t\right), t\right)\boldsymbol{B}\left(\boldsymbol{z}^{k}\left(t\right), t\right)\right\| \leq \beta_{m}$$

$$\sum_{m=1}^{N} \beta_{m} < 1$$
(20)

then the output of all following agents can track the trajectory of leader agent; that is, $\lim_{k\to\infty} x_i^k(t) = x_o(t)$ and $\lim_{k\to\infty} v_i^k(t) = v_o(t)$ as $t \in [0, T]$ for $i = 1, \dots, n$.

Proof. Considering (9) and (10), the tracking error is defined as

$$\boldsymbol{e}^{l}\left(t+\tau\right) = \boldsymbol{y}_{o}\left(t+\tau\right) - \boldsymbol{y}^{l}\left(t+\tau\right) = \delta \boldsymbol{g}^{l}\left(t+\tau\right)$$
(21)

where $\boldsymbol{y}_o(t+\tau) = \mathbf{1}_n \otimes g(\boldsymbol{z}_o(t+\tau), t+\tau), \, \boldsymbol{y}^l(t+\tau) = [\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{z}_1^l(t+\tau), t+\tau), \, \boldsymbol{y}_1^{\mathrm{T}}(\boldsymbol{z}_1^l(t+\tau), t+\tau)]^{\mathrm{T}}$, and $t \in [-\tau, T-\tau]$. Then, the derivative of (21) is

$$\dot{\boldsymbol{\varepsilon}}^{l}(t+\tau) = \dot{\boldsymbol{y}}_{o}(t+\tau) - \dot{\boldsymbol{y}}^{l}(t+\tau) = \boldsymbol{g}_{\boldsymbol{z}_{o}}(t+\tau) \dot{\boldsymbol{z}}_{o}(t+\tau) \dot{\boldsymbol{z}}_{o}(t+\tau) + \boldsymbol{g}_{t_{o}}(t+\tau) - \left(\boldsymbol{g}_{\boldsymbol{z}^{l}}(t+\tau) \dot{\boldsymbol{z}}^{l}(t+\tau) + \boldsymbol{g}_{\boldsymbol{z}_{o}}(t+\tau) - \left(\boldsymbol{g}_{\boldsymbol{z}^{l}}(t+\tau) \dot{\boldsymbol{z}}^{l}(t+\tau) + \boldsymbol{z}\right) + \boldsymbol{g}_{t_{o}}(t+\tau) + \boldsymbol{z}_{o}(t+\tau) + \boldsymbol{z}_{o}(t+\tau) + \boldsymbol{z}_{o}(t+\tau) + \boldsymbol{z}_{o}(t+\tau) - \boldsymbol{g}_{\boldsymbol{z}^{l}}(t+\tau) + \boldsymbol{z}_{o}(t+\tau) + \boldsymbol{z}_{o}(t+\tau) - \boldsymbol{g}_{\boldsymbol{z}^{l}}(t+\tau) + \boldsymbol{z}_{o}(t+\tau) + \boldsymbol{z}_{o}(t+\tau) + \boldsymbol{z}_{o}(t+\tau) - \boldsymbol{g}_{\boldsymbol{z}^{l}}(t+\tau) + \boldsymbol{z}_{o}(t+\tau) + \boldsymbol{z}_{o}($$

where $\mathbf{z}^{l}(t + \tau) = [(z_{1}^{l}(t + \tau))^{\mathrm{T}}, \cdots, (z_{n}^{l}(t + \tau))^{\mathrm{T}}]^{\mathrm{T}}, \mathbf{u}^{l}(t) = [u_{1}^{l}(t), \cdots, u_{n}^{l}(t)]^{\mathrm{T}}, \mathbf{u}_{o}(t) = \mathbf{1}_{n} \otimes u_{o}(t), \mathbf{f}^{l}(t + \tau) = [f_{1}^{\mathrm{T}}(z_{1}^{l}(t + \tau), t + \tau), \cdots, f_{n}^{\mathrm{T}}(z_{n}^{l}(t + \tau), t + \tau)]^{\mathrm{T}}, \mathbf{f}_{o}(t + \tau) = \mathbf{1}_{n} \otimes f_{o}(z_{o}(t + \tau), t + \tau), \mathbf{B}_{n}(z_{n}^{l}(t + \tau), t + \tau), \cdots, \mathbf{B}_{n}(z_{n}^{l}(t + \tau), t + \tau)], \mathbf{B}_{o}(t + \tau) = \mathbf{I}_{n} \otimes B_{o}(z_{o}(t + \tau), t + \tau), \text{and } f_{o}(z_{o}(t + \tau), t + \tau) = [z_{o}(t + \tau), f(z_{o}(t + \tau), t + \tau)]^{\mathrm{T}}, \mathbf{B}_{o}(z_{o}(t + \tau), t + \tau) = [0, B(z_{o}(t + \tau), t + \tau)]^{\mathrm{T}}.$

Substituting (21) and (22) into (17), then we have

$$\begin{split} \delta \boldsymbol{u}^{k+1}\left(t\right) &= \boldsymbol{u}_{o}\left(t\right) - \boldsymbol{u}^{k+1}\left(t\right) = \sum_{m=1}^{N} \boldsymbol{P}_{m}\left(t\right) \delta \boldsymbol{u}^{l}\left(t\right) \\ &- \sum_{m=1}^{N} \boldsymbol{Q}_{m}\left(t\right) \overline{\mathcal{W}} \delta \boldsymbol{g}^{l}\left(t+\tau\right) - \sum_{m=1}^{N} \boldsymbol{S}_{m}\left(t\right) \\ &\cdot \overline{\mathcal{W}} \int_{-\tau}^{t} \delta \boldsymbol{g}^{l}\left(s+\tau\right) ds - \sum_{m=1}^{N} \boldsymbol{R}_{m}\left(t\right) \\ &\cdot \overline{\mathcal{W}} \left\{ \boldsymbol{g}_{z^{l}}\left(t+\tau\right) \boldsymbol{B}^{l}\left(t+\tau\right) \delta \boldsymbol{u}^{l}\left(t\right) \\ &+ \boldsymbol{g}_{z_{o}}\left(t+\tau\right) \left(\boldsymbol{f}_{o}\left(t+\tau\right) + \boldsymbol{B}_{o}\left(t+\tau\right) \boldsymbol{u}_{o}\left(t\right)\right) \\ &- \boldsymbol{g}_{z^{l}}\left(t+\tau\right) \left(\boldsymbol{f}^{l}\left(t+\tau\right) + \boldsymbol{B}^{l}\left(t+\tau\right) \boldsymbol{u}_{o}\left(t\right)\right) \\ &+ \delta \boldsymbol{g}_{t^{l}}\left(t+\tau\right) \right\} = \sum_{m=1}^{N} \left\{ \boldsymbol{P}_{m}\left(t\right) \\ &- \boldsymbol{R}_{m}\left(t\right) \overline{\mathcal{W}} \boldsymbol{g}_{z^{l}}\left(t+\tau\right) \boldsymbol{B}^{l}\left(t+\tau\right) \right\} \delta \boldsymbol{u}^{l}\left(t\right) \\ &- \sum_{m=1}^{N} \boldsymbol{Q}_{m}\left(t\right) \overline{\mathcal{W}} \delta \boldsymbol{g}^{l}\left(t+\tau\right) - \sum_{m=1}^{N} \boldsymbol{R}_{m}\left(t\right) \\ &\cdot \overline{\mathcal{W}} \left\{ \delta \boldsymbol{g}_{z^{l}}\left(t+\tau\right) \left(\boldsymbol{f}_{o}\left(t+\tau\right) - \boldsymbol{f}^{N} \boldsymbol{R}_{m}\left(t\right) \\ &+ \delta \boldsymbol{g}_{t^{l}}\left(t+\tau\right) \right\} - \sum_{m=1}^{N} \boldsymbol{R}_{m}\left(t\right) \\ &\cdot \overline{\mathcal{W}} \left\{ \boldsymbol{g}_{z^{l}}\left(t+\tau\right) \left(\boldsymbol{f}_{o}\left(t+\tau\right) - \boldsymbol{f}^{l}\left(t+\tau\right)\right) \right\} \\ &- \sum_{m=1}^{N} \boldsymbol{S}_{m}\left(t\right) \overline{\mathcal{W}} \int_{-\tau}^{t} \delta \boldsymbol{g}^{l}\left(s+\tau\right) ds - \sum_{m=1}^{N} \boldsymbol{R}_{m}\left(t\right) \\ &\cdot \overline{\mathcal{W}} \left\{ \boldsymbol{g}_{z^{l}}\left(t+\tau\right) \left(\boldsymbol{g}_{o}\left(t+\tau\right) - \boldsymbol{g}^{l}\left(t+\tau\right)\right) \boldsymbol{u}_{o}\left(t\right) \right\} \end{split}$$

where $u_o(t) = \mathbf{1}_n \otimes u_o(t)$ and the condition (19) is considered. Taking norms on both sides of (23) yields

$$\begin{split} \left\| \delta \boldsymbol{u}^{k+1} \left(t \right) \right\| &\leq \sum_{m=1}^{N} \left\| \boldsymbol{P}_{m} \left(t \right) \right. \\ &- \boldsymbol{R}_{m} \left(t \right) \overline{\boldsymbol{\mathcal{W}}} \boldsymbol{g}_{\boldsymbol{z}^{l}} \left(t + \tau \right) \boldsymbol{B}^{l} \left(t + \tau \right) \right\| \left\| \delta \boldsymbol{u}^{l} \left(t \right) \right\| \\ &+ \sum_{m=1}^{N} \left\| \boldsymbol{Q}_{m} \left(t \right) \overline{\boldsymbol{\mathcal{W}}} \right\| \left\| \delta \boldsymbol{g}^{l} \left(t + \tau \right) \right\| + \sum_{m=1}^{N} \left\| \boldsymbol{R}_{m} \left(t \right) \overline{\boldsymbol{\mathcal{W}}} \right\| \\ &\cdot \left\| \delta \boldsymbol{g}_{\boldsymbol{z}^{l}} \left(t + \tau \right) \left(\boldsymbol{f}_{o} \left(t + \tau \right) + \boldsymbol{B}_{o} \left(t + \tau \right) \boldsymbol{u}_{o} \left(t \right) \right) \right. \\ &+ \left. \delta \boldsymbol{g}_{\boldsymbol{t}^{l}} \left(t + \tau \right) \right\| + \left. \sum_{m=1}^{N} \left\| \boldsymbol{R}_{m} \left(t \right) \overline{\boldsymbol{\mathcal{W}}} \right\| \end{split}$$

$$\cdot \left\| \boldsymbol{g}_{\boldsymbol{z}^{l}}\left(t+\tau\right) \left(\boldsymbol{f}_{o}\left(t+\tau\right)-\boldsymbol{f}^{l}\left(t+\tau\right)\right) \right\|$$

$$+ \sum_{m=1}^{N} \left\| \boldsymbol{S}_{m}\left(t\right) \overline{\boldsymbol{\mathscr{H}}} \right\| \int_{-\tau}^{t} \left\| \boldsymbol{\delta} \boldsymbol{g}^{l}\left(s+\tau\right) \right\| ds$$

$$+ \sum_{m=1}^{N} \left\| \boldsymbol{R}_{m}\left(t\right) \overline{\boldsymbol{\mathscr{H}}} \right\|$$

$$\cdot \left\| \boldsymbol{g}_{\boldsymbol{z}^{l}}\left(t+\tau\right) \left(\boldsymbol{B}_{o}\left(t+\tau\right)-\boldsymbol{B}^{l}\left(t+\tau\right)\right) \boldsymbol{u}_{o}\left(t\right) \right\|$$

$$\leq \sum_{m=1}^{N} \beta_{m} \left\| \boldsymbol{\delta} \boldsymbol{u}^{l}\left(t\right) \right\| + \sum_{m=1}^{N} b_{S} h_{g} \int_{-\tau}^{t} \left\| \boldsymbol{\delta} \boldsymbol{z}^{l}\left(s+\tau\right) \right\| ds$$

$$+ \sum_{m=1}^{N} \left\{ b_{Q} h_{g} \right\}$$

$$+ b_{R} \left(b_{D} b_{uo} h_{g_{z}} + h_{g_{t}} + b_{gz} h_{f} + b_{gz} b_{uo} h_{B} \right) \right\} \left\| \boldsymbol{\delta} \boldsymbol{z}^{l}\left(t\right)$$

$$+ \tau \right) \right\| = \sum_{m=1}^{N} \beta_{m} \left\| \boldsymbol{\delta} \boldsymbol{u}^{l}\left(t\right) \right\| + \sum_{m=1}^{N} \gamma_{1} \left\| \boldsymbol{\delta} \boldsymbol{z}^{l}\left(t+\tau\right) \right\|$$

$$+ \sum_{m=1}^{N} \gamma_{2} \int_{-\tau}^{t} \left\| \boldsymbol{\delta} \boldsymbol{z}^{l}\left(s+\tau\right) \right\| ds$$

$$(24)$$

where the condition (20) is applied; $h_g, h_{g_z}, h_{g_t}, h_f$, and h_B are Lipschitz constants of corresponding functions; $\delta z^{l}(t + \tau) =$ $\begin{aligned} \boldsymbol{z}_o(t+\tau) - \boldsymbol{z}^l(t+\tau), \boldsymbol{b}_D &= \max \sup_{k \to \infty, t \in [-\tau, T-\tau]} \|\boldsymbol{f}_o(t+\tau) + \boldsymbol{B}_o(t+\tau)\|, \boldsymbol{\gamma}_1 &= b_Q \boldsymbol{h}_{\boldsymbol{g}} + \boldsymbol{b}_R(\boldsymbol{b}_D \boldsymbol{b}_{uo} \boldsymbol{h}_{\boldsymbol{g}_z} + \boldsymbol{h}_{\boldsymbol{g}_t} + \boldsymbol{b}_{gz} \boldsymbol{h}_f + \boldsymbol{b}_{gz} \boldsymbol{b}_{uo} \boldsymbol{h}_B), \end{aligned}$ and $\gamma_2 = b_S h_g$. According to Assumption 7, one gets

$$\begin{split} \left\| \delta z^{l} (t+\tau) \right\| &= \left\| z_{o} (0) - z^{l} (0) \right\| + \int_{-\tau}^{t} \left\| \dot{z}_{o} (s+\tau) - \dot{z}^{l} (s+\tau) \right\| ds \\ &\leq \int_{-\tau}^{t} \left\{ \left\| f_{o} (s+\tau) - f^{l} (s+\tau) \right\| \\ &+ \left\| B_{o} (s+\tau) - B^{l} (s+\tau) \right\| \left\| u_{o} (s) \right\| \\ &+ \left\| B^{l} (s+\tau) \right\| \left\| \delta u^{l} (s) \right\| \right\} ds \\ &\leq \int_{-\tau}^{t} \left\{ \left(h_{f} + b_{uo} h_{B} \right) \left\| \delta z^{l} (s+\tau) \right\| \\ &+ b_{B} \left\| \delta u^{l} (s) \right\| \right\} ds = \int_{-\tau}^{t} \left\{ \gamma_{3} \left\| \delta z^{l} (s+\tau) \right\| \\ &+ b_{B} \left\| \delta u^{l} (s) \right\| \right\} ds \end{split}$$
(25)

where $\gamma_3 = h_f + b_{uo}h_B$.



FIGURE 1: Communication topology.

Applying Lemma 3, (25) is written as

$$\left\|\delta \boldsymbol{z}^{l}\left(t+\tau\right)\right\| \leq b_{B} \int_{-\tau}^{t} e^{\gamma_{3}\left(t-s\right)} \left\|\delta \boldsymbol{u}^{l}\left(s\right)\right\| ds \qquad (26)$$

Substituting (26) into (24), then we obtain

$$\begin{aligned} \left\| \delta \boldsymbol{u}^{k+1} \left(t \right) \right\| \\ &\leq \sum_{m=1}^{N} \beta_{m} \left\| \delta \boldsymbol{u}^{l} \left(t \right) \right\| \\ &+ \sum_{m=1}^{N} b_{B} \gamma_{1} \int_{-\tau}^{t} e^{\gamma_{3} \left(t-s \right)} \left\| \delta \boldsymbol{u}^{l} \left(s \right) \right\| ds \\ &+ \sum_{m=1}^{N} b_{B} \gamma_{2} \int_{-\tau}^{t} \int_{-\tau}^{\sigma} e^{\gamma_{3} \left(\sigma-s \right)} \left\| \delta \boldsymbol{u}^{l} \left(s \right) \right\| ds d\sigma \end{aligned}$$

$$(27)$$

Multiply both sides of (27) by $e^{-\lambda t}$, one has

$$\sup_{t \in [-\tau, T-\tau]} \left\{ e^{-\lambda t} \left\| \delta \boldsymbol{u}^{k+1}(t) \right\| \right\} \leq \sum_{m=1}^{N} \left(\beta_{m} + b_{B} \gamma_{1} \frac{1 - e^{(\gamma_{3} - \lambda)(t+\tau)}}{\lambda - \gamma_{3}} + b_{B} \gamma_{2} \frac{1 - e^{-\lambda(t+\tau)}}{\lambda} \frac{1 - e^{(\gamma_{3} - \lambda)T}}{\lambda - \gamma_{3}} \right)$$

$$\cdot \sup_{t \in [-\tau, T-\tau]} \left\{ e^{-\lambda t} \left\| \delta \boldsymbol{u}^{l}(t) \right\| \right\}$$
(28)

where Definition 1 is applied.

Hence, we get

$$\left\|\delta \boldsymbol{u}^{k+1}\left(t-\tau\right)\right\|_{\lambda} \leq \sum_{m=1}^{N} \overline{\beta}_{m} \left\|\delta \boldsymbol{u}^{l}\left(t-\tau\right)\right\|_{\lambda}$$
(29)

where $\overline{\beta}_m = \beta_m + b_B \gamma_1 ((1 - e^{(\gamma_3 - \lambda)T})/(\lambda - \gamma_3)) + b_B \gamma_2 ((1 - e^{-\lambda T})/\lambda)((1 - e^{(\gamma_3 - \lambda)T})/(\lambda - \gamma_3))$ and $t \in [0, T]$. Select λ sufficiently large such that $\sum_{m=1}^N \overline{\beta}_m < 1$. In view

of Lemma 4, we have

$$\lim_{k \to \infty} \left\| \delta \boldsymbol{u}^k \left(t - \tau \right) \right\| = 0 \tag{30}$$

Furthermore, we get from (26)

$$\left\|\boldsymbol{z}_{o}\left(t\right)-\boldsymbol{z}^{k}\left(t\right)\right\|_{\lambda} \leq b_{B}\frac{1-e^{\left(\gamma_{3}-\lambda\right)T}}{\lambda-\gamma_{3}}\left\|\delta\boldsymbol{u}^{k}\left(t-\tau\right)\right\|_{\lambda}$$
(31)

Hence, it is obviously obtained from (30) and (31) that $\lim_{k\to\infty} \|\boldsymbol{z}_o(t) - \boldsymbol{z}^k(t)\|_{\lambda} = 0.$ Considering the definitions of $z_o(t)$ and $z^k(t)$, we have $\lim_{k\to\infty} x_i^k(t) = x_o(t)$ and $\lim_{k\to\infty} v_i^k(t) = v_o(t)$ as $t \in [0,T]$ for $i = 1, \dots, n$, which means that all the following agents can track the trajectory of the leader on the interval [0, T]. The proof is completed.

5. Simulation Analysis

In this section, we will provide two cases to illustrate the effectiveness of theoretical results in this paper.

Consider a class of leader-following nonlinear multiagent systems with control delay. The directed communication topology consists of four following agents and one leader agent (labeled as 0) is shown in Figure 1.

The weighted adjacency matrices are

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix},$$
(32)
$$\mathcal{B} = \operatorname{diag} \{1, 1, 0, 0\}$$

The dynamics of four following agents are described as

$$\dot{z}_{i}^{k}(t) = \begin{bmatrix} v_{i}^{k}(t) \\ \cos\left(x_{i}^{k}(t)v_{i}^{k}(t)\right) \end{bmatrix} + \begin{bmatrix} 0 \\ \sin\left(x_{i}^{k}(t)\right) + 1 \end{bmatrix} u_{i}^{k}(t-\tau)$$

$$y_{i}^{k}(t) = \begin{bmatrix} x_{i}^{k}(t), v_{i}^{k}(t) \end{bmatrix}^{\mathrm{T}}$$
(33)

where τ is fixed delay time and i = 1, 2, 3, 4.

The dynamics of leader are given as

$$\dot{z}_{o}(t) = \begin{bmatrix} v_{o}(t) \\ \cos(x_{o}(t) v_{o}(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ \sin(x_{o}(t)) + 1 \end{bmatrix} u_{o}(t - \tau)$$

$$y_{o}(t) = \begin{bmatrix} x_{o}(t), v_{o}(t) \end{bmatrix}^{\mathrm{T}}$$
(34)

The parameters are set as follows: the initial position and velocity of four agents are given as $x(0) = [-0.6, 0.3, 1.3, -1.0]^{T}$ and $v(0) = [0.7, -1.2, -0.4, 1.5]^{T}$, respectively; the simulation time $t \in [0, 8]$ and the delay time $\tau = 0.2$; the initial states of leader agent are $x_o(0) = v_o(0) = 0$; the input of leader agent $u_o(t) = \sin(\pi t)$ and the iteration number $k_{\text{max}} = 50$.

In addition, we have from (20)

$$\boldsymbol{g}_{\boldsymbol{z}}\left(\boldsymbol{z}\left(t\right),t\right) = \frac{\partial \boldsymbol{g}\left(\boldsymbol{z}\left(t\right),t\right)}{\partial \boldsymbol{z}\left(t\right)} = \boldsymbol{I}_{8}$$
(35)

The following two cases are provided to check the validity of our results.

Case 1 (consensus analysis with N = 2). In this case, let $P_1 = 0.75I_4$, $P_2 = 0.25I_4$, $R_1 = 0.6(I_4 \otimes \mathbf{1}_{1\times 2})$, and $R_1 = 0.3(I_4 \otimes \mathbf{1}_{1\times 2})$.

Due to $P_1(t) + P_2(t) = 0.75I_4 + 0.25I_4 = I_4$, $\beta_1 = ||P_1(t) - R_1(t)\overline{\mathcal{H}}g_zB||_{\text{max}} = 0.5732$, $\beta_2 = ||P_2(t) - R_2(t)\overline{\mathcal{H}}g_zB||_{\text{max}} = 0.2748$, $\beta_1 + \beta_2 = 0.8480 < 1$, which implies that the convergence conditions (19) and (20) are satisfied. The simulation results for Case 1 at the 50th iteration are shown in Figures 2, 3, 4, and 5.

The tracking trajectories of position and velocity are shown in Figures 2 and 3, which indicate that the consensus problem of following-leader nonlinear multiagent systems with control delay (8) and (10) can be achieved by using the proposed high-order iterative learning control scheme (17). Although the control signal of multiagent systems is impacted by the time delay, the problem is solved via the control protocol proposed in this paper. The tracking errors of four following agents at the 50th iteration are given in Figure 4, and the control input curves at the 50th iteration are shown in Figure 5.

Case 2 (comparison analysis with N = 1 and N = 2). In this case, let $P_1 = I_4$ and $R_1 = 0.6(I_4 \otimes 1_{1\times 2})$ for N = 1. Due to $P_1 = I_4$ and $\beta_1 = ||P_1(t) - R_1(t)\overline{\mathcal{H}}g_z B||_{\text{max}} = 0.5732 < 1$, this means that the convergence conditions (19) and (20) are guaranteed. The simulation results for Case 2 at the 50th iteration are shown in Figure 6.

The comparison results of position and velocity are shown in Figure 6. Due to the consideration of control protocols with different orders, the control effects have changed to some extent. Compared with the first-order control protocol, the control effects of second-order control protocol are better.



FIGURE 2: Position tracking trajectories for Case 1.



FIGURE 3: Velocity tracking trajectories for Case 1.

Moreover, from Figure 6, it can be seen that the overshoot of position and velocity results using the second-order control protocol is smaller, and the convergence speed is faster.

6. Conclusions

In this paper, we addressed the consensus tracking problem of leader-following nonlinear control time-delay multiagent systems with iterative learning control. An improved



FIGURE 4: Tracking errors of four agents for Case 1.



FIGURE 5: Control input for Case 1.

high-order iterative learning control protocol was proposed. Then, a sufficient condition was given to guarantee all the following agents to track the trajectory of the leader. Through simulation analysis, the consensus problem of nonlinear multiagent systems with control delay can be achieved by using the presented high-order learning control scheme. Meanwhile, the simulation results also implied that the higher the order, the better the control effect.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.



FIGURE 6: State comparison results of four agents for Case 2.

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