Hindawi
Wireless Communications and Mobile Computing
Volume 2019, Article ID 3137927, 12 pages
https://doi.org/10.1155/2019/3137927

Research Article

Generalized Complex Quadrature Spatial Modulation

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Received 7 September 2018; Revised 6 February 2019; Accepted 24 February 2019; Published 28 April 2019

Guest Editor: Mohamad K. A. Rahim

Spatial modulation (SM) is a multiple-input multiple-output (MIMO) system that achieves a MIMO high spectral efficiency while maintaining the transmitter computational complexity and requirements as low as those of the single-input systems. The complex quadrature spatial modulation (CQSM) builds on the QSM scheme and improves the spectral efficiency by transmitting two signal symbols at each channel use. In this paper, we propose two generalizations of CQSM, namely, generalized CQSM with unique combinations (GCQSM-UC) and with permuted combinations (GCQSM-PC). These two generalizations perform close to CQSM or outperform it, depending on the system parameters. Also, the proposed schemes require much less transmit antennas to achieve the same spectral efficiency of CQSM, for instance, assuming 16-QAM, GCQSM-PC, and GCQSM-UC require 10 and 15 transmit antennas, respectively, to achieve the same spectral of CQSM which is equipped with 32 antennas.

1. Introduction

Multiple-input multiple-output (MIMO) techniques are capable of satisfying the continuous demand in high data rates in communication systems. Index modulation (IM) is a group of MIMO techniques that use the indices of a given resource(s) of the communication system to convey additional information, besides that carried by the signal symbols [1, 2]. These indices represent distinct antennas [3], spreading codes [4], polarities [5], subcarriers [6], and combinations of angles [7], among others. A virtual spatial modulation (VSM) was also proposed in [8], where index modulation is performed on the parallel channels resulting from the singular value decomposition of the MIMO channel matrix.

Spatial modulation (SM) has attracted an increasing interest from researchers due to its low-complexity and high data rates. In SM, information is transmitted through the signal symbols, drawn from any conventional constellation sets, and the index of the antenna from which the signal symbol is transmitted [3]. To achieve a high spectral efficiency, SM requires relatively high number of transmit antennas. A parallel SM, denoted by half-full transmit-diversity SM (HFTD-SM), is proposed recently, where antennas are grouped into two-antenna groups. The conventional SM is then performed using the same signal symbol in each group [9, 10].

A conventional generalized SM (GSM) transmits the same signal symbol from a combination of antennas, rather than one as in SM, leading to a reduction in the number of transmit antennas required to achieve a given spectral efficiency [11]. A different implementation of the GSM was proposed in [12], referred to as MA-SM, in which each activated antenna in a given combination transmits a different signal symbol. This improves the spectral efficiency at the cost of additional hardware requirements and computational complexity. GSM was extended to the multiuser scenario in [13].

Quadrature spatial modulation (QSM) is another technique that extends the spatial constellation into in-phase and quadrature dimensions that are used to transmit the real and imaginary part, respectively, of a single signal symbol [14]. In [15], a generalized QSM (GQSM) that combines the benefits of spatial multiplexing and QSM was proposed. Antennas are grouped and independent QSM modulation is performed in each group. The hardware design of the generalized quadrature space shift keying modulation (GQSSK) and generalized QSM (GQSM) using a single RF chain was investigated in [16].

Building on the QSM, a complex QSM (CQSM) improves the spectral efficiency by transmitting two signal symbols at each channel use from antennas whose indices also carry information [17]. When both signal symbols are transmitted from the same antenna, the resulting combination is drawn from the Minkowski sum of the two sets from which the symbols are drawn. The second modulation set is designed as a rotated version of the first, where the rotation angle is optimized to minimize the average bit-error rate. To avoid transmitting the two signal symbols from the same antenna, the transmitter is equipped with an additional antenna that is used to transmit the second symbol only when both symbols are supposed to be transmitted from the same antenna in CQSM. The proposed modification is named improved CQSM (ICQSM) in [18]. The modulation set design of the ICQSM was addressed in [19]. Notice that the CQSM can be conceptualized based on the conventional SM or the GSM, and not as an extension of the QSM. While this approach is correct, we built CQSM as an extension of the QSM where instead of transmitting the real and imaginary parts of a single signal symbol from the in-phase and quadrature spatial dimensions, two complex-valued signal symbols are transmitted through the two spatial dimensions. To be consistent with our previous work, we keep the same name that explicitly includes QSM.

Contributions. The contributions of this paper are summarized as follows:

(i) We propose two generalized CQSM schemes. The first is GCQSM with unique combinations (GCQSM-UC), where the spatial symbols are generated as in generalized GSM. Each resulting combination is then split into two subsets. The first subset of antennas is used to transmit the first symbol; the second subset is used to transmit the second signal symbol. The second scheme is named GCQSM with permuted combinations (GCQSM-PC). In GCQSM-PC the antenna combinations from which the first symbol is transmitted are generated as the full set of combinations of a given length. For each combination associated with the first symbol, the corresponding list of combinations for the second symbol is generated such that no overlap occurs between antennas used for transmitting the two symbols. This algorithm reduces the number of transmit antennas required to achieve a given spectral efficiency.

(ii) We also propose an analytical method to optimize the rotation angle for CQSM and ICQSM systems. The obtained rotation angle minimizes the upper-bound of pairwise error probability.

Based on the simulation results, the proposed schemes perform very close to CQSM, while requiring much smaller number of antennas to achieve the same spectral efficiency. For instance, CQSM, GCQSM-UC, and GCQSM-PC require 32, 15, and 10 transmit antennas to achieve a spectral efficiency of 14 bits per channel use (bpcu), assuming 2 bits per signal symbol.

Notations. We denote the spectral efficiency achieved by spatial and signal symbols ES_{spa} and ES_{sig}, respectively, and the total spectral efficiency $\mathrm{ES} = \mathrm{ES}_{sig} + \mathrm{ES}_{spa}$. The vector $\mathbf{e}_a^b$ is the ath column of the $b\times b$ identity matrix. $A = \{a_1, \ldots, a_K\}$ is a set, where the order of elements does not matter, and $B = (b_1, \ldots, b_K)$ is a tuple where order matters. In the following, $(\cdot)^T$ and $(\cdot)^H$ denote the matrix/vector transpose and Hermitian transpose, respectively. $Q(\cdot)$ is the Gaussian tail function, or simply the Q-function. A signal symbol is a complex element drawn from a quadrature amplitude modulation (QAM) or phase shift keying (PSK) set. The number of bits carried by each signal symbol is equal to q, where 2^q is the cardinality of the modulation set. A spatial symbol is the index(es) of the antenna(s) from which a single or several signal symbols are transmitted.

The rest of this paper is organized as follows. In Section 2, we describe the system model and review related works. In Section 3, we introduced the proposed generalized CQSM techniques and analyze their error performance in Section 4. In Section 5, we formulate the search of the optimal rotation angle for CQSM and ICQSM as an optimization problem that reduced the asymptotic upper-bound on the error probability. The optimization of the rotation angle of the proposed generalized schemes is addressed in Section 6. The simulation results are given in Section 7 and conclusions are drawn in Section 8.

2. System Model and Related Works

2.1. System Model. Consider a MIMO system with n_T transmit and n_R receive antennas. The system equation is given as follows.

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{H}$ and $\mathbf{n}$ are the channel matrix and the noise vector whose elements are i.i.d. centered circularly symmetric complex Gaussian and have a variance of one and σ_n^2, respectively, and $\mathbf{s}$ is the transmitted vector. In SM techniques, the vector $\mathbf{s}$ contains a few nonzero elements.

2.2. Spatial Modulation. In SM, both a signal symbol and the index of the antenna from which it is transmitted carry information. As such, SM achieves an improved spectral efficiency while keeping the transmitter as simple as that of the single-input communication systems. Accordingly, the SM system is modeled as follows:

$$\mathbf{y} = \mathbf{H}\mathbf{e}_i^{n_T} s_k + \mathbf{n} = \mathbf{h}_i s_k + \mathbf{n}. \tag{2}$$

The spectral efficiency, which is equivalent to the capacity at high signal-to-noise ratio (SNR), is equal to $q + N$ bits per channel use (bpcu), with $N = \log_2(n_T)$.

2.3. Generalized Spatial Modulation. In SM, the number of transmit antennas increases exponentially in terms of the number of bits carried by each spatial symbol. That is, $n_T = 2^N$, where N is the number of bits per spatial symbol. GSM reduces the number of transmit antennas by sending the same signal symbol from a combination of two or more antennas. Let n_U denote the number of active antennas at each channel use; then the number of spatial symbols, also referred to as combinations, for a given n_T is $\binom{n_T}{n_U}$. Since the number of spatial symbols should be a power of two, only $2^{\mathrm{ES}_{spa}}$ combinations can be used, where $\mathrm{ES}_{spa} = \lfloor \log_2 \binom{n_T}{n_U} \rfloor$. Assuming $n_U = 2$, GSM requires 7 transmit antennas to achieve $\mathrm{ES}_{spa} = 4$ bits per spatial symbol, whereas SM requires 16 antennas to achieve the same spectral efficiency.

Let $\boldsymbol{l} = \{l_1, \ldots, l_{n_U}\} \in \mathbf{L}$ be a spatial symbol, where $\mathbf{L}$ is the set of spatial symbols that can be used for transmission. Then the received vector in the case of GSM is given by

$$\mathbf{y} = \frac{s_k}{\sqrt{n_U}} \sum_{i \in \boldsymbol{l}} \mathbf{h}_i + \mathbf{n}. \tag{3}$$

2.4. Multi-Active Spatial Modulation. In contrast to GSM, in which a single signal symbol is transmitted from a combination of antennas, MA-SM transmits a different signal symbol from each activated antenna. Let n_U be the number of activated antennas; then the received vector is given by

$$\mathbf{y} = \sum_{i \in \boldsymbol{l}} \mathbf{h}_{l_i} s_i + \mathbf{n}, \tag{4}$$

where $\boldsymbol{l} = \{l_1, \ldots, l_{n_U}\} \in \mathbf{L}$, and $\mathbf{L}$ is the list of spatial symbols of MA-SM, which is equivalent to that of GSM. Additionally, a 3-dimensional constellation set is proposed in which each spatial symbol is associated with a unique rotation angle. Signal symbols transmitted from a given spatial symbol are rotated before transmission. According to this description, MA-SM benefits from the moderate computational complexity and low hardware requirements of SM and the high multiplexing gain of the vertical-Bell Labs layered space-time (V-BLAST) system. The spectral efficiency of MA-SM is equal to $(\mathrm{ES}_{spa} + q \times n_U)$, where ES_{spa} is defined in Section 2.3.

2.5. Complex Quadrature Spatial Modulation. QSM expands the spatial constellation into in-phase and quadrature dimensions. At each channel use, a single signal symbol is transmitted: the real part of the signal symbol is transmitted through the in-phase spatial dimension; the imaginary through the quadrature dimension. As such, the spectral efficiency of QSM is $q + 2\log_2(n_T)$, where $\mathrm{ES}_{spa} = 2\log_2(n_T)$ and $\mathrm{ES}_{sig} = q$.

CQSM transmits two signal symbols at each channel use, leading to a spectral efficiency of $2q + 2\log_2(n_T)$, where $\mathrm{ES}_{spa} = 2\log_2(n_T)$ and $\mathrm{ES}_{sig} = 2q$. The first signal symbol $s_{k_1} \in \Omega_a$ is transmitted from the i_1th transmit antenna, and the second signal symbol $s_{k_2'} \in \Omega_b$ is transmitted from the i_2th antenna. To make the distinction between s_{k_1} and $s_{k_2'}$ at the receiver side, Ω_b is obtained as follows:

$$\Omega_b = \left\{ s_{k_2'} \mid s_{k_2'} = s_{k_1} e^{j\theta}, s_{k_1} \in \Omega_a \right\}. \tag{5}$$

The rotation angle θ is optimized such that the probability of error is reduced. The received vector of the CQSM is given by

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = \mathbf{h}_{i_1} s_{k_1} + \mathbf{h}_{i_2} s_{k_2'} + \mathbf{n}, \tag{6}$$

which is expanded to

$$\mathbf{y} = \begin{cases} \mathbf{h}_{i_1} s_{k_1} + \mathbf{h}_{i_2} s_{k_2}' + \mathbf{n} & \text{if } i_1 \neq i_2 \\ \mathbf{h}_i \left(s_{k_1} + s_{k_2}' \right) = \mathbf{h}_i s_{k_3''}, & \text{if } i_1 = i_2 = i \end{cases} \tag{7}$$

where $s_{k_3''} \in \Omega_c$ and $\Omega_c = \Omega_a \oplus \Omega_b$, with $\oplus$ denoting the Minkowski sum [20]. The full modulation set $\Omega_d = \Omega_a \cup \Omega_b \cup \Omega_c$ is of size $(|\Omega_a| + |\Omega_b| + |\Omega_a| \times |\Omega_b|)$. For instance, assuming $|\Omega_a| = |\Omega_b| = 16$, the number of symbols in Ω_d becomes 288. This high density of symbols reduces the Euclidean distance among signal symbols, leading to degradation in the error performance.

2.6. Improved Complex Quadrature Spatial Modulation. In ICQSM, the transmitter is equipped with an additional antenna, indexed n_K with $n_K = n_T + 1$, which is used only when $i_1 = i_2$. In this case, the first signal symbol is transmitted from its designated antenna and the second symbol is transmitted from the additional antenna. Accordingly, the ICQSM system is modeled as follows:

$$\mathbf{y} = \begin{cases} s_{k_1} \mathbf{h}_{i_1} + s_{k_2'} \mathbf{h}_{i_2} + \mathbf{n}, & \text{if } i_1 \neq i_2 \\ s_{k_1} \mathbf{h}_{i_1} + s_{k_2'} \mathbf{h}_{n_K} + \mathbf{n}, & \text{if } i_1 = i_2 \end{cases} \tag{8}$$

This strategy reduces the size of the total modulation set Ω_d from $(|\Omega_a| + |\Omega_b| + |\Omega_a| \times |\Omega_b|)$ in the case of CQSM to only $(|\Omega_a| + |\Omega_b|)$ in ICQSM. This increases the Euclidean distance among the signal symbols and hence improves the error performance.

Based on the above descriptions, CQSM activates either one or two antennas, whereas ICQSM always has two active antennas. To perform a fair comparison between these two systems and MA-SM, we assume that $n_U = 2$. That is, the hardware requirements of the transmitter and the computational complexity of the detection algorithm are equivalent. Since both systems transmit two signal symbols at each channel use, the comparison is conducted in terms of the spectral efficiency achieved by spatial symbols. SM, on the other hand, transmits a single signal symbol at each channel use. Table 1 lists the number of transmit antennas required to achieve a given number of bits per spatial symbol (SE_{spa}) by several systems. For instance, CQSM and ICQSM require 3 and 2 antennas less than MA-SM to achieve the same SE_{spa} of 4 bpcu. This gap increases for higher SE_{spa}.

3. Generalized Complex Quadrature Spatial Modulation

Both CQSM and ICQSM require the same number of RF chains, and ICQSM requires one more physical transmit antenna compared to CQSM. Transmitting each signal symbol from a combination of antennas, instead of a single antenna, reduces the number of transmit antennas required to achieve a given SE_{spa}. In the following subsections, we introduce two generalizations of CQSM, namely, generalized CQSM with unique combinations (GCQSM-UC) and generalized CQSM with permuted combinations (GCQSM-PC).

Table 1: The number of transmit antennas required by several systems to achieve the same number of bits per spatial symbol (SE_{spa}).

SE_{spa}	SM	MA-SM	ICQSM	CQSM
2	4	4	3	2
4	16	7	5	4
6	64	12	9	8

Table 2: An example of the spatial symbols $(\boldsymbol{i}_1, \boldsymbol{i}_2)$ that can be used for transmission by GCQSM-UC, assuming $n_T = 6$ and $n_U = 2$.

$\boldsymbol{i}_1$	$\boldsymbol{i}_2$
$\{1,2\}$	$\{3,4\}$
$\{1,2\}$	$\{3,5\}$
$\{1,2\}$	$\{3,6\}$
$\{1,2\}$	$\{4,5\}$
$\{1,2\}$	$\{4,6\}$
$\{1,2\}$	$\{5,6\}$
$\{1,3\}$	$\{4,5\}$
$\{1,3\}$	$\{4,6\}$
$\{1,3\}$	$\{5,6\}$
$\{1,4\}$	$\{5,6\}$
$\{2,3\}$	$\{4,5\}$
$\{2,3\}$	$\{4,6\}$
$\{2,3\}$	$\{5,6\}$
$\{2,4\}$	$\{5,6\}$
$\{3,4\}$	$\{5,6\}$

In the following, we denote by $\boldsymbol{i}_1$ and $\boldsymbol{i}_2$ the set of antennas used for transmitting the first and second signal symbols, respectively. In the sequel, the tuple $A = (B, C)$ is composed of the two sets B and C, where the order of the sets in the tuple matters but the order of the elements in each set does not.

3.1. Generalized CQSM with Unique Combinations. Let n_U be the number of antennas from which each of the two signal symbols is transmitted; then for a given n_T, the number of combinations that can be used for transmission is $\binom{n_T}{2n_U}$. Assuming that $\boldsymbol{i} = \{\boldsymbol{i}_1, \boldsymbol{i}_2\}$ is a spatial symbol, i.e., a combination of antennas of length $2n_U$; then the first and second n_U antennas are used to transmit the first and second signal symbols, respectively. Table 2 depicts an example of the spatial symbols for $n_T = 6$ and $n_U = 2$. According to this description, there will be no overlap between the sets of antennas used to transmit the first and second signal symbols. Since the number of spatial symbols should be a power of two, then only $2^{SE_{spa}}$ signal symbols can be used for transmission, where $SE_{spa} = \lfloor \log_2 \binom{n_T}{2n_U} \rfloor$.

Let $s_{k_1} \in \Omega_a$ and $s_{k_2'} \in \Omega_b$ be the two signal symbols to be transmitted at a given channel use; then the received vector is given by

$$\mathbf{y} = \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_1} \mathbf{h}_i + \frac{s_{k_2'}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_2} \mathbf{h}_i + \mathbf{n}. \tag{9}$$

The receiver employs the maximum-likelihood principle to recover the two signal symbols and the spatial symbol as follows:

$$\begin{aligned} \left(\boldsymbol{i}^*, s_{k_1}^*, s_{k_2'}^*\right) &= \underset{\substack{s_{k_1} \in \Omega_a, s_{k_2'} \in \Omega_b \\ \boldsymbol{i} = (\boldsymbol{i}_1, \boldsymbol{i}_2) \in \mathbf{L}}}{\arg\min} \|\mathbf{y} - \mathbf{g}\|^2 \\ &= \underset{\substack{s_{k_1} \in \Omega_a, s_{k_2'} \in \Omega_b \\ \boldsymbol{i} = (\boldsymbol{i}_1, \boldsymbol{i}_2) \in \mathbf{L}}}{\arg\min} \|\mathbf{g}\|^2 - 2\Re\left(\mathbf{y}^H \mathbf{g}\right), \end{aligned} \tag{10}$$

where

$$\mathbf{g} = \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_1} \mathbf{h}_i + \frac{s_{k_2'}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_2} \mathbf{h}_i \tag{11}$$

GCQSM-UC can be easily extended to transmitting more than two signal symbols per channel use at the cost of requiring more RF chains at the transmitter side. This is possible because the error performance of the system does not depend on the rotation angle θ. This will be addressed in more detail in subsequent sections.

3.2. Generalized CQSM with Permuted Combinations. To reduce the number of transmit antennas required to achieve a given spectral efficiency, we use the structure of the transmitted vector to increase the number of spatial symbols. Let $\boldsymbol{i} = (\boldsymbol{i}_1, \boldsymbol{i}_2)$ be a spatial symbol, where the first signal symbol s_{k_1} is transmitted from the $\boldsymbol{i}_1$ combination of antennas and the second signal symbol $s_{k_2'}$ from the combination $\boldsymbol{i}_2$. For a given channel realization, the received noiseless vector $\mathbf{g}$ is given by

$$\begin{aligned} \mathbf{g} &= \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_1} \mathbf{h}_i + \frac{s_{k_2'}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_2} \mathbf{h}_i \\ &= \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_1} \mathbf{h}_i + \frac{s_{k_2}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_2} e^{j\theta} \mathbf{h}_i, \end{aligned} \tag{12}$$

where $s_{k_2'} = s_{k_2} e^{j\theta}$ and $s_{k_1}, s_{k_2} \in \Omega_a$. To obtain $\boldsymbol{i}_1$ and $\boldsymbol{i}_2$, we impose the following conditions:

(i) $\boldsymbol{i}_1 \cup \boldsymbol{i}_2 = \emptyset$: this implies that the set of antennas from which the first signal symbol and that from which the second symbol are transmitted do not overlap. This condition reduces the detection ambiguity at the receiver.

(ii) $(\boldsymbol{i}_1, \boldsymbol{i}_2) \neq (\boldsymbol{i}_2, \boldsymbol{i}_1)$: as described earlier, the first and second symbols are drawn from different constellation sets. Therefore, for given signal symbols' indices k_1 and k_2, the following two received lattice points—noiseless received vectors— are distinguishable:

$$\mathbf{g}_1 = \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_1} \mathbf{h}_i + \frac{s_{k_2}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_2} e^{j\theta} \mathbf{h}_i, \tag{13}$$

and

$$\mathbf{g}_2 = \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_2} \mathbf{h}_i + \frac{s_{k_2}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_1} e^{j\theta} \mathbf{h}_i. \tag{14}$$

Input: $A = \{1, \ldots, n_T\}, n_U, n_T$
Output: The set of spatial symbols $\mathcal{I}$
$\mathcal{C} = \mathrm{COMB}(A, n_U)$;
for $\boldsymbol{i}_1 \in \mathcal{C}$ **do**
  $B = A_{\overline{\boldsymbol{i}_1}}$;
  $\mathcal{D} = \mathrm{COMB}(B, n_U)$;
  for $\boldsymbol{i}_2 \in \mathcal{D}$ **do**
    $\mathrm{APPND}(\mathcal{I}, (\boldsymbol{i}_1, \boldsymbol{i}_2))$
  end
end

Algorithm 1: Generation of the spatial symbols, assuming n_T transmit antennas and that each signal symbol is transmitted from a combination of n_U antennas.

Table 3: An example of the spatial symbols $(\boldsymbol{i}_1, \boldsymbol{i}_2)$ that can be used for transmission by GCQSM-PC, assuming $n_T = 5$ and $n_U = 2$.

$\boldsymbol{i}_1$	$\boldsymbol{i}_2$
{1, 2}	{3, 4}, {3, 5}, {4, 5}
{1, 3}	{2, 4}, {2, 5}, {4, 5}
{1, 4}	{2, 3}, {2, 5}, {3, 5}
{1, 5}	{2, 3}, {2, 4}, {3, 4}
{2, 3}	{1, 4}, {1, 5}, {4, 5}
{2, 4}	{1, 3}, {1, 5}, {3, 5}
{2, 5}	{1, 3}, {1, 4}, {3, 4}
{3, 4}	{1, 2}, {1, 5}, {2, 5}
{3, 5}	{1, 2}, {1, 4}, {2, 4}
{4, 5}	{1, 2}, {1, 3}, {2, 3}

This is rendered possible using the rotation angle θ. The optimization of the rotation angle is addressed in a following section.

Algorithm 1 depicts a pseudocode of generating the set of spatial symbols $\mathcal{I}$ that can be used for transmission. Therein $\mathrm{COMB}(A, n_U)$ generates all combinations of length n_U of the elements of the set A. Also, $\mathrm{APPND}(\mathcal{I}, (\boldsymbol{i}_1, \boldsymbol{i}_2))$ appends the tuple $(\boldsymbol{i}_1, \boldsymbol{i}_2)$ to the set $\mathcal{I}$. $B = A_{\overline{\boldsymbol{i}_1}}$ is the difference of the two sets A and $\boldsymbol{i}_1$. Accordingly, the number of spatial symbols that can be used for transmission is given by

$$C_{\mathrm{spa}} = \binom{n_T}{n_U} \times \binom{n_T - n_U}{n_U}. \quad (15)$$

Table 3 depicts an example of the obtained spatial symbols, assuming $n_T = 5$ and $n_U = 2$. In this case, GCQSM-PC obtains 30 spatial symbols, whereas GCQSM-UC obtains only five.

Table 4 lists the number of transmit antennas required by each system to achieve a given spectral efficiency per spatial symbol. For instance, SM, GSM (and MA-SM), CQSM, GCQSM-UC, and GCQSM-PC require 1024, 46, 32, 15, and 10 transmit antennas, respectively, to achieve 10 bits per spatial symbol.

4. Performance Analysis of the Generalized CQSM

Let

$$\mathbf{g}_i = \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_1} \mathbf{h}_i + \frac{s_{k_2}}{\sqrt{n_U}} \sum_{i \in \boldsymbol{i}_2} e^{j\theta} \mathbf{h}_i \quad (16)$$

and

$$\mathbf{g}_k = \frac{s_{\tilde{k}_1}}{\sqrt{n_U}} \sum_{i \in \tilde{\boldsymbol{i}}_1} \mathbf{h}_i + \frac{s_{\tilde{k}_2}}{\sqrt{n_U}} \sum_{i \in \tilde{\boldsymbol{i}}_2} e^{j\theta} \mathbf{h}_i \quad (17)$$

be two noiseless received codewords corresponding to the transmitted symbol $(s_{k_1}, s_{k_2}, \boldsymbol{i}_1, \boldsymbol{i}_2)$ and $(s_{\tilde{k}_1}, s_{\tilde{k}_2}, \tilde{\boldsymbol{i}}_1, \tilde{\boldsymbol{i}}_2)$, where s_k is a signal symbol and $\boldsymbol{i} = \{l_1, \ldots, l_{n_U}\}$ is a spatial symbol. The corresponding received vectors are denoted by $\mathbf{y}_i$ and $\mathbf{y}_k$, respectively.

The conditional pairwise error probability (PEP) of the maximum-likelihood (ML) receiver is given by [21, 22]

$$\begin{aligned} \Pr[\mathbf{g}_i \longrightarrow \mathbf{g}_k \mid \mathbf{H}] &= Q\left(\sqrt{\frac{\|\mathbf{y}_i - \mathbf{y}_k\|^2}{2\sigma_n^2}}\right), \\ &= Q\left(\sqrt{\frac{\|\mathbf{H}(\mathbf{s}_i - \mathbf{s}_k)\|^2}{2\sigma_n^2}}\right), \end{aligned} \quad (18)$$

where $Q(\cdot)$ is the Gaussian tail probability function, or simply the Q-function, and $d_{i,k}^2 = \|\mathbf{y}_i - \mathbf{y}_k\|^2$ is the squared Euclidean distance between $\mathbf{s}_i$ and $\mathbf{s}_k$ at the receiver. Similarly, $d_{i,k(\mathrm{tx})}^2 = \|\mathbf{s}_i - \mathbf{s}_k\|^2$ is the squared Euclidean distance between $\mathbf{s}_i$ and $\mathbf{s}_k$ at the transmitter. Note that,

$$\begin{aligned} \mathbb{E}_{\mathbf{H}}\left\{d_{i,j}^2\right\} &= \mathbb{E}_{\mathbf{H}}\left\{(\mathbf{s}_i - \mathbf{s}_k)^H \mathbf{H}^H \mathbf{H} (\mathbf{s}_i - \mathbf{s}_k)\right\} \\ &= (\mathbf{s}_i - \mathbf{s}_k)^H (\mathbf{s}_i - \mathbf{s}_k) = d_{i,k(\mathrm{tx})}^2. \end{aligned} \quad (19)$$

The unconditional PEP (UPEP), assuming n_R receives antennas, which is obtained by taking the expectation of (18) over the channel $\mathbf{H}$ is given by

$$\Pr[\mathbf{g}_i \longrightarrow \mathbf{g}_k] = \mu_{i,j}^{n_R} \sum_{l=0}^{n_R - 1} \binom{n_R - 1 + l}{l} \left[1 - \mu_{i,j}\right]^l, \quad (20)$$

where

$$\mu_{i,j} = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma \cdot d_{i,k(\mathrm{tx})}^2}{4 + \gamma \cdot d_{i,k(\mathrm{tx})}^2}}\right), \quad (21)$$

and $\gamma = \sigma_s^2 / \sigma_n^2$ is the signal-to-noise ratio (SNR) with $\sigma_s^2 = \mathbb{E}(s^* s)$, the expected power of the signal symbol.

The union bound on the pairwise error probability is then given by averaging all the pairwise probabilities as follows;

$$\Pr[e] \leq \frac{1}{2^M} \sum_{i=1}^{2^M} \sum_{k=1}^{2^M} \Pr[\mathbf{g}_i \longrightarrow \mathbf{g}_k]. \quad (22)$$

Table 4: Examples of the number of transmit antennas required to achieve a given SE_{spa} for several spatial modulation systems.

SE_{spa}	SM	GSM	CQSM, QSM	GCQSM-UC	GCQSM-PC
6	64	12	8	8	6
8	256	24	16	11	8
10	1024	46	32	15	10
12	4096	92	64	20	13

Accordingly, the average bit-error rate (BER) is upper-bounded by

$$P_e \leq \frac{1}{M2^M} \sum_{i=1}^{2^M} \sum_{k=1}^{2^M} D_{\mathbf{s}_i,\mathbf{s}_k} \Pr\left[\mathbf{g}_i \longrightarrow \mathbf{g}_k\right] \tag{23}$$

where $D_{\mathbf{s}_i,\mathbf{s}_k}$, the hamming distance between $\mathbf{s}_i$ and $\mathbf{s}_k$, is the number of errors associated with the event $[\mathbf{g}_i \longrightarrow \mathbf{g}_k]$ and M is the spectral efficiency of the system. Note that in each transmitted vector symbol $\mathbf{s}$, there are exactly $(2 \times n_U)$ nonzero elements.

5. Optimization of the Rotation Angles of CQSM/ICQSM Revisited

In [17, 18], the optimal rotation angles applied to the second modulation set of the CQSM and ICQSM, respectively, are obtained through extensive Monte Carlo simulations. In this case, the obtained rotation angle corresponds to the minimum bit-error rate (BER) averaged over a large number of channel and noise realizations. Alternatively, the rotation angle can be obtained through minimizing the upper-bound of the unconditional pairwise error probability.

Let

$$\begin{aligned} \mathbf{g}_i &= \mathbf{H}\mathbf{s}_i = s_{k_1}\mathbf{h}_{i_1} + s'_{k_2}\mathbf{h}_{i_2} \\ \mathbf{g}_k &= \mathbf{H}\mathbf{s}_k = s_{\tilde{k}_1}\mathbf{h}_{\tilde{i}_1} + s'_{\tilde{k}_2}\mathbf{h}_{\tilde{i}_2}, \end{aligned} \tag{24}$$

where $\mathbf{y}_i = \mathbf{g}_i + \mathbf{n}$ and $\mathbf{y}_k = \mathbf{g}_k + \mathbf{n}$ are the corresponding received vectors for a given noise vector $\mathbf{n}$.

The PEP and UPEP of the CQSM and ICQSM can also be obtained using (18)–(21). The union bound on the pairwise error probability at high SNR values (a.k.a. asymptotic probability of error) is then given by

$$\begin{aligned} \Pr[e] &\leq \frac{1}{2^M} \sum_{i=1}^{2^M} \sum_{k=1}^{2^M} \Pr\left[\mathbf{g}_i \longrightarrow \mathbf{g}_k\right] \\ &= \binom{2n_R - 1}{n_R} \frac{\gamma^{-n_R}}{2^M} \sum_{i=1}^{2^M} \sum_{k=1}^{2^M} \left(d^2_{i,k(\text{tx})}\right)^{-n_R} \\ &= \binom{2n_R - 1}{n_R} \frac{\gamma^{-n_R}}{L^2} \sum_{i=1}^{B} f_i \Omega_i, \end{aligned} \tag{25}$$

where the maximum value of B equals 2^6 as shown in [19]. The term Ω_i is a function of the signal symbols $(s_{k_1}, s'_{k_2}, s_{\tilde{k}_1}, s'_{\tilde{k}_2})$ and n_R, and f_i is the frequency of Ω_i, where $\sum_{i=1}^{B} f_i = n_T^4$. It is shown in [19] that $B = 7$ in the case of ICQSM and it can be shown to be equal to 15 for CQSM. The rotation angle is then obtained by solving the following optimization problem:

$$\underset{0<\theta<\pi/2}{\operatorname{argmin}} \left(\sum_{i=1}^{B} f_i \Omega_i\right), \tag{26}$$

where $B = 15$ and 7 for CQSM and ICQSM, respectively.

Figure 1 depicts the cost function of (26) for CQSM and ICQSM and for several system configurations. The optimal rotation angle corresponds to the minimum value of the cost function. Assuming the case of CQSM and for the scenarios depicted in the figure, the obtained optimal rotation angles are *very close* to those obtained in [17]. Therefore, this small and tolerable degradation in the error performance comes at a high gain in the optimization time of the rotation angle. On the other hand, the obtained rotation angles for ICQSM are identical to those obtained in [18]. Assuming PSK modulation, the optimal rotation angle for the ICQSM is equal to π/L, with L denoting the cardinality of the constellation set.

6. Optimization of the Rotation Angles for the Generalized CQSM

6.1. Rotation Angle of the GCQSM-UC. As explained earlier and based on the design of the spatial symbols of the GCQSM-UC, if $(\boldsymbol{i}_1, \boldsymbol{i}_2)$ is a spatial symbol that can be used for transmission then $(\boldsymbol{i}_2, \boldsymbol{i}_1)$ is not a valid spatial symbol. This implies that the first symbol s_{k_1} and s'_{k_2} are distinguishable using the indices of the antennas from which they are sent. As such, both symbols can be drawn from the same modulation set and hence the rotation angle will have no impact on the performance of the system. In the following θ is set to zero for GCQSM-UC.

6.2. Rotation Angle of the GCQSM-PC. Opposed to the GCQSM-UC, both $(\boldsymbol{i}_1, \boldsymbol{i}_2)$ and $(\boldsymbol{i}_2, \boldsymbol{i}_1)$ are valid spatial symbols that can be used for transmission in GCQSM-PC. Therefore, the second symbol s'_{k_2} should be rotated so that it can be distinguished from the first signal symbol s_{k_1}. The formulation of the upper-bound of the GCQSM-PC as in (22) is a hard problem. Alternatively, we obtained the rotation angle using simulations. Figure 2 depicts the BER of GCQSM-PC versus the rotation angle: the upper subfigure assumes QPSK modulation and the lower subfigure assumes 16-QAM. The optimal rotation angle is about $\pi/4$ for all the simulated scenarios assuming QPSK modulation. Also, the optimal

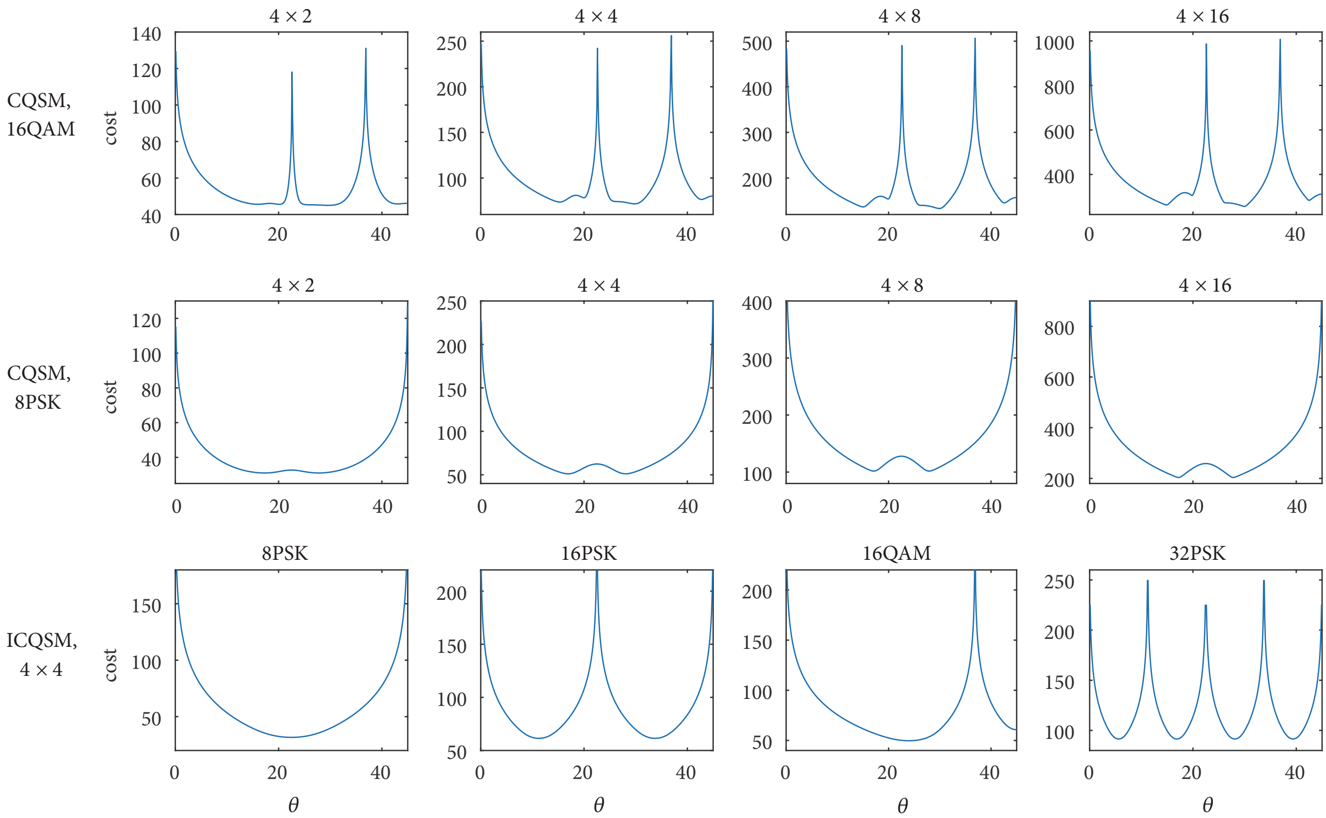


Figure 1: The cost function in (26) in dB scale versus the rotation angle for several system configurations. The subfigures in the first and second row are obtained for CQSM and those in the bottom row are for ICQSM.

rotation angle for the simulated scenarios using 16-QAM is approximately 23 degrees. These values of the optimal rotation angle are affected by the sets from which symbols are drawn, and the values of n_T and n_R. As a future work, we would like to analytically derive a generic formula to obtain the optimal angle for arbitrary system configurations.

7. Simulation Results

The following results are obtained assuming that the channel state information (CSI) is known only at the receiver. To make a fair comparison, the rotation angles for the simulated scenarios are optimized using simulations for both CQSM and the generalized CQSM. The obtained rotation angles using simulations and the optimization method described in Section 5 for CQSM scheme have very similar values.

Figure 3 depicts comparisons between the MA-SM and the CQSM for several system configurations of (n_T, q) and a fixed $n_R = 4$. We make the following comparisons.

(i) *For the same n_T and q and different spectral efficiency (Figure 3(a))*: in this case, MA-SM outperforms CQSM by about 2 dB for $n_T = 8$ and about 1.5 dB for $n_T = 16$. The gap is bridged between the two algorithms as n_T becomes large. This decrease in the performance gap is due to the decrease in the probability of transmitting the two symbols from the same transmit antenna in the CQSM. This probability is equal to $1/n_T$. The comparison here is not fair because the CQSM achieves a spectral efficiency 2 bpcu higher than that of the MA-SM.

(ii) *For the spectral efficiency and q and different n_T (Figure 3(b))*: the performance of the two algorithms is depicted for an equal spectral efficiency assuming $n_T = 8, 16$ and $n_T = 12, 24$ for CQSM and MA-SM, respectively. For these two scenarios, CQSM requires 4 and 8 transmit antennas less than MA-SM. This huge reduction in the number of transmit antennas comes at a negligible degradation in the SNR for high n_T. This implies that the CQSM is more attractive for massive MIMO systems.

(iii) *For the spectral efficiency and n_T and different q (Figure 3(c))*: finally, the performance of the two algorithms is evaluated for the same spectral efficiency and number of transmit antennas. In this case, q is set to 2 and 3 in the CQSM and MA-SM, respectively. At a target BER of 10^{-4}, CQSM outperforms MA-SM by 2 and 2.4 dB, respectively, for the two depicted system configurations.

According to this analysis and results depicted in Figure 3, we conclude that CQSM can be used to reduce the number of transmit antennas at a marginal cost in the SNR for high n_T or it can achieve an improvement in the SNR if the same number of antennas is deployed by the two systems.

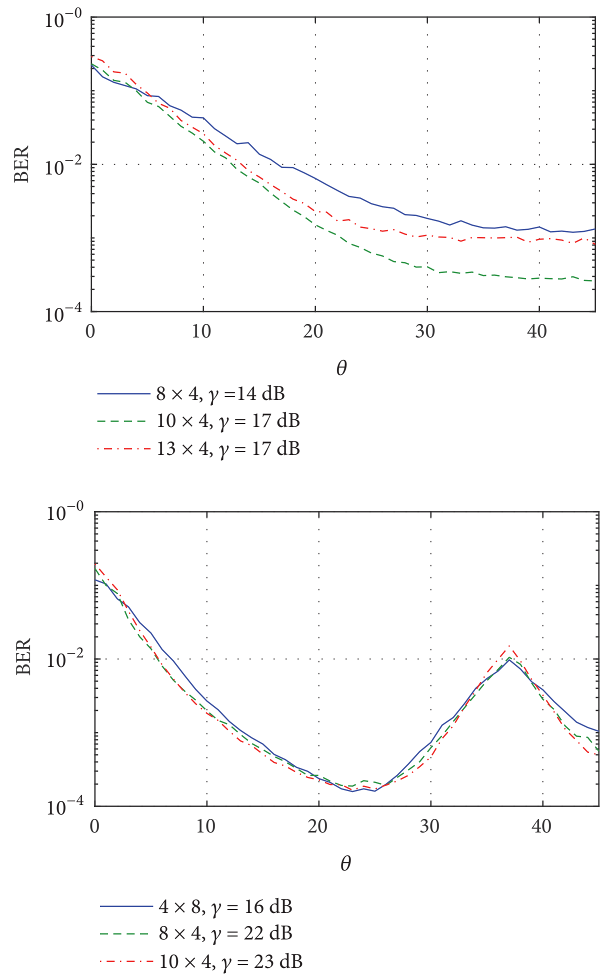


Figure 2: BER of the GCQSM-PU versus the rotation angle applied to the second signal symbol, using QPSK (first subfigure) and 16-QAM (second subfigure).

Since at high n_T CQSM requires one antenna less than ICQSM to achieve the same spectral efficiency at the cost of a moderate degradation in the BER performance, CQSM is used in the following comparisons.

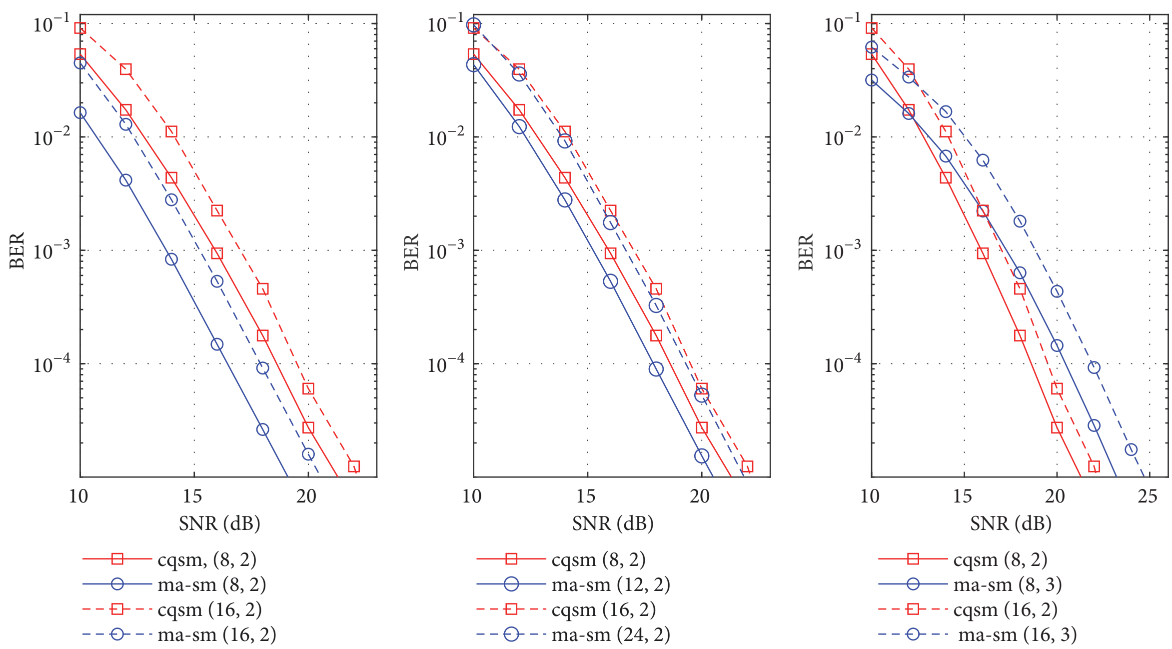
Figure 4 depicts the BER performance of CQSM, GCQSM-UC, and GCQSM-PC for several system scenarios and $n_U = 2$. The title of each subfigure is a tuple that includes the number of transmit antennas required by CQSM, GCQSM-UC, and GCQSM-PC, respectively, to achieve the same spectral efficiency. The spectral efficiency SE_{spa} achieved assuming the system's parameters in Figure 4(a) is 6, in 4(b) is 8, in 4(c) is 10, and in 4(d) is 12 bpcu. For each of the system configurations SE_{sig} is equal to 4 bpcu, leading to a total spectral efficiency $\mathrm{SE} = \mathrm{SE}_{spa} + \mathrm{SE}_{sig}$ of 10, 12, 14, and 16 bpcu, respectively. The range over which the rotation angle is optimized is $[0, \pi/4]$. The rotation angles applied to the second signal symbol for the scenarios depicted in Figure 4 are listed in Table 5. The rotation angles are optimized through Monte Carlo simulations. Based on the results depicted in Figure 4, we make the following remarks in terms of the BER performance, the rotation angles, and the number of employed transmit antennas.

(i) For $n_R = 2$, the three techniques achieve the same BER performance regardless of the number of transmit antennas.

(ii) For a given value of n_R, the BER performance is degraded as the number of transmit antennas increases. The degradation is relatively small as n_R increases. This is an expected behavior of SM techniques because the size of hypothesis set over which the ML detector performs the search increases as n_T increases.

(iii) In Figure 4(a), GCQSM-PC slightly outperforms the other two techniques for $n_R = 4$ and 8. As n_T gets larger, the probability that the two signal symbols are transmitted from the same antenna in CQSM reduces. Accordingly, the performance of CQSM and the two generalized algorithms almost coincide for $n_R = 4$ in Figures 4(b), 4(c), and 4(d). For $n_R = 8$, CQSM slightly outperforms GCQSM-UC and GCQSM-PU.

(iv) As shown in Figure 4(c), the generalized techniques perform close to the CQSM for $n_R = 4$. In this case, GCQSM-UC and GCQSM-PC require 17 and 22



(a) (b) (c)

FIGURE 3: A comparison between CQSM and MA-SM: (a) for the same n_T and different spectral efficiency, (b) for the same spectral efficiency and q, and different n_T, and finally (c) for the same spectral efficiency and n_T, and different q.

TABLE 5: Range of optimal rotation angles used to obtain the BER results depicted in Figure 4, assuming QPSK modulation.

SE_{spa}	n_R	θ_{CQSM}	$\theta_{GCQSM-PC}$
6	2	≥25	≥30
	4	≥35	≥41
	8	≥38	≥43
8	2	≥20	≥21
	4	≥34	≥35
	8	≥38	≥42
10	2	≥9	≥12
	4	≥30	≥32
	8	≥38	≥39
12	2	≥9	≥12
	4	≥25	≥30
	8	≥38	≥39

transmit antennas less than CQSM while achieving the same spectral efficiency.

Finally, we compare the BER performance of SM, GSM, QSM, CQSM, and the proposed generalized CQSM techniques. We assume that all the schemes achieve the same ES_{spa} and ES_{sig}, leading to the same total spectral efficiency of $ES = (ES_{spa} + ES_{sig})$. Also, the three generalized schemes use a combination of two antennas to transmit each signal symbol. Figures 5(a) and 5(b) depict the BER performance for a spectral efficiency of 16 and 18 bpcu, respectively. The number of transmit antennas and bits per signal symbol is represented as a tuple of the form (n_T, q) for each of the systems. The rotation angles applied to CQSM and GCQSM-PC are 20 and 23 degrees, respectively, to obtain the results in the two subfigures. We make the following two remarks:

(i) CQSM, GCQSM-PC, and GCQSM-UC apply 16-QAM modulation, whereas SM, GSM, and QSM use 256-QAM to achieve an ES_{sig} of 8 bpcu. The number of transmit antennas required to achieve the spectral efficiency of 16 and 18 bpcu is given in the second and third row, respectively, in Table 4. GCQSM-PC requires the least number of transmit antennas of 8 and 10 for the results in Figures 5(a) and 5(b), respectively. On the other hand, SM requires 256 and

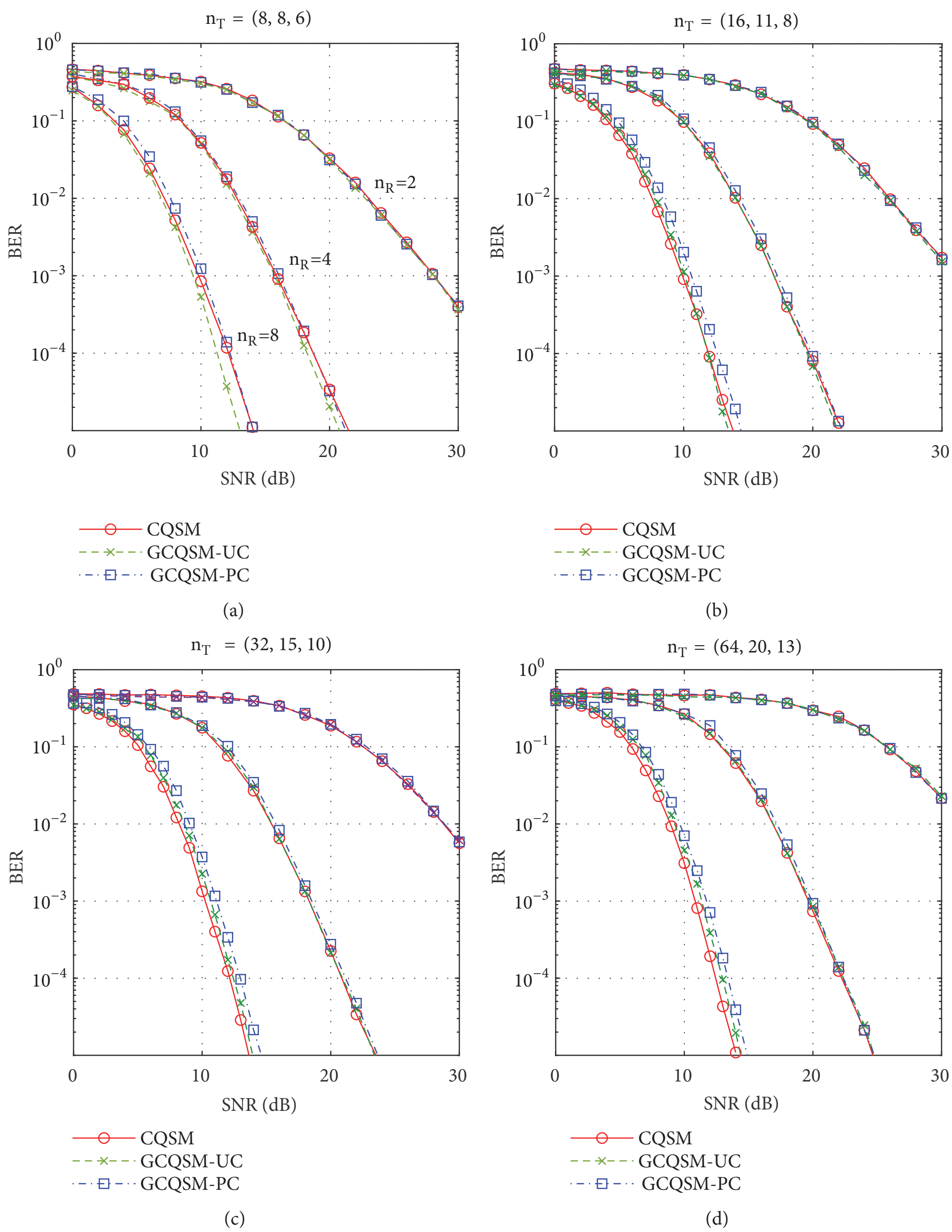


Figure 4: BER performance of CQSM, GCQSM-UC, and GCQSM-PC using QPSK modulation, where the spectral efficiency is equal to (a) 10, (b) 12, (c) 14, and (d) 16 bpcu. The title of each subfigure is a tuple of the number of antennas required by CQSM, GCQSM-UC, and GCQSM-PC, respectively.

1024 antennas, respectively, to achieve the same SE. Compared to CQSM, GCQSM-PC requires 8 and 22 less antennas to achieve the same SE_{spa} of 8 and 10 bpcu, respectively.

(ii) For both scenarios, GCQSM-UC achieves the best performance followed by GCQSM-PC and CQSM. In Figure 5(b), GCQSM-UC outperforms GCQSM-PC and CQSM by 1 and 1.6 dB, respectively. However, GCQSM-PC requires 5 less transmit antennas than GCQSM-UC. Therefore, this slight degradation in the SNR is tolerable in the case of GCQSM-PC given the high reduction in the number of transmit antennas used to achieve the same spectral efficiency.

Future Work. We would like to investigate the case of generalized CQSM where the number of transmit antennas used for transmitting each of the two signal symbols can be

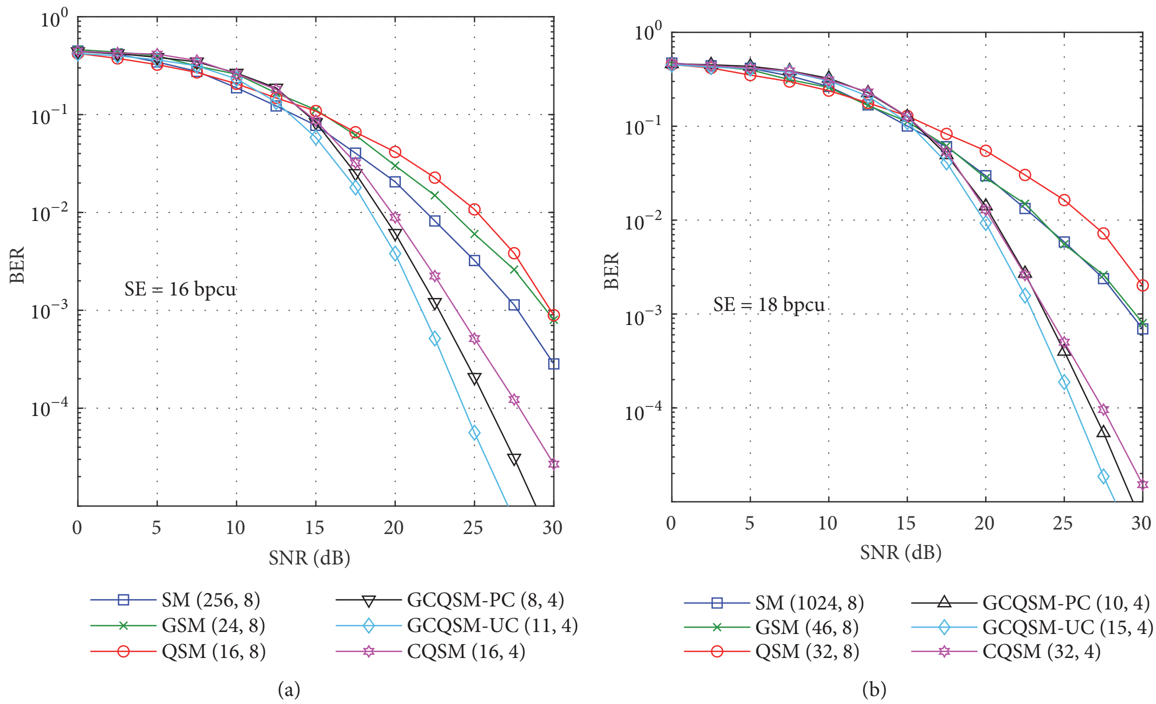


Figure 5: BER comparison between the generalized CQSM schemes, CQSM, SM, GSM, and QSM for spectral efficiency of (a) 16 and (b) 18 bpcu. CQSM, GCQSM-UC, and GCQSM-PC use 16-QAM and the remaining schemes use 256-QAM.

variable. Accordingly, the total number of spatial symbols will be increased, leading to higher spectral efficiency.

8. Conclusions

In this paper, we introduced two generalizations of the complex quadrature spatial modulation, namely, GCQSM with unique combinations (GCQSM-UC) and with permuted combinations (GCQSM-PC). While the former generalization uses the conventional spatial symbol generation of the GSM scheme, the later scheme relies on the fact that the second signal symbol is distinguishable from the first through the angle rotation. This allows expanding the set of antenna combinations that can be used for transmission, leading to the reduction in the number of transmit antennas required to achieve a given spectral efficiency. The two proposed generalizations perform close to CQSM using QPSK and outperform it using higher order modulation schemes. Also, GCQSM-PC requires 10 antennas to achieve a spectral efficiency of 10 bcpu per spatial symbol. To achieve the same efficiency, GSM requires 46, and CQSM and QSM require 32 transmit antennas.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this paper.

Acknowledgments

This work was supported by research stipend by Korea Tech in the period 2018-2019.

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