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Research Article

An Angle Estimation Method for Monostatic MIMO Radar Based on RCC-FLOM Algorithm

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The performance of the angle estimation algorithm based on the two-order or higher order cumulants in the impact noise background will decline sharply. Therefore, it is necessary to study the new algorithm to estimate target angle in the impact noise background. In order to solve the angle estimation problem of coherent sources in the impulse noise background, a conjugate rotation invariant subspace algorithm based on reduced order fractional lower order covariance matrix is proposed. Use the reduced dimension lower order fraction covariance matrix to reduce the impulse noise influence. And according to the conjugate rotation invariant subspace, the coherent source is decohered. The Monte-Carlo experiments show that the proposed algorithm has the advantages of high estimation probability and low root mean square error in the case of low signal-to-noise ratio, compared with the existing FLOM-MUSIC algorithm and FLOM-Unitary ESPRIT algorithm.

1. Introduction

The noise encountered in the practical application of Multiple-Input Multiple-Output (MIMO) radar often has certain impact characteristics, such as cosmic noise, atmospheric noise, and meteorological noise. In recent years, a large number of experimental data and simulation results [1–3] confirm that the impulse noise is consistent with the α stable distribution. Because the α stable distribution does not have two-order square or higher, this means that the performance of the angle estimation algorithm based on the two-order or higher order cumulants in the impact noise background will decline sharply. Therefore, it is necessary to study the new algorithm to estimate target angle in the impact noise background.

Tsakalides [4–6] and Liu [7] estimate the direction of arrival using Multiple Signal Classification (MUSIC) algorithm based on covariance and fractional lower order moment respectively, but all of them need to search spectral peak, and the amount of operation and storage is very large. Lv Zejun [8] and He Jin [9] estimate the arrival angle based on covariance in the impact noise environment, but the above methods need to estimate the appropriate fractional lower order parameters. Li Li [10] uses the Maximum Correntropy Criterion (MCC) to derive the ductile parallel factor algorithm for the impact noise environment, but has not solved the coherent source problem.

Based on the above research, we combine reduced dimension conjugate covariance (RCC) with fraction lower order matrix (FLOM) and then propose a RCC-FLOM algorithm for monostatic MIMO radar. The proposed algorithm can reduce the impulse noise influence and has the advantages of high estimation probability. The remainder of this paper is organized as follows. In Section 2, the monostatic MIMO radar echo model is established. Then, the angle estimation method based on RCC-FLOM is proposed in Section 3. Some simulations are conducted to verify the performance of the proposed method in Section 4. Finally, we conclude the paper in Section 5.

Notation: $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ denote transpose and conjugate-transpose operators; $\otimes$ denotes the Kronecker product; $E[\cdot]$ denotes the expected value; $(\cdot)^*$ denotes the conjugate operator; $\sum(\cdot)$ denotes the sum operator.

2. Monostatic MIMO Radar Echo Model

Considering the monostatic MIMO radar as shown in Figure 1, the transmitting and receiving antenna adopt M and N elements, respectively. The spacing of array elements is $d = \lambda/2$, λ is carrier wavelength. Assuming that the direction of arrival (DOA) and the direction of departure (DOD) both are θ_p. The echo signal processed by matching filtering as below,

$$\mathbf{y}(t) = \mathbf{A}\boldsymbol{\beta}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{A} = [\mathbf{a}_r(\theta_1)\otimes\mathbf{a}_t(\theta_1), \mathbf{a}_r(\theta_2)\otimes\mathbf{a}_t(\theta_2), \cdots, \mathbf{a}_r(\theta_P)\otimes\mathbf{a}_t(\theta_P)]$, the transmit steering vector of the *pth* target is $\mathbf{a}_t(\theta_p) = [1, \exp(-j\pi\sin\theta_p), \cdots, \exp(-j\pi(M-1)\sin\theta_p)]^{\mathrm{T}}$, and the receive steering vector is $\mathbf{a}_r(\theta_p) = [1, \exp(-j\pi\sin\theta_p), \cdots, \exp(-j\pi(N-1)\sin\theta_p)]^{\mathrm{T}}$, $\boldsymbol{\beta} = [\xi_1 e^{j2\pi f_{d1}t}, \xi_2 e^{j2\pi f_{d2}t}, \cdots, \xi_P e^{j2\pi f_{dP}t}]^{\mathrm{T}}$, ξ_p is the reflection coefficient of the *pth* target, f_{dp} is the normalized Doppler frequency of the *pth* target, and $\mathbf{n}(t)$ representation of noisy column vectors.

3. Angle Estimation Based on RCC-FLOM

3.1. Reduced Dimension Lower Order Fraction Covariance Matrix. If defined,

$$\mathbf{a}_r(\theta_p)\otimes\mathbf{a}_t(\theta_p) \triangleq \mathbf{F}\mathbf{b}(\theta_p) \tag{2}$$

In the form, $\mathbf{b}(\theta_p) = [1, \exp(-j\pi\sin\theta_p), \cdots, \exp(-j\pi(M+N-1)\sin\theta_p)]^{\mathrm{T}}$ and $\mathbf{F} \in \mathbf{C}^{MN\times(M+N-1)}$ is a dimensionality reduction matrix, which can be expressed as

$$\mathbf{F} = \begin{bmatrix} \left.\begin{matrix} 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & \cdots & 0 \end{matrix}\right\} M \\ \left.\begin{matrix} 0 & 1 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & \cdots & 0 \end{matrix}\right\} M \\ \vdots \\ \left.\begin{matrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{matrix}\right\} M \end{bmatrix} \in \mathbf{C}^{MN\times(M+N-1)} \tag{3}$$

Then the matrix $\mathbf{A}$ can be expressed as

$$\mathbf{A} = \mathbf{F}\mathbf{A}' \tag{4}$$

where $\mathbf{A}' = [\mathbf{a}'(\theta_1), \mathbf{a}'(\theta_2), \cdots, \mathbf{a}'(\theta_P)] \in \mathbf{C}^{(M+N-1)P}$, $\mathbf{a}'(\theta_p) = [1, a_p, \cdots, a_p^{I-1}]^{\mathrm{T}}$, $a_p = e^{j\pi\sin\theta_p}$, and it is a Vanermonde matrix. I is the virtual elements number after dimension reduction transformation, then $I = M + N - 1$.

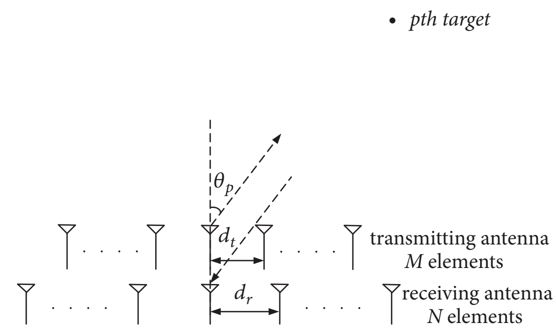


Figure 1: The structure of a monostatic MIMO radar.

According to formula (3), if formula $\mathbf{W} \triangleq \mathbf{F}^{\mathrm{H}}\mathbf{F}$ is defined, there are

$$\mathbf{W} = \operatorname{diag}\left(1, 2, \cdots, \underbrace{\min(M,N), \cdots, \min(M,N)}_{|M-N|+1}, \cdots, 2, 1\right) \tag{5}$$

If both sides of formula (1) are multiplied by $\mathbf{W}^{-1}\mathbf{F}^{\mathrm{H}}$, we can get

$$\mathbf{y}' = \mathbf{A}'\boldsymbol{\beta} + \mathbf{n}' \tag{6}$$

where $\mathbf{n}'(t_l) = \mathbf{W}^{-1}\mathbf{F}^{\mathrm{H}}\mathbf{n}(t_l)$ is a $(M+N-1)$ dimension additive noise vector.

When $\mathbf{n}'$ is impact noise, $\mathbf{y}'$ has no more than two-order square. According to [6, 7], the FLOM of $\mathbf{y}'$ can be constructed as

$$C_{uv} = E\left[y_u{}'(t)\left|y_v{}'(t)\right|^{p-2} y_v'^{*}(t)\right] \quad 1 < p < \alpha \le 2 \tag{7}$$

where $y_u{}'(t)$ and $y_v{}'(t)$ is the output of matched filtering after MIMO radar dimensionality reduction. C_{uv} is the (u,v) element of the fractional lower order covariance matrix $\mathbf{C}$ and satisfies $1 \le u, v \le I$.

Substituting formula (6) in formula (7), C_{uv} can be expressed as

$$\begin{aligned} C_{uv} = {} & \sum_{v=1}^{P} A'_{uv} \\ & \cdot E\left[\beta_v \left|\sum_{q=1}^{P} A'_{uv}\beta_q + n'_v\right|^{p-2} \left(\sum_{q=1}^{P} A'_{uv}\beta_q + n'_v\right)^{*}\right] \\ & + E\left[n_u \left|\sum_{q=1}^{P} A'_{uv}\beta_q + n'_v\right|^{p-2} \left(\sum_{q=1}^{P} A'_{uv}\beta_q + n'_v\right)^{*}\right] \end{aligned} \tag{8}$$

It can be written as a matrix as follows:

$$\mathbf{C} = \mathbf{A}'\boldsymbol{\Lambda}\mathbf{A}'^{\mathrm{H}} + \gamma\mathbf{I} \tag{9}$$

where $\Lambda_{uv} = \delta_{uv}E[\beta_v|\sum_{q=1}^{P} A'_{uv}\beta_q + n'_v|^{p-2}(\sum_{q=1}^{P} A'_{uv}\beta_q + n'_v)^{*}]$ and $\gamma = E[n_u|\sum_{q=1}^{P} A'_{uv}\beta_q + n'_v|^{p-2}(\sum_{q=1}^{P} A'_{uv}\beta_q + n'_v)^{*}]$.

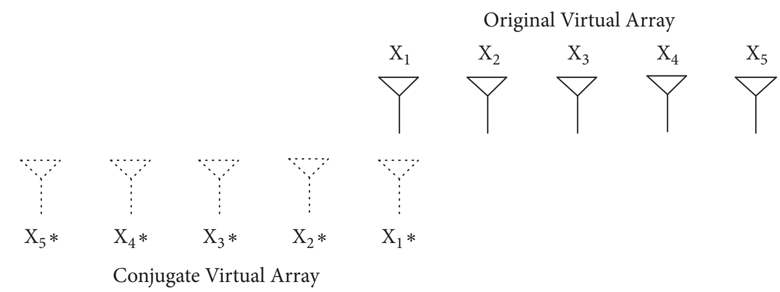


FIGURE 2: Monostatic MIMO radar conjugate virtual array.

After obtaining the reduced order fractional lower order covariance matrix formula (9), the eigenvalues of $\mathbf{C}$ according to [11, 12] are as follows:

$$\mathbf{C} = \mathbf{E}_S \boldsymbol{\Lambda}_S {\mathbf{E}_S}^{\mathrm{H}} + \mathbf{E}_N \boldsymbol{\Lambda}_N {\mathbf{E}_N}^{\mathrm{H}} \tag{10}$$

where $\mathbf{E}_{\mathrm{S}}$ is a subspace formed by eigenvectors corresponding to large eigenvalues, i.e., signal subspace, while $\mathbf{E}_{\mathrm{N}}$ is a subspace formed by eigenvectors corresponding to small eigenvalues, i.e., noise subspace.

Under ideal conditions, the signal subspace and the noise subspace are orthogonal to each other; that is, the steering vector in the signal subspace is also orthogonal to the noise subspace as

$$\mathbf{a}'^{\mathrm{H}}(\theta)\,\mathbf{E}_{\mathrm{N}} = 0 \tag{11}$$

The classical MUSIC algorithm is based on the above properties. However, considering the fact that the actual received data matrix is finite and there is noise, $\mathbf{a}'(\theta)$ and $\mathbf{E}_{\mathrm{N}}$ can not be completely orthogonal; that is to say, formula (11) does not hold. Therefore, in fact, DOA is realized by minimum optimization search; that is,

$$\theta_{MUSIC} = \arg\min_{\theta} \mathbf{a}'^{\mathrm{H}}(\theta)\,\mathbf{E}_{\mathrm{N}}\mathbf{E}_{\mathrm{N}}^{\mathrm{H}}\mathbf{a}'(\theta) \tag{12}$$

So the target angle can be estimated by the MUSIC algorithm's spectral search formula such as

$$P_{\mathrm{MUSIC}}(\theta) = \frac{1}{\mathbf{a}'^{\mathrm{H}}(\theta)\,\mathbf{E}_{\mathrm{N}}\mathbf{E}_{\mathrm{N}}^{\mathrm{H}}\mathbf{a}'(\theta)} \tag{13}$$

3.2. Conjugate Rotation Invariant Subspace Algorithm. Constructing the lower order fractional lower order covariance matrix can only solve the problem of impulse noise. When the multipath effect and electronic interference are affected, the angle estimation of coherent sources is also needed. In this paper, the conjugate rotation invariant subspace algorithm (C-ESPRIT) is used to solve the problem of coherent source angle estimation. Taking the MIMO radar with 5 virtual elements as an example, the virtual space array constructed is shown in Figure 2.

The target echo data obtained by virtual subarray 1 can be expressed as follows:

$$\mathbf{y}_{e1} = \begin{bmatrix} y_1 & y_2 & y_3 & \cdots & y_P \\ y_2 & y_3 & y_4 & \cdots & y_{P+1} \\ \vdots & \vdots & \vdots & & \vdots \\ y_{I-P} & y_{I-P+1} & y_{I-P+2} & \cdots & y_{I-1} \\ y_I^* & y_{I-1}^* & y_{I-2}^* & \cdots & y_{I-P+1}^* \\ \vdots & \vdots & \vdots & & \vdots \\ y_{P+2}^* & y_{P+1}^* & y_P^* & \cdots & y_3^* \\ y_{P+1}^* & y_P^* & y_{P-1}^* & \cdots & y_2^* \end{bmatrix}_{2(I-P)\times P} \tag{14}$$

According to formula (6), the matrix $\mathbf{y}_{e1}$ can be expressed as

$$\mathbf{y}_{e1} = \mathbf{R}\mathbf{A}'(\theta) + \mathbf{n}_{e1} \tag{15}$$

where

$$\mathbf{R} = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & \cdots & \beta_P \\ \beta_1 a_1 & \beta_2 a_2 & \beta_3 a_3 & \cdots & \beta_P a_P \\ \vdots & \vdots & \vdots & & \vdots \\ \beta_1 a_1^{I-P-1} & \beta_2 a_2^{I-P-1} & \beta_3 a_3^{I-P-1} & \cdots & \beta_P a_P^{I-P-1} \\ \beta_1^* a_1^{-(I-1)} & \beta_2^* a_2^{-(I-1)} & \beta_3^* a_3^{-(I-1)} & \cdots & \beta_P^* a_P^{-(I-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ \beta_1^* a_1^{-(P+1)} & \beta_2^* a_2^{-(P+1)} & \beta_3^* a_3^{-(P+1)} & \cdots & \beta_P^* a_P^{-(P+1)} \\ \beta_1^* a_1^{-P} & \beta_2^* a_2^{-P} & \beta_3^* a_3^{-P} & \cdots & \beta_P^* a_P^{-P} \end{bmatrix}_{2(I-P)\times P} \tag{16}$$

The target echo data obtained by virtual subarray 2 can be expressed as follows:

$$\mathbf{y}_{e2} = \begin{bmatrix} y_2 & y_3 & y_4 & \cdots & y_{P+1} \\ y_3 & y_4 & y_5 & \cdots & y_{P+2} \\ \vdots & \vdots & \vdots & & \vdots \\ y_{I-P+1} & y_{I-P+2} & y_{I-P+3} & \cdots & y_I \\ y_{I-1}^* & y_{I-2}^* & y_{I-3}^* & \cdots & y_{I-P}^* \\ \vdots & \vdots & \vdots & & \vdots \\ y_{P+1}^* & y_P^* & y_{P-1}^* & \cdots & y_2^* \\ y_P^* & y_{P-1}^* & y_{P-2}^* & \cdots & y_1^* \end{bmatrix}_{2(I-P)\times P} \tag{17}$$

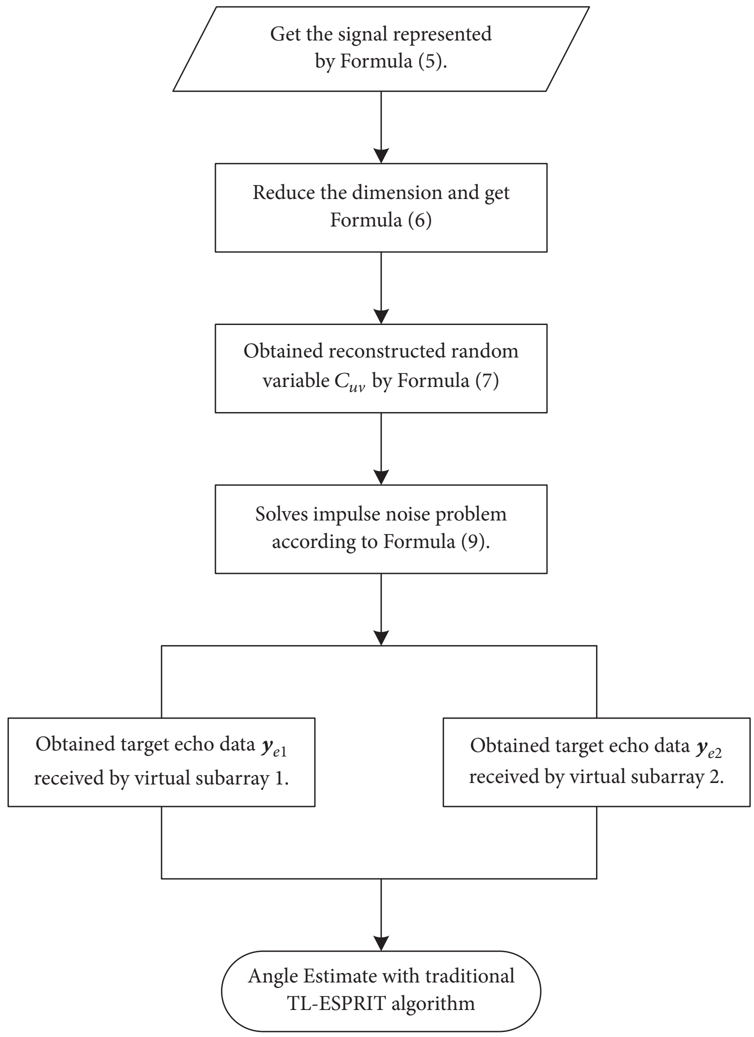


Figure 3: Flow chart of the RCC-FLOM algorithm.

According to formula (6), the matrix $\mathbf{y}_{e2}$ can be expressed as

$$\mathbf{y}_{e2} = \mathbf{R}\mathbf{\Phi}\mathbf{A}'(\theta) + \mathbf{n}_{e2} \quad (18)$$

where $\mathbf{\Phi}$ is a rotation factor and its expression is $\mathbf{\Phi} = \mathrm{diag}(a_1, a_2, \cdots, a_P)$.

The matrix $\mathbf{A}'(\theta)$ is a Vanermonde matrix, because the coherent source is located in different directions of space and linearly independent of each row, $\mathrm{rank}(\mathbf{A}'(\theta)) = P$.

When $2(I-P) \geq P$, the $\mathbf{R}$ of the provable matrix is linearly independent; that is, $\mathrm{rank}(\mathbf{R}) = P$. Because of $\mathrm{rank}(\mathbf{\Phi}) = P$, the rank of virtual subarray $\mathbf{y}_{e1}$ and $\mathbf{y}_{e2}$ is P. Therefore, the traditional TL-ESPRIT algorithm can be used to estimate P coherent sources.

3.3. The Steps of the RCC-FLOM Algorithm. Based on the above analysis, the operation steps of the RCC-FLOM algorithm can be summarized as follows.

Step 1. The echo signal of single pulse is matched filtered to get the signal represented by formula (5).

Step 2. Formula (6) is obtained for reducing the dimension of the signal represented by formula (5).

Step 3. According to formula (7), the fractional lower order covariance $\mathbf{y}'$ of the reconstructed random variable C_{uv} is obtained.

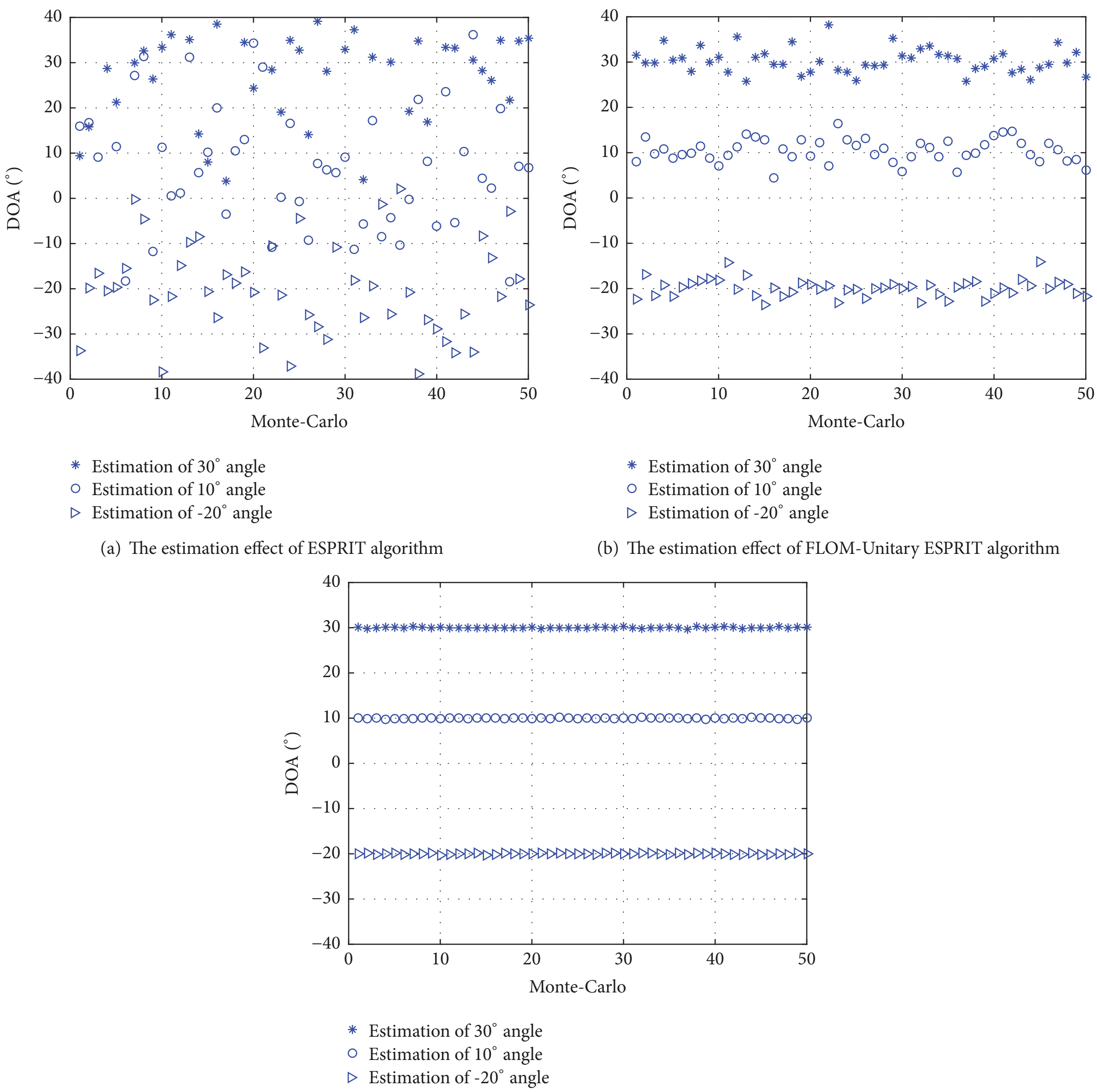
Step 4. The fractional lower order covariance C_{uv} is rewritten into the matrix form shown in formula (9), which solves the problem of impulse noise.

Step 5. According to formula (15), the target echo data $\mathbf{y}_{e1}$ received by virtual subarray 1 is obtained.

Step 6. According to formula (18), the target echo data $\mathbf{y}_{e2}$ received by virtual subarray 2 is obtained.

Step 7. Estimated the angle of P coherent sources based on the traditional TL-ESPRIT algorithm.

For a more intuitive presentation, the flow chart is drawn in Figure 3.



(a) The estimation effect of ESPRIT algorithm

(b) The estimation effect of FLOM-Unitary ESPRIT algorithm

(c) The estimation effect of RCC-FLOM algorithm

Figure 4: Angle estimation algorithms of coherent sources under impulse noise.

4. Computer Simulation Results

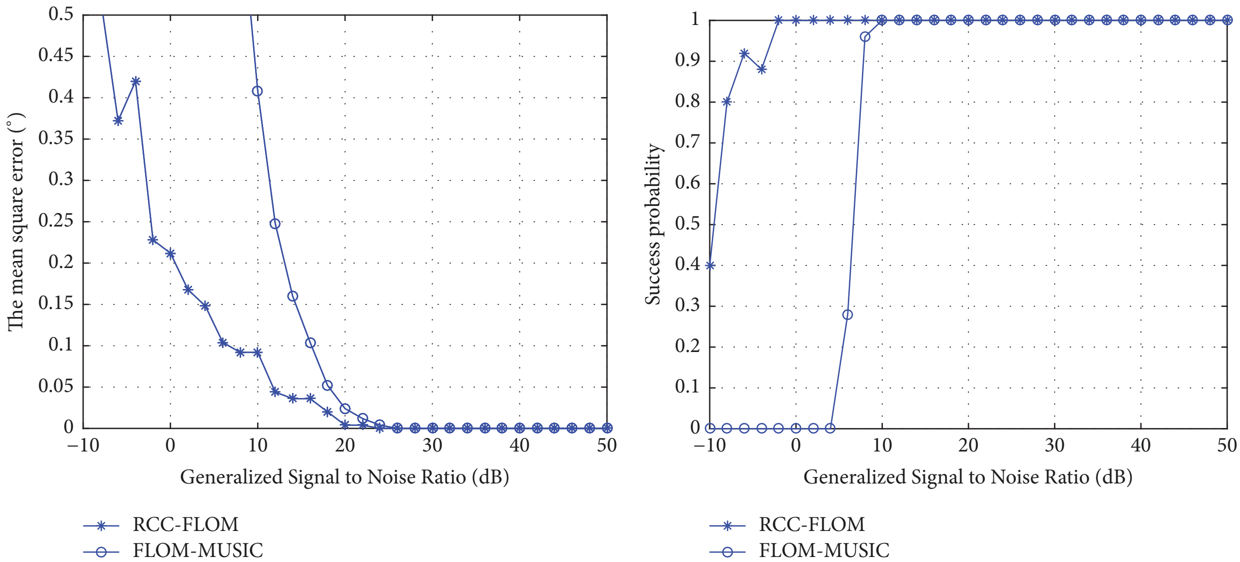
Assuming that the MIMO radar has 5 transmitting elements and 6 receiving elements, the number of virtual elements after dimension reduction is 10. The receiving and transmitting antennas are equal spaced uniform linear arrays with a spacing of 0.5λ. And we use MATLAB software to simulate the following experiments in this paper. The generalized signal-to-noise ratio (GSNR) is the ratio of average power to dispersion coefficient in simulation as follows:

$$GSNR = 10\lg\left(\frac{1}{\gamma N}\sum_{t=1}^{N}\left|s(t)^2\right|\right) \quad (19)$$

where N is the number of accumulated pulses. If $\alpha = 2$, the generalized signal-to-noise ratio is the same as the ordinary signal-to-noise ratio.

Experiment 1. Angle estimation of coherent sources under different impulse noise environments

Under the background of impact noise, the direction of arrival of 3 equal power coherent sources are 30°, 10°, −20°, respectively. The impulse noise of $S\alpha S$ distribution is $\alpha = 1.5$, and the generalized signal-to-noise ratio is 5 dB. Figures 4(a), 4(b), and 4(c), respectively, simulate the angle estimation of ESPRIT algorithm, FLOM-Unitary ESPRIT algorithm



(a) Success probability

(b) Root mean square error

Figure 5: Performance comparison of different algorithms under impact noise.

proposed in the paper [13], and RCC-FLOM algorithm in 50 simulation experiments.

The simulation results of Figure 4 can be seen as follows: In the background of impulse noise, the coherent sources' direction of arrival can not be estimated accurately by using the ESPRIT algorithm. It is shown that the conventional subspace based algorithm can not suppress impulse noise and coherent decoherence when applied to monostatic MIMO radar. The RCC-FLOM algorithm proposed in this paper can accurately estimate the direction of arrival of coherent sources in the background of impulsive noise. The simulation results verify the effectiveness of the proposed algorithm.

Experiment 2. Comparison of statistical performance between the proposed RCC-FLOM algorithm and other algorithms

The mean square error (RSME) of DOA estimation is defined as

$$\mathrm{RMSE}(\theta) = \sqrt{\frac{1}{L_m}\sum_{l_m=1}^{L_m}\left(\widehat{\theta}_p - \theta_p\right)^2} \tag{20}$$

where L_m represents the number of Monte-Carlo experiments and θ_p and $\widehat{\theta}_p$ represent the true angle and estimated angle, respectively.

Under the background of impact noise, the direction of arrival of 2 equal power coherent sources is 30° and −20°, respectively. The impulse noise of $S\alpha S$ distribution is $\alpha = 1.5$. Figures 5(a) and 5(b), respectively, simulate the statistical performance of FLOM-MUSIC algorithm [11] and RCC-FLOM algorithm in 50 Monte-Carlo experiments.

The simulation results of Figure 5 can be seen as follows: Under the impact noise background, the RCC-FLOM algorithm proposed in this paper is superior to the FLOM-MUSIC algorithm proposed in the literature [11] in terms of the success probability and the root mean square error. With the increase of the generalized signal-to-noise ratio, the success probability of the two algorithms is gradually increased and the root mean square error is gradually reduced. That is, the performance of the angle estimation is increased with the increase of the generalized signal-to-noise ratio. When the SNR is high to a certain extent, the performance of the two algorithms is basically the same.

5. Conclusions

In this chapter, the MIMO radar angle estimation problem of coherent sources in the impulse noise background is studied. A conjugate rotation invariant subspace (RCC-FLOM) algorithm based on reduced order fractional lower order covariance matrix is proposed. The influence of impulse noise is reduced by using the method of constructing lower order fractional lower order covariance matrix. Then the coherent source is decohered according to the conjugate rotation invariant subspace method. It solves the problem that the two or four order statistical models can not effectively estimate the target angle under the impact noise background. The 50 Monte-Carlo experiments show that the proposed algorithm has the advantages of high estimation probability and low root mean square error in the case of low signal-to-noise ratio, compared with the existing FLOM-MUSIC algorithm.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

No potential conflicts of interest were reported by the authors.

Authors' Contributions

Jian Gong and Yiduo Guo designed the algorithm scheme. Jian Gong performed the experiments and analysed the experiment results. Yiduo Guo and Huan Wang contributed to the manuscript drafting and critical revision. All authors read and approved the final manuscript.

Acknowledgments

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