

Research Article

Channel Allocation Policy for Distributed Wireless Network: Derivation and Analysis of Optimal Interference

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Received 17 April 2020; Revised 5 October 2020; Accepted 10 October 2020; Published 6 November 2020

Academic Editor: Pietro Manzoni

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Distributed wireless networks with smart users (independent and rational) are becoming popular, and researchers are studying distributed equilibrium solutions like Nash Equilibrium (NE) to analyze and predict the convergence of such networks. Our goal is to drive the distributed wireless network to NE with high total throughput. Study of the distribution of network metrics at NE with high total throughput shows that communication links still have significant amount of interference. Adding an interference-received term with an optimal weight (α_{opt}^*) to the link's payoff can push the distributed network to converge to NE with high total throughput. The channel allocation trend at NE with high total throughput is as follows: each of the $C - 1$ links occupies its own channel, and the remaining $N - C + 1$ links share the remaining one channel, where N is the number of links and C is the number of channels in the network. The links (transmitters and receivers) are randomly located and $C < N$ (limited resources). The transmitter of a link has a direct connection with the receiver of the link; hence, several links overlap. This leads to a dense network with considerable amount of interference especially for links sharing channels. A practical application of our work is when smart devices in a room, hall, or concert arena have a direct communication with other smart devices in the area using limited bandwidth. Using best response technique and definitions of NE, we derive and propose an approximate way to mathematically express α_{opt}^* (referred to as $\hat{\alpha}_{\text{opt}}$) along with its probability density function (PDF) for a specific scenario. Then, a generic equation for $\hat{\alpha}_{\text{opt}}$ is inferred for varying network sizes (links) and available resources (channels). Implementing such a policy enhances the total throughput of the distributed wireless network by up to 15%. In a more general setting, our distributed policy can achieve up to 75% of the maximum total throughput (benchmark value reached by centralized solution via exhaustive search) at a fraction of the time and computation resources.

1. Introduction

Wireless communication and networking is expanding exponentially [1]. Scientists are exploring new techniques to keep up with the escalated traffic. Various technologies in mobile communication (3G, 4G, and 5G) [2], wireless LAN (WiFi) [3], and personal area networking (Bluetooth) [4] are catering to the need of the present wireless communications, but new innovations and techniques are essential to meet up with the future trends and ever growing demand [5].

Fixed Resource Allocation (FRA) of power, spectrum, and transmission protocols are inefficient [6] as only a fraction of the allocated resources is utilized by the designated

users. By dynamically allocating the resources, the smart radios can enhance the efficiency of the network by several folds in comparison to FRA [7]. Intelligent radios are capable to sense the environment, reason based on the observations made [8], and adopt their transmission parameters to get higher efficiency and performance [9]. In this work, we focus on dynamically allocating the channels (spectrum) to the intelligent wireless users [10].

As radios are becoming more intelligent and have greater computational power, distributed networks are becoming popular [11]. Several, mobile devices may connect and share information with each other via Bluetooth or WiFi in the absence of a central entity [4]. Generally, in a distributed

network, each user independently makes the decision [12], i.e., each user acts selfishly to maximize its own payoff [13]. Although a central agent might ordain a global optimal strategy to these users, they will mostly drift away by choosing strategies that enhance their own payoff, consequently resulting in a lower overall network performance compared to the global optimal [14]. Centralized solutions via exhaustive search technique result in maximum total throughput; however, it requires massive amount of resources (computation and time) especially in larger networks. Hence, many networks like ad hoc and device to device (D2D) prefer decentralized solutions [15].

Nash Equilibrium (NE) is a natural and eventual equilibrium state, which the independent and selfish users converge to in distributed scenarios [16]. As each user tries to enhance its own payoff in every iteration, after considerable amount of iterations, the system converges to a Nash Equilibrium (NE). Depending on the starting point, the definition of a user's payoff, network size, resources available, etc., the system can converge to NE with varying performance [17]. We want to converge to a NE with high total throughput [18].

Several research groups have used game theory techniques to address the resource allocation problem in wireless distributed networks. The authors in [17] have defined the user's payoff to be the individual throughput. This is a distributed system and there is no information exchange between the users, but mostly, the system converges to NE solutions with poor performance [19]. In [20] the authors have defined the user's payoff to be the total throughput of all users to emulate a centralized system where each user works to maximize the total performance of the network. The performance is high; however, each user needs to know the throughput of all other users which is not viable for a practical distributed network. In [21], the authors have defined the user's payoff to be the received power minus interference received plus interference created to other users. The objective is to increase the received power and to minimize the interference received from other users and the interference a user generates to other users [22]. Here, the resource allocation schemes are performed in a distributed way, but complete information exchange among the users is essential, and the performance is only slightly better than its predecessors.

In [23, 24], the characteristics of the network metrics, link distance, individual throughput, and interference from/to, were studied. It was found that adding an optimal amount of interference-received term in the user's utility, the distributed wireless network can be driven towards NE with high total throughput [18]. There, α_{opt}^* is the optimal weight of the interference-received term for a specific network which is determined by running many random simulations for step size variations of α . In this work, we mathematically derive an approximate expression of α_{opt}^* (referred to as $\hat{\alpha}_{\text{opt}}$) along with its probability density function (PDF), for a random "10-link 4-channel" scenario. Then, a generic equation of $\hat{\alpha}_{\text{opt}}$ for an " N -link C -channel" scenario is inferred based on the trend of different network sizes. In our work, the links (transmitters and receivers) are randomly located and $C < N$

(limited resources). The transmitter of a link has a direct connection with the receiver of the link; hence, several links overlap. This leads to a dense network with considerable amount of interference especially for links sharing channels. A practical application of our work is when smart devices in a room, hall, or concert arena have a direct communication with other smart devices in the area using limited bandwidth. Implementing the policy enhances the total throughput of the distributed wireless network by up to 15%, and it can achieve up to 75% of the maximum total throughput (the benchmark value reached by the centralized solution via exhaustive search technique) at a fraction of time and computational resources.

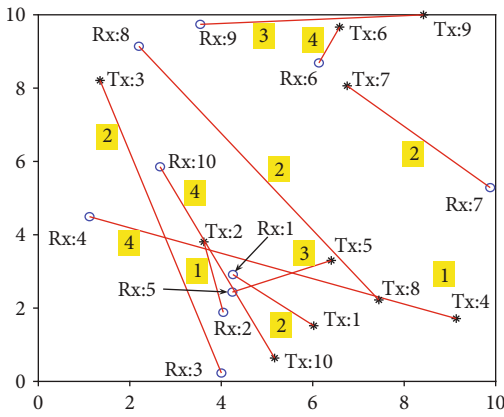
In Section 2, the system model of our distributed wireless network is explained. In Section 3, game theory is analyzed in detail. NE with high total throughput is defined, and how distributed solutions with high total throughput can be achieved by inserting the optimal weight (α_{opt}^*) of the interference-received term in the utility is explained. Section 4 mathematically derives the policy ($\hat{\alpha}_{\text{opt}}$) from NE with high total throughput constraints for a "10-link 4-channel" scenario and extends to incorporate a generic solution for varying network sizes and available resources. Section 5 compares the results from centralized and distributed networks. Using various illustrations, it is shown how implementing our policy enhances the performance of the distributed wireless network. Section 6 concludes the work.

2. System Model

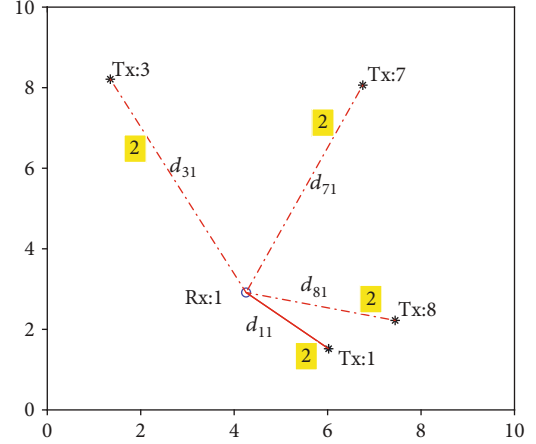
This section discusses the main components of the model. Initially, the links (pairs of receivers and transmitters) and channels in our model are described. Then, calculation of the received power, interference, and throughput at the links and network is explained.

2.1. Links and Channels. In our wireless network, there are N links. Each link comprises of a transmitter (Tx) and a receiver (Rx). There are $i \in 1, 2, \dots, N$ transmitters and $j \in 1, 2, \dots, N$ receivers. Link _{j} comprises a transmitter Tx _{i} which has a direct connection with a receiver Rx _{j} where $i = j$. We assume that the maximum communication distance of Link _{j} is the entire network area. For example, Link₁ is the communication between Tx₁ and Rx₁. Figure 1(a) shows a network with 10 links ($N = 10$) and 4 channels ($C = 4$). Symbols "*" and "o" represent the Tx _{i} and Rx _{j} , respectively. We can observe that several links overlap creating a dense network with considerable amount of interference especially for links sharing channels. The coordinates of the Tx _{i} and Rx _{j} are randomly and uniformly generated inside an area of 10 units by 10 units.

Limited resources are available in the wireless network, and hence, channels need to be shared by the links. The resources allocated to N links are C channels. An interesting case is when there are less channels than the number of links: $C < N$. A practical application of our work is when smart devices in a room, hall, or concert arena have direct communication with other smart devices in the area using limited



(a) There are a total of 10 links which share 4 channels (Link₁ is the communication between Tx₁ and Rx₁ which is using channel “2”). The coordinates of the Tx_{*i*} and Rx_{*j*} are randomly and uniformly generated



(b) The SINR of Link₁ is the ratio of signal power (from Tx₁ equaling $1/d_{11}^2$) over the interference power (from Tx₃, Tx₇, and Tx₈ equaling $1/d_{31}^2 + 1/d_{71}^2 + 1/d_{81}^2$) and noise power. These links use channel “2”

FIGURE 1: Distributed wireless network: random, “10-link 4-channel” scenario. The 10 links use a random channel configuration (initially) of “2124342234” (Link₁, Link₃, Link₇, and Link₈ use channel “2”).

bandwidth. We assume each link can occupy only one channel at a time.

2.2. Mathematical Formulations. The power at the receiver from a transmitter, using the free space ideal propagation model, can be expressed as

$$P_r = \frac{G_r G_t P_t}{(4\pi d/\lambda)^2} = \frac{A}{d^2}, \quad (1)$$

where G_r and G_t are the antenna gain at the receiving and transmitting sides, respectively. P_r and P_t are the power at the receiver and transmitter, respectively. The distance between the transmitter and the receiver is d , and λ is the wavelength of the electromagnetic wave used in the communication system [25]. In our work, the transmission power of all antennas and the gain of the antennas are constant. Similarly, as we are operating in a narrowband spectrum, we consider the wavelength of the electromagnetic wave used in the wireless network to be constant. These constants are combined into one constant A . Since the main idea is to explore the received power with the variation of the link distance, without loss of generality, we normalize A to unity. The received power at Rx_{*j*} from Tx_{*i*} can then be simplified to

$$p_{ij} = \frac{1}{d_{ij}^2}, \quad (2)$$

where d_{ij} is the distance between Tx_{*i*} and Rx_{*j*}. In order to avoid very high or infinite values of p_{ij} when the distance is very small, we consider the minimum value of d_{ij} to be 1 [26], avoiding near-field effects [27].

Then, the Signal to Interference plus Noise Ratio (SINR) of Link_{*j*} is

$$\text{SINR}_j = \frac{p_{ij}}{\sum_{j=1}^N p_{ij} + \sigma_n^2} = \frac{p_{ij}}{I_j^{\text{from}} + \sigma_n^2} \quad \text{where} \quad \begin{array}{l} i = j, \text{ same link,} \\ i \neq j, \text{ different links.} \end{array} \quad (3)$$

In Equation (3), σ_n^2 is the noise power and $\sum_{j=1}^N p_{ij}$ is the total interference received from other links to Link_{*j*} denoted as I_j^{from} . The equation assumes all links share the same channel; however, only a certain number of links share a channel and there is interference only between links sharing the same channel as illustrated in Figure 1(b). SINR₁ equals the ratio of the power of the signal (due to the Tx₁ which equals to $p_{11} = 1/d_{11}^2$) over the power of the interference and noise. The random channel configuration (initially) used in Figure 1 is “2124342234” (first number signifying the channel used by Link₁ and henceforth). Link₁, Link₃, Link₇, and Link₈ use channel “2”. Hence, for Link₁, there is interference from Tx₃, Tx₇, and Tx₈ which equals to $I_1^{\text{from}} = 1/d_{31}^2 + 1/d_{71}^2 + 1/d_{81}^2$. In this work, to make a concrete example, noise power (σ_n^2) is arbitrarily set at 0.001, which is 30 dB smaller than the transmit power.

The throughput of Link_{*j*} based on Shannon Capacity [28] with AWGN channel and SINR_{*j*} [29] is expressed as

$$T_j = \log_2(1 + \text{SINR}_j). \quad (4)$$

Here, the throughput is in bit/s/Hz, i.e., the throughput is normalized over the bandwidth.

The total throughput of a wireless network with N links is the sum of all the throughput of N links,

$$T_{\text{tot}} = \sum_{j=1}^N T_j. \quad (5)$$

T_{tot} varies as different links choose different strategies which will be further explained in Section 3. We define, $T_{\text{tot}}^{\text{max}}$ to be the maximum T_{tot} from all the different possible pure strategies obtained by exhaustive search of a random scenario.

2.3. Assumptions. Four notable assumptions have been made in this work, while focusing on distributed wireless networks, which are as follows:

First, the links compute the resource allocation schemes themselves in a distributed way and choose the strategies independently.

Second, the links are rational and selfish. Links will choose the strategies in order to increase their own payoff. In our work, utility is defined as in Equation (7) signifying a noncooperative network with independent and selfish users.

Third, there is no information sharing between links. Each link can measure the received power and interference it receives from other links assuming the number of links and channels available in the network are broadcasted in the beacon signal, like in cellular or WiFi networks [30].

Fourth, good error control techniques are implemented in the links. A system model as mentioned in Section 2.1 leads to a dense network with a significant amount of interference. In particular, links sharing channels face considerable amount of interference. We assume the links have good error control techniques which allow them to communicate with some small bit rate even in the presence high interference (as per Shannon's formula, Equation (4)). Over iterations, links could find other channels that have lower interference and decide to transmit at increased data rate as well.

Based on the system model and assumptions stated above, channel allocation policy in our distributed wireless network will be analyzed using game theory in the next section.

3. Game Formulation

Game theory is a mathematical tool that is used to analyze the strategic interaction of players, especially with conflicting interests. In recent years, game theory has been used in distributed wireless networks for resource allocation purposes.

A game can be symbolically expressed as a triplet, $\langle N, S_j, U_j(S_j, S_{-j}) \rangle$ such that there are N players (links) in the game, $j \in 1, 2, \dots, N$. S_j is the set of strategies of player j , and S_{-j} represents the strategies played by all players apart from j . The utility or payoff of player j (Link $_j$) is U_j , which is a function of the strategy chosen by all players [16]. In this work, the players are the links. Each link chooses to transmit in a particular channel resulting in a different payoff. Rational and independent users will try to maximize their U_j in

every iteration [31]. After certain iterations, they mostly converge to a Nash Equilibrium (NE) [32].

3.1. Nash Equilibrium (NE). Nash Equilibrium (NE) is a natural and eventual equilibrium state reached by rational and independent players in a distributed network. The players choose strategies that maximize their individual utility. At NE, each player will not be able to increase its utility by unilaterally changing its strategy if the strategies of other players remain unchanged [16], which is expressed mathematically as

$$U_j(S_j^*, S_{-j}^*) \geq U_j(S_j, S_{-j}^*) \quad \forall j \in 1, 2, \dots, N, \quad (6)$$

where S_j^* represents the best response strategy of link j and S_{-j}^* represents the best response strategy of all other links except j .

A centralized global solution may result in a higher overall performance, but once the rational and selfish players are allowed to make their independent choices, they will drift away from the centralized global solution. However, once the system converges to a NE, none of the rational players will change their strategies as, by doing so, they will get a lower U_j as indicated in Equation (6). Moreover, centralized solutions via an exhaustive search technique require a massive amount of resources (computational and time) especially in larger networks which is unfeasible. Hence, many practical systems prefer distributed solutions.

In the next section, we define the utility of a link such that the distributed system will converge to NE with higher performance.

3.2. Varying Utility to Converge to NE with High Performance. In [18, 24], total throughput of the network with varying links and channels was analyzed using an exhaustive search method. Distribution of network metrics (T_j , I_j^{from} , and d_{ij}) reflect that links have a significant amount of interference at NE with high total throughput.

In order to converge to NE solution with high total throughput, the utility of a link in this work is defined as

$$U_j = T_j + \alpha_{\text{opt}}^* \cdot I_j^{\text{from}} = \log_2 \left(1 + \frac{1/d_{ij}^2}{I_j^{\text{from}} + P_n} \right) + \alpha_{\text{opt}}^* \cdot I_j^{\text{from}}, \quad (7)$$

where T_j is the individual link's throughput as defined in Equation (4), I_j^{from} is the interference received from other links to Link $_j$ as defined in Equation (3), α is the weight of I_j^{from} , and d_{ij} is the distance between the transmitter and receiver of Link $_j$.

Adding an optimal interference term to the payoff of a link might sound contradictory; however, such characteristics are observed in different spectra of life as well. For example, let us consider a city with a certain population having a limited number of vehicles. An equilibrium point with better well-being can be achieved when the vehicles are shared

optimally [33]. By sharing the vehicles, the users have to experience inconveniences like longer ride time and route; however, the overall traffic congestion in the city can be reduced. Hence, the government can give benefits or other incentives to encourage the users in the city to share vehicles to enhance the well-being of the entire city. Similarly, when the wireless users in a distributed network voluntarily (or are hard-wired to) tolerate some level of interference in their payoff, the network can achieve solutions with higher performance. Another befitting example is of the tax paid by a citizen to the government of its country. No one really wants to pay taxes as its being cut off from the salary earned from hard work. Similarly, none of the links in a network wants to share the channel and the implicated interference, which thereby diminishes its throughput. However, all citizens have to pay taxes which are ideally used to run the government and provide social security/benefits like health care, education, and peace/security for the people of the country. Paying taxes facilitates the government to run better, so we can live in a better community [34]. Similarly, if each link in the network tolerates certain levels of interference, then the network can operate at a higher total throughput.

Figure 2 illustrates the convergence characteristics of the distributed system. The links use a best response technique to converge. In every iteration, each user chooses a strategy that maximizes its own utility, and eventually, the system converges to NE [32]. Generally, the best response technique converges to NE if the game is a potential function [35]. However, in our work, we start by assuming that the system converges to NE at a particular channel configuration (that results in high T_{tot}) as explained in Section 4.1. Then, from the NE constraints, we derive the PDF and mean value of α_{opt} which is the key parameter in the utility function (Equation (7)). So, using the best response technique in the derived utility pushes the distributed network to converge to NE (mostly) with high performance. In a general distributed system, each user tried to maximize its individual throughput, that is, $U_j = T_j$. However, the converged solution of the overall system is not so high. Adding an optimal amount of interference in the utility ($U_j = T_j + \alpha_{\text{opt}}^* \cdot I_j^{\text{from}}$, Equation (7)) pushes the system to converge to NE with high T_{tot} . In Section 4, we mathematically derive the approximate value of α_{opt}^* (referred to as $\hat{\alpha}_{\text{opt}}$) which gives similar performance at a fraction of the time and computation resources. Figure 2 shows the average $T_{\text{norm,cum}}^{\text{tot}}$ (normalized cumulative total throughput [24]) from 1000 random scenarios in the y -axis and the number of iterations (time) in the x -axis.

In the upcoming section, we formulate the policy and derive an approximate mathematical expression and PDF for α_{opt}^* (referred to as $\hat{\alpha}_{\text{opt}}$) from NE constraints that results in high T_{tot} for a random “10-link 4-channel” scenario and for varying links and channels.

4. Proposed Method for Approximating the Bounds and PDF of α_{opt}^*

In this section, we are going to approximately derive some bounds and PDF of α_{opt}^* (referred hereafter as $\hat{\alpha}_{\text{opt}}$) that need

to be in the individual link’s utility to converge to a NE solution with high T_{tot} for a random “10-link 4-channel” scenario then for scenarios with a varying number of links and channels. $\hat{\alpha}_{\text{opt}}$ is quite close to α_{opt}^* , but the main advantage of $\hat{\alpha}_{\text{opt}}$ is that it can be computed in a fraction of the time and computational resources required to compute α_{opt}^* .

To proceed with the analysis, shared utility is defined based on the definition of a link’s utility expressed in Equation (7): when a link gets a single channel, the utility of the link is U_{s1} which equals to $\log_2(1 + (1/d_a^2)/P_n)$. There is no interference received from other links for U_{s1} . When l links share a channel, the utility of the link is defined as U_{sl} which is expressed as

$$U_{sl} = \log_2 \left(1 + \frac{1/d_a^2}{\sum_{m=1}^{l-1} (1/d_{lm}^2) + P_N} \right) + \alpha_{\text{opt}}^* \cdot \left(\frac{1}{\sum_{m=1}^{l-1} d_{lm}^2} \right) \quad \text{for } N \geq 2. \quad (8)$$

Here, d_a is the distance of the signal between the transmitter and the receiver of the same link and d_{lm} is the distance from the m^{th} interfering link where $m = 1, 2, \dots, N - 1$.

The probability density function (PDF) of $1/d_a^2$ is illustrated in Figure 3. d_a is the distance between the transmitter (with coordinates x_2 and y_2) and the receiver (with coordinates x_1 and y_1) and can be mathematically expressed as $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. $x_1, x_2, y_1,$ and y_2 are random variables with uniform distribution between 0 and 10 units as illustrated in Figure 1(a). High probability mass occurs at the maximum value of $1/d_a^2 = 1$, as per our assumption in Equation (2). For simplicity of analysis, the distribution of the interfering links d_{lm} ($m = 1, 2, \dots, l - 1$) is assumed to have the same distribution as d_a . This distribution is used to compute the PDF of α_{opt}^* in the upcoming section.

4.1. The “10-Link 4-Channel” Scenario. For “10 links 4 channels,” it has been observed that the maximum T_{tot} occurs mostly ($\approx 85\%$) at channel configuration “1234444444” [18]. The left portion of Figure 4 considers the first link ($j = 1$) which is getting a “single channel,” and the right portion considers the fourth link ($j = 4$) which is “sharing channel” with other links. The first, second, and third links get a “single channel” and have a payoff of U_{s1} , as defined in Equation (8). Once the strategy is changed, the payoff of the link will be either U_{s2} if channel 2 or 3 is selected or U_{s8} if channel 4 is selected. Hence, the NE conditions are $U_{s1} \geq U_{s2}$ and $U_{s1} \geq U_{s8}$. The fourth to tenth links share channel 4, so the payoff is U_{s7} . Once the strategy is changed to channel 4, the payoff of the link will be U_{s2} . Hence, the NE condition is $U_{s7} \geq U_{s2}$.

Since the value of optimal α obtained from the condition $U_{s1} \geq U_{s2}$ is larger than the value obtained from the condition $U_{s1} \geq U_{s3}$ which is larger than the value obtained from the condition $U_{s1} \geq U_{s4}$ and so forth as illustrated in Figure 5, the upper bound for optimal α for the above mentioned conditions is exponentially decreasing. As $U_{s1} \geq U_{s2}$ is a subset of $U_{s1} \geq U_{s8}$, we get one unique upper bound

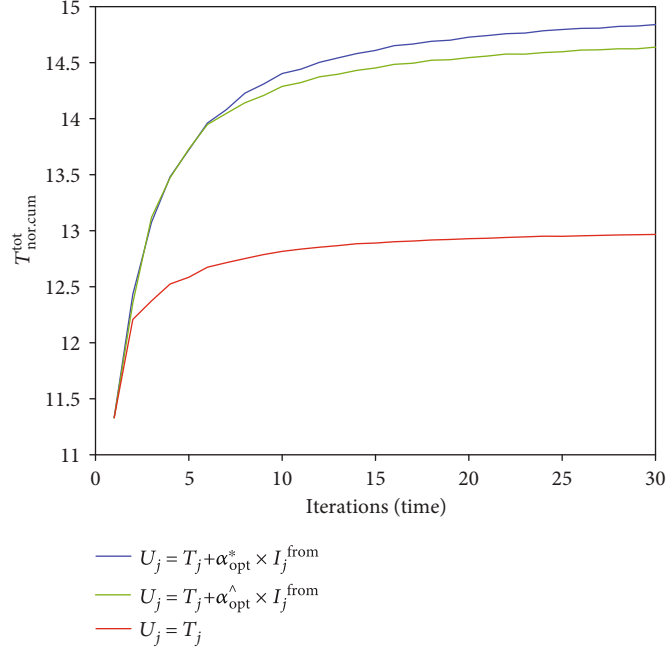


FIGURE 2: Convergence characteristics (iterations versus $T_{\text{nor.cum}}^{\text{tot}}$) of the distributed system for different definitions of utility. Best response technique is used to converge to NE. An average of 1000 random “10-link 4-channel” scenarios is considered.

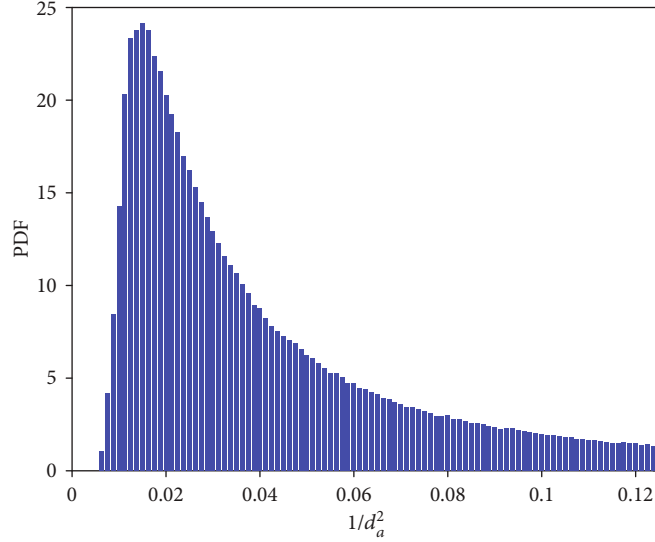


FIGURE 3: Probability density function (PDF) of $1/d_a^2$. d_a is the distance of the signal between transmitter and receiver of the same link.

(UB) condition: $U_{s1} \geq U_{s8}$, and one lower bound (LB) condition: $U_{s7} \geq U_{s2}$.

UB condition: solving for $U_{s1} \geq U_{s8}$, using the definition of shared utility from Equation (8) when $l = 1$ and 8 results in

$$\log_2 \left(1 + \frac{1/d_a^2}{P_N} \right) \geq \log_2 \left(1 + \frac{1/d_a^2}{1/d_{l1}^2 + \dots + 1/d_{l7}^2 + P_N} \right) + \alpha \left(\frac{1}{d_{l1}^2} + \dots + \frac{1}{d_{l7}^2} \right),$$

$$\alpha_{\text{opt}}^{\text{UB}} \leq \frac{[\log_2(1 + (1/d_a^2)/P_N) - \log_2(1 + (1/d_a^2)/(1/d_{l1}^2 + \dots + 1/d_{l7}^2 + P_N))]}{(1/d_{l1}^2 + \dots + 1/d_{l7}^2)}.$$

(9)

LB condition: solving for $U_{s7} \geq U_{s2}$, using the definition of shared utility from Equation (8) when $l = 7$ and 2 results in

$$\log_2 \left(1 + \frac{1/d_a^2}{1/d_{l1}^2 + \dots + 1/d_{l6}^2 + P_N} \right) + \alpha \left(\frac{1}{d_{l1}^2} + \dots + \frac{1}{d_{l6}^2} \right) \geq \log_2 \left(1 + \frac{1/d_a^2}{1/d_{l1}^2 + P_N} \right) + \alpha \left(\frac{1}{d_{l1}^2} \right),$$

$$\alpha_{\text{opt}}^{\text{LB}} \geq \frac{[\log_2(1 + (1/d_a^2)/(1/d_{l1}^2 + P_N)) - \log_2(1 + (1/d_a^2)/(1/d_{l1}^2 + \dots + 1/d_{l6}^2 + P_N))]}{(1/d_{l2}^2 + \dots + 1/d_{l6}^2)}.$$

(10)

$\alpha_{\text{opt}}^{\text{UB}}$ is larger than $\alpha_{\text{opt}}^{\text{LB}}$ because the first term of $\alpha_{\text{opt}}^{\text{UB}}$ is much larger than the first term of $\alpha_{\text{opt}}^{\text{LB}}$. The absence of $1/d_{l1}^2$

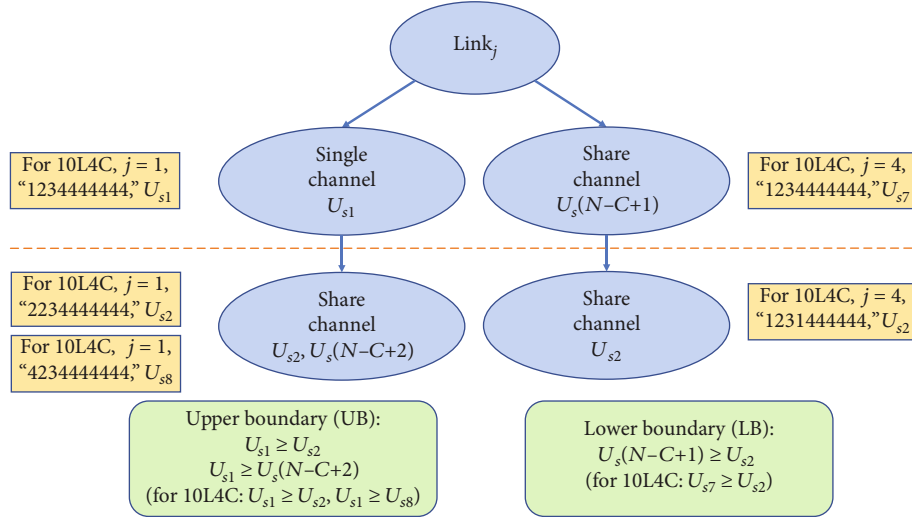


FIGURE 4: At NE with high T_{tot} , Link $_j$ can use a “single channel” or “share channel” with other links. Based on the channel configuration at NE $_{\text{Good}}$, the possible outcome after changing the strategy can be categorized. Applying the definition of NE, we can come up with the upper bound (UB) and lower bound (LB) conditions of optimal α .

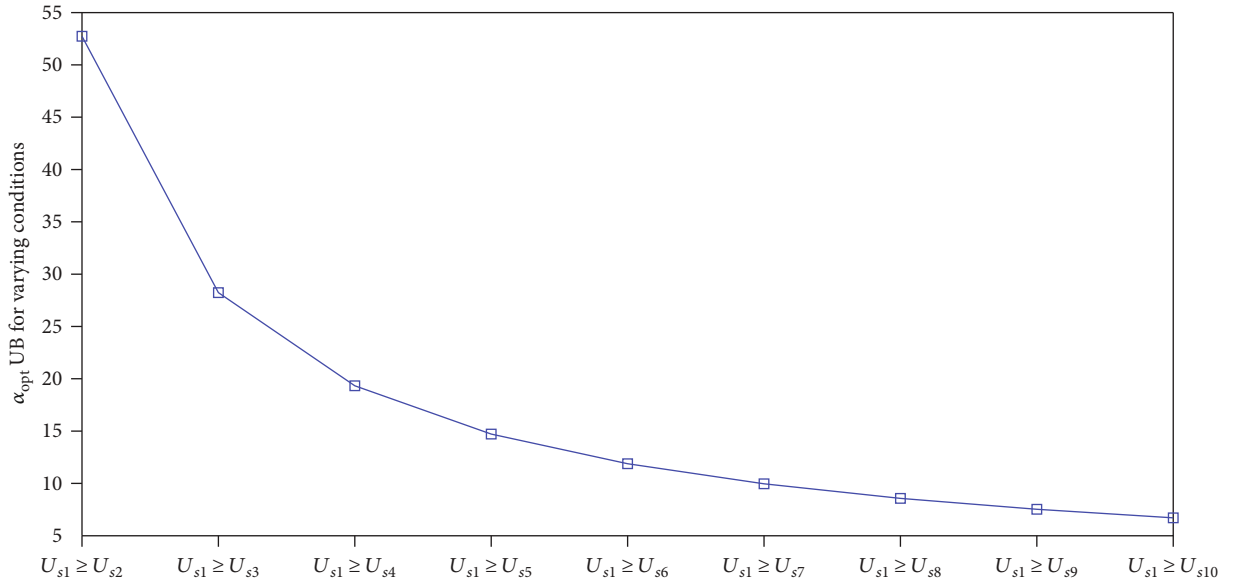


FIGURE 5: Upper bound (UB) of optimal α for varying conditions.

interfering expression (which is much larger than P_N) in the denominator of the first term of $\alpha_{\text{opt}}^{\text{UB}}$ produces such a result, $[\log_2(1 + (1/d_a^2)/P_N) \gg \log_2(1 + (1/d_a^2)/(1/d_{I1}^2 + P_N))]$. This holds true for the “ N -link C -channel” scenario presented in the next section as well. $\alpha_{\text{opt}}^{\text{UB}}$ and $\alpha_{\text{opt}}^{\text{LB}}$ are computed using the PDF of $1/d_a^2$ and $1/d_{\text{Im}}^2$. For a random “10-link 4-channel” scenario, the PDF of $\hat{\alpha}_{\text{opt}}$ is computed by taking the average of $\alpha_{\text{opt}}^{\text{UB}}$ and $\alpha_{\text{opt}}^{\text{LB}}$, which is illustrated in Figure 6 and has a mean value of 7.03 units. α_{opt}^* for this network is determined as such: the value of α is stepwise varied from 0 to 30 with an increment of 0.1. For each value of α , many random simulations are run and the average value of T_{tot} is stored. The α value that results in maximum T_{tot} is defined

as α_{opt}^* [18]. For this network α_{opt}^* is equal to 6.8 units which is close to the value of $\hat{\alpha}_{\text{opt}}$. In the next section, we are going to generalize the mathematical expression of $\hat{\alpha}_{\text{opt}}$ for varying network sizes.

4.2. The “ N -Link C -Channel” Scenario. In this section, we are going to deduce an expression of $\hat{\alpha}_{\text{opt}}$ for different network sizes, with varying numbers of links (N) and channels (C).

Generalizing the findings from the the Section 4.1, NE with high T_{tot} mostly occurs when $(C - 1)$ links get a single channel each and the remaining channel is shared by $(N - C + 1)$ links. As illustrated in Figure 4, Link $_j$ can be sharing a channel or using it alone at NE with high T_{tot} and

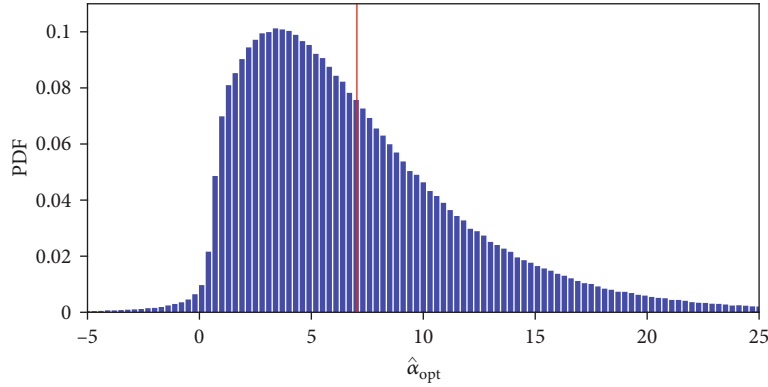


FIGURE 6: Probability density function (PDF) of $\hat{\alpha}_{\text{opt}}$ for “10-link 4-channel” scenario; the mean is 7.03 units.

changing its strategy should yield in lower utility. The left portion of the flow chart shows that initially Link_{*j*} is using the strategy “single channel.” Its payoff is U_{s1} as it is not sharing its channel with any other link. When the link changes its strategy to “share channel,” it results in having two different payoffs. First, it will have to share a channel with a link which was previously getting a single channel, then its payoff will be U_{s2} . Secondly, it will share a channel with links that were previously sharing a channel with $(N - C + 1)$ links, then its new payoff will be $U_{s(N-C+2)}$. Hence, the UB condition to achieve NE with high T_{tot} are $U_{s1} \geq U_{s2}$ and $U_{s1} \geq U_{s(N-C+2)}$. The first

condition is the subset of the second condition which was explained earlier and illustrated in Figure 5. The right portion of the flow chart shows that initially, Link_{*j*} is using the strategy “shared channel.” Its payoff is $U_{s(N-C+1)}$ as it is sharing its channel with $(N - C + 1)$ links. When the link changes the strategy, it will share a channel with the link which was previously using a “single channel.” So, its new payoff will be U_{s2} . Hence, the LB condition is $U_{s(N-C+1)} \geq U_{s2}$.

An expression to compute the expected value of $\hat{\alpha}_{\text{opt}}$ for the “ N -link C -channel” scenario can be computed by taking an expected average of $\alpha_{\text{opt}}^{\text{UB}}$ and $\alpha_{\text{opt}}^{\text{LB}}$ as follows:

$$\hat{\alpha}_{\text{opt}} = \frac{\left[\log_2 \left(1 + (1/d_a^2)/P_N \right) - \log_2 \left(1 + (1/d_a^2) / \left(\sum_{n=1}^{N-C+1} (1/d_{in}^2) + P_N \right) \right) \right] / \sum_{n=1}^{N-C+1} (1/d_{in}^2) + \left[\log_2 \left(1 + (1/d_a^2) / (1/d_{i1}^2) + P_N \right) - \log_2 \left(1 + (1/d_a^2) / \left(\sum_{n=1}^{N-C} (1/d_{in}^2) + P_N \right) \right) \right] / \sum_{n=1}^{N-C-1} (1/d_{in}^2)}{2} \quad (11)$$

Equation (11) holds true for varying values of N and C , but better results are obtained when $N/C \geq 1.8$. In the next section, we will compare the results and analyze the performance while implementing our policy: α_{opt}^* and its approximate mathematical expression $\hat{\alpha}_{\text{opt}}$ along with benchmark schemes.

5. Results: Comparing Centralized and Distributed (Basic, α_{opt}^* and $\hat{\alpha}_{\text{opt}}$) Schemes

In this section, we are going to compare the results obtained by implementing our policy with a standard centralized and basic distributed network. Figure 7 plots the T_{tot} for four different schemes in the y -axis and varying network sizes in the x -axis. The maximum value of T_{tot} for each scenario is obtained from the centralized solution via an exhaustive search technique. In the exhaustive search method, all possible channel configurations are listed and T_{tot} is sorted to obtain $T_{\text{tot}}^{\text{max}}$. This is denoted by the blue line. For large networks, it is not possible to perform the exhaustive search method. For example, for the “13-link 6-channel” scenario, the total number of channel configurations is 6^{13} which is

over 13 billion. So, the exhaustive search results for “13L6C” and larger scenarios is omitted. The lowest T_{tot} is obtained when $U_j = T_j$, which represents a basic distributed wireless network (gray line). The value of T_{tot} obtained by implementing the policy $U_j = T_j + \alpha_{\text{opt}}^* \times I_{\text{from}}$ is illustrated by the red line. α_{opt}^* is the optimal value of α , which is obtained by varying the value of α for a wide range (here, from 0:0.2:50) and choosing the value of α that results in maximum T_{tot} . In this work, we proposed a mathematical expression that approximates the real optimal α which is referred to as $\hat{\alpha}_{\text{opt}}$, represented by the orange line. Computing the optimal α via the proposed approximation is much faster and consumes only a fraction of computational resources, and the performance is quite close to α_{opt}^* , as illustrated in Table 1. Implementing the policy can enhance the total throughput of a distributed network by up to 15% as illustrated in Figure 7.

Table 1 enlists seven different network sizes with varying links (N) and channels (C). As N and C of a network increases, the total possible channel configurations drastically increase to C^N , which is shown in the second column of the table. The 3rd column lists the time consumed to run

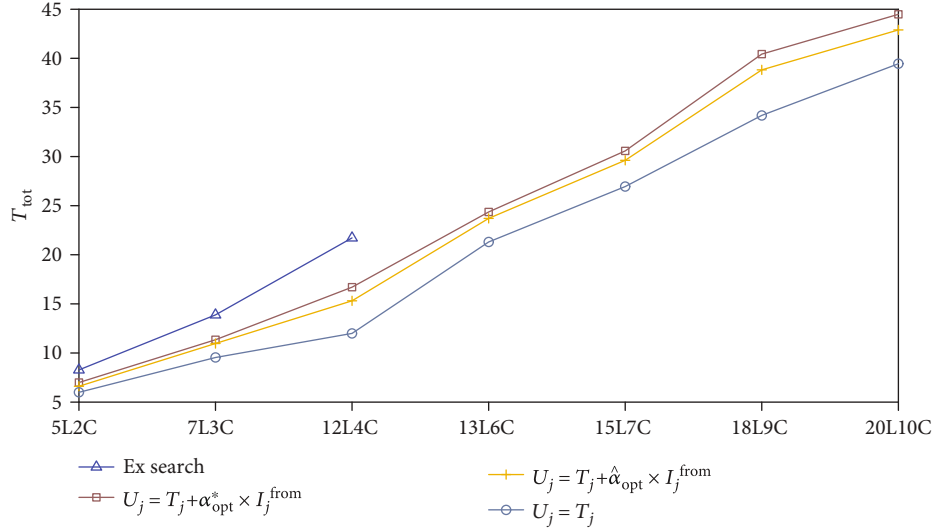


FIGURE 7: Illustration of how the distributed policy (adding $\hat{\alpha}_{\text{opt}} \times I_{\text{from}}$ term in the utility) increases the T_{tot} by up to 15% for distributed wireless networks. α_{opt}^* is the value of optimal α obtained from many random simulations for a wide range of α . $\hat{\alpha}_{\text{opt}}$ is obtained from a mathematical expressions that approximates the α_{opt}^* and can be obtained at a fraction of computational and time resources. Although, the centralized solution from exhaustive search is the ideal solution, it is not feasible to adopt it especially for larger networks, due to its tremendous consumption of time and computational resources.

TABLE 1: Comparing the resources (time and computation) needed and the performance of the converged solution by centralized scheme via exhaustive search technique and our distributed policy (adding $\hat{\alpha}_{\text{opt}} \times I_{\text{from}}$ term in the utility).

Network	Centralized solution via exhaustive search		Distributed solution, policy ($\hat{\alpha}_{\text{opt}}$)
	Total channel configurations	Time (estimated) consumed	Time consumed
5L2C	$2^5 = 3.2 \times 10^1$	32 τ milliseconds	39 τ ms
7L3C	$3^7 \approx 2.19 \times 10^3$	8.74 τ seconds	42 τ ms
12L4C	$4^{12} \approx 1.68 \times 10^7$	18.64 τ hours	48 τ ms
13L6C	$6^{13} \approx 1.31 \times 10^{10}$	20 τ months	50 τ ms
15L7C	$7^{15} \approx 4.75 \times 10^{12}$	602 τ years	53 τ ms
18L9C	$9^{18} \approx 1.50 \times 10^{17}$	$1.90 \times 10^7 \tau$ years	56 τ ms
20L10C	$10^{20} \approx 1.00 \times 10^{20}$	$1.27 \times 10^{10} \tau$ years	60 τ ms

all the C^N channel configurations and find $T_{\text{tot}}^{\text{max}}$. Computation time for single configuration is around $\tau = 4$ ms (considered as a unit time) in our desktop computer. The fourth column shows the time consumed (computing $\hat{\alpha}_{\text{opt}}$ plus running best response technique) using the distributed policy proposed in this work. From the table, it is evident that as the link size increases, it is impossible (due to time and computational resources required) to go through each channel configuration and obtain $T_{\text{tot}}^{\text{max}}$. Implementing the policy, the system can reach to a distributed NE solution with $\approx 75\%$ of the $T_{\text{tot}}^{\text{max}}$ consuming a fraction of the time and computational resources in comparison to the general distributed system ($U_i = T_i$).

6. Conclusion

With the increment of smart users (independent and rational), distributed and decentralized wireless networks are

gaining prominence. In such networks, there is no central entity that supervises/decides the strategies of each user, and the users do not share information with each other as well. Our network has many links (with a direct communication between a transmitter and receivers) within a limited area. Hence, links generally overlap, and when resources are limited ($C < N$), there is high interference especially to links sharing channels. If the communication distance is limited to a certain range and channels are spatially reused, a different solution is expected (we plan to explore it further in our future work). The smart user in these networks tries to maximize its own utility in every iteration, and eventually, the network converges to an equilibrium solution called the Nash Equilibrium (NE). However, not all NE have high performance. From the analysis of different network metrics, it was found that communication links still have significant amount of interference values at NE with high T_{tot} . Adding

weighted interference-received term in the payoff of a link can drive the decentralized system to converge to a solution with higher T_{tot} . In this work, we derived an approximate mathematical expression along with the PDF for α_{opt}^* , referred as $\hat{\alpha}_{\text{opt}}$ for a random “10-link 4-channel” scenario. Then, we extracted the general trend of channel configuration at NE with high T_{tot} and proposed a mathematical expression for $\hat{\alpha}_{\text{opt}}$ for varying links (N) and channels (C).

Some limitations of our work are the value of noise power, area/dimension of the network, and the wireless path loss exponent is fixed. In reality, these values can vary which may result in different channel allocation at NE with high total throughput. This might cause a shift in the value of α_{opt}^* and pdf of $\hat{\alpha}_{\text{opt}}$. When we derived a generic equation for $\hat{\alpha}_{\text{opt}}$, we obtained the NE conditions from the channel allocation: “each of the $C - 1$ links occupies its own channel and the remaining $N - C + 1$ links share the remaining one channel” because, most of the time, NE with high T_{tot} occurs at the channel configuration. However, at times especially when the ratio of N/C is less than 1.8, NE with high T_{tot} occurs at other channel allocations. We plan to address these issues in our future work.

In this work, we implement the policy and addition of an optimally weighted ($\hat{\alpha}_{\text{opt}}$) interference-received term in the payoff of a communication link. This enhances the performance of the distributed wireless network by up to 15%, and the system can achieve up to 75% of the maximum total throughput (benchmark value reached by centralized solution via exhaustive search technique) at a fraction of the time and computation resources.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors would like to thank Asst. Prof. Dr. Somsak Kitipiyakul, Dr. Attaphongse Taparugssanagorn, Assoc. Prof. Dr. Steven Gordon, and Assoc. Prof. Dr. Komwut Wipusitwarakun for their valuable suggestions and comments. This research is partially supported by the Excellent-Foreign-Students (EFS) scholarship from the Sirindhorn International Institute of Technology, Thammasat University.

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