A Switching-Based Interference Control for Booster Separation of Hypersonic Vehicle

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1. Introduction

The near space air-breathing hypersonic vehicle must have certain height, speed, and dynamic pressure, only in this way can the scramjet engine be reliably ignited. Whether launching in the air or from the ground, there is a process of accelerating to scramjet ignition window at low speed. Then, the booster is separated from the vehicle, the scramjet is ignited, and the hypersonic vehicle flies autonomously. The scramjet can be ignited at a relatively high speed and under high dynamic pressure; therefore, the external flow field will generate strong interference to the hypersonic air-craft in its stage separation process. When the vehicle is statically stable, the control system can control it again after the vehicle flies out of the interference flow field. In this process, the separation interference only affects the vehicle attitude and has no mutual coupling effect with the control system. However, with the development of the weaponization of vehicles, hypersonic vehicles increasingly present larger characteristics of static instability. After stage separation, the vehicle control system must be activated to control the statically unstable projectile body in the interference flow field area. Therefore, the stage separation interference flow field will not only affect the attitude of the projectile body...
2. Methodology

2.1. Description of the Hypersonic Vehicle Stage Separation Process. The stage separation process between the tandem booster and the hypersonic vehicle is shown in Figure 1. When the hypersonic vehicle reaches the separation window, the attitudes of its front body and afterbody remain the same, and there is no mutual flow field interference. After the vehicle receives the stage separation instruction, the explosive bolt will be detonated, and the two bodies will be separated. Due to the impulse interference of the explosion bolt and the relative movement of the attitude between the precursors after separation, the attitude changes of the two bodies cause complex flow field change; especially, the rear body will generate antishock wave, which produces a torque to the front body. At the same time, the rudder surface control efficiency of the front body will be affected, resulting in antimanipulation.
For stage separation, this paper adopts the local mesh reconstruction technology under nonstructured mesh, CFD/6DoF coupling calculation, the six-degree-of-freedom unsteady method suitable for solving multibody separation problems with complex configurations, and the overlapping mesh technology to accelerate convergence.

The following are the main states of stage separation:

(a) Mach number: 5.6, altitude: 19 km, angle of attack: 2.0°, sideslip angle: 0.0°, and separation: 200 mm
(b) Mach number: 5.6, altitude: 19 km, angle of attack: 0.0°, sideslip angle: 0.0°, and separation: 1900 mm

Figures 2–5 show the pressure nephogram of the fuselage during the booster separation process.

It can be seen from the figure that in the separation process, when the booster stage reverses injection engine works and the distance between stages is small, there will be a certain afterbody effect. The forward jet will produce a certain aerodynamic interference to the precursor aircraft and interact with the aircraft tail, thus affecting the stability of the precursor aircraft. In the process of interstage separation, the middle section is a high-pressure area, which makes the resistance of the aircraft decrease and the resistance of the booster increase. The aerodynamic effect will accelerate the process of interstage separation. Because the interstage separation studied in this paper is controlled by the reverse injection engine, the working of the reverse injection engine will have a certain aerodynamic interference impact on the attitude of the booster stage, forming a blocking effect on the incoming flow in front of the nozzle, resulting in a large range of high-pressure area, and a low-pressure area after the nozzle due to the injection of high-pressure air flow. Under the joint action of the two, the force and attitude of the first-class aircraft are affected.

Figure 6 shows the variation curve of the pitching moment coefficient of the hypersonic vehicle with the angle of attack and deflection angle of control surface during the separation process.

It can be clearly seen from Figures 6 and 7 that due to the influence of the interference flow field, the pitching moment coefficient of the vehicle decreases, and the steerage shows aileron reversal in the process of stage separation. During the boost separation of hypersonic vehicle, the tail wing of the vehicle is affected by the interference flow field, and the control effect of the control surface will be weakened. If the controller is designed based on the aerodynamic data as shown in Figure 6, the control system will diverge when the aerodynamic characteristics of the vehicle appear...
anticontrol phenomenon under the influence of the interference flow field (as shown in Figure 7).

2.2. Dynamic Modeling of the Hypersonic Vehicle. Mainly considering the longitudinal motion of hypersonic aircraft, the dynamic model of the hypersonic vehicle in the process of stage separation is given below:

\[
\begin{align*}
\frac{mdV}{dt} &= -X - mg \sin \theta, \\
I_{z} \frac{d\omega_{z}}{dt} &= M_z, \\
\frac{d\theta}{dt} &= \omega_{z}, \\
\theta &= \theta + \alpha, \\
X &= \frac{1}{2} \rho V^2 S \bar{C}_D, \\
Y &= \frac{1}{2} \rho V^2 S \bar{C}_m, \\
M_z &= \frac{1}{2} \rho V^2 S L \bar{C}_m,
\end{align*}
\]

where \( \rho \) is the atmospheric density, \( \bar{S} \) is the reference area, and \( L \) is the reference length. The aerodynamic model of the longitudinal channel drag coefficient \( C_D \), lift coefficient \( C_L \), and pitching moment coefficient \( C_{M_{yy}} \) on angle of attack, angle of pitch rudder deflection, and Mach number is

\[
\begin{align*}
\rho &= \rho_0 \exp \left( \frac{(H - H_0)}{H_s} \right), \\
C_D &= C_D^0 + C_D^a \alpha + C_D^\delta \delta e + C_D^{Ma} Ma + C_D^{Ma a} Ma \cdot \alpha + C_D^{Ma \delta} Ma \cdot \delta e + C_D^{Ma^2} Ma^2, \\
C_L &= C_L^0 + C_L^a \alpha + C_L^\delta \delta e + C_L^{Ma} Ma + C_L^{Ma a} Ma^2 + C_L^{Ma \delta} Ma \cdot \delta e + C_L^{Ma^2} Ma^2, \\
C_{M_{yy}} &= C_{M_{yy}}^0 + C_{M_{yy}}^a \alpha + C_{M_{yy}}^\delta \delta e + C_{M_{yy}}^{Ma a} Ma + C_{M_{yy}}^{Ma \delta} Ma \cdot \delta e + C_{M_{yy}}^{Ma^2} Ma^2 + C_{M_{yy}}^{Ma^2 \delta} Ma^2 \cdot \delta e,
\end{align*}
\]

where \( M_z \) is the Mach number, \( \rho_0 \) is the standard atmospheric density and \( H_0 \) is its corresponding height, and \( 1/ H_s \) is the attenuation law of atmospheric density, \( \delta e \) is elevator deflection angle, and \( C_D, C_L, \) and \( C_{M_{yy}} \) are the aerodynamic coefficients related to the angle of attack, the angle of elevator, and the Mach number, respectively.

For the separation time of hypersonic vehicle is very short, the difference between the disturbed motion parameters near the separation point and the undisturbed motion parameters is very small. At the separation time, the rolling motion and lateral motion amplitude of the aircraft are very small, and the influence on the attitude of the aircraft can be ignored. Therefore, small disturbance linearization can be used in the modeling process.

By linearizing the above equation with small disturbance, the longitudinal dynamic model of hypersonic vehicle can be obtained as follows:

\[
\begin{align*}
\dot{x} &= Ax + Bu, \\
y &= Cx + Du,
\end{align*}
\]

where \( x = [\alpha \ \omega_z]^T \).

\[
A = \begin{bmatrix} -a_1 & 1 \\ -a_2 & -a_1 \end{bmatrix}, B = \begin{bmatrix} -a_3 \\ -a_3 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
\]

Make the dynamical model as subsystem 1 when the hypersonic vehicle is in the separation interference flow field:

\[
\begin{align*}
\dot{x}_1 &= A_1 x + B_1 u, \\
\dot{x}_1 &= A_1 x + B_1 u.
\end{align*}
\]

Make the dynamical model as subsystem 2 after the hypersonic vehicle flies out of the separation interference flow field:

\[
\begin{align*}
\dot{x}_2 &= A_2 x + B_2 u, \\
y_2 &= C_2 x + D_2 u.
\end{align*}
\]
the sliding mode switching control principle and designing a sliding mode switching controller based on a sliding switching surface, so as to realize the attitude stabilization of the hypersonic vehicle in the process of stage separation.

2.3.1. Consider a Nonlinear System.

\[
\dot{x}(t) = A_i x(t) + B_i u_i + f_i(x, t, \sigma_t),
\]

\[
y = C_i x(t),
\]

where \(x(t) \in \mathbb{R}^n\) is the system’s state vector, \(u_i \in \mathbb{R}^m\) is the control input vector, and \(y \in \mathbb{R}^p\) is the output vector. \(\sigma_t\) is the switching signal. \(\sigma_t = I\) indicates that the \(i\)-th subsystem is activated.

The system can be expressed as

\[
\dot{x}(t) = A_i x(t) + B_i u_i,
\]

\[
y = C_i x(t).
\]

Assumption 1. The matrix pairs \((A_i, B_i)\) and \((A_i, C_i)\) are controllable and observable, respectively.

From the observability of \((A_i, C_i)\), it can be known that there is the matrix \(L_i\) that makes \((A_i - L_i C_i)\) stable. Therefore, for any positive definite matrix \(Q_i\), the following Lyapunov equation has a unique positive definite symmetric solution \(P_i\):

\[
(A_i - L_i C_i)^T P_i + P_i (A_i - L_i C_i) = -Q_i.
\]

This paper assumes that the output matrix satisfies \(C_i = [I_r, 0]\), so systems (7) and (8) can be expressed as follows:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
A_{i11} & A_{i12} \\
A_{i21} & A_{i22}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
B_{i1} \\
B_{i2}
\end{bmatrix} u_i,
\]

\[
y(t) = x_1,
\]

where \(x_1 \in \mathbb{R}^r, A_{i11} \in \mathbb{R}^{rxr}, \) and \(B_{i1} \in \mathbb{R}^{r \times m}\).

Assumption 2. There is a continuous function \(M_i\) that makes \(f_i(x, t) < B_i M_i, \|M_i\| \leq \beta_i \eta_i,\) and \((y, x_1), (y, x_2)\) satisfy \(\|B_i - \eta_i\| \leq \mu \|x_1(t) - x_2(t)\|,\) where \(\cdot \) expresses Euclidean norm.

For systems (11) and (12), there is \(K_i\) that makes \((A_{i22} + K_i A_{i12})\) stable. It can be learnt from formula (10) and \(C_i = [I_r, 0]\) that \(K_i = P_{i12}^{-1} P_{i21}\), where

\[
P_i =
\begin{bmatrix}
P_{i11} & P_{i12} \\
P_{i21} & P_{i22}
\end{bmatrix},
\]

\[
Q_i =
\begin{bmatrix}
Q_{i11} & Q_{i12} \\
Q_{i21} & Q_{i22}
\end{bmatrix}.
\]
For all i, the following Lyapunov equation holds:

\[
(A_{i22} + P_{i22}^{-1} P_{i21} A_{i12})^T P_{i22} + P_{i22} (A_{i22} + P_{i22}^{-1} P_{i21} A_{i12}) = -Q_{i22}.
\]  

(14)

Construct the state observer:

\[
\begin{aligned}
\dot{z}_2 &= (A_{i22} + P_{i22}^{-1} P_{i21} A_{i12}) z_2 \\
&\quad + \left[ A_{i21} + P_{i22}^{-1} P_{i21} A_{i11} - (A_{i22} + P_{i22}^{-1} P_{i21} A_{i12}) P_{i22}^{-1} P_{i21} \right] y \\
&\quad + (B_{i2} + P_{i22}^{-1} P_{i21} B_{i1}) u_i,
\end{aligned}
\]

(15)

\[
\tilde{z}_2 = \bar{z}_2 - P_{i22}^{-1} P_{i21} y.
\]

(16)

**Theorem 3.** For any initial states \(x(0)\) and \(\bar{x}(0)\), there exists a constant \(a > 0\) in state observers (15) and (16) that makes \(||\bar{x}_2(t) - \bar{z}_2(t)|| \leq a\), where \(\bar{x}_2(t)\) is the observation state of \(\bar{x}_2(t)\).

**Proof.** Introduce linear transformation \(z = T_ix\), and \(T_i\) can be defined as

\[
T_i = \begin{bmatrix}
I_r & 0 \\
-P_{i22}^{-1} P_{i21} & I_{n-r}
\end{bmatrix}.
\]

(17)

From formulas (11), (12), and (17), the following dynamic systems can be obtained:

\[
\begin{aligned}
\dot{z}_2 &= (A_{i22} + P_{i22}^{-1} P_{i21} A_{i12}) z_2 \\
&\quad + \left[ A_{i21} + P_{i22}^{-1} P_{i21} A_{i11} - (A_{i22} + P_{i22}^{-1} P_{i21} A_{i12}) P_{i22}^{-1} P_{i21} \right] y \\
&\quad + (B_{i2} + P_{i22}^{-1} P_{i21} B_{i1}) u_i,
\end{aligned}
\]

(18)

\[
y = z_1.
\]

(19)

It can be obtained from \(z = T_ix\) and formula (16) that \(z_2 = x_2 + P_{i22}^{-1} P_{i21} y\). According to the definition \(\bar{x}_2 = \bar{z}_2 + P_{i22}^{-1} P_{i21} y\), it can be obtained that \(\bar{x}_2 - \bar{z}_2 = z_2 - \bar{z}_2\). Assume the observation error \(e = \bar{x}_2 - \bar{z}_2\); it can be obtained from formulas (15) and (18) that

\[
\dot{e} = (A_{i22} + P_{i22}^{-1} P_{i21} A_{i12}) e.
\]

(20)

For the error system (19), consider the following Lyapunov function:

\[
V_e = \sum_{i=1}^{N} e^T_i P_{i22} e.
\]

(21)

Calculate the derivative of time for \(V_e\), we can get

\[
\dot{V}_e = \sum_{i=1}^{N} e^T_i P_{i22} e < 0.
\]

(22)

According to the stability theory of Lyapunov, the error system (19) is asymptotically stable, so there is a positive constant \(a\) which satisfies \(||\bar{x}_2(t) - \bar{z}_2(t)|| \leq a\) for the motion in any initial state.

The proof completes. \(\square\)

Design the sliding mode control to ensure that the state of the system reaches the sliding mode area and performs sliding mode motion. The reachable condition for the sliding state of systems (7) and (8) is

\[
V = \sum_{i=1}^{N} s_i^T W_i s_i < 0,
\]

(23)

where \(W_i\) is symmetric positive definite matrix and \(s_i\) is the sliding mode function of each subsystem. The definition is as follows:

\[
s_i = F_{i1} y + F_{i2} \bar{z}_2.
\]

(24)

In the new coordinate \((x, e)\), formula (24) can be expressed as

\[
s_i = F_{i1} x - F_{i2} e,
\]

(25)

where \(F_i = \begin{bmatrix} F_{i1} & F_{i2} \end{bmatrix}^T\) is the parameter to be designed and \(F_{i1} \in \mathbb{R}^{m \times r}\). \(F_i\) should satisfy that \(A_i - B_i (F_i B_i)^{-1} F_i A_i\) has \(n - m\) eigenvalues with a negative real part; meanwhile, \(F_i B_i\) is nonsingular.

The derivative of the sliding mode function obtained from formulas (20) and (25) is as follows:

\[
\dot{s}_i = F_{i1} x + B_i u_i - F_{i2} \left( A_{i22} + P_{i22}^{-1} P_{i21} A_{i12} \right) e.
\]

(26)

When the system enters the sliding mode dynamic area, \(\dot{s}_i = 0\) can get equivalent control:

\[
u_{eq}(t) = -(F_i B_i)^{-1} \left[ F_i A_i x - F_{i2} \left( A_{i22} + P_{i22}^{-1} P_{i21} A_{i12} \right) e \right].
\]

(27)

Express the formula \(\dot{V}\) as follows:

\[
\dot{V} = \sum_{i=1}^{N} \left( W_i s_i \right)^T W_i s_i.
\]

(28)
Make $\pi_i = -m_i \delta_i$, and $m_i > 0$. The definition is:

$$
\pi_i = \begin{cases} 
-m_i \frac{W_i \delta_i}{\|W_i \delta_i\|}, \\
0, \\
\delta_i \neq 0.
\end{cases} 
$$

(29)

The equivalent control $u_{eq}$ satisfies that $\delta_i = 0$; therefore,

$$
F_i(A_i x + B_i u_{eq}) - F_i(A_{i2} + P_{i2}^{-1} P_{i1} A_{i12}) e = 0.
$$

(30)

Subtracting formula (30) from formula (26), it can obtain

$$
\dot{\delta}_i = F_i B_i (u_i - u_{eq}) = 0.
$$

(31)

Putting formula (27) into formula (31), we can get

$$
u_i = -(F_i B_i)^{-1} \left[ F_i A_i x - (A_{i22} + P_{i2}^{-1} P_{i1} A_{i12}) e - \pi_i \right].
$$

(32)

Formula (32) needs to change its form as it contains the nonlateral state vector $\bar{s}_i$.

**Theorem 4.** Consider the definition controller of switching systems (7) and (8):

$$
u_i = -(F_i B_i)^{-1} \left[ (F_i A_{i12} + F_i A_{i2}) y + (F_i A_{i12} + F_i A_{i2}) \bar{s}_i + K_i(y, t) + \|F_i B_i\| |\beta \delta_i| - \pi_i \right].
$$

(33)

where $\pi_i$ can be defined as formula (29).

$$
K_i > a(\|F_i A_{i12} - F_i A_{i2} P_{i2}^{-1} P_{i1} A_{i12}\| + \|F_i B_i\| |\beta \delta_i|).
$$

(34)

Under the effect of the control (33), systems (7) and (8) enter the sliding mode dynamic zone and perform sliding mode motion.

**Proof.** When $\delta_i = 0$, the system has entered the sliding mode area for sliding mode motion. In the case of $\delta_i \neq 0$, substitute formula (33) into formula (26) and formula (23) gives

$$
V = \sum_{i=1}^{N} s_i^T W_i [F_i A_i x - (F_i A_{i11} + F_i A_{i21}) y - (F_i A_{i12} + F_i A_{i2}) \bar{s}_i - \|F_i B_i\| |\beta \delta_i| - K_i(t)] - F_i (A_{i2} + P_{i2}^{-1} P_{i1} A_{i12}) e.
$$

(35)

After simplification, we can know that

$$
F_i A_i x - (F_i A_{i11} + F_i A_{i21}) y - (F_i A_{i12} + F_i A_{i2}) \bar{s}_i = (F_i A_{i12} + F_i A_{i21}) e.
$$

(36)

It can be known from Assumption 2 that

$$
F_i f_i - ||F_i B_i|| |\beta \eta_i| = F_i B_i M_i(t) - ||F_i B_i|| ||M_i|| |\beta \eta_i| \leq |\beta \eta_i| ||F_i B_i|| e.
$$

(37)

Substituting formulas (36) and (37) into formula (35), we can get

$$
V = \sum_{i=1}^{N} s_i^T W_i [\pi_i + (F_i A_{i11} + F_i A_{i21}) e + \|F_i B_i\| |\beta \eta_i| - K_i(t)]
$$

$$
\leq \sum_{i=1}^{N} s_i^T W_i [\|F_i A_{i12} + F_i A_{i2} P_{i2}^{-1} P_{i1} A_{i12}\| e + |\beta \eta_i| ||F_i B_i|| e]
$$

$$
- K_i(t) < \sum_{i=1}^{N} s_i^T W_i \pi_i < -\sum_{i=1}^{N} m_i s_i^T W_i \|W_i \delta_i\| < 0.
$$

(38)

Therefore, the sliding mode arrival conditions are satisfied, and under the action of control (33) and a given switching rate, systems (7) and (8) can enter the sliding mode area and perform sliding mode motion.

The proof completes. □

**3. Results and Discussion**

3.1. **Simulation Verification.** Assume that the hypersonic vehicle flies at an altitude of 19 km with the flying speed of 6.0 Ma. When stage separating, the initial attack angle $\alpha = 0^\circ$, and the initial angular velocity $\omega_z = 1^\circ/s$. During the stage separation, systems of the hypersonic vehicle in and out of the interference flow field are subsystem 1 and subsystem 2, respectively, switching from subsystem 1 to subsystem 2 at 200 mm seconds. Only considering the antieffect of the surface control without regard to the interference in the separation process, the dynamic system matrix in the process of stage separation can be calculated as follows:

$$
A_1 = \begin{bmatrix}
-0.3563 & 23.7567 \\
1 & -0.0955
\end{bmatrix},
A_2 = \begin{bmatrix}
-0.3632 & 9.2644 \\
1 & -0.2039
\end{bmatrix},
B_1 = \begin{bmatrix}
-25.4341 \\
0
\end{bmatrix},
B_2 = \begin{bmatrix}
-74.0901 \\
-0.0355
\end{bmatrix},
Q_1 = Q_2 = 6I_2.
$$

(39)

Let $L_1 = [12, 4], L_2 = [14, 2]$; solving formula (10) gives a positive definite symmetric matrix:

$$
P_1 = \begin{bmatrix}
0.2709 & -0.1157 \\
-0.1157 & 2.6255
\end{bmatrix},
P_2 = \begin{bmatrix}
0.2235 & -0.2107 \\
-0.2107 & 5.1401
\end{bmatrix}.
$$

(40)

The coefficient matrix $F_1 = [5, 3], F_2 = [6, 5]$ can be
obtained according to the requirements of $F_1$, and response diagrams of pitch angle and attack angle are shown as Figures 8, 9.

Assume that the hypersonic vehicle is also affected by the interference torque during the separation process, whose form of interference is as follows:

$$f_1 = B_1 M_1 (y) = B_1 \sin x_1, f_2 = B_2 M_2 (y) = B_2 \sin x_2.$$  

(41)

The interference satisfies

$$\|M_1\| \leq |x_1|, \|M_2\| \leq |x_2|, \beta_1 = \beta_2 = 1, \eta_1 = |x_1|, \eta_2 = |x_2|. \quad (42)$$

Let $K_1 = K_2 = 1$, and the state response diagrams are shown in Figures 10, 11.

3.2. Discussion. Figure 12 shows the response curve of the conventional controller when there is stage separation interference. At the beginning of separation, the aircraft precursor is in the interference flow field; the control surface shows reversal effect. The angular velocity increases with the increase of rudder deflection angle until the rudder surface is saturated. At this time, the angular rate continues to increase under the influence of interference; after the front body of flight is out of the interference area, the rudder surface is not saturated, producing the suppression effect on interference. Therefore, the control system is at risk of losing control when using a single-mode control method. While adopting the switching control based on sliding mode, the control system switches the control mode according to the state during separation, which can effectively suppress the separation interference and prevent the antimanipulation of the rudder surface.

It can be seen from the simulation results shown in Figures 8–11 that when the flow field interference during stage separation makes the rudder surface ineffective, sliding mode switching control can not only realize reliable separation of the hypersonic aircraft from the booster under a high dynamic pressure but also quickly suppress the interference of the separation process on the vehicle attitude. Under the
action of interference, the designed controller can converge the angle of attack response of the aircraft to the equilibrium angle of attack within 5 seconds, and the amplitude is no more than 5°. The pitch angular velocity converges within 3 seconds, and the amplitude does not exceed 6°/s. The dynamic characteristics of the system meet the design requirements.

4. Conclusion

Aiming at the interference suppression of hypersonic stage separation under a high dynamic pressure, this paper proposes a method of hypersonic stage separation control based on sliding mode switching control. The flight state of the hypersonic separation process in and out of the interference flow field region is regarded as a subsystem with switching characteristics. A sliding mode switching control system is designed to realize the attitude stabilization in different states during the stage separation process. Simulation results show that the designed control system can realize the reliable stage separation under a high dynamic pressure and make the effective suppression on the interference in the separation process, with the advantages of stability and rapidity.

Data Availability

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Conflicts of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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