

## *Retraction*

# **Retracted: Dynamic Index Optimal Investment Strategy Based on Stochastic Differential Equations in Financial Market Options**

### **Wireless Communications and Mobile Computing**

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

- (1) Discrepancies in scope
- (2) Discrepancies in the description of the research reported
- (3) Discrepancies between the availability of data and the research described
- (4) Inappropriate citations
- (5) Incoherent, meaningless and/or irrelevant content included in the article
- (6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

### **References**

- [1] J. Zhang, "Dynamic Index Optimal Investment Strategy Based on Stochastic Differential Equations in Financial Market Options," *Wireless Communications and Mobile Computing*, vol. 2021, Article ID 5545956, 9 pages, 2021.

## Research Article

# Dynamic Index Optimal Investment Strategy Based on Stochastic Differential Equations in Financial Market Options

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With the gradual development and improvement of the financial market, financial derivatives such as futures and options have also become the objects of competition in the financial market. Therefore, how to make the most favorable and optimized investment and consumption when options are included? It has become a problem facing investors. Aiming at the optimal investment problem of investors, this paper studies the calculation of an optimal investment strategy in stochastic differential equations in financial market options on the basis of fuzzy theory. Now, stochastic calculus has become an important branch of stochastic analysis, finance, control, and other fields. The study of introducing stochastic differential equations is mainly to solve the stochastic control problem, which is the principle of the stochastic maximum. In finance, some hedging or pricing problems of contingent rights can eventually be transformed into a series of stochastic differential equations. Based on the historical data of five aspects of bank deposits, bonds, funds, stocks, and real estate of four listed insurance companies, the paper analyzes the optimization strategy of the capital investment of listed insurance companies based on the investment yield of the domestic investment market. According to the final results, the historical proportion of bank deposits of the superior company is 27%, and the optimal proportion given by the model is 25%; the total proportion of funds and stocks is 15%, and the optimal proportion of funds analyzed in the model is 20% and the optimal proportion of stocks is 10%. Therefore, the final results show that the investment efficiency of listed insurance companies can actually increase investment in stocks and funds and reduce the proportion of bank deposits to obtain greater investment returns.

## 1. Introduction

*1.1. Background Meaning.* In the era of economic globalization, the important position of finance in the economy has been further highlighted. The issue of optimal investment and consumption of financial derivatives in the financial market has always been a hot issue in the current financial investment market, as well as a problem that investors have been paying attention to. The main problem of finance is resource allocation. Time and random uncertainty are the key factors that affect investors' financial and economic behavior in the financial market. Scholars across multiple disciplines and policy makers from multiple institutions have been looking for economic, political, and institutional foundations to improve the strength of the financial market [1]. Therefore, in random financial markets, there is research on how market participants (individuals or institutions) treat

limited wealth or resources which will be allocated to various (financial) assets, such as stocks, cash accounts, bonds, and capital market consumption, so as to achieve timely optimal allocation of resources to achieve reasonable cash flow and risk/return characteristics to meet the financial needs of market participants. Its main content is to maximize the use of investors' interests as the decision criterion to obtain the optimal investment portfolio and optimal consumption plan for the wealth distribution of participants. Econometrics, financial econometrics, mathematical finance, and empirical financial methods have made significant contributions to the study of financial econometrics. Derivatives include market-based stochastic volatility model estimation, the fine structure of equity index option dynamics, and multifactor Wishart option pricing based on stochastic volatility, etc. [2]. Option, as one of the derivative securities in the financial market, is the same as any other derivative product. Its value

depends on the underlying securities. For example, the value of stock options depends on the stock price, which has always been at the forefront of research in financial mathematics. The stochastic differential equation has become a very practical tool in the financial field, especially in the problem of optimal investment in options.

The purpose of this paper is to consider the optimal investment strategy of options in the financial investment market under the premise of an incomplete market closer to the actual situation, so that investors can maximize their investment benefits. Therefore, on the basis of previous studies, this paper uses stochastic differential equations to calculate the dynamic index of financial market options on the basis of fuzzy theory, gives the expression of the optimal investment strategy in an incomplete market, and studies the optimal investment solution in the case of options.

## 2. Proposed Method

**2.1. Related Work.** Wang et al. studied several linear quadratic optimal control problems of the forward and backward stochastic differential equations of the mean field [3]; this control problem is different from the existing literature on the optimal control of the mean field stochastic system, and it has more applications; a closed-form optimal solution can be obtained under detailed conditions. Yang and Zhao applied it to the special type of forward and backward stochastic differential equations (FB-SDE) that appeared in finance and stochastic control and provided convergence analysis for the recently proposed multistep scheme [4], but this method is more difficult to solve. Tardelli uses reverse stochastic differential equations (BSDE) and filtering techniques to study stochastic control problems [5], but this method has certain limitations; it requires a sequence of functions that converge to a value function. Wang et al. use the method and Monte Carlo method to study the optimal investment problem of DC pension under inflation and loss avoidance [6], but this method has a large amount of calculation and slower improvement in accuracy. Li et al. studied the optimal investment strategy based on derivatives based on the asset-liability management (ALM) problem of the mean-variance criterion in the presence of random volatility [7]. Xu et al. use the dynamic programming method to discuss the best investment and reinsurance policies of an insurance company with a classic earnings process [8], but this method has prominent spatial contradictions and difficult data storage. Kim and Lee used the Black–Scholes–Merton model to study the best investment using real option valuation and fuzzy logic [9], but the derivation process of this model is unacceptable. Ibrahim studied the three-stage investment ratio and the impact of key factors on the real option investment threshold [10]. Wang and Huang use a multiple jump-diffusion (MJD) model to provide an optimal asset allocation strategy that can improve risk management performance in the face of financial crises [11]; this model requires careful analysis of the optimization process.

This article applies theoretical research and experimental research, adopts a combination of quantitative and qualitative theoretical and numerical calculation methods, and uses

a variety of calculation methods, such as theoretical analysis, mathematical modeling, analytical subtraction, and numerical calculation. Based on the dynamic optimal investment strategy, we conduct more systematic and in-depth research and discussion. Moreover, this article contains basic theories such as financial economics, financial mathematics, financial stochastic analysis, modern securities investment, and stochastic differential equations, which can analyze the problems of this article in a more comprehensive and detailed manner. In the research of this article, the only information that investors can obtain is the information generated by asset prices, and the unobservable process will be modeled by stochastic differential equation models [12].

**2.2. Stochastic Differential Equation.** Stochastic differential equation (SDE) is now an important branch of mathematics. Since its development, it has been gradually improved after many scholars' research and discussion. Stochastic differential equation theory inherits many ideas and methods in deterministic theory. Its research content includes existence and uniqueness, constraints, solution evaluation, solution stability and branching, and martingale and Markov properties in stochastic analysis. Stochastic differential equations can be used to obtain the autocorrelation function of the output noise voltage and other solution statistics such as the mean and variance [13]. In recent years, due to the profound mathematical and physical background of stochastic stability and the stochastic branch, many researchers have paid great attention to it, and it has become an important part of the theoretical research of stochastic dynamical systems. At present, although stochastic stability has obtained more theoretical results, it is still very small compared with the stability theory of deterministic equations, and there are still many unsolved problems. The theoretical system needs to be further improved and perfected. The research on stochastic separation theory is based on stochastic stability and is still in its infancy, especially for high-dimensional and time-delay differential systems.

Stochastic differential equations are usually ordinary differential equations, whose coefficients are random variables and differential equations driven by random processes. Stochastic differential equations can be used to describe various natural phenomena, economic phenomena, financial phenomena, etc. These phenomena have a similarity, that is, reducing the input to continuous, very fast random operations, that is, using a Brown as the system driving force. The formula of stochastic differential equation is generally

$$d\xi_t = A(t, \xi_t)dt + B(t, \xi_t)dB_t, \quad (1)$$

where  $A(t, \xi_t)B(t, \xi_t)$  is a given function, with more specific and normal characteristics; this stochastic differential equation can be integrated to get its integral formula:

$$\xi_t = \xi_0 + \int_0^t A(t, \xi_t)dt + \int_0^t B(t, \xi_t)dB_t. \quad (2)$$

So it is also called a stochastic integral equation, where the first integral is the integral using the time parameter as

a regular function, and the second integral is the Ito integral, and  $\xi_0$  is a random initial value independent of Brownian motion  $\{B_t : t \geq 0\}$ .

Stochastic differential equations can be divided into many types, of which backward stochastic differential equations are widely used in finance. In option pricing, the backward stochastic differential equation (BSDE) has a wide range of applications, and the Black-Scholes model is one of the classic pricing models [14]. First, consider simple deterministic systems and ordinary differential equations. In order to solve specific results, first give a definite solution condition for the solution within a period of time. When  $0 \leq t \leq T$ , there is a positive condition that gives the definite solution condition at the initial time  $t = 0$ . To ordinary differential equations,

$$\begin{cases} dx(t) = f(x(t))dt, \\ x(0) = x_0. \end{cases} \quad (3)$$

Then, we consider the backward ordinary differential equation and still give the definite solution condition. The backward ordinary differential equation based on the fixed solution condition at the end time  $t = T$  is as follows:

$$\begin{cases} dW(t) = g(W(t))dt, \\ W(T) = W_T. \end{cases} \quad (4)$$

Then, we consider the complex nondeterministic system, use  $B(t)$  to represent Brownian motion, and assume that there are  $d$  mutually independent interference sources, that is, we use  $d$  to represent the dimension and still give the initial conditions when  $t = 0$  and get the positive Stochastic differential equation:

$$\begin{cases} dx(t) = f(x(t))dt + \sigma(x(t))dB(t), \\ x(0) = x_0. \end{cases} \quad (5)$$

We use  $\eta$  to denote the fixed value at  $T$ , and  $W(t)$  and  $H(t)$  are two adaptation processes that represent the backward stochastic differential equations that can be obtained in the initial state according to the law of motion of the differential equation:

$$\begin{cases} -dW(t) = g(W(t), H(t))dt - H(t)dB(T), \\ W(T) = \eta. \end{cases} \quad (6)$$

**2.3. Fuzzy Logic.** Fuzzy logic is developed on the mathematical basis of fuzzy set theory founded by Professor L.A. Zadeh of the Electrical Engineering Department of the University of California, Berkeley. Fuzzy theory refers to the application of the basic concepts of imprecise sets or adhesion functions, which are mainly divided into five branches: fuzzy mathematics, fuzzy logic and artificial intelligence, fuzzy systems, uncertain information and theory, and fuzzy decision-making.

In the real world, many uncertain technical rules are expressed through physical rules (such as uncertain words and languages), and the recent fuzzy reasoning mechanism can solve this problem well. Uncertain finance is a new field in which the risk process is described by the uncertain process. Good uncertainty predicts that future economic activities, such as consumption, output, and investment, will increase and are positively correlated with the valuation ratio, while bad uncertainty predicts that economic growth will decline and depress asset prices [15]. Uncertainty and competition will have an important impact on the value of investment projects [16]. The existence of inevitable uncertainty brings some difficulties to the numerical processing of optimization tasks [17]. In finance, the current technical analysis using fuzzy theory is mainly through assigning different input variables and parameters to the fuzzy system. For example, T. Chavarnakul and D. Enke constructed a neuro-fuzzy inference system with trading volume as input variables and then used genetic algorithms to adjust the membership functions of the input variables in the inference system, thereby building a multi-intelligent algorithm for securities trading system; T. Anbalagan and S.U. Maheswari used technical indicators Simple Moving Average (SMA), Moving Average Convergence/Divergence (MACD), and Relative Strength Index (RSI) as input variables of the fuzzy decision-making system for decision-making; for A.D. Ijegwa et al., MACD, RSI, Stochastic Oscillator (SO), and Balance Volume (OBV) indicators are used as the input variables of the fuzzy decision system to determine the trading time; and G. Rubell Marion Lincy and C. Jessy John use less used earnings per share (EPS) information, as well as closing prices, profit fluctuations, and investor risk behavior, as input variables. Finally, three different fuzzy systems of buying, selling, and holding are established, and decisions are made based on the fuzzy decision values obtained by the three fuzzy systems. However, if a large number of secondary indicators are selected as input variables, duplication of information and calculation troubles may occur.

**2.4. Option.** Options are financial products derived from underlying securities, which refer to the right to buy or sell financial products or futures contracts at an agreed price within a specified time limit. Options are important financial derivatives, and their risk management characteristics have been widely accepted by investors [18]. The first batch of option transactions in history was only used for physical commodities and precious metals, and they were all spot options. Subsequently, the emergence of the world's first option house caused a huge change in the option trading market, which brought trading options into a new stage of fully integrated and standardized management. Standardized functions make the transaction process simple, and the exchange also guarantees the final performance. The emergence of exchanges introduced optional transactions from over-the-counter trading to on-exchange trading, which led to a sharp increase in the trading volume at the bottom of the transaction; the standardization of option contracts increased market liquidity, improved transaction efficiency, and reduced transaction costs. Options are the right of the

buyer and the obligation of the seller. Options have the following characteristics: (1) the buyer must pay the seller corresponding fees to obtain the right, (2) the buyer obtains the right on a specific date or within a specific time period in the future, (3) the purchase or sale of the item in the future is specific, and (4) the price of the basic material purchased or sold by the option buyer in the future is predetermined. The buyer can choose whether to implement it and can flexibly choose according to the price of the expired asset.

The most important feature of option trading is to allow the holders of expected rights to have a certain degree of flexibility in decision-making so that they can flexibly use the possibility of various changes in the financial market and control market risks such as interest rates and exchange rates to maximize the percentage, without losing potential profit opportunities. A call option is the right to purchase an asset at the current fixed price at a specific time in the future, not a burden. A put option is the right to sell an asset at a predetermined price at a specific time in the future, but if the trading option shows an asymmetric ability, it is impossible to predict the analyst's suggestion to modify the underlying stock [19]. From a different perspective, options are divided into many types, such as European options and American options. European options are delivered at a specified future time  $T$ , and American options are delivered at any time before time  $T$ . The European call parity condition is used to estimate the preexercise premium for US currency options traded on the Philadelphia Stock Exchange [20]. For European stock purchase options, the holder has the power, but is not obliged, to sell the stock at a specific price in the future  $T$  time, for example, sell the stock for 10 yuan. If the stock price rises by more than 10 yuan at time  $T$ , the holder of the call option will buy the stock for 10 yuan. If the price is less than 10 yuan, then it will obviously give up the implementation of the agreement (no obligation to pay for damages), then the price of the right is zero.

Option pricing is an important field of financial research [21]. At present, there are mainly the following methods for option pricing:

(1) *Black-Scholes Option Pricing Method*. At time 0, the values  $c$  and  $p$  of European call and European put options that do not pay dividends are

$$c = SN(d_1) - Ke^{-rT}N(d_2), \quad (7)$$

$$p = Ke^{-rT}N(-d_2) - SN(-d_1), \quad (8)$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T}. \quad (9)$$

Among them,  $N(x)$  is the distribution function of the standard normal distribution;  $S$  and  $K$  are the current stock price and execution price, respectively;  $r$  is the risk-free interest rate; and  $\sigma$  is the asset volatility rate. When the number of options to be calculated is small, this method is more convenient, but it can only calculate the analytical solution of European options and cannot be applied to American options.

(2) *Binomial Tree Option Pricing Method*. In this model, the effective period of the option is divided into several equal cell intervals, and the duration of each small period is  $\Delta t = T/n$ . Taking European call options as an example, the option value is

$$f = \frac{\sum_{i=0}^n (n!/(n-i)!i!)p^i(1-p)^{n-i} \max(Su^i d^{n-i} - X, 0)}{(1+r)^n}. \quad (10)$$

If other factors (such as transaction taxes) are not taken into account, then if a lucrative option is not redeemed before the expiry date, it will definitely be redeemed on the expiry date. For an option, we can set its intrinsic value. Assuming that  $A$  represents the strike price and  $X_T$  represents the price of the underlying asset at the expiry date, the intrinsic value of the long call option is defined as

$$\max(X_T - A, 0). \quad (11)$$

This means that if  $X_T > A$ , the option will be executed; otherwise, the option will not be executed. Similarly, the intrinsic value of a put option is defined as

$$\max(A - X_T, 0). \quad (12)$$

This means that if  $X_T < A$ , the option will be executed; otherwise, the option will not be executed. But no matter what, the intrinsic value of options is nonnegative.

For an option, whether it is profitable, in addition to considering the intrinsic value of the option, the purchase period of the option should also be considered. For call options, suppose the price of each option is  $Y$ , and by the intrinsic value of the call option, the buyer's income per share of the call option is

$$\max(X_T - A, 0) - Y = \begin{cases} -Y, & X_T \leq A, \\ X_T - A - Y, & X_T > A. \end{cases} \quad (13)$$

The seller's return per call option is

$$Y - \max(X_T - A, 0) = \begin{cases} Y, & X_T \leq A, \\ -X_T + A + Y, & X_T > A. \end{cases} \quad (14)$$

Similarly, the corresponding formula can be obtained for put options.

*2.5. Optimal Investment Strategy*. The research on the optimal investment strategy was first mentioned in Markowitz's "Securities Portfolio Theory." He used the mean variance model to describe the risk of insurance companies. Samuelson extended his model and used the principle of stochastic dynamic programming to obtain an optimal investment strategy. Later, after the research and promotion of many researchers, the optimal investment strategy model was

gradually improved and widely used in the financial investment market. In addition, according to whether the investment portfolio changes dynamically over time, the investment portfolio can be divided into a one-period portfolio model and a multiperiod portfolio model. The one-year portfolio model means that investors keep their portfolio unchanged from the beginning to the end of the period. If it is believed that the one-period portfolio model cannot cope with the rapidly changing market, then a dynamic multiperiod model is needed to simulate the portfolio problem. Martingale theory is also a powerful tool for studying optimal investment problems. Previous scholars used the martingale method and duality theory to conduct detailed research on the investment problem of maximizing the utility function. Using the martingale method, they studied the option pricing problem in underdeveloped financial markets and discussed the optimal investment with options. The problem of high-dimensional optimal investment and consumption is difficult to deal with, especially because it is difficult to characterize the value function [22].

To study the optimal investment problem, the decision-making between investors also influences each other. Therefore, when considering this problem, we should start from two aspects: the optimal investment choice of a single investor and the optimal investment choice of multiple investors. The research of a single investor's dynamic portfolio model has produced a lot of research results. The collection of investment opportunities determines the investment market environment of the financial market facing investors and therefore further determines the stochastic economic model faced by investors and determines the type and quantity of financial assets that investors can choose. Existing research on the potential optimal consumption and investment portfolio of two or more economic entities still has the following problems to be solved: First, when defining investment criteria, most models only consider the relative distance between the investors' respective wealth levels. Mathematically speaking, they usually directly target the relative distance between two levels of wealth. In addition, participants mainly involve investors and virtual opponents of the "market."

### 3. Experiment Setup

**3.1. Test Subject.** This article uses stochastic differential equations and Markowitz's model to calculate and analyze the optimal investment strategies of listed insurance companies in terms of fixed deposits, bonds, funds, stocks, and other factors of four listed insurance companies from 2015 to 2019. The research mainly focuses on risk-free investment and risk investment and establishes six variables: bank deposit yield, treasury bond yield, fund yield, stock yield, and real estate investment yield to solve the company's optimal investment strategy.

#### 3.2. Experimental Data

**3.2.1. Bond Investment.** The investment in bonds of listed insurance companies mainly includes government bonds,

TABLE 1: Bond investment proportion table.

Year	Company A	Company B	Company C	Company D	Average value
2015	0.69	0.59	0.59	0.53	0.6
2016	0.66	0.57	0.46	0.52	0.55
2017	0.55	0.57	0.43	0.52	0.52
2018	0.47	0.50	0.51	0.50	0.495
2019	0.41	0.45	0.46	0.47	0.45

TABLE 2: Proportion of stock investment.

Year	Company A	Company B	Company C	Company D	Average value
2015	0.045	0.047	0.014	0.01	0.029
2016	0.084	0.076	0.06	0.073	0.073
2017	0.063	0.055	0.043	0.051	0.053
2018	0.074	0.08	0.037	0.06	0.063
2019	0.08	0.085	0.04	0.047	0.063

TABLE 3: Fund investment proportion table.

Year	Company A	Company B	Company C	Company D	Average value
2015	0.0561	0.0341	0.0641	0.0297	0.046
2016	0.0423	0.0297	0.0415	0.0531	0.042
2017	0.0416	0.0258	0.0721	0.0251	0.041
2018	0.0347	0.0372	0.0591	0.0463	0.044
2019	0.0619	0.0395	0.0362	0.0347	0.043

financial bonds, and corporate bonds. The yield of bond investment is higher than that of bank deposits, and its risk is lower than that of stock investment. Regardless of whether it is government bonds, financial bonds, or corporate bonds, there are corresponding credit entities. As a guarantee, under certain risk control, insurance companies can obtain stable investment income satisfactorily, maintain insurance funds, and increase value. It is to an insurance company the best investment option. The bond investment proportions of the four listed insurance companies from 2015 to 2019 are shown in Table 1.

According to the data in Table 1, the bond investment proportion of listed insurance companies has a downward trend, but it is basically stable at about 50%. Bond investment has become the preferred investment channel for insurance companies to maintain and increase their value.

**3.2.2. Stock Investment.** Since the China Insurance Regulatory Commission clearly stated that insurance funds can be directly invested in the stock market, investors in many insurance companies have rushed because of the high rate of return that stocks can obtain. Table 2 shows the proportion of the four companies' investment in stocks.

It can be seen from the data in the table that the share of stock investment of the four companies fluctuates and fluctuates greatly each year. But in the end, the company's average

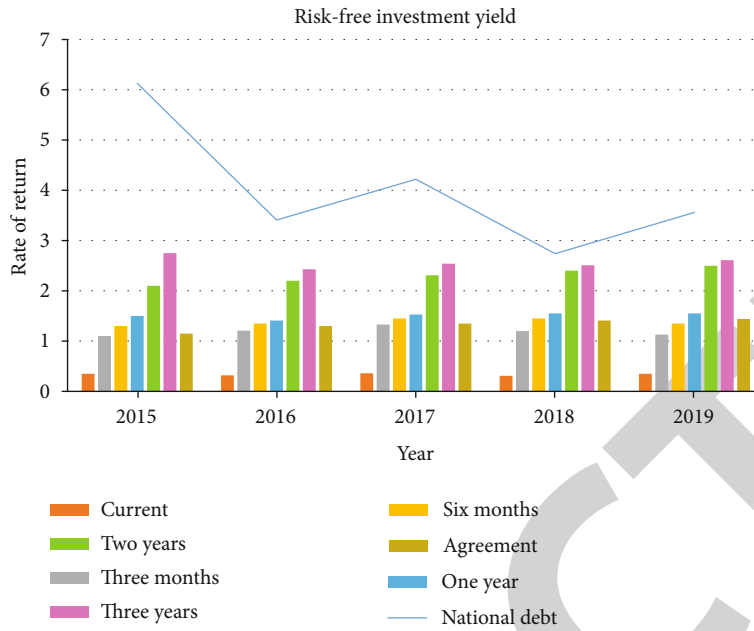


FIGURE 1: Risk-free investment yield chart.

investment ratio was basically stable at around 0.063. Although the share of listed insurance companies is small, they have a greater impact on investment returns and investment robustness. In addition, the incomplete development of the author’s country’s capital market has made the guesses of insurance companies more serious. In the past two years, the capital market has fluctuated frequently, and frequent reports of insurance funds have undoubtedly become the hottest topic in the capital market recently.

**3.2.3. Fund Investment.** As an important part of the financial market, funds have become an important participant in asset allocation. They are matched with a large amount of insurance funds, with a long term and continuous stability. They have optimized the asset allocation of listed insurance companies and improved the long-term stability of investment returns, playing an active role in diversifying investment risks. Table 3 shows the proportion of the investment of the four listed insurance companies in the fund.

According to the fund investment proportion data in Table 3, it can be seen that the average investment proportion of insurance companies in funds is basically around 0.04. The proportion of investment funds of listed insurance companies is relatively low and needs to be improved. Mutual funds can bring higher return on investment for listed insurance companies. The rate of return on investment and the rate of return on the market are positively correlated: when stocks are expected to grow, listed insurance companies can choose funds with higher risk factors to obtain higher rates of return. When the stock market is expected to fall, you can choose a mutual fund with a lower risk factor to reduce risk. The listed insurance company chooses the appropriate amount of capital investment according to the risk tolerance and market risk level.

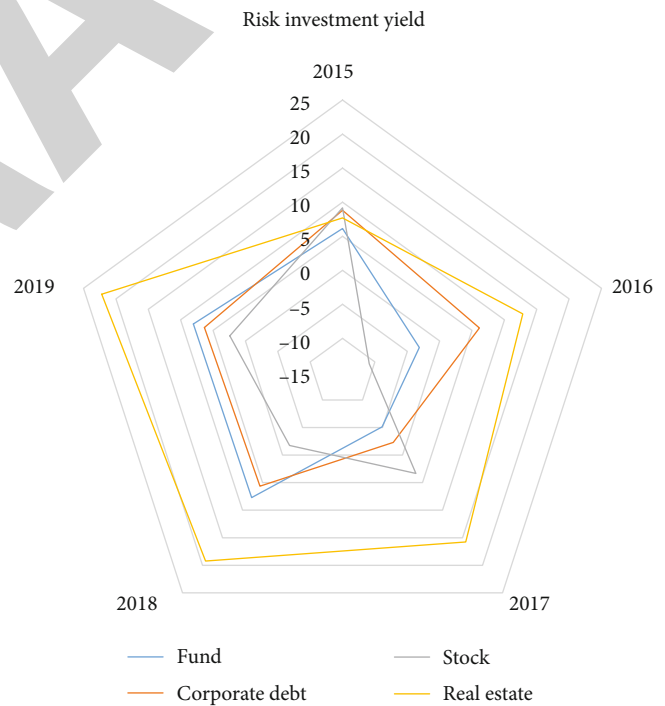


FIGURE 2: Risk investment yield chart.

**4. Discussion of Result**

**4.1. Risk-Free Investment Income.** This article regards bank deposits and government bonds as investment risk returns of listed insurance companies. The value of treasury bonds is determined by the stock market based on the weighted value of treasury bonds issued by listed companies, which can reflect the overall fluctuations in the bond market; bank

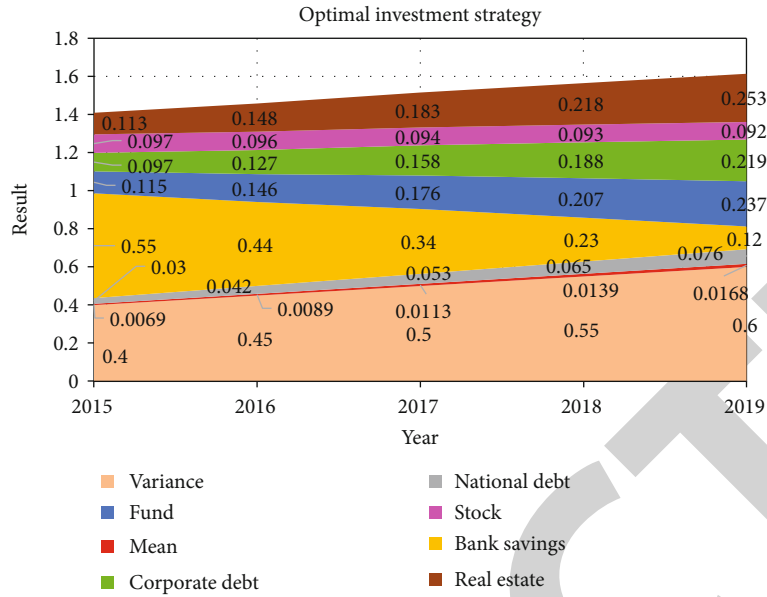


FIGURE 3: Diagram of optimal investment strategy.

deposits are based on the current deposit interest rate on the official website of the Bank of China from 2015 to 2019, 3 months, 6. The sum of monthly, 1-year, 2-year, and 3-year fixed interest rates and agreed deposit rates finds the average return on deposit rates for each year. The upper and lower limits of interest rates are popular interest rate derivatives in financial markets [23]. Finally, the data of treasury bonds and bank deposits are sorted and calculated to get a risk-free investment income chart as shown in Figure 1.

According to the graphical trend in Figure 1, it can be seen that the yield of government bonds is much higher than the yield of bank deposits, and the demand yield of bank deposits is the lowest; the three-year yield from 2015 to 2019 is higher than the two-year yield and the one-year yield. The annual yield of bank deposits is basically stable. The order of high and low is as follows: three-year yield > two-year yield > one-year yield > six-month yield > agreement yield > three-month yield > demand deposits.

**4.2. Risk Investment Yield.** The venture capital in this article mainly includes corporate bonds, capital, stocks, and real estate. Select corporate bonds are based on the weighted Shanghai Stock Exchange bond issuance ratio. The selected fund is the Shanghai Stock Exchange Mutual Fund Index, which reflects the price change index of the capital market; the stock is the Shanghai Composite Index, and its stock samples are all listed stocks, including A shares and B shares. According to the data of the opening and closing prices of each index in 2015-2019, the annual average return rate of each risk asset is calculated as shown in Figure 2.

It can be seen from the trend of risk investment yield in Figure 2 that the investment yield of real estate is rising year by year, and it is predicted that there will be an upward trend in the next few years; the investment yield of stocks fluctuates the most, with the highest risk, reaching below -10% at the lowest. The yield of funds and corporate bonds has gradually

stabilized in 2018, basically stabilizing between 5% and 10%. The investment yield of the fund is higher than the corporate bond yield and stock yield after 2018.

**4.3. Optimal Investment under Different Risks and Returns.** Due to the restrictions of the China Insurance Regulatory Commission, insurance companies have established a major asset supervision ratio for their insurance funds. The bank deposit limit ratio is 5% or more, the total investment ratio of funds and stocks is limited to 30% or less, the real estate investment ratio is limited to 30% or less, and the investment ratio of government bonds and corporate bonds is unlimited. Using the above data, we calculate the mean and variance of each investment through the Solver function and get the optimal investment strategy of listed insurance companies. The specific data trend is shown in Figure 3.

From the empirical results of the effective investment portfolio of listed insurance companies in Figure 3, it can be seen that as the rate of return continues to increase, the risks faced have also increased, which is more in line with the principle of negative correlation between risk and rate of return. As the yield increases, the percentage of bank deposits in the portfolio gradually decreases, while the percentage of capital and corporate bonds gradually increases as the yield increases. Since the total investment limit of funds and stocks cannot exceed 30%, the best combination can be achieved when the rate of return is 5.5% and the fluctuation is 0.0139. At present, the bank deposit investment ratio is 23%, government bond investment ratio is 6.5%, funds account for 20.7%, corporate bonds account for 18.8%, stocks account for 9.3%, and real estate investment account for 21.8%. It can be concluded from empirical analysis that bank deposits are a necessary and important investment field for listed insurance companies in the game of risk control and returns. In addition, they can increase their investment in corporate bonds and funds.



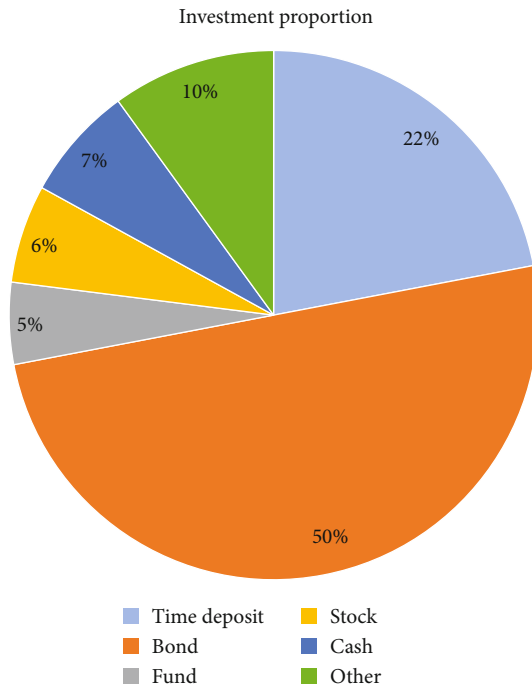


FIGURE 4: Historical investment ratio chart.

**4.4. Analysis of Optimal Investment Strategy.** In the actual situation, the investment channels of listed insurance companies are not limited to these six categories, and the risk asset investment rate of return in this article is determined based on the average value of each risk index of the Shanghai Stock Exchange. The investment yields of risk assets selected in the application of insurance capital investment are different, and the deviation between theory and reality will bring errors to the accuracy of the empirical results. The historical investment ratio of listed insurance companies is shown in Figure 4.

According to the historical investment proportion data of listed insurance companies in Figure 4, it can be seen that the total investment proportion of funds and stocks is 11%, but the optimal investment proportion of funds obtained by the analysis of the model results is 20.7%; the optimal investment of stocks accounted for 9.3%. This shows that in fact, the investment efficiency of listed insurance companies can be improved by increasing the allocation of capital and stock investment, but since the control of the regulatory ratio reduces the actual investment income, under certain risk control, listed insurance companies can appropriately increase the allocation equity in order to obtain a higher return on investment.

## 5. Conclusions

In recent years, due to the development and improvement of the financial market, people are full of great interest and enthusiasm in financial investment, financial transactions, and other businesses, especially the transactions of financial derivatives. People have invested a lot of assets and energy in the transactions of financial derivatives, especially in

financial transactions with options. However, investment transactions in this area have great benefits and also have great risks. Therefore, how to make investors obtain greater benefits with less risk is a problem that everyone has been discussing all the time.

This paper calculates the optimal investment strategy based on the historical investment yields of the four listed insurance companies in various aspects and the function of the data. The investment efficiency of listed insurance companies can be improved by increasing capital allocation and stock investment. The China Insurance Regulatory Commission's control of the investment regulatory index actually reduces the assessment of the actual investment efficiency of insurance companies. As a leader in the insurance industry, listed insurance companies need to hedge risks. The stability of investment in the insurance industry requires listed insurance companies to invest to cover costs and achieve surpluses. This requires listed insurance companies to have a strong market crisis and investment opportunities. Under specific risk management and control, they can increase owners' equity. The distribution of assets is expected to have a higher return on investment. The two Brownian motions in the investor's surplus process and the risky asset price process are related, and they describe the correlation or dependence between the insurance market and the financial market [24].

In today's world of continuous financial fluctuations [25], financial markets are developing rapidly, financial innovations are constantly emerging, financial services have entered the era of big data, digital financial decision-making has become a trend, and quantitative investment and data-based algorithmic transactions have been widely implemented abroad. The research is biased towards theoretical research, and the conclusions drawn are consistent with economic intuition. In the next step, we will conduct more in-depth theoretical and empirical research on the best high-frequency trading strategies and asset pricing based on high-frequency data in the financial market.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

There are no potential competing interests in our paper. And all authors have seen the manuscript and approved to submit to your journal.

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