Research Article

A Source Number Estimation Algorithm Based on Data Local Density and Fuzzy C-Means Clustering

Na Wu ^{1,2}, Ke Wang ³, Liangtian Wan ⁴, and Ning Liu ²

¹Jiangsu Key Laboratory of Broadband Wireless Communication and Internet of Things, Nanjing University of Posts and Telecommunications, Nanjing 210003, China

²School of Internet of Things, Nanjing University of Posts and Telecommunications, Nanjing 210003, China

³Electronic and Information School, Yangtze University, Jingzhou 434023, China

⁴Key Laboratory for Ubiquitous Network and Service Software of Liaoning Province and School of Software, Dalian University of Technology, Dalian 116620, China

Correspondence should be addressed to Ning Liu; liuning@njupt.edu.cn

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An advanced source number estimation (SNE) algorithm based on both fuzzy C-means clustering (FCM) and data local density (DLD) is proposed in this paper. The DLD of an eigenvalue refers to the number of eigenvalues within a specific neighborhood of this eigenvalue belonging to the data covariance matrix. This local density essentially as the one-dimensional sample feature of the FCM is extracted into the SNE algorithm based on FCM and can enable to improve the probability of correct detection (PCD) of the SNE algorithm based on the FCM especially for low signal-to-noise ratio (SNR) environment. Comparison experiment results demonstrate that compared to the SNE algorithm based on the FCM and other similar algorithms, our proposed algorithm can achieve highest PCD of the incident source number in both cases of spatial white noise and spatial correlation noise.

1. Introduction

Superresolution spatial spectrum estimation algorithm for the direction of arrival (DOA) estimation has been applied in various fields including passive detection and location, communication, sonar detection, multiple input multiple output (MIMO) radar, and communication [1–11]. Almost all spatial spectrum estimation algorithms are constructed on the basis of the known incident source numbers to further enabling the accurate DOA estimation. Unknown or erroneous source numbers will cause the very bad accuracy or even ineffective of the spatial spectrum estimation algorithms, which is contributed to that the noise subspace and signal subspace are staggered and overlapped in this case [12]. The source number estimation (SNE) is thus the first and key process of the spatial spectrum estimation algorithm, and the accurate SNE is the primary prerequisite for the following DOA estimation. Therefore, how to rapidly and accu-

rately retrieve the unknown incident source number from the array observation data containing the complex space noise and receiver noise thus becomes an issue of practical significance.

In the last few decades, SNE algorithms have been extensively studied by worldwide scholars. To be more specific, SNE algorithms can be divided into the following two stages, namely, the classical SNE algorithms. The classical algorithms proposed during this period mainly include the SNE algorithms based on hypothesis testing [13] and information theory criteria [14]. Due to the limitations of these classical algorithms in terms of estimation performance and stability, a series of improved SNE algorithms based on mAIC criteria [15] and mMDL criteria [16] for source number estimation under spatial correlation noise environment and a robust improved MDL criterion [17] for algorithm stability have been proposed. These improved algorithms achieve the better estimation performance of the classical SNE algorithms.

However, almost none of them can accurately estimate the incident source number when encountering the complex spatial environment, such as the poor signal-to-ratio (SNR) cases and intense unknown correlated noise scenarios.

Under the powerful impetus of array signal processing technology, more and more advanced theories and strategies have been used to the SNE and then strongly pushed the SNE algorithm into the next stage. Abundant SNE algorithms have been constantly emerging, such as canonical correlation technology [18, 19] and MDL criterion based on the random matrix theory [20], although these SNE algorithms have their respective superiorities in terms of the accuracy, robustness, operation cost, and antijamming capability. However, they are limited in practical application more or less. SNE algorithms based on the regularization correlation technique have a severe requirement on the array design for obtaining enough accuracy. SNE algorithms based on the MDL criterion by using random theory have to require the receiver array featuring many enough elements to enable better estimation performance. SNE algorithms based on the information theory criterion by introducing diagonal loading have a higher estimation accuracy under the color noise background, but a poor accuracy in the cases of the white noise.

In recent years, the artificial intelligence technology has been explosively researched and has achieved exciting progresses. Also, artificial intelligence algorithms have been further applied in passive direction finding systems for accurately estimating the number of incident sources [21–23]. One of the unsupervised algorithms named fuzzy logic theory can effectively deal with the unknown and uncertain factors in the actual system and shown good performance in SNE, such as the representative SNE algorithm based on fuzzy C-means clustering (FCM) [24]. This algorithm has superior estimation capability in retrieving the number of the incident sources under both the spatially white noise scenarios and the spatially correlated color noise ones. However, the sample features of existing SNE based on FCM algorithms are obtained according to the value of eigenvalues rather than the overall distribution characteristics of eigenvalues, which thus limits its further application in the real passive FCM algorithm systems, especially for low SNR, arbitrary, and complex electromagnetic environment.

To tackle this issue and further improve the accuracy and applicability of the SNE algorithms in the actual system, we extract the distribution density characteristics of the eigenvalues from the received data covariance matrix and take it as one-dimensional (1D) characteristic information of the fuzzy C-means clustering and further construct an advanced SNE algorithm based on both the FCM and the data local density (DLD). The DLD potentially increases the relationship between by adding 1D information and may have a good expectation that our proposed SNE algorithm can obtain a high probability of correct detection (PCD) even for low SNR and few snapshots cases.

The following contents of this paper are organized as follows. The array signal model for a uniform linear array used in source number estimation algorithm is rigorously formulated in Section 2. Section 3 introduces the basic theory of the FCM algorithm in brief. Then, the SNE algorithm based

on the FCMC and the DLD is proposed in the Section 4. Section 5 demonstrates the accuracy and applicability of our proposed SNE algorithm by numerical simulation. Finally, Section 6 is the conclusion of this paper.

2. Array Signal Model

Assume that there are K stationary random far-field narrow-band independent signals in space. The receiving array is a uniform linear array composed of $N(N \geq K)$ scalar arrays; assume that the wavelength of the incident source is λ , and the array spacing of the receiving array is $d = \lambda/2$. Let the incident angle of the first incident signal be θ_k . The first element in the array is taken as the reference, when the k th signal impinging on the array, the delay between the n th ($1 \leq n \leq N$) element and the reference element is

$$\tau = \frac{(n-1)d \sin(\theta_k)}{c} = \frac{(n-1)\lambda \sin(\theta_k)}{2c}. \quad (1)$$

The corresponding phase difference is

$$\phi_k = 2\pi f \tau = 2\pi \left(\frac{c}{\lambda}\right) \left(\frac{(n-1)\lambda \sin(\theta_k)}{2c}\right) = (n-1)\pi \sin(\theta_k). \quad (2)$$

Then, the array guide vector is

$$\mathbf{a}_k = [1, \pi \sin(\theta_k), \dots, (N-1)\pi \sin(\theta_k)]^T. \quad (3)$$

Therefore, the array receiving data vector is

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N}, \quad (4)$$

where $\mathbf{X} = [x_1, \dots, x_n, \dots, x_N]^T \in \mathbb{C}^{N \times 1}$ is the array receiving data vector, $\mathbf{A} = [\mathbf{a}_1, \dots, \mathbf{a}_k, \dots, \mathbf{a}_K]^T \in \mathbb{C}^{N \times K}$ is array popularity matrix, $\mathbf{S} = [s_1, \dots, s_k, \dots, s_K]^T \in \mathbb{C}^{K \times 1}$ is the array receiving signal vector, and $\mathbf{N}(t) = [n_1, \dots, n_n, \dots, n_N]^T \in \mathbb{C}^{N \times 1}$ is the array receiving noise vector.

Assuming that the spatial noise received by the array is zero-mean stationary random Gaussian white noise of power is σ^2 and the noise is independent of the incident signal, the covariance matrix of the data vector which received by array can be obtained as:

$$\mathbf{R}_X = \mathbb{E}[\mathbf{X}\mathbf{X}^H] = \mathbf{A}\mathbf{R}_S\mathbf{A}^H + \mathbf{R}_N = \mathbf{A}\mathbf{R}_S\mathbf{A}^H + \sigma^2\mathbf{I}, \quad (5)$$

where $\mathbf{R}_S$ is the incident signal covariance matrix and $\mathbf{R}_N$ is the covariance matrix of noise received by the array.

By eigenvalue decomposition of Equation (5), the following formula can be obtained as:

$$\mathbf{R}_X = \sum_{i=1}^M \lambda_i \mathbf{u}_i \mathbf{u}_i^H = \sum_{i=1}^K \lambda_i \mathbf{u}_i \mathbf{u}_i^H + \sum_{i=K+1}^N \lambda_i \mathbf{u}_i \mathbf{u}_i^H, \quad (6)$$

where λ_i is the i th eigenvalue of the array receiving data covariance matrix, $\mathbf{u}_i$ is the eigenvector corresponding to λ_i .

FIGURE 1: Distribution of eigenvalues versus SNR.

In an ideal situation, arranged the eigenvalues in descending order, the result is:

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_K > \lambda_{K+1} = \dots = \lambda_N = \sigma^2. \quad (7)$$

However, array receiving data for signal processing are actually obtained through limited sampling of system in actual direction finding system. Therefore, the array receiving data covariance matrix is usually obtained by the maximum likelihood algorithm; when the number of snapshots is M , the ML estimate is:

$$\hat{\mathbf{R}}_X = \frac{1}{M} \sum_{l=1}^M X(l)X(l)^H. \quad (8)$$

By eigenvalue decomposition of Equation (8) and arranging the eigenvalues in descending order, the following formula can be obtained as:

$$\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \dots \geq \hat{\lambda}_K > \hat{\lambda}_{K+1} \geq \dots \geq \hat{\lambda}_N \geq \sigma^2. \quad (9)$$

The reason why the noise eigenvalues in formula (9) no longer meet the equal relationship is that there will be errors which cannot be avoided in the actual system and the estimation process. Obviously, it is impossible to estimate incident source number from the identical eigenvalues number. So it is necessary to carry out the study of the estimation algorithm of source number, i.e., the SNE algorithm.

3. Fuzzy C-Means Clustering

Clustering is a kind of unsupervised learning. The key of clustering is the selection of sample characteristics to divide the samples. FCM [24] is a soft clustering method, which takes into account the uncertainty of data to be clustered. Compared with the hard clustering algorithm represented by K-mean clustering, fuzzy C-mean clustering can better deal with the uncertainty in the system.

Let $X = \{x_1, x_2, \dots, x_M\} \in \mathbb{C}^S$ express a given set of samples. For the incident SNE problem, it is the value of the relevant feature of the eigenvalue of array received data covariance matrix, such as the value and the local density of eigenvalues. Where S represents the sample space dimension, M represents the sample number, that is the eigenvalue number or the elements number in receiving antenna array. For the problem of source number estimation, the purpose of clustering is to divide the eigenvalues into two categories, which belong to signal subspace and noise subspace, respectively, so the number of cluster centers is fixed, and the value is 2. A general FCM model is shown as:

$$\begin{aligned} \min_{\mathbf{U}, \mathbf{V}} \quad & J_{FCM}(\mathbf{U}, \mathbf{V}) = \sum_{k=1}^c \sum_{i=1}^M u_{ki}^m d_{ki}^2 \\ \text{s.t.} \quad & \sum_{k=1}^c u_{ki} = 1, \quad \sum_{i=1}^M u_{ki} > 0, \quad u_{ki} \in [0, 1], \\ & 1 \leq k \leq c, \quad 1 \leq i \leq M \end{aligned} \quad (10)$$

where $\mathbf{U} = \{u_{ki}\}$ with dimension $c \times M$ is the fuzzy membership matrix, u_{ki} represents the membership value of the i th sample $\mathbf{x}_i$ belonging to the k th class; here, c is the number of cluster centers, and its value is 2 in source number estimation. $\mathbf{V} = [v_1, v_2, \dots, v_c] \in \mathbb{C}^{S \times c}$ is the matrix composed of c cluster center vectors. $m > 1$ is the weighted index of fuzzy clustering; $d_{ki} = \|\mathbf{x}_i - \mathbf{v}_k\|$ is the distance from the sample $\mathbf{x}_i$ to the center $\mathbf{v}_k$.

According to formula (10), the updating formulas of fuzzy membership u_{ki} and cluster center $\mathbf{v}_k$ are as follows:

$$u_{ki} = \sum_{l=1}^c \left(\frac{\|\mathbf{x}_i - \mathbf{v}_k\|^2}{\|\mathbf{x}_i - \mathbf{v}_l\|^2} \right)^{-1/(m-1)}, \quad (11)$$

$$\mathbf{v}_k = \frac{\sum_{i=1}^M u_{ki}^m \mathbf{x}_i}{\sum_{i=1}^M u_{ki}^m}. \quad (12)$$

4. Proposed Algorithm

Suppose there are two signal incidents on a uniform array composed with 10 elements, the distribution of eigenvalues of the array received data covariance matrix versus different SNR is shown in Figure 1; red and blue points in the figure represent signal eigenvalues and noise eigenvalues, respectively.

The result of Figure 1 shows that compared to the eigenvalues of signal, the value difference of the eigenvalues of noise is small; in another word, the density is larger in a certain neighborhood. The concept of local density and its calculation formula are proposed in [25]. The local density of a sample is the number of samples within a field of that sample. According to the result shown in Figure 1 and the concept of local density, we can divide each eigenvalue of array receiving data into noise eigenvalue and signal eigenvalue according to its local density and then get the incident source number. So, in this paper, we construct 1D input samples of

FIGURE 2: Clustering results of LDFCM.

FCM according to the local density of eigenvalues, especially in low SNR environments.

Firstly, the local density of each eigenvalue is calculated by the following formula:

$$\rho_k = \sum_{k \neq l} \chi(d_{kl} - d_c), \quad (13)$$

where $\chi(x)$ is a piecewise function:

$$\chi(x) = \begin{cases} 1, & x < 0, \\ 0, & x \geq 0. \end{cases} \quad (14)$$

d_{kl} indicates the distance between the k th and the l th eigenvalue; d_c indicates a truncated distance, which specifies the neighborhood range of each eigenvalue when calculating the local density. In this paper, the calculation formula of d_c is:

$$d_c = \frac{1}{M} \sum_{k=1}^M \lambda_k. \quad (15)$$

It can be seen from formula (13) that the local density of the k th eigenvalue ρ_k is equal to the eigenvalue number distributed in the neighborhood with radius of d_c of the k th eigenvalue. Figure 1 shows that with the increase of SNR, the difference between signal eigenvalue and noise eigenvalue also increases. The value of d_c has a great influence on the algorithm performance. The value of d_c cannot always satisfy $\lambda_{\max} \leq d_c \leq \lambda_{\min}$ with the change of SNR. Therefore, we use fuzzy C-means clustering to cluster the sample features and then get the number of the incident source.

It can be seen from formulas (13)–(15) that the calculation of local density in the proposed algorithm can be com-

pleted by simple addition calculation; therefore, compared with the eigen decomposition operation, the influence of the calculation of local density on the algorithm complexity can be ignored.

The main steps of the source number estimation algorithm based on data local density and fuzzy C-means clustering (LDFCM) are summed up as follows:

Step 1. Calculate the array receiving data covariance matrix and extract the eigenvalue from eigenvalue decomposition.

Step 2. Calculate the local density of all eigenvalues according to formulas (13) and (14).

Step 3. The eigenvalues and their data local density are used as input samples of fuzzy C-means clustering.

Step 4. The maximum and minimum values of eigenvalues are taken as the initial values of the two clustering centers, respectively, and the termination conditions are set.

Step 5. Fuzzy membership and clustering center are updated by using formulas (11) and (12) until the termination condition is satisfied.

Step 6. Judge the number of information sources according to the final result.

5. Simulation Verification

Firstly, we will show the validity of LDFCM algorithm through the first experiment; assuming that the number of array elements is 10, the DOAs of 2 incident signals with equal power are $[\theta_1, \theta_2] = [35^\circ, 80^\circ]$, SNR is 0, and the number of snapshots is 100. It can be seen from Figure 2 that the LDFCM algorithm can correctly classify the eigenvalues belonging to the noise subspace and the signal subspace, where the number of eigenvalues belonging to the signal subspace is the number of sources.

Then, we will compare the PCD of incident source number use LDFCM with MDL, AIC, mMMDL, mAIC, and FCM in white noise and spatial correlation noise, respectively. 5000 Monte-Carlo runs are taken in each experiment for performance comparison. We consider two narrowband and equal-amplitude stationary Gaussian signals illuminating on a uniform linear array at intervals of half-wavelength between elements.

Figure 3 shows the relationship between the PCD and the SNR. The numbers of antennas and snapshots are 10 and 50, respectively, in this experiment. DOAs of the two incident sources with the same power are $[\theta_1, \theta_2] = [35^\circ, 80^\circ]$. It can be seen that the LDFCM, FCM, MDL, mAIC, and mMMDL methods converge to 1 in PCD when the SNR tends to infinity. Since AIC is not a consistent estimate, it does not converge to 1. The LDFCM Algorithm has the highest PCD under low SNR; the reason is that in the case of low SNR, the difference between noise eigenvalue and minimum signal eigenvalue is small, but the local density difference between them is large. Therefore, we make the local density of

FIGURE 3: PCD versus SNR in white noise.

FIGURE 4: The PCD versus the number of snapshots in white noise.

eigenvalue as 1D sample feature used in FCM, which is equivalent to 1D additional information to the FCM algorithm, so the PCD of improved algorithm proposed in this paper is higher than FCM algorithm, especially for the low SNR cases. For example, when SNR is -10 dB, the PCD of LDFCM algorithm is higher than 70%, and the PCD of FCM algorithm is less than 10%.

In the third experiment, assume that the number of antennas is 10. DOAs of the two incident sources with equal power are $[\theta_1, \theta_2] = [35^\circ, 80^\circ]$; SNR is 0. The PCD versus the number of snapshots are provided in Figure 4. It is observed that the LDFCM, FCM, MDL, mAIC, and mMDL methods converge to 1 in PCD as the number of snapshots goes to infinity. AIC is still not converge to 1. The PCD of LDFCM and FCM methods are close and are better than other com-

parison algorithms. The comparison results show that SNE algorithm based on LDFCM and FCM have a low requirement for the number of snapshots.

Figures 5 and 6 show the PCD of algorithms versus the number of snapshots and the SNR in the condition of the spatial correlation noise.

Numerical simulation results demonstrate that the MDL and AIC methods fail in the case of the spatial correlation noise, and LDFCM has the highest PCD compared with the other three algorithms. It also can be seen that when the SNR reaches saturation which means that the PCD of LDFCM and FCM algorithms both reaches 100%. The PCD of algorithms is less affected by the number of snapshots.

We can compare the PCD of LDFCM algorithm in white noise with that in spatial correlation noise by seeing Figures 3

FIGURE 5: PCD versus SNR in spatial correlation noise.

FIGURE 6: PCD versus number of snapshots in spatial correlation noise.

and 5. LDFCM algorithm has the better performance while in noise environment than that in spatial correlation noise when the SNR is below -5 dB, but when the SNR is better than -5 dB, the PCD of proposed algorithm both can get 1 in both two different noise environments. In addition, the LDFCM algorithm has the highest PCD among all contrast algorithm in both two kinds of noise environments.

6. Conclusion

In this paper, we propose an advanced source number estimation algorithm named LDFCM. This algorithm adds 1D sample which named data local density of eigenvalues, which

increase the amount of information available for FCM algorithm from the perspective of information theory. Data local density of eigenvalues expresses the relative distribution relation among multiple eigenvalues; therefore, when the SNR is low, the difference between the maximum noise eigenvalue and the minimum signal eigenvalues are small; the proposed algorithm can distinguish these two eigenvalues into two different kinds by using the local density. The numerical simulation results demonstrate that the LDFCM algorithm can provide higher accuracy of the SNE than the FCM algorithm in both cases, i.e., the white noise and the spatial correlation noise.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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