Research Article

Scaling Performance Analysis and Optimization Based on the Node Spatial Distribution in Mobile Content-Centric Networks

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Content-centric networks (CCNs) have become a promising technology for relieving the increasing wireless traffic demands. In this paper, we explore the scaling performance of mobile content-centric networks based on the nonuniform spatial distribution of nodes, where each node moves around its own home point and requests the desired content according to a Zipf distribution. We assume each mobile node is equipped with a finite local cache, which is applied to cache contents following a static cache allocation scheme. According to the nonuniform spatial distribution of cache-enabled nodes, we introduce two kinds of clustered models, i.e., the clustered grid model and the clustered random model. In each clustered model, we analyze throughput and delay performance when the number of nodes goes to infinity by means of the proposed cell-partition scheduling scheme and the distributed multihop routing scheme. We show that the node mobility degree and the clustering behavior play the fundamental roles in the aforementioned asymptotic performance. Finally, we study the optimal cache allocation problem in the two kinds of clustered models. Our findings provide a guidance for developing the optimal caching scheme. We further perform the numerical simulations to validate the theoretical scaling laws.

1. Introduction

During recent years, wireless traffic is undergoing explosively increase due to the subscribers’ enormous data demands (such as video streaming). Content-centric networks (CCNs) [1] have emerged as a promising solution to deal with the increasing traffic, which shifts the traditional host-oriented communication pattern to the novel content-oriented communication pattern. In CCNs, nodes or user terminals are allowed to cache and forward contents based upon their names rather than the host addresses. This enables users request desired contents by local communications, without communicating with backhaul links to the core networks, which also reduces the delivery time of desired contents. In this context, as the number of users continually grows, the scaling performance of content-centric networks has attracted research interests, which is important to help us understand the scalability of CCNs.

In the pioneer work of Gupta and Kumar [2], they first study the scaling behavior of large-scale wireless ad hoc networks. In a static unit network consisting of $n$ randomly distributed nodes, Gupta and Kumar [2] shows the asymptotic throughput of each node scales as $\Theta(1/\sqrt{n \log n})$. Given two nonnegative functions $f(n)$ and $g(n)$: $f(n) = O(g(n))$ means there exists a constant $c$ such that $f(n) \leq cg(n)$ for $n$ large enough; $f(n) = \Omega(g(n))$ if $g(n) = O(f(n))$; $f(n) = \Theta(g(n))$ means both $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$; $f(n) = o(g(n))$ means $\lim_{n \to \infty} f(n)/g(n) = 0$; and $f(n) = \omega(g(n))$ means $\lim_{n \to \infty} g(n)/f(n) = 0$, which indicates the poor scalability of wireless networks as the number of nodes increases. In [3], Franceschetti et al. apply the percolation theory to improve the asymptotic performance of wireless networks and per-node throughput is achieved as $\Theta(1/\sqrt{n})$. Grossglauser and Tse [4] first take the node mobility into consideration, and they propose a two-hop relaying policy to obtain the constant per-node throughput, which costs a vast transmission...
delay. Subsequently, a series of researches focus on the scaling laws of various wireless ad hoc networks. Talak et al. [5] investigate the broadcast capacity and transmission delay in highly mobile wireless networks. In [6], Lin et al. investigate the optimal throughput-delay tradeoff under the i.i.d mobility model for mobile ad hoc networks. Jia et al. [7] introduce the correlated mobility into the analysis of throughput and delay. They find that strong correlation of node mobility results in poor asymptotic performance. The nature of different mobility models [6–9] affects the network performance remarkably. In addition, there are several technics to improve the performance of wireless networks, such as directional antennas [10–12], infrastructure support [13–15], secrecy analysis [16, 17], and reinforcement learning [18, 19].

In light of the asymptotic analysis of traditional wireless ad hoc networks, the throughput and delay scaling behaviors of large-scale cache-enabled content-centric networks have also received wide attention in recent years. In a static square network, Gitzenis et al. [20] formulate a joint optimization problem for content replication and transmission. They derive the minimum link capacity by utilizing a Zipf popularity law. In [21], by assuming that the content cached time in a node is finite, Azimdoost et al. investigate the throughput and delay of content-centric networks for the static grid network model and the static random network model, respectively. Jeon et al. [22] study the per-node throughput for wireless static device-to-device networks based on a decentralized caching scheme. In [23], Mahdian et al. analyze the scaling laws for pure static content-centric networks and heterogeneous content-centric networks, respectively. The authors further consider the optimal cache strategy for two kinds of networks. Zhang et al. [24] investigate the capacity of static hybrid content-centric wireless networks. In [25, 26], the authors study how the content popularity impact the network throughput. Contrast to the static networks, [27, 28] investigate the optimal throughput-delay tradeoff for mobile content-centric networks. Do et al. in [27] adopt a decoupling approach to achieve the optimal caching allocation in the hybrid mobile content-centric networks. In [29], Alfano et al. investigate the throughput and delay performance of mobile content-centric networks with limited cache space. They find that the stronger mobility degree of nodes results in poor network performance. In [30], Luo et al. introduce fast and slow mobility models into mobile content-centric networks according to different time scales. Then, the authors analyze the asymptotic performance with an arbitrary content popularity distribution.

The distribution of all nodes in the aforementioned literatures is uniform regardless of the traditional ad hoc networks or the content-centric networks. However, the node distribution or the mobility degree is nonuniform in the real world. For example, the density of nodes around hot spots or home points is relatively intensive. The mobile terminals are more likely to move around the airports or the tourist attractions. Motivated by the above considerations, the researchers begin to investigate the nonuniform wireless traffic [31–34]. Alfano et al. [31] first analyze the upper bounds to the per-node throughput of inhomogeneous content-centric networks according to different time scales. Then, the authors analyze the asymptotic performance with an arbitrary content popularity distribution.

(i) Firstly, we construct a novel system model in the content-centric networks where each cache-enabled node moves around its corresponding home point. Based on the deployment of home point, we formalize the spatial distribution of nodes into the two clustered models.

(ii) Secondly, we devise a cell-partition-based TDMA scheduling scheme to maximize the concurrent transmissions and develop a distributed multihop routing scheme. On this basis, we derive the asymptotic performance of the two kinds of clustered models and further establish the closed form of the throughput-delay tradeoff.

(iii) Thirdly, we design the optimal cache allocation and investigate optimal throughput and delay performance by utilizing Lagrangian relaxation method under the assumption of a Zipf content popularity distribution. Moreover, massive numerical simulations are conducted to validate aforementioned theoretical results.
The rest of the paper is organized as follows. In Section 2, we describe the system models and outline some definitions. In Section 3, we analyze the throughput and delay performance in both clustered grid model and clustered random model. In Section 4, we introduce the optimal cache allocation. Finally, we conclude the paper in Section 5.

2. System Models and Definitions

In this paper, we study a mobile content-centric network composed of \( n \) mobile nodes. We assume that \( n \) nodes move over a square region \( \mathcal{O} \) of area \( n \) with the wrap-condition, to eliminate border effects. Note that, under aforementioned assumption, we adopt an extended network model that the node density over the square area remains constant as the number of mobile nodes increases. In the following subsections, we first describe the mobility model, content request model, and interference model. Then, we give some important definitions and notations used in this paper.

2.1. Mobility Model. In this paper, we adopt a bidimensional i.i.d mobility model for each mobile node, and time is divided into slots of equal duration. At the beginning of each slot, every node moves to a new location, which is independent of other nodes, and stays in the new location for the remaining duration of the slot. Let \( X_i(t) \) denote the location of node \( i \) at time \( t \), and \( d_{ij} = ||X_i(t) - X_j(t)|| \) denotes the distance between node \( i \) and node \( j \).

To characterize the spatial distribution of nodes, we assume that each node \( i \) is associated with a home point \( H_i \), which is uniformly and independently selected over the square region \( \mathcal{O} \). We consider that a node moves independently around its home point following a general ergodic process, which can be described by a rotationally invariant spatial distribution \( \phi(d) \). Here, \( d \) denotes the Euclidean distance between the mobile node and its corresponding home point.

We further assume that \( \phi(d) \) is an arbitrary nonincreasing function that decays as a power law of exponent \( \delta \), i.e.,

\[
\phi(d) \sim d^{-\delta}, \quad \delta \geq 0.
\]

We take function \( s(d) = \min(1,d^{-\delta}) \) and consider the following normalized probability density function (PDF) over the whole mobility area to avoid convergence problems in proximity of the home point.

\[
\phi(d) = \frac{s(d)}{\int_0^\infty s(d) \, \text{d}d} = \begin{cases} \Theta(s(d)n^{-(\delta-2)/2}), & 0 \leq \delta < 2, \\ \Theta(s(d)/\log n), & \delta = 2, \\ \Theta(s(d)), & \delta > 2. \end{cases} \tag{1}
\]

Exponent \( \delta \) reflects the mobility degree of nodes, that is, the probability that each node moving to one point of the network area and stay at that point at a given time slot is not uniform. According to the PDF \( \phi(d) \), it indicates that each node moves in a limited region, differing from the global mobility. In addition, the probability for the node moving in the proximity of its corresponding home point is larger than the probability that the node moves to the relatively far network area.

However, exponent \( \delta \) just describes the individual mobility behavior and cannot reflect the mobile node density over the network area at a time slot. In real mobile world, the mobile node density is tightly related with the number of home points. Mobile nodes are more likely to move around the hotspots [36] or social spots [37]. For example, the density of mobile terminals (e.g., phones and ipads) is relatively dense in the office buildings or the tourist attractions while the density of mobile terminals in the suburb is reversely sparse due to the less home points. That is, the number of home points will affect the spatial distribution of mobile nodes. Motivated by this fact, clustering behavior has been found in [32, 33, 38] based on the long-term observations. In our work, we introduce the clustered model combined with the distribution of home points. First, we assume \( m \) clusters and each cluster has a middle point (the center of the cluster), which are distributed over the area \( \mathcal{O} \) independently. Then, each home point randomly chooses one of the clusters with equal opportunity. Finally, the home points of the same cluster are belonged to a disk of radius \( R \) centered at the cluster middle point.

Considering the asymptotic performance, we assume that \( m = n^\nu, \, 0 < \nu < 1 \) and the cluster density over the whole network area \( \rho_c = n^\nu /n = n^{\nu-1} \). The average distance between two cluster middle points is \( d_i = n^{(1-\nu)/2} \). For the ease of analysis, we do not consider the cluster overlapping behavior. Hence, the cluster radius should be satisfied by \( R = o(d_i) \).

2.2. Content Request Model. In the mobile content-centric networks, there are \( M \) distinct content objects of same size, where \( M = n^\gamma, \, 0 < \gamma < 1 \). We assume each mobile node is equipped with an equal-sized local cache, which can store \( K \) content objects (\( K \) is a positive constant). For the problem to be not trivial, we assume \( K < M \), that is, each node has to decide which kinds of content to cache. We refer to a mobile node requesting a desired content \( k, 1 \leq k \leq M \) at any time slot as the requester of content \( k \). We call a mobile node carrying a content \( k \) in its local cache the holder of content \( k \).

In this paper, a caching scheme consists of two phases: content placement and content retrieve [22]. In the content placement phase, each node randomly and independently chooses contents to be stored in its local cache. Let \( \mathcal{M}_k \) denote the set of nodes that cache content \( k \in M \) in their local caches, where \( N_k = |\mathcal{M}_k| \). Thus, the probability that the content \( k \) is cached by a mobile node in its local cache is \( N_k/n \). In order to achieve a feasible cache allocation, the total cache constraint should be satisfied.

\[
\sum_{k=1}^{M} N_k \leq nK. \tag{2}
\]

In the content retrieve phase, each holder decides whether to deliver the requested content to the corresponding requester. During the retrieve phase, each node requests its desired content independently according to a Zipf
popularity distribution [39], i.e., the request probability $p_k$ of content $k \in M$ is satisfied by

$$p_k = \frac{k^{-\alpha}}{H_{\alpha}(M)},$$  \hspace{1cm} (3)

where $\alpha > 0$ is the Zipf's law exponent and $H_{\alpha}(M) = \sum_{k=1}^{M} k^{-\alpha}$ is a normalization constant and is given by

$$H_{\alpha}(M) = \begin{cases} \Theta(1), & \alpha > 1, \\ \Theta(\log M), & \alpha = 1, \\ \Theta(M^{1-\alpha}), & \alpha < 1. \end{cases}$$  \hspace{1cm} (4)

### 2.3. Interference Model

In this paper, to avoid multiuser simultaneous transmission interference, we adopt the protocol model in [2]. Moreover, we assume that the transmission range of mobile node $i$ is $T_i$. If the content $k$ is transmitted from mobile node $i$ to node $j$ successfully, then the following two conditions should be held:

1. The distance between the transmitter $i$ and the receiver $j$ is no more than $T_i$, i.e.,

$$\|X_i(t) - X_j(t)\| \leq T_i.$$  \hspace{1cm} (5)

2. Other transmitter $l$ delivering different contents at the same time slot does not interfere the receiver $j$, i.e.,

$$\|X_i(t) - X_j(t)\| \geq (1 + \Delta)T_i.$$  \hspace{1cm} (6)

Here, $\Delta > 0$ denotes a constant guard factor. $X_i(t)$ denotes the location of a mobile node at the time slot $t$. $\|\cdot\|$ denotes the Euclidean distance between the transmitter and the receiver. We further assume that each mobile node can deliver the contents at a constant rate $W$ bits/sec.

### 2.4. Definitions

**Definition 1.** For a given scheduling and routing scheme, let $F(i, t)$ be the total number of bits of the requested contents received by mobile node $i$ up to time $t$. We define the long-term throughput of mobile node $i$ as

$$\liminf_{t \to \infty} \frac{1}{t} F(i, t).$$  \hspace{1cm} (7)

The average throughput over all nodes is given as

$$\frac{1}{n} \sum_{i=1}^{n} \liminf_{t \to \infty} \frac{1}{t} F(i, t).$$  \hspace{1cm} (8)

The throughput is defined as the expectation of the average throughput over all mobile nodes,

$$\lambda(n) = \mathbb{E}\left[\frac{1}{n} \sum_{i=1}^{n} \liminf_{t \to \infty} \frac{1}{t} F(i, t)\right].$$  \hspace{1cm} (9)

**Definition 2.** The delay of a content retrieve process is the moment the interest packet leaves node $i$ until the requested content arrives at node $i$ from the closest holder. For a given scheduling and routing scheme, let $D(i, d)$ be the delay of the $d$th requested content of mobile node $i$. We define the long-term delay of mobile node $i$ as

$$\limsup_{t \to \infty} \frac{1}{r} \sum_{d=1}^{r} D(i, d).$$  \hspace{1cm} (10)

The average delay over all nodes is given as

$$\frac{1}{n} \sum_{i=1}^{n} \limsup_{t \to \infty} \frac{1}{r} \sum_{d=1}^{r} D(i, d).$$  \hspace{1cm} (11)

The delay is defined as the expectation of the average delay over all mobile nodes.

$$D(n) = \mathbb{E}\left[\frac{1}{n} \sum_{i=1}^{n} \limsup_{t \to \infty} \frac{1}{r} \sum_{d=1}^{r} D(i, d)\right].$$  \hspace{1cm} (12)

### 3. Throughput and Delay Analysis

For better understanding of content placement and retrieve process in the clustered random model, we first investigate the throughput and delay performance in the clustered grid model [33] in Section III-A, that is, the home points are placed regularly, which can be considered as a special instance of the distribution of the home points. In the real world, the home points can be deployed regularly, for example, the base stations (BSs) and the road side units (RSUs). In general, the BSs are deployed in the center of the cells, and the RSUs are deployed along the road (i.e., one RSU every 50 meters), which can be regarded as the examples of the grid clustered model. In Section III-B, we analyze the case of clustered random model. Specifically, when $\delta \leq 2$ and $n$ goes to infinity, the spatial distribution $\phi(i, d)$ tends to 0; we leave to the analysis of the network performance in this case for future work.

#### 3.1. Clustered Grid Model

In the clustered grid model, we divide the whole network region into equal-sized cells of area $S_a = d_c^2$. The distribution of clusters should satisfy following two conditions:

1. The middle point of each cluster is located at the center of each cell, and the distance between two adjacent cluster middle points is $d_c$
Note that when \( n = \log \log n \), Lemma 3.

**Proof.** For an arbitrary cell, in a given slot, the probability that one node moves to any cell is

\[
\rho_c \left( \frac{1}{\log n} \right) \leq 1 - e^{-n^{-\nu}},
\]

where \( I_{X_i \in S_\nu} \) is an i.i.d Bernoullian random variable, and \( I_{X_i \in S_\nu} = 1 \) if node \( i \) is in the cell \( S_\nu \); otherwise, \( I_{X_i \in S_\nu} = 0 \). Hence,

\[
E[N(S_\nu)] = E \left[ \sum_{i=1}^{n} I_{X_i \in S_\nu} \right] = n \cdot E[I_{X_i \in S_\nu}] = |S_\nu| = \Theta(1).
\]

We apply Chernoff bounds and have

\[
P \left\{ N(S_\nu) < \frac{1}{2} E[N(S_\nu)] \right\} \leq e^{-\frac{|S_\nu|}{8}},
\]

\[
P \left\{ N(S_\nu) > 2E[N(S_\nu)] \right\} \leq e^{-\frac{|S_\nu|}{3}} < e^{-\frac{|S_\nu|}{8}}.
\]

Then, letting \( \epsilon = \Theta(\log \log n / \log n) \), for any \( 0 < \nu < 1 - \epsilon \), we have \( |S_\nu| = n^{-\nu} \geq 16 \log n \). Thus, we have

\[
P \left\{ \frac{1}{2} E[N(S_\nu)] \leq N(S_\nu) \leq 2E[N(S_\nu)] \right\} \geq 1 - 2e^{-\frac{|S_\nu|}{8}} \geq 1 - 2e^{-2 \log n} = 1 - 2n^{-2},
\]

which tends to 1 when \( n \) goes infinity. Hence, we obtain the expected number of nodes in each cell is \( \Theta(n^{-\nu}) \).

### 3.1.1. Scheduling Schemes. At a given slot, a scheduling scheme enables the contents retrieve between transmitter-receiver pairs not to be interfered. Based on the mobility model and the spatial distribution of mobile node, we adopt a cell-partition-based TDMA scheduling scheme to avoid the multiuser simultaneous transmission interference and maximize the number of noninterfering transmission pairs at a time slot. Figure 1(a) illustrates a general real-world node distribution case, which shows mobile terminals are relatively dense near the home point, and there are fewer mobile terminals far away from home point. Figure 1(b) illustrates home points are deployed regularly and can be regarded as a clustered grid model. Figure 1(c) shows a cell-partition-based TDMA scheduling scheme corresponding to the time slot division, which guarantees multithop transmission in the clustered grid model.

We first divide each time slot into two half slots with equal length. Then, in the first half slot, we further divide the first half slot into several subslots with equal length. We partition each cell \( S_\nu \), into squares with same area \( S_\nu \). (\( S_\nu \) will be defined later). We call a square is *active* if a node in this square can transmit an interest packet or a content during the subslot. Based on the square partition, we assume that a node in a square can transmit an interest packet or a content to the node in the same square or the adjacent eight squares. We define the transmission radius of the node \( i \) as \( T_i = \Theta(d_i) \). Hence, we obtain the following lemma according to the protocol model, which is essential to describe the noninterfering transmission process.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>The number of mobile nodes, the area of network region ( \Theta )</td>
</tr>
<tr>
<td>( m )</td>
<td>The number of clusters</td>
</tr>
<tr>
<td>( M )</td>
<td>The number of content objects</td>
</tr>
<tr>
<td>( K )</td>
<td>The size of a node’s local cache</td>
</tr>
<tr>
<td>( d_i )</td>
<td>The distance between two cluster middle points</td>
</tr>
<tr>
<td>( \rho_c )</td>
<td>The cluster density over the whole network area</td>
</tr>
<tr>
<td>( S_\nu )</td>
<td>The area of the cell in the clustered grid model</td>
</tr>
<tr>
<td>( S_\nu' )</td>
<td>The area of the cell in the clustered random model</td>
</tr>
<tr>
<td>( p_k )</td>
<td>The request probability of content ( k )</td>
</tr>
<tr>
<td>( T_i )</td>
<td>The transmission radius of a mobile node</td>
</tr>
<tr>
<td>( h )</td>
<td>The maximum hops from the requester to the closest holder</td>
</tr>
</tbody>
</table>

(ii) The radii of all clusters equal to 0, i.e., the home points of the same cluster are gathered at their corresponding cluster’s middle point (or the cell center).

Before introducing scheduling and routing schemes, we premise a necessary lemma that guarantees each cell has at least one mobile node at a time slot and the expected number of nodes in each cell is \( \Theta(n^{-\nu}) \) which guarantees the network connectivity.

**Lemma 3.** In the clustered grid model, assuming \( \epsilon = \Theta(\log \log n / \log n) \), for any \( 0 < \nu < 1 - \epsilon \), each cell has at least one mobile node and the expected number of nodes in each cell \( S_\nu \) is \( N(S_\nu) = \Theta(n^{-\nu}) \) in a given slot with high probability.

**Proof.** For an arbitrary cell, in a given slot, the probability that one node moves to any cell is \( d^2_i/n \), i.e., \( n^{-\nu} \). Hence, the probability \( p_c \) that each cell has at least one mobile node is

\[
p_c = 1 - (1 - n^{-\nu})^n = 1 - e^{n^{\nu} \log (1 - n^{-\nu})} \geq 1 - e^{-n^{-\nu}},
\]

where the last inequality follows that \( \log (1 + x) \leq x \) for \( x \geq -1 \). Let \( p_c' = 1 - e^{-n^{-\nu}} \) and \( p_c \leq p_c' \). By assuming \( \epsilon = \Theta(\log \log n / \log n) \), for any \( 0 < \nu < 1 - \epsilon \), we have \( n^{-\nu} \geq \log n \). Note that when \( n \) goes infinity, \( \epsilon \) tends to 0; the upper bound of \( \nu \) is 1. Then, we have

\[
p_c' = 1 - e^{-n^{-\nu}} \geq 1 - e^{-\log n} = 1 - \frac{1}{n},
\]

which tends to 1 when \( n \) goes infinity. Hence, with high probability, each cell has at least one mobile node in a given slot.

Let \( N(S_\nu) \) denote the number of nodes in the cell \( S_\nu \), by definition:

\[
N(S_\nu) = \sum_{i=1}^{n} I_{X_i \in S_\nu},
\]
Lemma 4. Each square can be active at most every \((1 + c_1)\) time subslots, and the achievable rate at the active square is \(W/(1 + c_1)\).

Proof. According to the interference model, we consider the distance between transmitter \(X_i\) and receiver \(X_j\) is bounded by \(T_i = \Theta(d_i)\). If there is another simultaneously transmitting node \(X_l\) at the same time subslot and the transmission distance between \(X_l\) and \(X_j\) is less than \((2 + \Delta)T_i\), then \(X_l\) will cause the interference with \(X_j\). Hence, the area of the total interference region is bounded by \((2(2 + \Delta)T_i + 3T_i)^2\). We obtain that each square has at most \(c_1 = (2(2 + \Delta)T_i + 3T_i)^2 / T_i^2 = (2(2 + \Delta) + 3)^2\) interference neighbouring squares, which is a constant and independent of \(n\).

Each square gets a noninterfering transmission opportunity and becomes active in every \((1 + c_1)\) time subslots. Since each transmitter-receiver pair can send \(W\) bits in a successful transmission, the achievable transmission rate at the active square is \(W/(1 + c_1)\).

From Lemma 4, we can construct \((1 + c_1)\) subsets of regularly spaced, simultaneously transmitting squares without interference.

Next, we calculate the area of each square \(S_c\). From Lemma 3, we obtain the average number of nodes in each cell.
$S_i$ is $\Theta(n^{-1-\gamma})$. We assume there exists point $A$ in the cell that the distance between $A$ and the home point is $d$. The mobility process of each node is independent at each slot, and the probability that the node moves to point $A$ is $\Theta(d^{-\delta})$ at each slot according to the node distribution. Hence, when $\delta > 2$, the density of nodes at point $A$ is $\Theta(n^{1-\gamma})$ at each slot. Applying Lemma 4 in [33], we can obtain the maximum density of achievable transmitter-receiver pairs (T-R pairs) when $d = d_t/2$. Thus, the density of T-R pairs at any point of the network scales as $\phi = \Theta(n^{-1-\gamma/d_t^2}) = \Theta(n^{1-\gamma/(1-\delta/2)})$. Therefore, the area of each square is equal to $1/\phi$, i.e., $S_i = \Theta(n^{-1-\gamma/(1-\delta/2)})$.

At a given time slot, each cell can schedule the number of concurrent noninterfering transmissions $\chi$ is

$$\chi = \Theta\left(\frac{d_c^2}{(1 + c_1)(1/\phi_c)}\right) = \Theta\left(n^{1-\gamma/(2-\delta/2)}\right). \quad (19)$$

Since the area of each square is $\Theta(n^{1-\gamma/(1-\delta/2)})$, we can acquire that the probability that at least one node in a subcell is a constant by applying Lemma 3.

3.1.2. Routing Schemes. Considering the mobility model of nodes and the clustering behavior, we propose a multihop transmission scheme $\Pi$ for the content transmission between the requester-holder pairs. We assume the content request and delivery process of each node is conducted in one time slot. According to the maximum hops (derived in the following section) in the clustered grid model, the value of the one time slot can be set as $\Omega(n^{\nu/2})$ to guarantee the multihop transmission within one time slot.

$\Pi$-1: according to the TDMA scheduling phase, each time slot is divided into two half slots with equal length.

$\Pi$-2: during the first half slot, we further divide the first half slot into several subslots with equal length. Each cell is divided into squares with the same area, the requested content in the same square (or the same cluster), the requester $i$ directly forwards an interest packet to the holder

(1) if there is one requested content in the same square (or the same cluster), the requester $i$ directly forwards an interest packet to the holder

(2) if there is no any requester-holder pair in the same square (or the same cluster), the requester $i$ sends an interest packet to the relay node in the adjacent square at the beginning of subslot; multihop is used for all relay nodes until the interest packet reached the nearest holder. Furthermore, we assume that the first hop towards the adjacent subcell is on the horizon path (if possible) and then on the vertical path

$\Pi$-3: during the second half slot, we continue dividing the second half slot into several subslots with equal length,

(1) if there is one requested content in the same square (or the same cluster), the requested content will be directly forwarded back to the requester $i$. In this case, the requested content will be achieved in one hop

(2) if there is no any requester-holder pair in the same square (or the same cluster), the nearest holder receives the interest packet in the first half slot, and then, the nearest holder in the second half slot sends the requested content back to its corresponding requester in the reverse direction

**Lemma 5.** In the clustered grid model, for any node requesting content $k$, the probability $P(k)$ that an interest packet for content $k$ is satisfied by one hop is $\min(\Theta(N_k n^{1-\gamma/(\delta/2)-1}), 1)$.

*Proof.* For an arbitrary square, the probability that there is at least one requested content $k$ within the same square or the adjacent squares of the requester is $9S_i/n = 9n^{1-\gamma/(\delta/2)-1}/n = 9n^{1-\gamma/(\delta/2)-1}$. Thus,

$$P(k) = 1 - \left(1 - 9n^{1-\gamma/(\delta/2)-1}\right)^{N_k} \leq 9N_k n^{1-\gamma/(\delta/2)-1}.$$

The inequality follows that $(1 + x)^n \geq 1 + nx$ for any $x > 0$. If $9N_k n^{1-\gamma/(\delta/2)-1} = o(1)$, then $P(k) = \Theta(N_k n^{1-\gamma/(\delta/2)-1})$; otherwise, $P(k) = 1$.

In the clustered grid model, due to the transmission radius of each node is $\Theta(d_t)$, we can derive the maximum number of hops from a content requester to the closest holder is $h = \Theta(n^{\nu/2})$.

**Lemma 6.** In clustered grid model, for any mobile node requesting content $k$, the average number of hops needed to transmit an interest packet along the path from the content requester to the closest content holder, denoted by $E(H_k)$, is

$$E(H_k) = \begin{cases} \Theta(1), & N_k = \Theta\left(n^{(1-\gamma)/(\delta/2)-1}\right), \\ \Theta\left(n^{\nu/2}\right), & N_k = \Theta\left(n^{(1-\gamma)/(\delta/2)-1}\right). \end{cases}$$

(21)

*Proof.* Let $H_k$ denote the number of hops along the requesting path, and we have

$$E(H_k) = \sum_{i=1}^{h-1} \left(i \cdot P(k)(1 - P(k))^{i-1} \right) + h(1 - P(k))^{h-1} = \sum_{i=1}^{h} (1 - P(k))^{i-1} = \frac{1 - (1 - P(k))^h}{P(k)}.$$

Based on Lemma 5 and the value of $h$, we consider three different conditions to further derive $E(H_k)$.

(1) If $P(k) = \Theta(1)$, then $E(H_k) = \Theta(1)$.

(2) If $P(k) = o(1)$ and $hP(k) = O(1)$, then $1 - (1 - P(k))^h \geq 1 - e^{-hP(k)} = \Theta(1)$. Hence, we obtain that $E(H_k) = \Theta(1/(P(k)))$.

(3) If $P(k) = o(1)$ and $hP(k) = o(1)$, applying the equivalent infinitesimal, we obtain that $1 - (1 - P(k))^h \approx 1 - (1 - hP(k)) \approx \Theta(hP(k))$. Hence, we get that $E(H_k) = \Theta(h)$. This completes the proof.
Theorem 7. In the clustered grid model, the delay and throughput of each mobile node, denoted by $D(n)$ and $\lambda(n)$, are given by w.h.p.

$$D(n) = \Theta \left( \sum_{m=1}^{M} p_k E(H_k) \right),$$  \hspace{1cm} (23)

$$\lambda(n) = \Theta \left( \frac{n^{1-(1-\nu)((1-\delta)/2)}}{\sum_{m=1}^{M} p_k E(H_k)} \right).$$  \hspace{1cm} (24)

Proof. First, we consider the delay performance. Based on the scheduling and routing scheme, each square can be active in every $1 + c_1$ time sublots, where $c_1$ is a constant. Thus, the time spent in each hop is a constant fraction of time. In Lemma 6, we obtain the average number of hops from a requester to the closest holder for content $k$ is $E(H_k)$. In addition, the requested content forwarded back to the requester takes the same route as its corresponding interest packet in reverse direction, the total number of hops for content $k$ is $2E(H_k)$. Taking the request probability $p_k$ for the content $k$ into consideration, we get the delay for all contents over the network as (6).

Next, we derive the throughput performance. Since there are $O(n^\nu)$ cells over the network area, and the total number of bits that active squares in a cell can transmit is $(\mathcal{W}/(1 + c_1))n^{1-(1-\nu)((1-\delta)/2)}$. Lemma 6 indicates the average number of hops for content $k$ is $E(H_k)$. Considering the request probability $p_k$ for the content $k$, we obtain the average number of bits transmitted in the whole network is $n\lambda(n)\sum_{m=1}^{M} p_k E(H_k)$. Then, we have

$$n\lambda(n) \sum_{m=1}^{M} p_k E(H_k) \leq \frac{\mathcal{W}}{(1 + c_1)}n^{(1-\nu)((1-\delta)/2)} \cdot n^\nu,$$  \hspace{1cm} (25)

$$\lambda(n) \leq \frac{n^{1-(1-\nu)((1-\delta)/2)}}{\sum_{m=1}^{M} p_k E(H_k)}.$$  \hspace{1cm} (26)

The achievable throughput of each mobile node is determined by the scheduling and routing scheme. Hence, we obtain the throughput performance as (24).

Corollary 8. The throughput and delay tradeoff in the clustered grid model is given by

$$\lambda(n) = \Theta \left( \frac{n^{1-(1-\nu)((1-\delta)/2)}}{D(n)} \right).$$  \hspace{1cm} (26)

3.2. Clustered Random Model. In this section, we analyze the throughput and delay of each node in the clustered random model, where clusters are distributed randomly and independently in the network, that is, the home points are deployed in a random manner.

We first introduce an important lemma given in [31]. This lemma indicates the appropriate partition of the whole network area can guarantee that the average number of clusters in each cell is $\Theta(\log n)$, which makes the multihop transmission in the clustered random model possible.

Lemma 9. Consider a set of $m$ points independently distributed over a bidimensional domain $\mathcal{D}$ of area $n$, with density $\rho = m/n$. The domain $\mathcal{D}$ is partitioned by regular tessellations, and $A_s$ denotes the tiles over the tessellations with area $|A_s| \geq 16(\log m/\rho)$, $\forall s$. Let $N(A_s)$ be the number of nodes falling within the tiles $A_s$. Then, uniformly over the tessellation, $N(A_s)$ is contained between $\rho|A_s|/2$ and $2\rho|A_s|$ with high probability, i.e.,

$$\frac{\rho|A_s|}{2} < \inf_{s} N(A_s) \leq \sup_{s} N(A_s) < 2\rho|A_s|.$$  \hspace{1cm} (27)

This lemma can be proved by the Chernoff bound in [40]; we neglect the proof process for simplicity.

Considering Lemma 9 and the cluster density $\rho_c = n^{1-\nu}$, we partition the whole network into equal-sized cells with area $S_n = 16(\log n^n/\rho)^{-1} = \Theta(n^{1-\nu}\log n)$. Note that we choose the equal-sized cell in the clustered random model in order to make sure that nodes in each cluster has the opportunity to retrieve the content; in fact, we can choose different size area only if $S_n > \Theta(n^{1-\nu}\log n)$. Thus, we can obtain the expected number of clusters in each cell is $\Theta(\log n)$. We assume that the transmission radius of each mobile node in the clustered random model is $T_n = \Theta(n^{1/2})$, which guarantees the nodes can communicate with the nodes in their neighbor cells. Since the maximum distance between the requester and the closest holder is $\Theta(n^{1/2})$, the maximum number of hops from a content requester to the closest holder is $h = \Theta(n^{1/2}\sqrt{\log n})$.

Based on the cell partition, we obtain the following lemma that guarantees the average number of nodes in every cluster is $\Theta(n^{1-\nu})$.

Lemma 10. In the clustered random model, the average number of node in every cluster is $\Theta(n^{1-\nu})$.

Proof. The proof process of this lemma is similar to that in Lemma 3 by applying he Chernoff bound.

The density of T-S pairs at any point of the network scales as $\phi' = \Theta(n^{1-\nu}/(n^{(1-(1-\nu)(\delta - 1/2)/2)} \log^{\delta/2} n)) = \Theta(n^{1-\nu}(1-(\delta/2)) \log^{\delta/2} n)$. We further partition each cell into squares with area $1/\phi'_c$, i.e., $S'_c = \Theta((1-\nu)(\delta/2)-1 \log^{\delta/2} n)$. Hence, each active cell can schedule the number of concurrent noninterfering transmissions $\chi'$ is

$$\chi' = \Theta \left( \frac{S}{(1 + c_2)(1/\phi'_c)} \right) = \Theta \left( \frac{n^{1-\nu}\log n}{n^{(1-\nu)(\delta/2)-1} \log^{\delta/2} n} \right)$$

$$= \Theta \left( \frac{n^{1-\nu}(2-\delta)}{\log^{1-(\delta/2)} n} \right).$$  \hspace{1cm} (28)
Hence, a noninterference scheduling scheme in the clustered random model can be designed as following steps. First, we divide the whole network into equal-sized cells with area $\Theta(n^{1-\nu} \log n)$, which makes the transmission connectivity possible. Then, we partition each cell into equal-sized squares with area $\Theta((n^{1-\nu} \log n)^2)$, which guarantees the content transmission free from interference. Figure 2 shows the cell partition and the content request process in the clustered random model. Finally, a routing scheme similar with that in the cluster grid model can be utilized to achieve the content multihop transmission at a given slot.

**Lemma 11.** In the clustered random model, for any node requesting content $k$, the probability $P'(k)$ that an interest packet for content $k$ is satisfied by one hop is $\min(\Theta(N_k n^{1-\nu}((\log 2)/2)-1) \log^{\delta_k/2} n, 1)$.

\[
\mathbb{E}'(H_k) = \begin{cases} 
\Theta(1), & N_k = \Omega\left(\frac{n^{1-\nu}((\log 2)/2)-1)}{\log^{\delta_k/2} n}\right), \\
\Theta\left(\frac{n^{(1/2)-(1-\nu)((\log 2)/2)-(3/2))}{N_k \log^{\delta_k/2} n}\right), & N_k = o\left(\frac{n^{1-\nu}((\log 2)/2)-1)}{\log^{\delta_k/2} n}\right), \\
\Theta\left(\frac{n^{n/2}}{\sqrt{\log n}}\right), & N_k = \Theta\left(\frac{n^{(1/2)-(1-\nu)((\log 2)/2)-(3/2))}}{\log^{\delta_k/2} n}\right). 
\end{cases}
\]

**Proof.** According to Lemma 6, we have

\[
\mathbb{E}'(H_k) = \frac{1 - (1 - P'(k))^h}{P'(k)}. 
\]

Based on the value of $h$ and $P'(k)$, we consider three different conditions to further derive $\mathbb{E}'(H_k)$.

(1) If $P'(k) = \Theta(1)$, then $\mathbb{E}'(H_k) = 1$

(2) If $P'(k) = o(1)$ and $hP'(k) = \Theta(1)$, then $(1 - (1 - P'(k))^h \geq 1 - e^{-hP'(k)} = \Theta(1)$. Hence, we obtain that $\mathbb{E}'(H_k) = \Theta(1/(P'(k))) = \Theta(n^{(1/2)-(1-\nu)((\log 2)/2)-(3/2)})/N_k \log^{\delta_k/2} n)$.

(3) If $P'(k) = o(1)$ and $hP'(k) = o(1)$, applying the equivalent infinitesimal, we obtain that $1 - (1 - P'(k))^h \approx 1 - e^{-hP'(k)} = \Theta(hP'(k))$. Hence, we get that $\mathbb{E}'(H_k) = \Theta(h) = \Theta(n^{n/2}/\sqrt{\log n})$.

This completes the proof.

**Theorem 13.** In the clustered random model, the delay and throughput of each mobile node, denoted by $D(n)$ and $\lambda(n)$, are given by w.h.p.

**Proof.** For an arbitrary square, the probability that there is at least one requested content $k$ is within the same square or the adjacent squares of the requester is $(9S^2/n) = 9 n^{1-\nu}((\log 2)/2)-1 \log^{\delta_k/2} n$. Thus, we have

\[
P'(k) = 1 - \left(1 - 9 n^{1-\nu}((\log 2)/2)-1 \log^{\delta_k/2} n\right)N_k 
\leq 9N_k n^{1-\nu}((\log 2)/2)-1 \log^{\delta_k/2} n. 
\]

The inequality follows that $(1 + x)^n \geq 1 + nx$ for any $x > -1$. If $9N_k n^{1-\nu}((\log 2)/2)-1 \log^{\delta_k/2} n = o(1)$, then $P'(k) = \Theta(N_k n^{1-\nu}((\log 2)/2)-1 \log^{\delta_k/2} n)$. Otherwise, $P'(k) = 1$.

**Lemma 12.** In the clustered random model, for any mobile node requesting content $k$, the average number of hops needed to transmit an interest packet along the path from the content requester to the closest content holder, denoted by $E'(H_k)$, is

\[
D(n) = \Theta\left(\sum_{m=1}^{M} p_m \mathbb{E}'(H_k)\right), \\
\lambda(n) = \Theta\left(\frac{n^{1-\nu}((\log 2)/2)}{\sum_{m=1}^{M} p_m \mathbb{E}'(H_k)}\right).
\]

**Proof.** The proof of delay performance is similar to that of Theorem 7, and we do not repeat it for simplicity.

We analyze the throughput performance of each node. Based on the cell partition of the network, we can obtain the total number of cells is $\Theta(n^{\nu}/\log n)$. Considering the scheduling scheme and routing scheme, the total number of bits that active square in a cell can transmit is $(W/(1 + c_z)) n^{1-\nu}((\log 2)/2)) \log^{\delta_k/2} n$. Moreover, Lemma 12 indicates the average number of hops for content $k$ is $E'(H_k)$. Since the request probability of the content $k$ is $p_k$, the average number of bits transmitted in the whole network can be calculated as $n\lambda(n)\sum_{m=1}^{M} p_m \mathbb{E}'(H_k)$. The rest proof is similar to the Theorem 7; we neglect it here for simplicity.

From Theorem 13, we can conclude that the difference of throughput and delay performance between the clustered random model and the clustered grid model is the logarithm factor and the average transmission hops.
Corollary 14. The throughput and delay tradeoff in the clustered random model is given by

\[
\lambda(n) = \Theta\left(\frac{n^{(1-\gamma)(1-\delta/2)}}{D(n) \log^{\delta/2} n}\right),
\]

(33)

which differs from Corollary 8 by a factor \(\log^{-\delta/2} n\).

4. Optimal Cache Allocation

In this section, we analyze the optimal throughput and delay performance of the mobile content-centric network with respect to optimal cache allocation strategy. To achieve the optimal goal, we need to select appropriate \(\{N_k\}_{k=1}^M\) based on the cache constraints for the clustered grid model and clustered random model, respectively.

4.1. Clustered Grid Model. From Corollary 8, it indicates that minimizing the transmission delay is equivalent to maximizing the throughput performance. In order to achieve the minimum delay, we formulate the following optimization problem:

\[
\begin{aligned}
\text{minimize} & \quad \sum_{k=1}^M \lambda_k E(H_k) \\
\text{subjectto} & \quad \sum_{k=1}^M N_k \leq nK, \\
& \quad 1 \leq N_k \leq n^{1-(1-\gamma)(\delta/2)-1},
\end{aligned}
\]

(34)

where \(E(H_k)\) is given by

\[
E(H_k) = \begin{cases} \Theta(1), & N_k = D\left(n^{(1-\gamma)(\delta/2)-1}\right), \\ \Theta\left(\frac{1}{n^{(1-\gamma)(\delta/2)-1}}\right), & N_k = o\left(n^{(1-\gamma)(\delta/2)-1}\right) \& N_k = D\left(n^{(1-\gamma)(\delta/2)-1}\right), \\ \Theta(n^{(1-\gamma)(\delta/2)-1}), & N_k = o\left(n^{(1-\gamma)(\delta/2)-1}\right). \end{cases}
\]

(35)

The first constraint in (34) comes from the feasible cache allocation in (2), and the second constraint guarantees that for different types of contents, there is at most one copy content in each square. Note that the second derivatives of the objective function in (34) is always positive, which can be considered as a strictly convex optimization problem. Hence, we apply Lagrangian relaxation method to find the unique optimal solution.

Based on the Zipf distribution law, it shows that \(p_k\) decreases as \(k\) increases and so is \(N_k\). For the convenient and tractable analysis, we first define three sets \(K_1, K_2,\) and \(K_3\) according to the size of \(N_k\), respectively. That is,

(i)let \(K_1 = \{1, 2, \ldots, k_1 - 1\}\) be the set of contents such that \(N_k = n^{1-(1-\gamma)(\delta/2)-1}\)

(ii)let \(K_2 = \{k_1, k_1 + 1, \ldots, k_2 - 1\}\) be the set of contents such that \(n^{1-(1-\gamma)(\delta/2)-1} < N_k < n^{1-(1-\gamma)(\delta/2)-1}\)

(iii)let \(K_3 = \{k_2, k_2 + 1, \ldots, M\}\) be the set of contents such that \(N_k = 1\)

Next, we take the Lagrangian multiplier \(\lambda \in R^+\) for the first constraint in (34) and combine with the second constraint; the necessary conditions for the minimal \(D(n)\) are given as

\[
\frac{\partial D(n)}{\partial N_k} = - \frac{p_k}{N_k n^{1-(1-\gamma)(\delta/2)-1}} \begin{cases} \leq -\lambda, & \forall k \in K_1, \\ = -\lambda, & \forall k \in K_2, \\ \geq -\lambda, & \forall k \in K_3. \end{cases}
\]

(36)

When \(k \in K_2\), we obtain

\[
N_k = \frac{p_k^{1/2}}{\lambda^{1/2} n^{1-(1-\gamma)(\delta/2)-1}}. 
\]

(37)

By adding up \(N_k\) in (37) for \(k \in K_2\), we have

\[
\lambda^{1/2} = n^{1-(1-\gamma)(\delta/2)-1} \sum_{k=k_1}^{k_2-1} \frac{p_k^{1/2}}{\sum_{k=k_1}^{k_2-1} p_k}. 
\]

(38)

Combined (37) and (38), we obtain

\[
N_k = \frac{p_k^{1/2}}{\sum_{k=k_1}^{k_2-1} p_k} \sum_{k=k_1}^{k_2-1} N_k = \frac{p_k^{1/2}}{\sum_{k=k_1}^{k_2-1} p_k} n K', 
\]

(39)

where \(K' = K - ((k_1 - 1)n^{1-(1-\gamma)(\delta/2)-1}) - ((M - k_2 + 1)/n)\). Therefore, we obtain the optimal number of content \(k\), denoted by \(N_k^*\). (In this paper, we apply symbol \(\oplus\) to denote the optimal value.), i.e.,
Proof. By applying the condition \( \{k_1, k_2 - 1\} \in K_2 \), we derive that

\[
\frac{k_1}{k_2} = n^{(2a)/(1-v)((d/2)-1)(2a)/2}.
\]

When \( n \to \infty \), we obtain that \( K' \to K - ((k_1 - 1)/n^{(1-v)((d/2)-1)}) \). Since \( k_1 \in K_2 \), according to (40), we have \( N_{k_2} < n^{-(1-v)((d/2)-1)} \). Hence, it follows that

\[
n^{(1-v)((d/2)-1)}K' < k_2^{u/2}[H_{au2}(k_2) - H_{au2}(k_1 - 1)].
\] (44)

From the fact that \( k_1 \) is the smallest index in set \( K_2 \) such that \( N_{k_1} < n^{-(1-v)((d/2)-1)} \), we decrease \( k_1 \) by one, which will result in \( N_{k_1-1} \geq n^{-(1-v)((d/2)-1)} \). Hence, we obtain that

\[
n^{(1-v)((d/2)-1)}K' \geq (k_1 - 1)^{u/2}[H_{au2}(k_2) - H_{au2}(k_1 - 2)].
\] (45)

Combining (44) and (45), for \( k_1 > 1 \), we obtain the approximation value of \( k_2 \) scales as

\[
n^{(1-v)((d/2)-1)}K' \approx (k_1 - 1)^{u/2}[H_{au2}(k_2) - H_{au2}(k_1 - 1)].
\] (46)

Similarly, we can derive the approximation value of \( k_2 \). We know that \( k_2 \) is the smallest index in set \( K_3 \) such that \( N_{k_2} < n^{(11/2)-(1-v)((d/2)-(3/2))} \), which result in

\[
n^{(1/2)+(1-v)((d/2)-(3/2))}K' < k_2^{u/2}[H_{au2}(k_2) - H_{au2}(k_1 - 1)].
\] (47)

By substituting (40) into (34), we achieve the minimum delay, denoted by \( D(n)^{w} \).

\[
D(n)^{w} = \Theta \left( \sum_{k=1}^{n} p_k + \left( \sum_{k=k_1}^{k_2} p_k \right)^{2} + \sum_{k=k_2}^{M} n^{\nu/2}p_k \right). \quad (41)
\]

**Lemma 15.** In the clustered grid model, for \( n \to \infty \), the values of \( k_1 \) and \( k_2 \) is given as follows

\[
k_1 = \begin{cases} \Theta \left( n^{(1-v)((d/2)-1)} \right), & \alpha > 2, \\ \Theta \left( n^{(1-v)((d/2)-1)} / \log M \right), & \alpha = 2, \\ \min \left\{ M, \Theta \left( n^{(2a)/(1-v)((d/2)-1)} \right) \right\}, & 0 < \alpha < 2. \end{cases}
\]

\[
k_2 = \begin{cases} \min \left\{ M + 1, \Theta \left( n^{(a-2)/(a(1-v)((d/2)-1)+(2a)} \right) \right\}, & \alpha > 2, \\ M + 1, & \alpha \leq 2. \end{cases}
\] (42)

According to (43), \( k_2 \) can be obtained by

\[
k_2 = \frac{\alpha - 2}{2} n^{(1-v)((d/2)-1)}. \quad (52)
\]

(2) For \( \alpha = 2 \), we assume \( k_2 \leq M \), and (49) can be simplified as

\[
n^{(1/2)+(1-v)((d/2)-(3/2))} K - \frac{(k_1 - 1)}{n^{(1-v)((d/2)-1)}} \approx (k_1 - 1) \log k_2. \quad (53)
\]

which follows that

\[
(k_2 - 1) \left[ 1 + n^{(1-v)((d/2)-(3/2))-1/2} \right] \approx \frac{n^{(1/2)+(1-v)((d/2)-(3/2))} K}{\log k_2}, \quad (54)
\]

resulting in \( k_2 = nK \). This contradicts \( k_2 \leq M = n^y \), \( 0 < y < 1 \). Thus, we obtain \( k_2 = M + 1 \).

Assuming \( k_1 > 1 \), we can simplify (46) as

\[
n^{(1-v)((d/2)-1)} K - \frac{(k_1 - 1)}{n^{(1-v)((d/2)-1)}} = (k_1 - 1) \log k_2, \quad (55)
\]

by using (43), we obtain

\[
k_1 = \Theta (n^{(1-v)((d/2)-1)/\log M}).
\]
For $0 < \alpha < 2$: Assuming $1 \leq k_1 < k_2 \leq M$, we can simplify (49) as
\[
n^{(1/2)+1-(1/\nu)(1/(\nu+2)-1/2)} \left( K - \frac{k_1 - 1}{n^{(1-\nu)(1/(\nu+2)-1/2)}} \right) \approx (k_2 - 1)^{\alpha/2} \frac{(k_2 - 1)^{1-(\alpha/2)}}{1 - (\alpha/2)},
\]
which results in
\[
k_2 = \frac{k_1 - 1}{n^{(1-\nu)(1/(\nu+2)-1/2)}} K.
\]
This contradicts $k_2 \leq M$. Hence, we have $k_2 = M + 1$.

By simplifying (46), we have
\[
n^{(1-\nu)(1/(\nu+2)-1)} \left( K - \frac{k_1 - 1}{n^{(1-\nu)(1/(\nu+2)-1)}} \right) = (k_1 - 1)^{\alpha/2} M^{1-(\alpha/2)},
\]
resulting in $k_1 = \Theta(n^{(2\alpha)/(1-\nu)(1/(\nu+2)-1/2)})$.

**Theorem 16.** In the clustered grid model, letting $\omega = (1 - \nu) \times (\nu+2) - 1)$, according to the Zipf distribution, the optimal delay and throughput are given by w.h.p.

\[
D(n)^{\text{av}} = \frac{1}{M^\nu} \begin{cases} 
\Theta(1), & \alpha > 2, \\
\Theta\left(\frac{\log^2 M}{n^\nu}\right), & \alpha = 2, \\
\Theta\left(\frac{M^{2-\alpha}}{n^\nu}\right), & 1 < \alpha < 2, \\
\Theta\left(\frac{M}{n^\nu \log M}\right), & \alpha = 1, \\
\Theta\left(\frac{M}{n^\nu}\right), & 0 < \alpha < 1,
\end{cases}
\]

\[
\lambda(n)^{\text{av}} = \Theta(n^{1-\nu}), \quad \alpha > 2,
\]

\[
\Theta\left(\frac{1}{\log M}\right), \quad \alpha = 2,
\]

\[
\Theta\left(\frac{1}{M^{2-\nu}}\right), \quad 1 < \alpha < 2,
\]

\[
\Theta\left(\frac{\log M}{M}\right), \quad \alpha = 1,
\]

\[
\Theta\left(\frac{1}{M}\right), \quad 0 < \alpha < 1.
\]

**Proof.** We first prove the optimal delay based on the values of $k_1$ and $k_2$. Then, the optimal throughput can be easily derived by the tradeoff relation given by (26). Substituting the value of $p_\nu$ into (41), we have

\[
D(n)^{\text{av}} = \Theta\left(\frac{H_{\nu}(k_1 - 1)}{H_{\nu}(M)} + \frac{H_{\nu}(k_2 - 1) - H_{\nu}(k_1 - 1)}{n^{1-\nu}(1/(\nu+2)-1/2) K^\nu H_{\nu}(M)} + \frac{n^{\nu/2} H_\nu(M) - H_\nu(k_2 - 1)}{H_\nu(M)}\right).
\]

We divide the RHS of (60) into three terms, which are denoted by $D_1, D_2,$ and $D_3$, respectively. Then, we have following five conditions according to the values of $\alpha, k_1,$ and $k_2$.

1. For $\alpha > 2$: We have $D_1 = \Theta(1)$, and $D_2 = o(1)$. If $k_2 = M + 1$, we have $D_3 = 0$. Hence, we have $D^\nu = \Theta(1)$. If $k_2 < M + 1$, we have $D_3 = \Theta((\log^2 M)/n^{1-\nu}(1/(\nu+2)-1/2))$. Since $k_2 = M + 1$, we have we have $D_3 = 0$. Hence, we have $D^\nu = \Theta((\log^2 M)/n^{1-\nu}(1/(\nu+2)-1/2))$.

2. For $\alpha = 2$: We have $D_1 = \Theta(1)$, and $D_2 = \Theta((\log^2 M)/n^{1-\nu}(1/(\nu+2)-1/2))$. Since $k_2 = M + 1$, we have we have $D_3 = 0$. Hence, we have $D^\nu = \Theta((\log^2 M)/n^{1-\nu}(1/(\nu+2)-1/2))$.

3. For $1 < \alpha < 2$: We have $D_1 = \Theta(1)$, and $D_2 = \Theta(M^{2-\alpha}/n^{1-\nu}(1/(\nu+2)-1/2))$. $D_2$ is same with the condition (2), i.e., $D_3 = 0$. Hence, we have $D^\nu = \Theta(M^{2-\alpha}/n^{1-\nu}(1/(\nu+2)-1/2))$.

4. For $\alpha = 1$: We have $D_1 = \Theta(1)$, and $D_2 = \Theta(M/(n^{1-\nu}(1/(\nu+2)-1/2)) \log M)$. $D_3$ is same with the condition (2), i.e., $D_3 = 0$. Hence, we have $D^\nu = \Theta(M/n^{1-\nu}(1/(\nu+2)-1/2)) \log M$.

5. For $0 < \alpha < 1$: We have $D_1 = \Theta(1)$, and $D_2 = \Theta(M/n^{1-\nu}(1/(\nu+2)-1/2))$. $D_3$ is same with condition (2), i.e., $D_3 = 0$. Hence, we have $D^\nu = \Theta(M/n^{1-\nu}(1/(\nu+2)-1/2))$.

In Figure 3(a), we have plotted the optimal delay performance of the clustered grid model for different values of $\alpha$ according to the Theorem 16. Similarly, the optimal throughput results for various values of $\alpha$ are plotted in Figure 3(b). We adopt $\nu = 0.5$ and $\gamma = 0.8$ for both figures. We observe that the delay curves appear the ascending tendency while the throughput curves present descending tendency as the number of mobile nodes $n$ increases. We further find that when the number of nodes is fixed, the optimal delay shows a decreasing trend while the optimal throughput increases as $\alpha$ increases. From the simulations, we can conclude that the most popular contents are mainly cached and transmitted by mobile nodes when $\alpha$ is large, which reduces the content transmission time and increases the number of simultaneous transmissions. That is, the advantage of caching is more obvious as $\alpha$ increases.

### 4.2. Clustered Random Model

In the clustered random model, we first formulate the optimal delay problem according to Theorem 13, and then, we design the optimal cache allocation strategy to achieve the minimum delay. Combining Theorem 13 and the cache constraint, the objective function of minimum delay is given by

\[
\text{minimize} \sum_{k=1}^{M} p_k E'(H_k), \\
\text{subjectto} \sum_{k=1}^{M} N_k \leq nK, \\
1 \leq N_k \leq n^{1-(1-\nu)/(\nu+2)-1/2} \log^{-(\nu+2)} n,
\]
where $E'(H_k)$ is given by

$$E'(H_k) = \begin{cases} 69(1), & N_k = \Omega \left( \frac{n^{1-(1-L)/(6/2)-1}}{\log n} \right), \\ 69 \left( \frac{n^{1-(1-L)/(6/2)-1}}{N_k \log n} \right), & N_k = \Omega \left( \frac{n^{1-(1-L)/(6/2)-1}}{\log^2 n} \right), \\ 69 \left( \frac{n^{(1/2)-1-L/2}}{\sqrt{\log n}} \right), & N_k = \Omega \left( \frac{n^{(1/2)-1-L/2}}{\log^2 n} \right). \end{cases}$$

The optimization process is similar with the clustered grid model. Hence, we give a brief description.

We first define three sets $K'_1, K'_2$, and $K'_3$ according to the size of $N_k$, respectively. Let $K'_1 = \{1, 2, \cdots, k_1 - 1\}$ be the set of contents such that $N_k = n^{1-(1-L)/(6/2)-1}/\log^{6/2} n$, let $K'_2 = \{k_1, k_1 + 1, \cdots, k_2 - 1\}$ be the set of contents such that $N_k \in \left[n^{1/2} - (1-L)/(6/2) - (3/2)/\log^{6/2} n, n^{1-(1-L)/(6/2)-1}/\log^{6/2} n\right]$. And let $K'_3 = \{k_2, k_2 + 1, \cdots, M\}$ be the set of contents such that $N_k = 1$.

Then, we take the Lagrangian multiplier $\lambda \in \mathbb{R}^+$ for the first constraint in (61) and combine with the second
constraint; the necessary conditions for the minimal \( D(n) \) are given as:

\[
\frac{\partial D(n)}{\partial N_k} = -\frac{p_k n^{1-(1-\nu)/(\delta/2)-1}}{N_k^2 \log^{\delta/2} n} \begin{cases} 
-\lambda, & \forall k \in K_1', \\
=\lambda, & \forall k \in K_2', \\
\geq-\lambda, & \forall k \in K_3', 
\end{cases}
\]

(63)

Next, we apply the similar calculation method to obtain the values of \( N' \). Hence, the minimum delay in the clustered random model is given by

\[
D(n)^{\alpha_0} = \Theta\left( \sum_{k=1}^{k-1} p_k + \frac{\left(\sum_{k=1}^{k-1} p_k^{1/2}\right)^2}{n^{(1-\nu)(\delta/2)-1} K' \log^{\delta/2} n} + \sum_{k=k_1}^{M} n^{\nu/2} p_k \right).
\]

(64)

Before obtaining the minimum delay \( D(n)^{\alpha_0} \), we need to estimate the value of \( k_1 \) and \( k_2 \) by the following lemma.

**Lemma 17.** In the clustered random model, for \( n \rightarrow \infty \), the values of \( k_1 \) and \( k_2 \) is given as follows

\[
k_1 = \begin{cases} 
\Theta\left(n^{(1-\nu)/(\delta/2)-1} \log^{(\delta/2)} n\right), & \alpha > 2, \\
\Theta\left(n^{(1-\nu)/(\delta/2)-1} \log^{\delta/2} n \log^{\delta/2} n\right), & \alpha = 2, \\
\min\left\{ \Theta\left(n^{(1-\nu)/(\delta/2)-1} \log^{\delta/2} n\right), \Theta\left(Mn^{(2/\alpha)(1-\nu)/(\delta/2)-1} \log^{\delta/2} n\right) \right\}, & 0 < \alpha < 2, 
\end{cases}
\]

(65)

\[
k_2 = \begin{cases} 
M + 1, & \alpha > 2, \\
M + 1, & \alpha = 2, \\
\min\left\{ M + 1, \Theta\left(n^{(2-\nu)/(\delta/2)-1} \log^{\delta/2} n\right) \right\}, & 0 < \alpha < 2. 
\end{cases}
\]
The proof of this theorem is similar to that in Lemma 15. We omit it for simplicity.

Combining Lemma 17 and the different values of $\alpha$, the optimal delay and throughput in the clustered random model are given as follows:

**Theorem 18.** In the clustered random model, letting $\omega = (1 - \nu)((\delta/2) - 1)$, according to the Zipf distribution, the optimal delay and throughput are given by w.h.p.

\[
D(n)^{\alpha} = \begin{cases} 
\Theta(1), & a > 2, \\
\Theta\left(\frac{\log^2 M}{n^a \log^2 n}\right), & a = 2, \\
\min \left\{ \Theta\left(\frac{M^{\alpha}}{n^a \log^2 n}\right), \Theta\left(\frac{n^{\alpha}}{\log^2 n}\right) \right\}, & 1 < a < 2, \\
\min \left\{ \Theta\left(\frac{M^2}{n^{\alpha}}\right), \Theta\left(\frac{M^{\alpha}}{n^4 \log^2 n}\right) \right\}, & a = 1, \\
\min \left\{ \Theta\left(\frac{M}{n^{\alpha}}\right), \Theta\left(\frac{M^{\alpha}}{n^{4a} \log^2 n}\right) \right\}, & 0 < a < 1, 
\end{cases}
\]

\[
\lambda(n)^{\alpha} = \begin{cases} 
\Theta\left(\frac{n^{\alpha}}{\log^2 n}\right), & a > 2, \\
\Theta\left(\frac{1}{\log^2 M}\right), & a = 2, \\
\max \left\{ \Theta\left(\frac{1}{M^{\alpha}}\right), \Theta\left(\frac{\log^2 M}{n^{\alpha}}\right) \right\}, & 1 < a < 2, \\
\max \left\{ \Theta\left(\frac{\log M}{M}\right), \Theta\left(\frac{\log M \log^2 n}{M^{\alpha}}\right) \right\}, & a = 1, \\
\max \left\{ \Theta\left(\frac{1}{M}\right), \Theta\left(\frac{M^{1-a} \log^2 (\delta/2)^2 n}{n^{\alpha} \log^2 n}\right) \right\}, & 0 < a < 1. 
\end{cases}
\]

**Proof.** The proof of this theorem is similar to that in Theorem 16. Hence, we neglect it for simplicity.

In Figure 4, we depict the optimal delay and throughput performance of the clustered random model for different values of $\alpha$. We also adopt $\nu = 0.5$ and $\gamma = 0.8$ for both figures. In particular, for $a < 2$, we choose the smaller value of the two delay and we choose the larger value of the two throughput. In Figure 5, we plot the comparison results of the scaling performance of the different clustering models for $\alpha = 1.5$. We observe that the delay performance is better in the clustered random model than that in the clustered grid model while the throughput degrades in the clustered random model. This can be explained that the transmission range in the clustered random model is larger than that in the clustered grid model, which leads to the decreased transmission hops in the clustered random model accordingly. On the contrary, the larger transmission range covers more network areas, which results in the decrease concurrent transmissions.

### 5. Conclusions and Future Works

In this paper, we analyzed the throughput and delay performance of mobile content-centric networks, where the node spatial distribution of nodes is not uniform. We adopted a cell-partition TDMA scheduling scheme and proposed a distributed multihop routing scheme to achieve the throughput-delay tradeoff in the clustered grid model and clustered random model, respectively. Moreover, according to the content popularity distribution, we applied Lagrangian relaxation method to optimize the cache allocation in two kinds of clustered models. Finally, we obtained the optimal throughput and delay in the mobile content-centric networks. Our theoretical results were validated by the numerical simulations.

There are several problems left for future research, for example, the scaling laws and the caching optimization for mobile content-centric networks with infrastructures support. In addition, the multicast capacity of nonuniform mobile content-centric networks has not been studied. Finally, it is meaningful to optimize cache allocation strategy when the distribution of contents is unknown.

### Data Availability

The data used to support the findings of this study are available from the authors upon request.

### Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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