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*Research Article*

# Active Fault-Tolerant/Active Passive Intrusion-Tolerant $H_\infty$ Cooperative Control of Discrete NCS under the Background of Big Data

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For the linear discrete networked control system (NCS) which may suffer DoS attack on both sides of the controller, when the actuator has time-varying failure, the intelligent sensor unit uses wireless sensors to collect data. According to the large amount of data collected, the active fault tolerance/active passive capacity of linear discrete NCS under the discrete event-triggered communication mechanism (DETCS) is studied. The problem of cooperative controller design is discussed. Firstly, a linear discrete NCS model integrating DETCS, actuator fault, and network attack is established. Then, based on the idea of integral sliding mode control, an active fault-tolerant/attack active passive intrusion-tolerant cooperative controller is designed, and the actuator attack side network attack and sensor side network attack are extended to the state to obtain a new state vector. Then, an adaptive Kalman filter estimator (AKF) estimates the fault and attack information and then adjusts the initial fault-tolerant/intrusion-tolerant cooperative controller in real time according to the estimated information obtained by the adaptive Kalman filter estimator; finally, the MATLAB simulation example is used to verify the improvement of system performance by the designed control law and the saving of network resources by the introduction of DETCS.

## 1. Introduction

Compared with the traditional control system, NCS has the unique properties of remote operation, remote monitoring, security detection and resource sharing due to the existence of networks. However, due to the existence of networks, the structure of the control system is more complex, the data transmission will be affected to a certain extent, the system is more prone to component failure, and network transmission is more vulnerable to attack, so the physical fault tolerance is very important The research of intrusion control and network attack tolerance control has great practical significance and practical application value [1–3].

With the rapid development of various wireless sensor technologies, people can gradually perceive the real world through wireless sensors in every corner of the world, even in places where people rarely visit. These wireless sensors transmit the data to the computer on people's worktable through the network, which is convenient for scholars to carry out the corresponding research on all kinds of data. Generally speaking, fault-tolerant control (FTC) technology is divided into passive fault-tolerant control (PFTC) [4–6] and active fault-tolerant control (AFTC) [7–10]. PFTC uses a controller to make the closed-loop system insensitive to specific faults and maintain the stability and performance of the system. On the basis of acquiring fault information, AFTC designs a fault-tolerant controller to ensure the performance and stability of the system. For the actuator fault of the system, Zhang et al. proposed an adaptive observer [8, 9] by which the system state and fault can be obtained and the fault can be adjusted. Reference [10] studies the problem of adaptive fault-tolerant control for uncertain actuator fault compensation of linear time invariant NCS with unknown plant parameters and actuator fault parameters. Intrusion tolerance is developed on the basis of fault tolerance, but it is very different from fault tolerance. It is mainly reflected in that network attacks are malicious in motivation and difficult to judge in form. Denial of service (DoS) attack is a kind of attack means by weakening and preventing legitimate users from using legitimate network resources, which can affect the normal use of the network. Reference [11] studies the stability of control and measurement packets transmitted over a communication network under DoS attack. Reference [12] studies the event-triggered attack-tolerant tracking control problem of nonlinear NCS under sudden DoS attack. In Reference [13], the progress of security control and attack detection of industrial information physical fusion system is reviewed from the perspective of control theory. Although many achievements have been made in fault tolerance and attack tolerance of NCS, most of them are limited to independent design. For NCS with actuator failure and network attack coexisting, the research on comprehensive security control of fault tolerance and intrusion tolerance is rarely involved, and only literature [14] and literature [15] can be consulted at present. However, in actual NCS, it is inevitable that faults and attacks occur simultaneously.

In view of this, aiming at the actuator fault and DoS network attack on both sides of the controller, this paper establishes a closed-loop linear NCS fault/attack coexistence model by constructing the active fault-tolerant/active passive intrusion-tolerant system structure of NCS under DETCS with the help of the joint modeling method of network attack and controlled object. In addition, adaptive Kalman filter estimators are designed, respectively, for online monitoring of state, attack, and fault. The active fault-tolerant control is used to compensate the actuator network attack with the active intrusion-tolerant strategy, and the passive intrusion-tolerant mechanism is used to be robust to the sensor network attack, so as to realize the state feedback and active fault-tolerant and active passive intrusion-tolerant cooperative control.

## 2. Material and Method

*2.1. System Model.* Controlled object model:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Ef(k) + N_a a_a(k) + D_1 w(k), \\ y(k) = Cx(k) + N_s a_s(k) + D_2 v(k). \end{cases} \tag{1}$$

Among them, $x(k) \in R^n$, $y(k) \in R^m$, and $u(k) \in R^l$ are the state, output, and control input of the system, respectively; $f(k) \in R^p$ is the time-varying fault vector of the actuator; $a_a(k) \in R^q$ is the constant network attack vector suffered by the actuator side, referred to as the actuator attack vector; $a_s(k) \in R^q$ is the constant network attack vector suffered by the sensor side, referred to as the sensor attack vector; $A \in R^{n\times n}$, $B \in R^{n\times l}$, $C \in R^{m\times n}$, $E \in R^{n\times p}$, and $N_s \in R^{n\times q}$ are the coefficient matrices of proper dimension; and $w(k) \in R^p$ and $v(k) \in R^p$ are the perturbations of bounded energy. They are Gaussian white noise sequences with 0 mean value and independent of each other. Their covariance matrices are $Q \in R^{p\times p}$ and $R \in R^{p\times p}$; $D_1 \in R^{n\times p}$ and $D_2 \in R^{m\times p}$ are the perturbations of proper dimension.

Referring to the idea of state augmentation, the system state $x(k)$, actuator attack $a_a(k)$, and sensor attack $a_s(k)$ are augmented into a new state $\eta(k)$, which is called the augmented state. The augmented system model is

$$\begin{cases} \eta(k+1) = \tilde{A}\eta(k) + \tilde{B}u(k) + \tilde{E}f(k) + \tilde{D}_1\tilde{w}(k), \\ y(k) = \tilde{C}\eta(k) + D_2 v(k), \end{cases} \tag{2}$$

where

$$\eta(k) = \begin{bmatrix} x^T(k) \\ a_a^T(k) \\ a_s^T(k) \end{bmatrix}, \tilde{w}(k) = \begin{bmatrix} w^T(k) \\ b_a^T(k) \\ b_s^T(k) \end{bmatrix}, \tilde{A} = \begin{bmatrix} A & N_a & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\tilde{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \tilde{E} = \begin{bmatrix} E \\ 0 \\ 0 \end{bmatrix}, \tilde{D}_1 = \begin{bmatrix} D & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \tilde{C} = \begin{bmatrix} C \\ 0 \\ N_s \end{bmatrix}^T. \tag{3}$$

*2.2. DETCS Trigger Conditions.* This paper adopts the most representative discrete event trigger condition [16]:

$$[x\wedge(k) - x\wedge(t_k)]^T \mathbf{\Phi}[\hat{x}(k) - \hat{x}(t_k)] \le \sigma x\wedge^T(t_k)\mathbf{\Phi}\hat{x}(t_k). \tag{4}$$

Among them, $\hat{x}(k)$ is the estimated value of the state at the current sampling time, $\hat{x}(t_k)$ is the latest estimated value of the state at the network transmission time that satisfies equation (4) at the previous time, $\mathbf{\Phi}$ is the event trigger matrix with positive definite symmetry, and $\sigma \in [0, 1)$ is the event trigger parameter scalar, which can be set in advance and is related to the expected performance of the system. Therefore, only when the sampling data $\hat{x}(k)$ meets the trigger condition of equation (4) can it be transmitted from the sensor network to the controller. A lot of network resources is saved, so as to improve the utilization of the network. The system control structure is shown in Figure 1.

Define delay function $\tau(k) = k - t_k, k \in [t_k, t_{k+1})$, and $0 \le \tau(k) \le \tau_M$. Among them, $\tau_M \triangleq T_{k\max} + \tilde{\tau}$ are the maximum nonuniform transmission periods of data filtering, and $\tilde{\tau}$ is the upper bound of delay function $\tau(k)$ at $t_k + 1$.

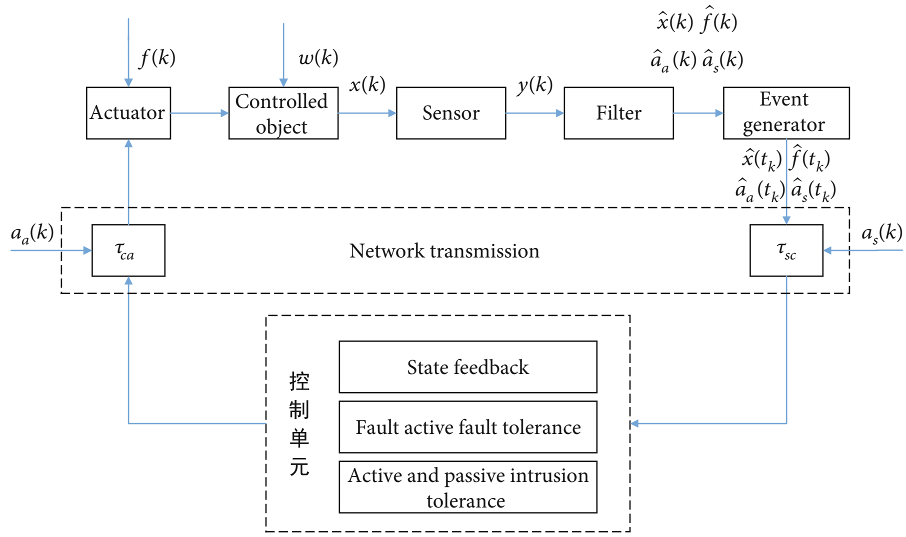


Figure 1: System control structure diagram.

*2.2.1. Related Lemmas*

**Lemma 1** (Schur complement lemma [17]). *For a given symmetric matrix*

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^T & Z_{22} \end{bmatrix}, \tag{5}$$

*the following three conditions are equivalent:*

$$\begin{gathered} Z < 0, \\ Z_{11} < 0, Z_{22} - Z_{12}^T Z_{11}^{-1} Z_{12} < 0, \\ Z_{22} < 0, Z_{11} - Z_{12} Z_{22}^{-1} Z_{12}^T < 0. \end{gathered} \tag{6}$$

**Lemma 2** (Wirtinger inequality in discrete form [18, 19]). *For a given positive definite matrix $N \in R^{n\times n}$, scalar $0 \le \rho_1 \le \rho_2$ and vector function $\eta : [-\rho_2, \rho_1] \longrightarrow R^{n\times n}$ satisfy the following inequalities:*

$$-(\rho_2-\rho_1)\sum_{s=k-\rho_2}^{k-\rho_1-1}\eta^T(s)N\eta(s) \le -\begin{bmatrix} \mathbf{\Omega}_1 \\ \mathbf{\Omega}_2 \end{bmatrix}^T \begin{bmatrix} N & 0 \\ 0 & 3N \end{bmatrix} \begin{bmatrix} \mathbf{\Omega}_1 \\ \mathbf{\Omega}_2 \end{bmatrix}. \tag{7}$$

*Among them,*

$$\begin{aligned} \eta(s) &= x(s+1) - x(s), \\ \mathbf{\Omega}_1 &= x(k-\rho_1) - x(k-\rho_2), \\ \mathbf{\Omega}_2 &= x(k-\rho_1) + x(k-\rho_2) - \frac{2}{\rho_2-\rho_1+1}\sum_{s=k-\rho_2}^{k-\rho_1} x(s). \end{aligned} \tag{8}$$

*2.3. Preliminary Design of Active Fault-Tolerant/Active Passive Intrusion-Tolerant Cooperative Control Law.* For system (1), the control law can be designed as follows:

$$u(k) = u_n(k) + u_{fa}(k). \tag{9}$$

Among them, $u_n(k)$ is the nominal control law when there is no fault and no attack, and $u_{fa}(k)$ is the compensation control law when fault and attack occur simultaneously.

Considering the time delay problem, the sliding mode function can be designed as follows:

$$S(k) = Gx(k-\tau(k)) + \sum_{i=k-\tau(k)}^{k-1} GBu(i) + \mathbf{\Gamma}(k). \tag{10}$$

Among them, $G$ is a constant matrix with proper dimension, and $GB$ is a nonsingular matrix; $\mathbf{\Gamma}(k)$ is a sliding mode discrete compensator, which satisfies the following requirements: $\mathbf{\Gamma}(k+1) = \mathbf{\Gamma}(k) - G(A-BK)x(k-\tau(k)) + Gx(k-\tau(k))$, where $K$ is the gain matrix of the controller; the sliding mode of the system can be obtained by introducing $\sum_{i=k-\tau(k)}^{k-1} GBu(i)$.

Omitting $w(k)$ and $v(k)$, combining formula (1) and formula (10), we can get

$$\begin{aligned} \Delta S(k) &= S(k+1) - S(k)Gx(k+1-\tau(k+1)) + \sum_{i=k+1-\tau(k)}^{k} GBu(i) \\ &\quad + \Gamma(k+1) - Gx(k-\tau(k)) + \sum_{i=k-\tau(k)}^{k-1} GBu(i) + \Gamma(k) \\ &= GAx(k-\tau(k)) + GBu(k-\tau(k)) + GEf(k-\tau(k)) \\ &\quad + GN_a a_a(k-\tau(k)) - Gx(k-\tau(k)) + GBu(k) \\ &\quad - GBu(k-\tau(k)) + Gx(k-\tau(k)) - GAx(k-\tau(k)) \\ &\quad + GBKx(k-\tau(k)) = GBKx(k-\tau(k)) + GBu(k) \\ &\quad + GEf(k-\tau(k)) + GN_a a_a(k-\tau(k)). \end{aligned} \tag{11}$$

If the order is $\Delta S(k)=0$, there will be

$$u(k)=-(GB)^{-1}[GBKx(k-\tau(k))+GEf(k-\tau(k))+GN_a a_a(k-\tau(k))]. \tag{12}$$

Furthermore, the sliding mode control law $u(k)$ based on the reaching law method is

$$\begin{aligned}u(k)=&-Kx(k-\tau(k))-(GB)^{-1}[qS(k)+\varepsilon\ \text{sgn}\ S(k)]\\&-(GB)^{-1}[GEf(k-\tau(k))+GN_a a_a(k-\tau(k))]\\&+N_s a_s(k-\tau(k)),\end{aligned} \tag{13}$$

where $q, \varepsilon$ is a constant scalar and $q>0, \varepsilon>0$.

When there is no fault and no attack, the nominal control law $u_n(k)$ is

$$u_n(k)=-Kx(k-\tau(k))-(GB)^{-1}(qS(k)+\varepsilon\ \text{sgn}\ S(k)). \tag{14}$$

When faults and attacks occur simultaneously, the compensation control law $u_{fa}(k)$ is

$$u_{fa}=-(GB)^{-1}[GEf(k-\tau(k))+GN_a a_a(k-\tau(k))]. \tag{15}$$

**Theorem 3.** *When constant $q, \varepsilon$ satisfies $q>0, \varepsilon>0$, respectively, the system (1) satisfies the existence and reachability conditions of the sliding mode.*

*Proof.* Select Lyapunov function: $V(k)=S^T(k)S(k)$.

Combining formulas (1) and (10), we can get

$$\begin{aligned}\Delta V(k)&=S^T(k)\Delta S(k)=S^T(k)[S(k+1)-S(k)]\\&=S^T(k)[GBKx(k-\tau(k))+GBu(k)\\&\quad+GEf(k-\tau(k))+GN_a a_a(k-\tau(k))].\end{aligned} \tag{16}$$

According to the control law (13),

$$\Delta V(k)=S^T(k)[-qS(k)-\varepsilon\ \text{sgn}\ S(k)]. \tag{17}$$

When the constants $q, \varepsilon$ satisfy $q>0, \varepsilon>0$, there are

$$\Delta V(k)=-S^2(k)-\varepsilon S(k)\le 0. \tag{18}$$

Therefore, the reachability condition of the sliding surface is satisfied. So, the system state can reach the sliding surface in finite time.

When the system is on the sliding mode surface, i.e., $S(k)=0$, the equivalent control term of the sliding mode control law can be obtained by substituting it into equation (13):

$$\begin{aligned}u_{eq}(k)=&-Kx(k-\tau(k))-(GB)^{-1}[GEf(k-\tau(k))\\&+GN_a a_a(k-\tau(k))]+N_s a_s(k-\tau(k)).\end{aligned} \tag{19}$$

By substituting equation (19) into equation (1), the closed-loop model of NCS on a sliding surface can be obtained:

$$\begin{cases}x(k+1)=Ax(k)-Kx(k-\tau(k))-B(GB)^{-1}[GEf(k-\tau(k))+GN_a a_a(k-\tau(k))]+BN_s a_s(k-\tau(k))+Ef(k)+N_a a_a(k)+D_1 w(k),\\ y(k)=Cx(k)+N_s a_s(k)+D_2 v(k),\\ x(k)=\phi(k), -\tau_M\le k\le 0,\end{cases} \tag{20}$$

where $\phi(k)$ is a real valued initial function on $[-\tau_M, 0]$. □

*2.4. Design of Adaptive Kalman Filter Fault/Attack Estimator.* For augmented system (2), the following adaptive Kalman filter fault/attack estimator is designed.

The prediction part of the adaptive Kalman filter fault/attack estimator is as follows:

$$\begin{cases}\hat{\eta}(k+1\mid k)=\tilde{A}\hat{\eta}(k)+\tilde{B}u(k)+\tilde{E}\hat{f}(k),\\ P(k+1\mid k)=\tilde{A}P(k-1\mid k-1)\tilde{A}^T+\tilde{D}_1 Q\tilde{D}_1^T,\\ \tilde{y}(k+1)=y(k+1)-\tilde{C}\hat{\eta}(k+1\mid k).\end{cases} \tag{21}$$

Among them, $P\in R^{n\times n}$ is the state error covariance matrix, and $\tilde{y}(k)$ is the error between the system output and the estimated output.

The gain matrix of adaptive Kalman filter fault/attack estimator is as follows:

$$\begin{cases}\Sigma(k)=\tilde{C}P(k\mid k-1)\tilde{C}^T+D_2RD_2^T,\\ K_1(k)=P(k\mid k-1)\tilde{C}^T\Sigma^{-1}(k).\end{cases} \tag{22}$$

Among them, $K_1(k)$ is the gain matrix of Kalman filter.

The error gain matrix of adaptive Kalman filter fault/attack estimator is as follows:

$$\begin{cases} \mathbf{\Omega}(k) = \tilde{C}\gamma(k) + \tilde{C}\tilde{E}, \\ \gamma(k+1) = \left(\tilde{A} - K_1(k)\tilde{C}\right)\gamma(k) + \tilde{E}, \\ F(k) = J^T(k-1)\mathbf{\Omega}^T(k)\mathbf{\Lambda}^T(k), \\ J(k) = \dfrac{1}{\lambda}\left[I - J(k-1)\mathbf{\Omega}^T(k)\mathbf{\Lambda}(k)\mathbf{\Omega}(k)\right]J(k-1), \\ \mathbf{\Lambda}(k) = \left[\lambda\Sigma(k) + \mathbf{\Omega}(k)J(k-1)\mathbf{\Omega}^T(k)\right]^{-1}. \end{cases} \tag{23}$$

Among them, $\gamma(k)$, $F(k)$ is the error gain matrix, and $\lambda \in [0, 1)$ is the forgetting factor.

The update part of adaptive Kalman filter fault/attack estimator is as follows:

$$\begin{cases} \hat{\eta}(k+1) = \hat{\eta}(k+1 \mid k) + K_1(k)\tilde{y}(k) + \gamma(k+1)\left(\hat{f}(k+1) - \hat{f}(k)\right), \\ \hat{f}(k+1) = \hat{f}(k) + F(k)\tilde{y}(k), \\ P(k+1 \mid k+1) = \left[I - K_1(k)\tilde{C}\right]P(k+1 \mid k). \end{cases} \tag{24}$$

Suppose 4. Matrix $[\tilde{A}, \tilde{C}]$ is completely observable, $[\tilde{A}, Q^{1/2}]$ is completely controllable, if matrix $\xi(k)$ satisfies

$$\xi(k+1) = \left(\tilde{A} - K_1(k)\tilde{C}\right)\xi(k). \tag{25}$$

Then, for Kalman gain matrix $K_1(k)$, $\xi(k)$ are exponentially convergent.

Suppose 5. Under initial condition $J(0) = \omega I (\omega > 0)$, matrix $J(k)$ is strictly positive definite.

System error definition:

$$\begin{cases} \tilde{\eta}(k) = \eta(k) - \hat{\eta}(k), \\ \tilde{f}(k) = f(k) - \hat{f}(k), \\ z(k) = \tilde{\eta}(k) - \gamma(k)\tilde{f}(k). \end{cases} \tag{26}$$

From equations (2), (21), and (26),

$$z(k+1) = \left(\tilde{A} - K_1(k)\tilde{C}\right)z(k) - K_1(k)D_2\nu(k) + \tilde{D}_1\tilde{w}(k). \tag{27}$$

Further, we can get

$$E[z(k+1)] = \left(\tilde{A} - K_1(k)\tilde{C}\right)E[z(k)], \tag{28}$$

where $E[z(k)]$ is the mean of $z(k)$.

From hypothesis 1 and equation (28), we can see that 14 is exponentially convergent.

From equations (21) and (24),

$$\tilde{f}(k+1) = (I - F(k)\mathbf{\Omega}(k))\tilde{f}(k) - F(k)\left(\tilde{C}z(k) + D_2\nu(k)\right). \tag{29}$$

Because of $E[z(k)] = 0$, $E[\nu(k)] = 0$,

$$E\left[\tilde{f}(k+1)\right] = [I - F(k)\mathbf{\Omega}(k)]E\left[\tilde{f}(k)\right]. \tag{30}$$

From hypothesis 2, we can see that $J(k)$ is strictly positive definite.

**Theorem 6.** *When matrix $J(k)$ is strictly positive definite and $E[\tilde{\eta}(k)]$ exponentially approaches 0, the adaptive Kalman filter is convergent. Note $M(k) = J^{-1}(k)$.*

*Proof.* Define Lyapunov function:

$$V(k+1) = \left(E\left[\tilde{f}(k+1)\right]\right)^T M(k)E\left[\tilde{f}(k+1)\right]. \tag{31}$$

From equation (23), we can get

$$J(k) = \frac{1}{\lambda}[I - F(k)\mathbf{\Omega}(k)]J(k-1). \tag{32}$$

Then,

$$M(k) = \lambda M(k-1)[I - F(k)\mathbf{\Omega}(k)]^{-1}. \tag{33}$$

From equations (32) and (33),

$$\begin{aligned} V(k+1) &= \left(E\left[\tilde{f}(k)\right]\right)^T [I - F(k)\mathbf{\Omega}(k)]^T \lambda M(k-1)E\left[\tilde{f}(k)\right] \\ &= \lambda\left(E\left[\tilde{f}(k)\right]\right)^T M(k-1)E\left[\tilde{f}(k)\right] \\ &\quad - \lambda\left(E\left[\tilde{f}(k)\right]\right)^T \Xi E\left[\tilde{f}(k)\right]. \end{aligned} \tag{34}$$

In equation (34),

$$\begin{aligned} \Xi(k) &= \mathbf{\Omega}^T(k)\mathbf{\Gamma}^T(k)M(k-1) \\ &= \mathbf{\Omega}^T(k)\mathbf{\Lambda}(k)\mathbf{\Omega}(k)J(k-1)J^{-1}(k-1) \\ &= \mathbf{\Omega}^T(k)\mathbf{\Lambda}(k)\mathbf{\Omega}(k). \end{aligned} \tag{35}$$

Since $\mathbf{\Lambda}(k)$ is a positive definite matrix, then $\Xi(k)$ is also a positive definite matrix. So,

$$V(k+1) \le \lambda\left(E\left[\tilde{f}(k)\right]\right)^T M(k)E\left[\tilde{f}(k)\right] \le \lambda V(k). \tag{36}$$

It can be seen from equation (36) that $V(k)$ is exponentially close to 0. Since matrix $M(k)$ is strictly positive definite, $E\tilde{f}(k)$ is exponentially close to 0.

From equation (26),

$$E[\tilde{\eta}(k)] = E[z(k)] + \gamma(k)E\left[\tilde{f}(k)\right]. \tag{37}$$

□

Then, $E[\tilde{\eta}(k)]$ is also an index close to 0, so the adaptive Kalman filter is convergent.

## 3. Analysis

The online fault-tolerant/intrusion-tolerant control for system (1) can be described as follows: using the estimated value of system state $\hat{x}(k)$, actuator fault $\hat{f}(k)$, and network attack $\hat{a}_a(k)$ on the actuator side, considering the event triggering conditions, substituting the estimated value satisfying the conditions into control law $u(k)$ (13), the adjusted fault-tolerant/intrusion-tolerant control law $u(k)$ is

$$\begin{aligned} u(k) = &-K\hat{x}(t_k) - (GB)^{-1}[qS(k) + \varepsilon \operatorname{sgn} S(k)] \\ &- (GB)^{-1}\left[GE\hat{f}(t_k) + GN_a\hat{a}_a(t_k)\right] + N_s a_s(t_k). \end{aligned} \tag{38}$$

Suppose 7. Actuator fault $f(k)$ is bounded, i.e., $\|f(k)\| \le \rho_1$; $a_a(k)$ is bounded actuator attack, i.e., $\|a_a(k)\| \le \rho_2$, where $\rho_1, \rho_2$ are known functions greater than 0.

The existence condition and global reachability condition of sliding mode are

$$S^T(k)\Delta S(k) < 0. \tag{39}$$

By substituting equation (38) into equation (39), we find that

$$\begin{aligned} S^T(k)\Delta S(k) &= S^T(k)[GBKx(t_k) + GBu(k) + GEf(t_k) + GN_a a_a(t_k)] \\ &= S^T(k)\Big[GBKx(t_k) + GEf(t_k) + GN_a a_a(t_k) \\ &\quad + GB\Big(-K\hat{x}(t_k) - (GB)^{-1}\Big(qS(k) + \varepsilon \operatorname{sgn} S(k) \\ &\quad + GE\hat{f}(t_k) + GN_a\hat{a}_a(t_k)\Big)\Big)\Big] \\ &= S^T(k)\Big[GBK(x(t_k) - \hat{x}(t_k)) + GE\Big(f(t_k) - \hat{f}(t_k)\Big) \\ &\quad + GN_a(a_a(t_k) - \hat{a}_a(t_k)) - qS(k) - \varepsilon \operatorname{sgn} S(k)\Big]. \end{aligned} \tag{40}$$

From hypothesis 3,

$$\left\|f(t_k) - \hat{f}(t_k)\right\| \le \rho_1, \|a_a(t_k) - \hat{a}_a(t_k)\| \le \rho_2 \tag{41}$$

can be obtained.

Then,

$$\begin{aligned} S^T(k)\Delta S(k) \le &-qS^2(k) - \varepsilon\left|S^T(k)\right| + \left|S^T(k)\right|(GBKe \\ &+ GE\rho_1 + GN_a\rho_2). \end{aligned} \tag{42}$$

When the constant $q, \varepsilon$ satisfies the following inequality:

$$\varepsilon \ge (GBKe + GE\rho_1 + GN_a\rho_2)\,;\, q > 0. \tag{43}$$

Then, $S^T(k)\Delta S(k) \le -qS^2(k) \le 0$.

Therefore, the existence and reachability conditions of the sliding mode of the system (20) are satisfied, which means that the starting point from any initial state other than the sliding mode surface $S(k) = 0$ can return to the sliding mode surface in a finite time.

When the system is on the sliding surface, the equivalent control term of the controller can be obtained by combining equation (54) with equation $S(k) = 0$:

$$u_{eq}(k) = -K\hat{x}(t_k) + N_s a_s(t_k) - (GB)^{-1}\left[GE\hat{f}(t_k) + GN_a\hat{a}_a(t_k)\right]. \tag{44}$$

By substituting equation (44) into equation (1), the state space model of the closed-loop system on the sliding surface can be obtained as follows:

$$\begin{cases} x(k+1) = Ax(k) + Ef(k) + N_a a_a(k) + D_1 w(k) + B, \\ \left[-K\hat{x}(t_k) - (GB)^{-1}\left(GE\hat{f}(t_k) + GN_a\hat{a}_a(t_k)\right)\right] + N_s a_s(t_k), \\ y(k) = Cx(k) + N_s a_s(k) + D_2 v(k), \\ x(k) = \phi(k), k \in [-\tau_M, 0], \end{cases} \tag{45}$$

where $\phi(k)$ is a real valued initial function on $[-\tau_M, 0]$.

Since the system has satisfied the reachability condition of the sliding mode surface, in order to ensure the stability, the sliding mode described by system (45) can be stabilized by designing an appropriate gain matrix $K$:

$$\begin{bmatrix} X_1 & * & * & * & * & * & * & * & * & * & * & * & * \\ Y_1 & -Q_1 & * & * & * & * & * & * & * & * & * & * & * \\ 2S_1 & 0 & -4S_1 & * & * & * & * & * & * & * & * & * & * \\ 6S_1 & 0 & 6S_1 & -12S_1 & * & * & * & * & * & * & * & * & * \\ \tau_M^2(BK)^T & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * & * & * & * \\ \tau_M^2(BK)^T & 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * & * & * \\ \tau_M^2(BK)^T & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * & * \\ \tau_M^2(BK)^T & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma\Phi & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Phi & * & * & * \\ \tau_M A S_1 & -\tau_M B K_1 & 0 & 0 & -\tau_M B K_1 & -\tau_M E & -\tau_M N_a & \tau_M D_1 & 0 & 0 & -S_1 & * & * \\ A S_1 & -B K_1 & 0 & 0 & -B K_1 & -E & -N_a & D_1 & 0 & 0 & 0 & -2S_1 + P_1 & * \\ S_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0. \tag{46}$$

Among them,

$$\begin{cases} Y_1 = \tau_M^2 K_1^T B^T - 2S_1, \\ X_1 = -P_1 + Q_1 + R_1 + \left(\tau_M^2 - 4\right) S_1 - \tau_M^2 S_1 A^T - \tau_M^2 A S_1, \end{cases} \tag{47}$$

and $K_1 = K\tilde{S}$.

**Theorem 8.** *Given scalar $\sigma > 0, \tau_M > 0, \gamma > 0$, if there are matrix $K$, symmetric positive definite matrix $P_1, Q_1, R_1, S_1$, and event triggering matrix $\Phi$ satisfying the following matrix inequalities (46), then the sliding mode (45) of the closed-loop system is asymptotically stable at $H_\infty$ disturbance rejection level $\gamma$.*

It is proved that the stability of mode (45) of the closed-loop system is investigated in the presence of external disturbance $w(k)$, actuator failure $f(k)$ and network attack $a_a(k)$. The asymptotic stability condition and robust performance index of the closed-loop system are as follows

$$\sum_{k=0}^{\infty} x^T(k)x(k) < \gamma^2 \sum_{k=0}^{\infty} \left(w^T(k)w(k) + (t_{k+1} - t_k)\left(e_x^T(k)e_x(k) + e_f^T(k)e_f(k) + e_a^T(k)e_a(k)\right)\right). \tag{48}$$

The Lyapunov-Krasovskii function is constructed as

$$V(k) = \sum_{i=1}^{4} V_i(k). \tag{49}$$

Among them,

$$\begin{cases} V_1(k) = x^T(k)\tilde{P}x(k), \\ V_2(k) = \displaystyle\sum_{i=k-\tau(k)}^{k-1} x^T(i)\tilde{Q}x(i), \\ V_3(k) = \displaystyle\sum_{i=k-\tau_M}^{k-1} x^T(i)\tilde{R}x(i), \\ V_4(k) = \tau_M \displaystyle\sum_{i=-\tau_M}^{-1} \sum_{j=k+i}^{k-1} \xi^T(j)\tilde{S}\xi(j), \end{cases} \tag{50}$$
$$\xi(k) \triangleq x(k+1)\text{-}x(k).$$

For $k \in [t_k, t_{k+1})$, define $\Delta V(k) = \sum_{i=1}^{4} \Delta V_i(k)$, where

$$\Delta V_1(k) = x^T(k+1)\tilde{P}x(k+1) - x^T(k)\tilde{P}x(k), \tag{51}$$

$$\begin{aligned} \Delta V_2(k) &= \sum_{i=k+1-\tau(k+1)}^{k} x^T(i)\tilde{Q}x(i) - \sum_{i=k-\tau(k)}^{k-1} x^T(i)\tilde{Q}x(i) \\ &= x^T(k)\tilde{Q}x(k) - x^T(k-\tau(k))\tilde{Q}x(k-\tau(k)), \end{aligned} \tag{52}$$

$$\begin{aligned}\Delta V_3(k) &= \sum_{i=k+1-\tau_M}^{k} x^T(i)\tilde{R}x(i) - \sum_{i=k-\tau_M}^{k-1} x^T(i)\tilde{R}x(i) \\ &= x^T(k)\tilde{R}x(k) - x^T(k-\tau_M)\tilde{R}x(k-\tau_M),\end{aligned} \tag{53}$$

$$\begin{aligned}\Delta V_4(k) &= \tau_M \sum_{i=-\tau_M}^{-1} \sum_{j=k+1+i}^{k} \xi^T(j)\tilde{S}\xi(j) - \tau_M \sum_{i=-\tau_M}^{-1} \sum_{j=k+i}^{k-1} \xi^T(j)\tilde{S}\xi(j) \\ &= \tau_M \sum_{i=-\tau_M}^{-1} \left( \sum_{j=k+1+i}^{k} \xi^T(j)\tilde{S}\xi(j) - \sum_{j=k+i}^{k-1} \xi^T(j)\tilde{S}\xi(j) \right) \\ &= \tau_M \sum_{i=-\tau_M}^{-1} \left( \xi^T(k)\tilde{S}\xi(k) - \xi^T(k+i)\tilde{S}\xi(k+i) \right) \\ &= \tau_M^2 \xi^T(k)\tilde{S}\xi(k) - \tau_M \sum_{j=k-\tau_M}^{k-1} \xi^T(j)\tilde{S}\xi(j).\end{aligned} \tag{54}$$

According to Wirtinger's inequality, it can be concluded that

$$\begin{aligned}&-\sum_{j=k-\tau_M}^{k-1} \tau_M[x(j+1)-x(j)]^T\tilde{S}[x(j+1)-x(j)] \\ &\quad \le [x(k)-x(k-\tau_M)]^T\tilde{S}[x(k)-x(k-\tau_M)] \\ &\quad - 3\left[x(k)+x(k-\tau_M) - \frac{2}{\tau_M+1}\sum_{j=k-\tau_M}^{k} x(j)\right]^T \tilde{S}\left[x(k) \right. \\ &\quad \left. + x(k-\tau_M) - \frac{2}{\tau_M+1}\sum_{j=k-\tau_M}^{k} x(j)\right].\end{aligned} \tag{55}$$

The event triggers the condition, when $k \in [t_k + \tau_{t_k}, t_{k+1} + \tau_{t_{k+1}})$, there is

$$e_l^T(k)\mathbf{\Phi}e_l(k) \le \sigma x\wedge^T(t_k)\mathbf{\Phi}\hat{x}(t_k). \tag{56}$$

Among them, $e_l(k) = \hat{x}(k) - \hat{x}(t_k)$ is the state estimation error.

Note $\varphi^T(k) = [x^T(k)\, x^T(t_k)\, x^T(k-\tau_M)\, \psi^T(k)\, e_x^T(t_k)\, e_f^T(t_k)\, e_a^T(t_k)\, w(k)\, x\wedge^T(t_k)\, e_l^T(k)]$ .

Among them, $\psi(k) = 1/\tau_M + 1\sum_{j=k-\tau_M}^{k} x(j)$ can be obtained by combining formulas (51)–(55) and (45):

$$\begin{aligned}&\Delta V(k) + x^T(k)x(k) - \gamma^2\left[w^T(k)w(k) + \sum_{k=0}^{\infty}(t_{k+1}-t_k) \right. \\ &\quad \left. \cdot \left(e_x^T(t_k)e_x(t_k) + e_f^T(t_k)e_f(t_k) + e_a^T(t_k)e_a(t_k)\right)\right] \\ &\quad + \sigma x\wedge^T(t_k)\mathbf{\Phi}\hat{x}(t_k) - e_l^T(k)\mathbf{\Phi}e_l(k) \le \varphi^T(k)\mathbf{\Theta}\varphi(k).\end{aligned} \tag{57}$$

Among them,

$$\mathbf{\Theta} = \begin{bmatrix} \mathbf{\Theta}_{11} & * & * & * & * & * & * & * & * & * \\ \mathbf{\Theta}_{21} & \mathbf{\Theta}_{22} & * & * & * & * & * & * & * & * \\ 2\tilde{S} & 0 & -4\tilde{S} & * & * & * & * & * & * & * \\ 6\tilde{S} & 0 & 6\tilde{S} & -12\tilde{S} & * & * & * & * & * & * \\ \mathbf{\Theta}_{51} & \mathbf{\Theta}_{52} & \mathbf{\Theta}_{53} & \mathbf{\Theta}_{54} & \mathbf{\Theta}_{55} & * & * & * & * & * \\ \mathbf{\Theta}_{61} & \mathbf{\Theta}_{62} & \mathbf{\Theta}_{63} & \mathbf{\Theta}_{64} & \mathbf{\Theta}_{65} & \mathbf{\Theta}_{66} & * & * & * & * \\ \mathbf{\Theta}_{71} & \mathbf{\Theta}_{72} & \mathbf{\Theta}_{73} & \mathbf{\Theta}_{74} & \mathbf{\Theta}_{75} & \mathbf{\Theta}_{76} & \mathbf{\Theta}_{77} & * & * & * \\ \mathbf{\Theta}_{81} & \mathbf{\Theta}_{82} & \mathbf{\Theta}_{83} & \mathbf{\Theta}_{84} & \mathbf{\Theta}_{85} & \mathbf{\Theta}_{86} & \mathbf{\Theta}_{87} & \mathbf{\Theta}_{88} & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma\mathbf{\Phi} & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{\Phi} \end{bmatrix},$$

$$\begin{aligned}&\mathbf{\Theta}_{11} = A^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)A - \tilde{P} + \tilde{Q} + \tilde{R} + \tau_M^2\tilde{S} - \tau_M^2A^T\tilde{S} - \tau_M^2\tilde{S}A - 4\tilde{S} + I, \\ &\mathbf{\Theta}_{21} = -(BK)^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)A + \tau_M^2(BK)^T\tilde{S} - 2\tilde{S}, \\ &\mathbf{\Theta}_{51} = -(BK)^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)A + \tau_M^2(BK)^T\tilde{S}, \\ &\mathbf{\Theta}_{61} = -E^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)A + \tau_M^2E^T\tilde{S}, \\ &\mathbf{\Theta}_{71} = -N_a^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)A + \tau_M^2N_a^T\tilde{S}, \\ &\mathbf{\Theta}_{71} = D_1^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)A - \tau_M^2D_1^T\tilde{S}, \\ &\mathbf{\Theta}_{22} = (BK)^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)BK - \tilde{Q}, \\ &\mathbf{\Theta}_{52} = (BK)^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)BK, \mathbf{\Theta}_{62} = E^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)BK, \\ &\mathbf{\Theta}_{72} = N_a^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)BK, \mathbf{\Theta}_{82} = D_1^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)BK, \\ &\mathbf{\Theta}_{53} = \mathbf{\Theta}_{63} = \mathbf{\Theta}_{73} = \mathbf{\Theta}_{83} = 0, \mathbf{\Theta}_{54} = \mathbf{\Theta}_{64} = \mathbf{\Theta}_{74} = \mathbf{\Theta}_{84} = 0, \\ &\mathbf{\Theta}_{55} = (BK)^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)BK - \gamma^2I, \\ &\mathbf{\Theta}_{65} = E^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)BK, \mathbf{\Theta}_{75} = N_a^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)BK, \\ &\mathbf{\Theta}_{85} = -D_1^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)BK, \mathbf{\Theta}_{66} = E^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)E - \gamma^2I, \\ &\mathbf{\Theta}_{76} = N_a^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)E, \mathbf{\Theta}_{86} = -D_1^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)E, \\ &\mathbf{\Theta}_{77} = N_a^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)N_a - \gamma^2I, \mathbf{\Theta}_{87} = -D_1^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)N_a, \\ &\mathbf{\Theta}_{88} = -D_1^T\left(\tilde{P}+\tau_M^2\tilde{S}\right)D_1 - \gamma^2I,\end{aligned} \tag{58}$$

supposing that $\Delta V(k) + x^T(k)x(k) - \gamma^2[w^T(k)w(k) + \sum_{k=0}^{\infty}(t_{k+1}-t_k)(e_x^T(t_k)e_x(t_k) + e_f^T(t_k)e_f(t_k) + e_a^T(t_k)e_a(t_k))] < 0, \mathbf{\Theta}$

$<0$. By using Lemma 1 twice, we can get the following results:

$$\begin{bmatrix} X & * & * & * & * & * & * & * & * & * & * & * \\ Y & -\tilde{Q} & * & * & * & * & * & * & * & * & * & * \\ 2\tilde{S} & 0 & -4\tilde{S} & * & * & * & * & * & * & * & * & * \\ 6\tilde{S} & 0 & 6\tilde{S} & -12\tilde{S} & * & * & * & * & * & * & * & * \\ \tau_M^2(BK)^T\tilde{S} & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * & * & * \\ \tau_M^2 E^T\tilde{S} & 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * & * \\ \tau_M^2 N_a^T\tilde{S} & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * \\ \tau_M^2 D_1^T\tilde{S} & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma\mathbf{\Phi} & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{\Phi} & * & * \\ \tau_M\tilde{S}A & -\tau_M\tilde{S}BK & 0 & 0 & -\tau_M\tilde{S}BK & -\tau_M\tilde{S}E & -\tau_M\tilde{S}N_a & \tau_M\tilde{S}D_1 & 0 & 0 & -\tilde{S} & * \\ \tilde{P}A & -\tilde{P}BK & 0 & 0 & -\tilde{P}BK & -\tilde{P}E & -\tilde{P}N_a & \tilde{P}D_1 & 0 & 0 & 0 & -\tilde{P} \end{bmatrix} < 0. \tag{59}$$

Among them,

$$\begin{cases} Y = \tau_M^2(BK)^T\tilde{S} - 2\tilde{S}, \\ X = -\tilde{P} + \tilde{Q} + \tilde{R} + \left(\tau_M^2 - 4\right)\tilde{S} - \tau_M^2 A^T\tilde{S} - \tau_M^2\tilde{S}A + I. \end{cases} \tag{60}$$

If (59) is multiplied by diag $\{\tilde{S}^{-1}, \tilde{S}^{-1}, \tilde{S}^{-1}, \tilde{S}^{-1}, \tilde{S}^{-1}, \tilde{S}^{-1}, \tilde{S}^{-1}, I, I, I, \tilde{P}^{-1}, \tilde{P}^{-1}\}$ on the left and the right, then Lemma 1 is applied again to the inequality obtained. If $K_1 = K\tilde{S}^{-1}$, $S_1 = \tilde{S}^{-1}$, $P_1 = \tilde{S}^{-1}\tilde{P}\tilde{S}^{-1}$, $Q_1 = \tilde{S}^{-1}\tilde{Q}\tilde{S}^{-1}$, $R_1 = \tilde{S}^{-1}\tilde{R}\tilde{S}^{-1}$ is used at the same time, then

$$\begin{bmatrix} X_1 & * & * & * & * & * & * & * & * & * & * & * & * \\ Y_1 & -Q_1 & * & * & * & * & * & * & * & * & * & * & * \\ 2S_1 & 0 & -4S_1 & * & * & * & * & * & * & * & * & * & * \\ 6S_1 & 0 & 6S_1 & -12S_1 & * & * & * & * & * & * & * & * & * \\ \tau_M^2(BK)^T & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * & * & * & * \\ \tau_M^2 E^T & 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * & * & * \\ \tau_M^2 N_a^T & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * & * \\ \tau_M^2 D_1^T & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma\mathbf{\Phi} & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{\Phi} & * & * & * \\ \tau_M AS_1 & -\tau_M BK_1 & 0 & 0 & -\tau_M BK_1 & -\tau_M E & -\tau_M N_a & \tau_M D_1 & 0 & 0 & -S_1 & * & * \\ AS_1 & -BK_1 & 0 & 0 & -BK_1 & -E & -N_a & D_1 & 0 & 0 & 0 & -\tilde{P}^{-1} & * \\ S_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0. \tag{61}$$

Among them,

$$\begin{cases} Y_1 = \tau_M^2 K_1^T B^T - 2S_1, \\ X_1 = -P_1 + Q_1 + R_1 + \left(\tau_M^2 - 4\right)S_1 - \tau_M^2 S_1 A^T - \tau_M^2 A S_1. \end{cases} \tag{62}$$

Similarly, for symmetric positive definite matrix $\tilde{P}, \tilde{S}$, we have

$$\tilde{S}\tilde{P}^{-1}\tilde{S} - 2\tilde{S} + \tilde{P} = \left(\tilde{S} - \tilde{P}\right)\tilde{P}^{-1}\left(\tilde{S} - \tilde{P}\right) \geq 0. \tag{63}$$

Further, we can get

$$-\tilde{P}^{-1} \leq -2S_1 + P_1. \tag{64}$$

Combining formulas (61) and (64), formula (46) holds. If (46) is true, then

$$\Delta V(k) + x^T(k)x(k) - \gamma^2\left[w^T(k)w(k) + \sum_{k=0}^{\infty}(t_{k+1} - t_k) \cdot \left(e_x^T(t_k)e_x(t_k) + e_f^T(t_k)e_f(t_k) + e_a^T(t_k)e_a(t_k)\right)\right] < 0. \tag{65}$$

We can go further:

$$\sum_{k=0}^{\infty}\left[x^T(k)x(k) - \gamma^2\left(w^T(k)w(k) + (t_{k+1} - t_k)\left(e_x^T(t_k)e_x(t_k) + e_f^T(t_k)e_f(t_k) + e_a^T(t_k)e_a(t_k)\right)\right)\right] < -\sum_{k=0}^{\infty}\Delta V(k) = -V(k) \leq 0. \tag{66}$$

Obviously,

$$\sum_{k=0}^{\infty}x^T(k)x(k) < \gamma^2\sum_{k=0}^{\infty}\left(w^T(k)w(k) + (t_{k+1} - t_k)\left(e_x^T(t_k)e_x(t_k) + e_f^T(t_k)e_f(t_k) + e_a^T(t_k)e_a(t_k)\right)\right). \tag{67}$$

Namely,

$$\|x(t)\|_2 < \gamma_{\min}\left(\|w(t)\|_2 + (t_{k+1}\text{-}t_k)\left(\|e_x(t_k)\|_2 + \left\|e_f(t_k)\right\|_2 + \|e_a(t_k)\|_2\right)\right). \tag{68}$$

In other words, when equation (46) holds, sliding mode (45) of the closed-loop system is asymptotically stable and has $H_\infty$ disturbance rejection performance.

Theorem 8 is proved.

## 4. Result and Discussion

In this paper, the control system model given in Reference [20] is taken as the research object:

$$A = \begin{bmatrix} 0.9879 & 0.0098 \\ -0.0837 & 0.9908 \end{bmatrix}, B = \begin{bmatrix} -0.0029 & -0.0005 \\ -0.1919 & -0.0378 \end{bmatrix},$$
$$E = \begin{bmatrix} -0.0029 \\ -0.1919 \end{bmatrix}, N_a = \begin{bmatrix} -0.1341 \\ -0.1243 \end{bmatrix}, N_s = \begin{bmatrix} -0.0051 \\ -0.0078 \end{bmatrix},$$
$$D_w = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}, D_v = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{69}$$

In this simulation, we make the following assumptions about actuator failure and network attack.

Actuator failure:

$$f(k) = 2\sin 0.01\pi k + 2, \quad 230 \leq k \leq 500. \tag{70}$$

Network attack:

$$\begin{cases} a_a(k) = 2, & 100 \leq k \leq 500, \\ a_s(k) = -2, & 100 \leq k \leq 500. \end{cases} \tag{71}$$

The expanded coefficient matrices are as follows:

$$A_{11} = \begin{bmatrix} 0.9879 & 0.0098 & -0.0029 & 1 \\ -0.0837 & 0.9908 & -0.1919 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$
$$B_{11} = \begin{bmatrix} -0.0029 & -0.0005 \\ -0.1919 & -0.0378 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, E_{11} = \begin{bmatrix} -0.0029 \\ -0.1919 \\ 0 \\ 0 \end{bmatrix},$$

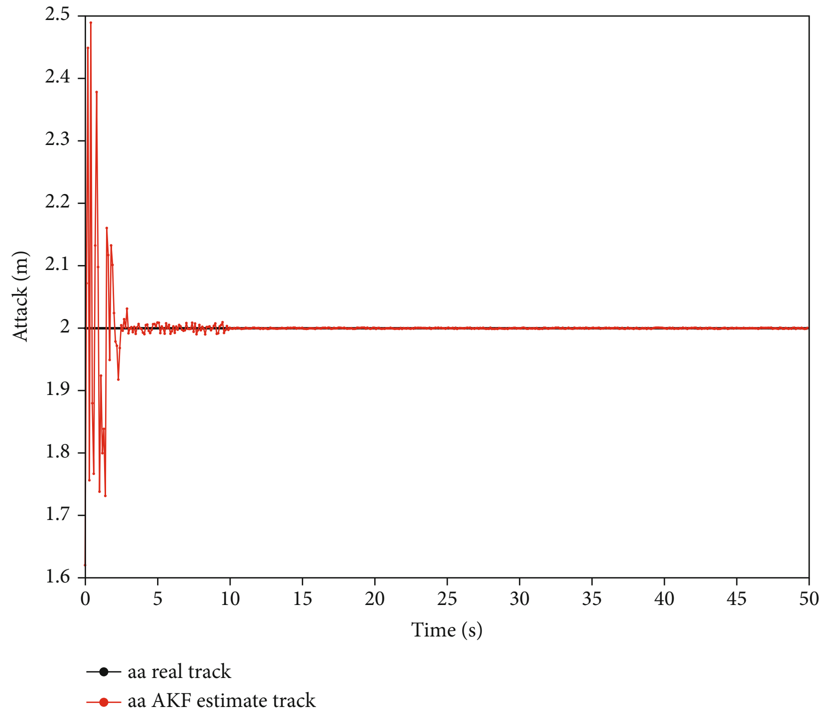


FIGURE 2: Actuator attack and its estimation for active fault-tolerant/active passive intrusion-tolerant systems.

$$D_{w11} = \mathrm{diag}\,\{D_w, I, I\} = \begin{bmatrix} 0.1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, D_{v11} = D_v = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}. \tag{72}$$

Let the initial condition of the system be $P(0 \mid 0) = I_4$, $\lambda = 0.9$, $x(0) = \hat{x}(0) = [-30 \quad 20 \quad 2 \quad -2]^T$, $R = 0.0025$, $r(k) = 0_{2\times1}$, $Q = 0.0001$.

From equation (26),

$$K_1 = \begin{bmatrix} 1.1555 & -0.1168 \\ 1.6243 & -0.1598 \\ 0.4433 & -0.0447 \\ -0.0156 & -0.0018 \end{bmatrix}. \tag{73}$$

In Theorem 6, let the maximum delay upper bound $\tau_M = 1.2$, trigger parameter $\sigma = 0.85$, take $q = 5, \varepsilon = 2, T = 0.1$. Based on the sliding mode trigger matrix, we can get the event gain matrix $G$ and $\mathbf{\Phi}$, respectively,

$$G = \begin{bmatrix} -5.6995 & 1.0126 \\ -0.656 & 0.8053 \end{bmatrix}, \mathbf{\Phi} = \begin{bmatrix} 1.5812 & 0.2811 \\ -0.2657 & 2.6766 \end{bmatrix}. \tag{74}$$

The state feedback gain matrices are

$$K_{c1} = \begin{bmatrix} 63.72 & -48 \\ 23.28 & 23 \end{bmatrix}, K_{c2} = \begin{bmatrix} -273.6 & -998.25 \\ 832.23 & 389.58 \end{bmatrix}. \tag{75}$$

The simulation results are shown in Figures 2–11. Figures 2 and 3 show the attack and its estimation curve and attack estimation error diagram, respectively. Figures 4 and 5 show the actuator fault and its estimation curve and error diagram, respectively. Figures 6–9 show the state of the system and its estimation curve and state estimation curve, respectively. Figure 10 shows the output response curve of the system when actuator failure and network attack occur simultaneously. Figure 11 shows the data transmission time and the transmission interval of the system nonuniform transmission NCS.

It can be seen from Figures 2–9 that the adaptive Kalman filter fault/attack estimator can well estimate the state, fault, and attack of the system. It can be seen from Figure 10 that when the system encounters actuator

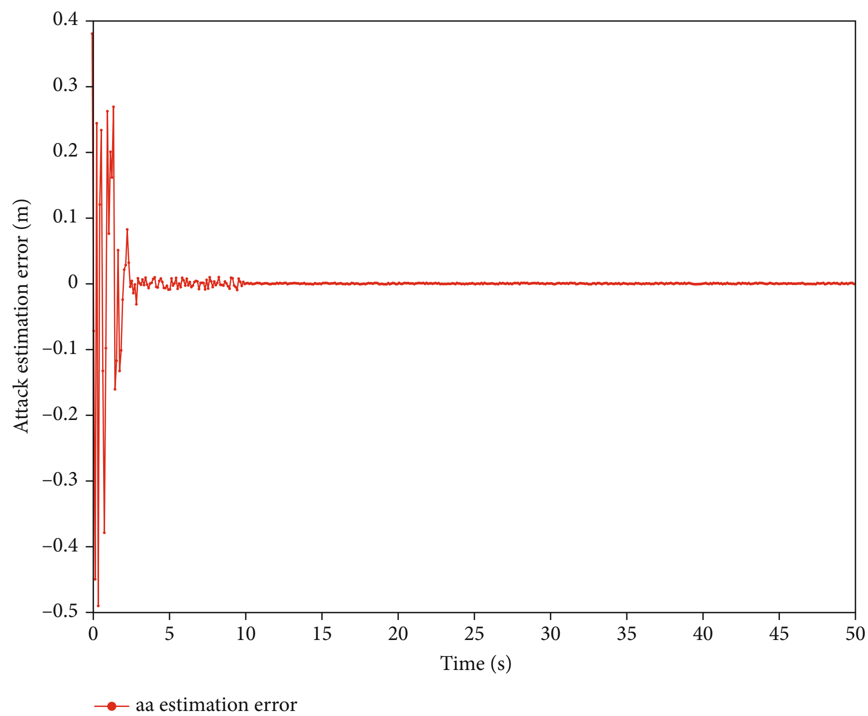


Figure 3: Actuator attack estimation error of active fault-tolerant/active passive intrusion-tolerant system.

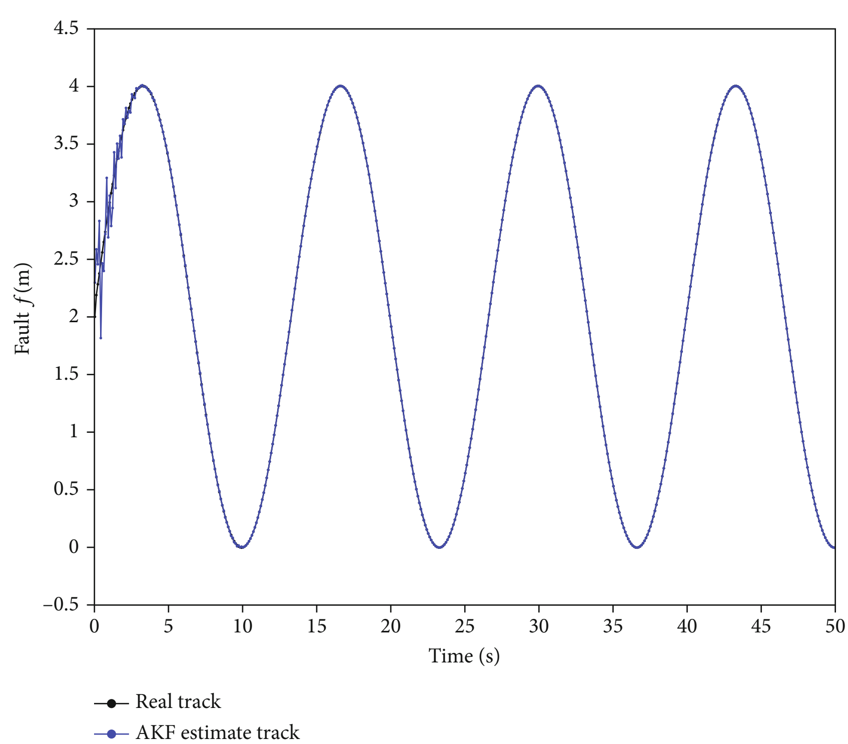


Figure 4: Fault estimation of active fault-tolerant/active passive intrusion-tolerant system.

failure and network attack, the vibration is gradually attenuated and tends to balance by using the cooperative controller designed in this paper and the compensation strategy of failure and attack. Simulation results show that the active fault-tolerant/active passive intrusion-tolerant cooperative controller designed in this paper is

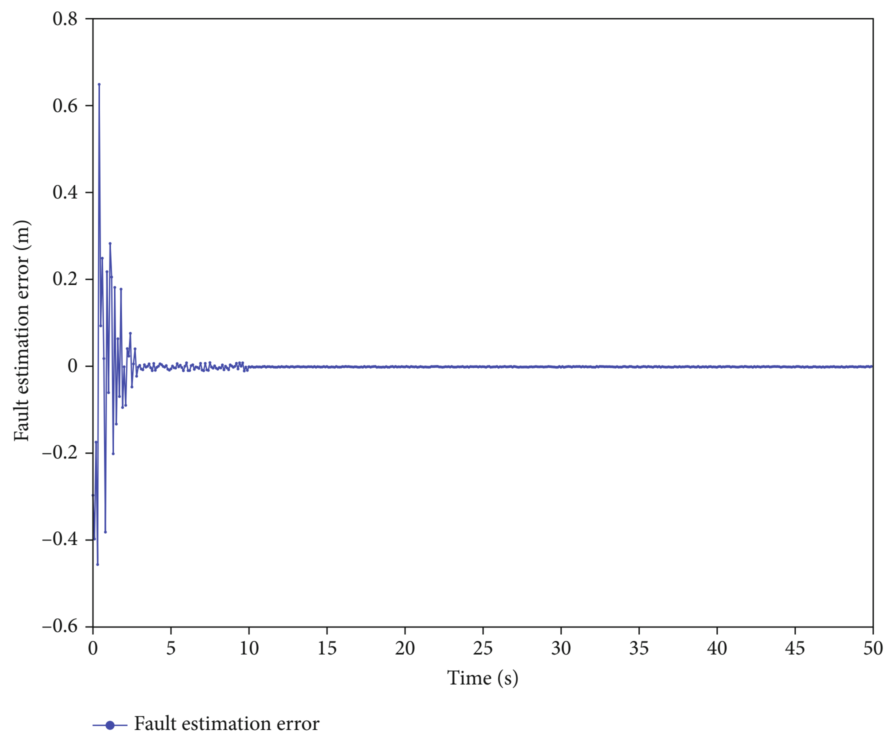


Figure 5: Fault estimation error of active fault-tolerant/active passive intrusion-tolerant system.

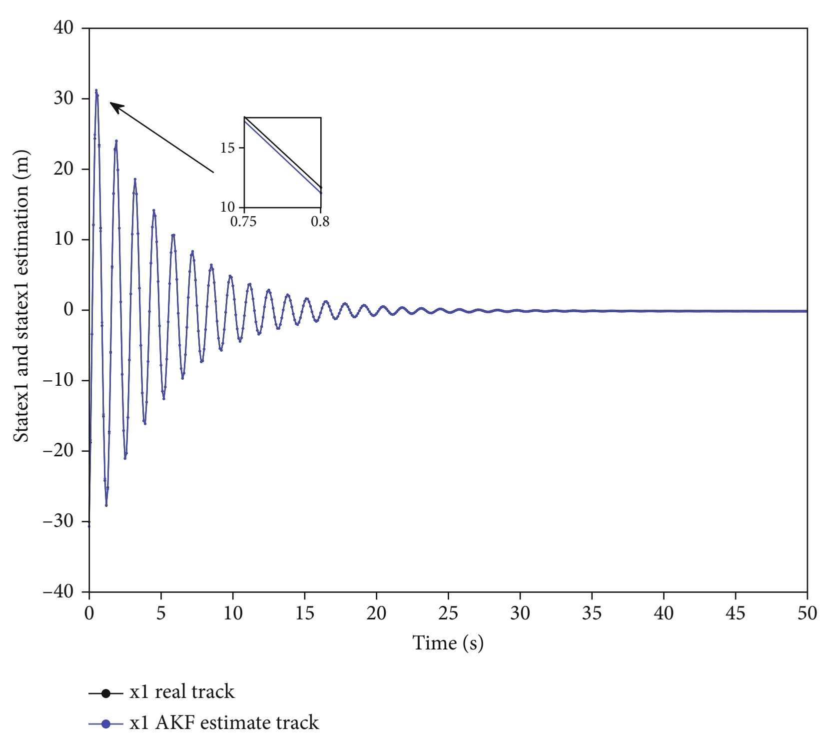


Figure 6: State 1 and estimation of active fault-tolerant/active passive intrusion-tolerant system.

effectively fault-tolerant and active and passive intrusion-tolerant for network attacks and also suppresses the influence of disturbance and noise.

According to the analysis in Figure 11, when the event trigger parameter is $\sigma = 0.85$, compared with the traditional PPTCS which needs to transmit 500 data in the

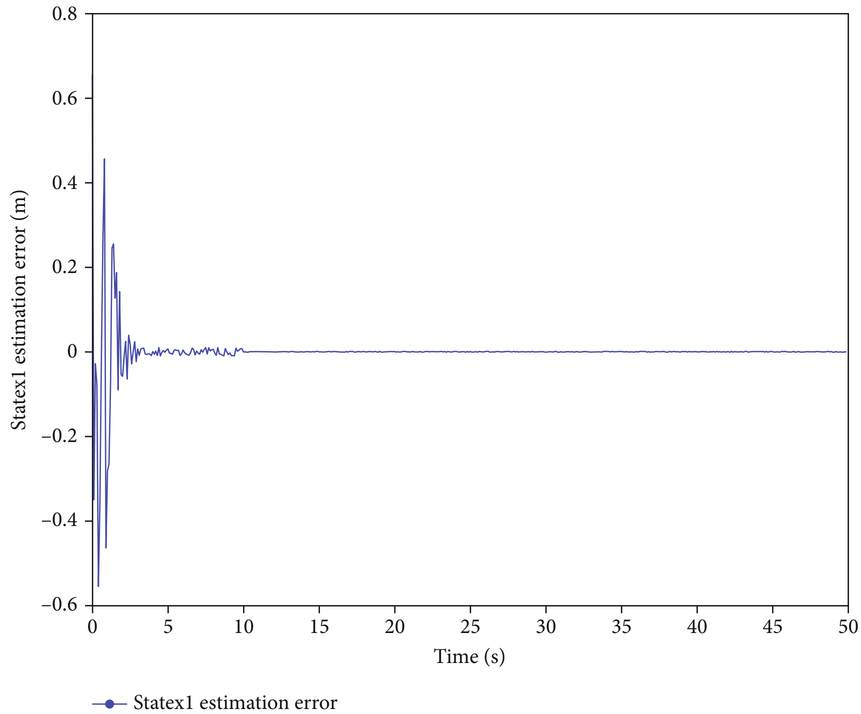


Figure 7: State 1 estimation error of active fault-tolerant/active passive intrusion-tolerant system.

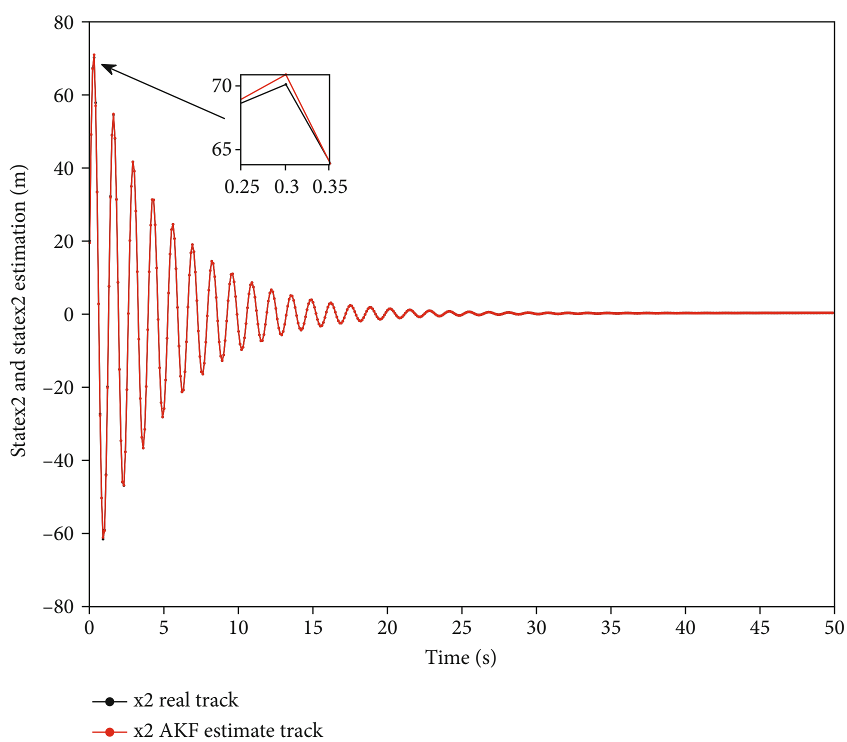


Figure 8: State 2 and estimation of active fault-tolerant/active passive intrusion-tolerant system.

simulation time of 50 s, 249 data are transmitted in the DETCS, the data transmission rate is 49.8%, the average transmission cycle is $\bar{T} = 0.2006$ s, and the maximum transmission cycle is $T_{\max} = 0.6$ s. This shows that the control method proposed in this paper not only ensures the excellent performance of the system but also effectively saves the network communication resources and improves the efficiency of network utilization.

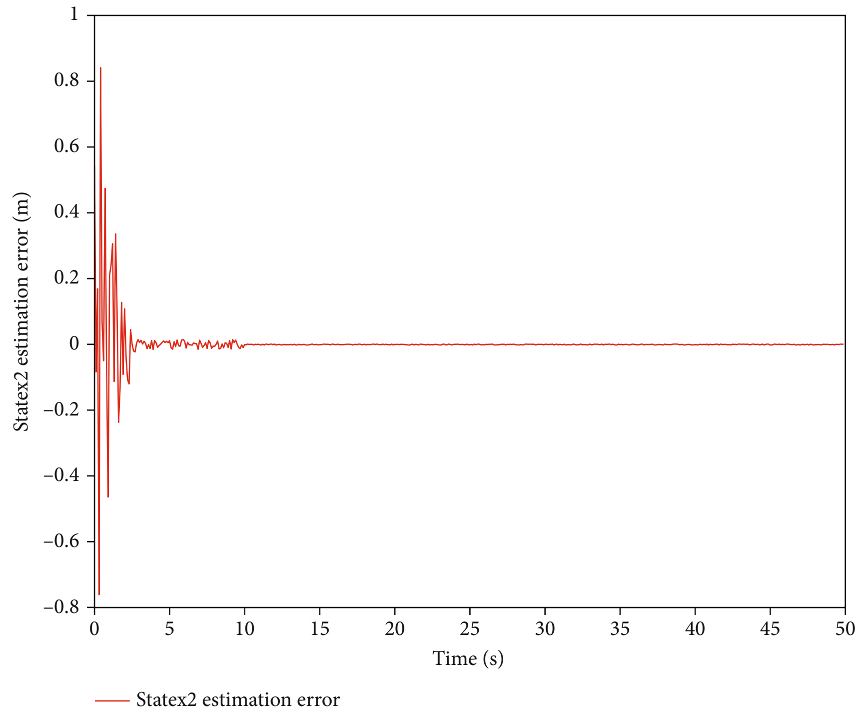


Figure 9: State 2 estimation error of active fault-tolerant/active passive intrusion-tolerant system.

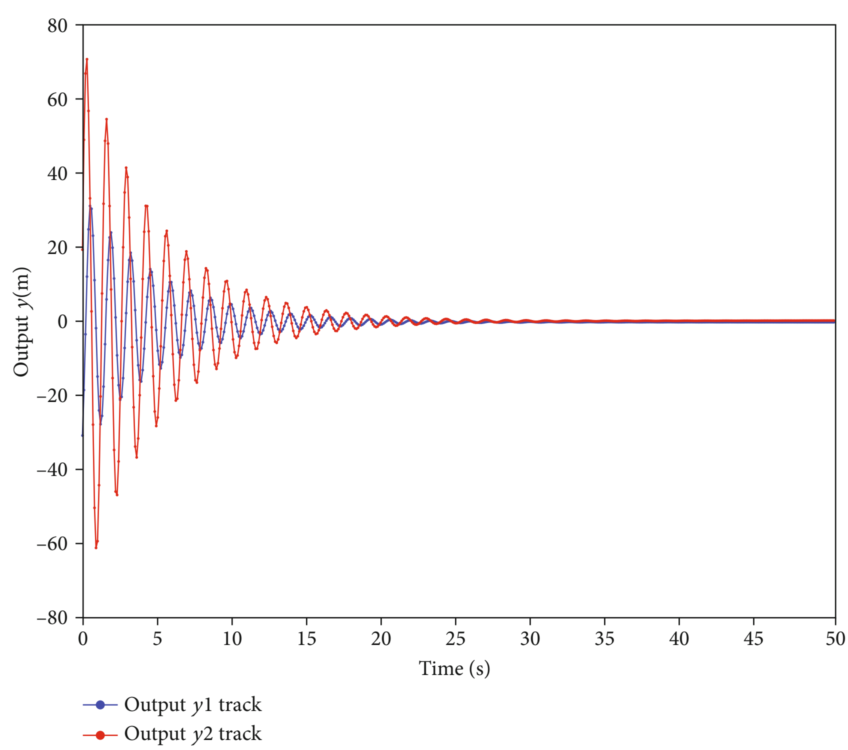


Figure 10: Output response of active fault-tolerant/active passive intrusion-tolerant system.

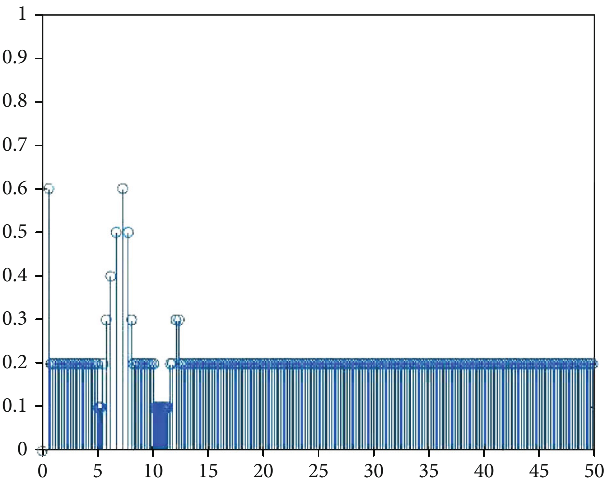


FIGURE 11: Transmission time and interval of nonuniform transmission.

## 5. Conclusion

In this paper, an active fault-tolerant/active passive intrusion-tolerant cooperative controller design method is proposed for linear discrete NCS with time-varying delay based on discrete event triggering mechanism when actuator failure and network attack occur. In this method, the attack is extended to the state, and the adaptive Kalman filter is used to estimate the state, fault, and attack. Then, the integral sliding mode control method is used to design the active fault-tolerant/active passive intrusion-tolerant cooperative controller, so that the system can keep normal operation and have $H_\infty$ disturbance rejection performance in the case of actuator failure and network attack. Finally, a simulation example is given to illustrate the effectiveness and applicability of the proposed method.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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