Alternate Distributed Beamforming with Power Allocation for Decode-and-Forward Multirelay Systems Using Buffers

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In this paper, a novel transmission policy referred as alternate distributed beamforming (ADB) for buffer-aided half-duplex (HD) multirelay systems is proposed, in which the relays are divided into two groups and activated alternately in their transmitter and receiver modes, with one group receiving the same information broadcast from the source while the other group simultaneously transmitting the common messages to the destination via distributed beamforming in each time slot. It is worth noting that the relays used for reception and transmission are determined without the instantaneous channel state information (CSI). Theoretical analysis of the achievable throughput of the proposed scheme in Rayleigh fading channels is provided, and the approximate closed-form expressions are derived for multiantenna relay scenario, with single-antenna relay scenario as a special case. Moreover, the asymptotic expressions for the maximum achievable throughput of the proposed scheme and corresponding optimal power allocation in the high- and low-SNR regimes are derived as well. Numerical results in accordance with theoretical analysis demonstrate that the proposed policy achieves a significant improvement in the maximum achievable throughput compared with benchmark schemes. It is shown that for the ADB scheme, the almost or exact symmetric relay grouping may achieve the best performance in the high-SNR regime. It is also shown that increasing the number of antennas equipped at each relay is better than increasing the number of relays equipped with a single antenna when the total number of antennas at the relays is fixed.

1. Introduction

With the further improvement of people’s demand for coverage and throughput of wireless networks, cooperative communications have been an important building block for wireless communication systems, in which the communication between a source node and a destination node is accomplished via the help of a number of relay nodes [1]. With the help of relays, alternative and independent transmission paths are offered, and the diversity gain of the network can be obtained. Also, distributed beamforming gain can be expected [2].

Profiting from the higher node density in the next generation wireless network, the enthusiasm to tap the potential of cooperative communications continues unabated. To better utilize the benefits provided by multiple relays, various relay selection schemes have been proposed [3–10]. The conventional relay selection (CRS) scheme selected a single relay that provides the strongest end-to-end path between the source and destination [3]. The selected relay immediately forwards the received data in the next time slot to the destination, so the ability of the relays to store at least a limited number of data packets is not incorporated. This relay selection policy is considered as the optimal selection scheme for conventional relaying system without buffers.

Meanwhile, the adoption of buffer-aided relays can provide both throughput and diversity gain by adaptive link selection [11]. Storing packets and transmitting them in more favorable wireless conditions increases the network’s resiliency, throughput, and diversity and has therefore been widely
applied in cooperative networks to improve the system performance [4–10]. For instance, a space full-duplex max-max relay selection (SFD-MMRS) scheme was introduced in [4], which mimics full-duplex (FD) relaying with half-duplex (HD) relays via link selection. In addition, a modified max-link relay selection scheme has also been proposed, in which the direct link between the source and destination was exploited to achieve a significant performance gain in terms of diversity and delay [5]. In [6], a priority-based max-link relay selection scheme was proposed for buffer-aided decode-and-forward cooperative networks, in which the best relay node is selected corresponding to the link having the highest channel gain among the links within a priority class. The authors in [7] designed the buffer-aided adaptive transmission scheme for the buffer-aided wireless powered cooperative nonorthogonal multiple-access (NOMA) relaying network. A novel probabilistic buffer-aided relay selection scheme was proposed in [8], in which the link selection outcome depends not only on the wireless channel condition and the buffer state but also on additional randomness. In [9], NOMA, buffer-aided full-duplex (FD) relaying and broadcasting were combined to improve the relay selection with low complexity. Two asynchronous learning algorithms for joint hybrid NOMA/OMA relay selection and power allocation in buffer-aided delay-constrained networks were proposed in [10]. It is worth noting that the relay selected with the adaptive link selection policies varies with the instantaneous channel state information (CSI), which may introduce more complexities in practical implementation.

Note that a relay usually operates in either FD or HD mode. In FD relaying, the relays transmit and receive at the same time and frequency, at the cost of hardware complexity [12, 13]. In HD relaying, relays are incapable of transmitting and receiving simultaneously, thus leading to reduced capacity of the whole network. In order to recover the HD loss, several successive relaying schemes have been proposed [14–19], the main idea of which is to adopt two different relays acting as the receiving and transmitting relay simultaneously. In [14], a two-way relaying protocol and a two-path (successive) relaying protocol were proposed to avoid the prelog factor one-half. It was shown that both protocols recover a significant portion of the HD loss for different relaying strategies like decode-and-forward (DF) and amplify-and-forward (AF). A successive relaying protocol is studied for a two-relay wireless network in [15], in which simple repetition coding is applied at the relays. In [16], the authors considered a two-relay network with multiple antennas at the destination. The infrastructure-based relays with highly directional antennas were used to avoid IRI. A factor graph representing the joint a posteriori probability of the coded symbols and the channels in the frequency domain was introduced in [17], and the AF successive relaying protocol was used in a two-relay network to overcome the HD limitation [18] considering the throughput optimization of an energy harvesting assisted two-relay network using a buffer-aided successive relaying protocol. In [19], the authors developed an adaptive cooperative transmission scheme with relays harvesting wireless energy, where two relays are adopted to alternately forward source data to the destination using the two-path successive relaying (TPSR) protocol. A hybrid algorithm aiming at enhancing buffer-aided relaying for supporting delay-sensitive applications was presented in [20], which switches between successive relaying and HD operation. The authors in [21] proposed and analyzed a novel threshold-based relaying scheme for buffer-aided two-way relaying systems. Note however that a two-hop network relying either on a single two-way relay or a pair of relay nodes with successive relaying protocol is investigated in most of the aforementioned works.

In this paper, we propose a novel fixed scheduling transmission protocol named alternate distributed beamforming (ADB) for the buffer-aided HD multirelay systems, where the relays are divided into two groups and activated alternately in their transmitter and receiver modes in each time slot to create a virtual FD system. It is worth noting that in contrast to the relay selection schemes, the receiving and transmitting relays of the proposed scheme are predetermined and do not vary with the CSI. We assume half-duplex DF relays with infinite-size buffers. We assume that there is no direct link between the source and the destination or between any relay pair due to path loss and shadowing. We analyze the achievable throughput of the proposed scheme in Rayleigh fading channels and derive the closed-form expressions. We also derive the asymptotic expressions for the maximum achievable throughput and corresponding optimal power allocation in the high- and low-SNR regimes, respectively. Numerical results in accordance with the theoretical analysis corroborate the superiority of the proposed scheme in buffer-aided dual-hop cooperative relay networks.

The contributions of this paper can be summarized as follows:

1. We propose an ADB scheme for the buffer-aided HD DF cooperative communication systems, which combats the HD loss with successive relay group and is independent of the instantaneous CSI.

2. We derive the closed-form expressions for the achievable throughput of the proposed scheme, and we also obtain the asymptotic expressions for the maximum achievable throughput and corresponding optimal power allocation in the high- and low-SNR regimes, respectively.

3. Through numerical results, we show that the proposed fixed scheduling protocol can further improve the achievable throughput compared with benchmark schemes. We also show that with the ADB scheme, increasing the number of antennas equipped at each relay is better than increasing the number of relays equipped with a single antenna when the total number of antennas at the relays is fixed.

The remainder of this paper is organized as follows. Section 2 describes the system model. Section 3 revisits several existing transmission protocols for multiantenna relay scenario. Section 4 gives the main results of this paper including the operation of the ADB scheme, comprehensive analysis of
the achievable throughput, and the asymptotic closed-form expressions for the high- and low-SNR regimes. Section 5 provides the numerical results. Finally, conclusions are drawn in Section 6 with some lengthy proofs in the Appendices.

Notations. Throughout this paper, $(\cdot)^T$ and $(\cdot)^H$ denote the matrix transpose and conjugate transpose, respectively. $\|\cdot\|$ is the Euclidean or $L_2$ vector norm. $F_y(\cdot)$ and $f_y(\cdot)$ represent the cumulative distribution function (CDF) and the probability density function (PDF) of random variable (RV) $y$, respectively. $E[\cdot]$ denotes the expectation operation. $\mathcal{M}(x, m, \Omega)$ denotes the distribution of a Nakagami-$m$ RV $x$ with shape parameter $m$ and scale parameter $\Omega$, and $\sim$ stands for “distributed as.”

2. System Model and Preliminaries

2.1. System Model. We consider a relay network consisting of one source node $S$, a set of $L$ decode-and-forward (DF) relays $R_1, \ldots, R_L$, and one destination node $D$, as shown in Figure 1. We assume that the source node $S$ and the destination node $D$ are equipped with a single antenna, and the relays are equipped with $N_R$ antennas. We assume that all nodes operate in the HD mode, i.e., they cannot transmit and receive data simultaneously. We assume that there is a buffer of infinite length at each relay such that each relay can store the information received from the source and transmit it in later time.

We assume that there is no direct link between the source and destination, and the communications can be established only via relays, i.e., all the information that the destination receives has been processed by the relays. We also assume that there is no interfering link between any relay pair [17, 18]. In practice, this assumption is valid if the relays are located far away from each other or if fixed infrastructure-based relays with directional antennas are used [22, 23]. We use $h_{SR_i}$ and $h_{R_iD}$ for $i \in \{1, \ldots, L\}$ and $j \in \{1, \ldots, N_R\}$ to denote the channel coefficients of $S-R_j$ and $R_j-D$ links, respectively, where $R_{ij}$ denotes the $j$-th antenna of the relay $R_i$. The channel is assumed to be stationary and ergodic. We consider the block fading channels, in which the channel coefficients remain constant during one time slot and vary independently from one to another. In addition to fading, all wireless links are impaired by additive white Gaussian noise (AWGN).

We assume Rayleigh fading for the channel coefficients and the variances of $h_{SR_i}$ and $h_{R_iD}$ to be $\sigma_{h_{SR_i}}^2$ and $\sigma_{h_{R_iD}}^2$, respectively. Throughout this paper, we consider the case of independent and identically distributed (i.i.d.) fading for both $S-R_i$ and $R_i-D$ links, i.e., $\sigma_{h_{SR_i}}^2 = \sigma_g^2$ and $\sigma_{h_{R_iD}}^2 = \sigma_n^2$, $i \in \{1, \ldots, L\}, j \in \{1, \ldots, N_R\}$, which is a typical assumption to facilitate analysis [4, 24].

2.2. Two-Hop Transmission. The transmission between $S$ and $D$ through relay $R_i$ is divided into two hops. In the first hop, relay $R_i$ receives data from $S$ and decodes with the maximal ratio combining (MRC) protocol. Hence, the channel output after MRC at the relay $R_i$ is given by

$$y_{SR_i} = w_{SR_i} (h_{SR_i} x_S + n_R),$$

where $x_S$ is the signal transmitted by $S$ with an average power $P_R$. $h_{SR_i} = [h_{SR_1}, h_{SR_2}, \ldots, h_{SR_{\nu_S}}]^T$ denotes the channel vectors between $S$ and $R_i$, with $i \in \{1, \ldots, \nu_S\}$. $w_{SR_i} = h_{SR_i}^H / \| h_{SR_i} \|$ is the receiving vector at $R_i$. $n_R$ is the additive white Gaussian noises (AWGNs) at $R_i$ with zero mean and variance $\sigma_R^2$. The instantaneous received signal-to-noise ratio (SNR) at relay $R_i$ is given by $\gamma_{SR_i} = P_R \| h_{SR_i} \|^2 / \sigma_R^2$.

Similarly, in the second hop, the relay $R_i$ transmits data to $D$ with the maximal ratio transmission (MRT) protocol. The received signal at $D$ is given as

$$y_{R_iD} = h_{R_iD} w_{R_iD} x_R + n_D,$$

where $x_R$ is the signal transmitted by $R_i$ with an average power $P_R$. $h_{R_iD} = [h_{R_1D}, h_{R_2D}, \ldots, h_{R_{\nu_D}D}]^T$ denotes the channel vector between $R_i$ and $D$, where $i \in \{1, \ldots, \nu_D\}$. $w_{R_iD} = h_{R_iD}^H / \| h_{R_iD} \|$ is the transmit beamforming vector at $R_i$. $n_D$ is the AWGN at $D$ with zero mean and variance $\sigma_D^2$. And the instantaneous received SNR at the destination from relay $R_i$ is given by $\gamma_{R_iD} = P_R \| h_{R_iD} \|^2 / \sigma_D^2$. Without loss of generality, we assume that the noise power at the receiving nodes is equal to one, i.e., $\sigma_R^2 = \sigma_D^2 = 1$.

3. Existing Relaying Protocols

In this section, we revisit several existing relaying protocols in a multiantenna relay scenario.

3.1. Conventional Relay Selection (CRS). The conventional relay selection protocol selects the relay which provides the strongest end-to-end path between the source and destination [3]. The source transmits in the first time slot, and the
selected relay forwards the data received from the source towards the destination in the second time slot. The best relay \( R_j \) is selected based on

\[
j = \arg \max_{i \in \{1, \ldots, L\}} \left\{ \min \left\{ Y_{SR}, Y_{R,D} \right\} \right\}.
\]

The second best relay for transmission \( R_t \) is selected based on

\[
r_t = \arg \max_{i \in \{1, \ldots, L\}} \left\{ Y_{SR} \right\}.
\]

The achievable throughput is given by

\[
C_k = \frac{1}{2} \log_2 \left( 1 + \max_{1 \leq k \leq L} \left( \frac{P_s \| h_{SR_k} \|^2}{C_0 + C_1 + C_2} \right) \right).
\]

Then, the achievable throughput is given by \( \mathbb{E}[C_k] \). Throughout this text, the unit for the throughput is bps/Hz.

### 3.2. Space Full-Duplex Max-Max Relay Selection (SFD-MMRS)

This protocol chooses different relays for reception and transmission, according to the quality of the channels, so that the relay selected for reception and the relay selected for transmission can work simultaneously [4]. The best relay for reception \( R_r \) and the best relay for transmission \( R_t \) are selected, respectively, based on

\[
r_r = \arg \max_{i \in \{1, \ldots, L\}} \left\{ Y_{SR} \right\}.
\]

The second best relay for reception \( R_{r2} \) and the second best relay for transmission \( R_{t2} \) are selected, respectively, according to

\[
r_{r2} = \arg \max_{i \in \{1, \ldots, L\} \setminus \{r_r\}} \left\{ Y_{SR} \right\}.
\]

Then, in SFD-MMRS, the relays selected for reception \( R_{r1} \) and transmission \( R_{t1} \) are chosen as

\[
(R_{r1}, R_{t1}) = \begin{cases} (R_{r2}, R_{t1}), & \text{if } r_1 \neq t_1, \\ (R_{r1}, R_{t2}), & \text{if } r_1 = t_1 \text{ and } \min \left( Y_{SR}, Y_{R,D} \right) > \min \left( Y_{SR_{r2}}, Y_{R_{r2},D} \right), \\ (R_{r2}, R_{t2}), & \text{otherwise}. \end{cases}
\]

Let \( C_{SR} \) and \( C_{RD} \) denote the instantaneous capacities of the \( S \rightarrow R \) and \( R \rightarrow D \) links, respectively, i.e.,

\[
C_{SR} = \log_2 \left( 1 + P_s \| h_{SR} \|^2 \right),
\]

\[
C_{RD} = \log_2 \left( 1 + P_r \| h_{R,D} \|^2 \right).
\]

The achievable throughput is given by \( \mathbb{E}[C_{SR}], \mathbb{E}[C_{RD}] \) [4, 25].

### 3.3. Decode and Forward (DF)

In DF [2], each relay must decode the common message transmitted by the source node and beamform their transmissions to the destination, which is also performed in two time slots. Then, the instantaneous rate for the overall system is given by

\[
C_k = \frac{1}{2} \log_2 \left( 1 + \max_{1 \leq k \leq L} \left( \frac{P_s \min \left\{ \| h_{SR_k} \|^2, \| h_{R_k,D} \|^2 \right\}}{C_{SR} + C_{RD}} \right) \right).
\]

The achievable throughput is given by \( \mathbb{E}[C_k] \).

### 4. Alternate Distributed Beamforming Policy

In this section, we first propose the ADB scheme and then derive the approximate closed-form expressions for the achievable throughput. Based on the overall system power constraint, we define the maximum achievable throughput and obtain the asymptotic closed-form expressions in the high- and low-SNR regimes, respectively. Subsequently, the special case of single-antenna relays is provided as well.

#### 4.1. The ADB Policy

The operation of the ADB can be seen in Figure 2, which has two patterns. Time is slotted into discrete equal-size time slots. We divide \( L \) relays into two groups, i.e., group 1 with \( M \) relays, \( \mathcal{R}_1 = \{R_1, \ldots, R_M\} \) and group 2 with \( L - M \) relays, \( \mathcal{R}_2 = \{R_{M+1}, \ldots, R_L\} \) (Due to the i.i.d. assumption, the relays can be divided arbitrarily. In case of different fading statistics, the relay grouping will be another interesting problem). In pattern I, the source broadcasts messages to the relays in group \( \mathcal{R}_1 \) for each \( t_1 \)-th time slot while at the same time, the relays in group \( \mathcal{R}_2 \) beamform the same message to the destination. It is assumed that the relays are synchronized through signaling. By contrast, in pattern II, the relays in group \( \mathcal{R}_2 \) must decode the message transmitted by the source node and stores the packet in their buffers for each \( t_2 \)-th time slot while the relays in group \( \mathcal{R}_1 \) transmit the previously received packets through distributed beamforming to the destination. Denote \( \mathcal{T}_1 = \{t_1\}, \mathcal{T}_2 = \{t_2\} \) as the set of time indices for the relays in groups 1 and 2 receiving data from the source, respectively. Note that \( \mathcal{T}_1 \cup \mathcal{T}_2 = \{1, 2, \ldots, N\} \), where \( N \) is the total number of time slots. We assume that the cardinality of \( \mathcal{T}_1 \) and \( \mathcal{T}_2 \) is \( |\mathcal{T}_1| = |\mathcal{T}_2| = N/2 \). Denote \( R_m \) as the average arrival rate at the relay buffer and \( R_{out} \) as the departure rate considering the queue of the buffer.
Note that $R_{in} \geq R_{out}$ is always valid due to the buffered relaying protocol, and when $R_{in} > R_{out}$, the queue of the buffer is said to be in the absorbing state. One can always improve the policy to increase the throughput until it reaches the point where $R_{in} = R_{out}$, i.e., under the optimal policy, the queue stays at the edge of nonabsorption where we have $R_{in} = R_{out}$. And the probability that the queue is empty is close to zero when $R_{in} = R_{out}$ [26]. Moreover, the relays do not have to transmit when the queue is empty at the initial state, and it would not affect the overall performance in the long run.

It is evidently that with this protocol, a virtual FD system is created with successive relay group, and the HD loss of conventional relays can hence be recovered and distributed beamforming gain can also be expected. And it is worth noting that compared with the selective protocols, there is no instantaneous CSI requirement for the proposed scheme when selecting the relays for transmission and reception, i.e., the receiving and transmitting relays are predetermined in advance of data transmissions, which makes it easier to implement in practice.

4.2. Achievable Throughput Analysis. Here we analyze the achievable throughput of the proposed ADB scheme and derive the approximate closed-form expressions. First, we have the following results.

**Proposition 1.** Given the transmit power levels $P_S$ and $P_R$, the achievable throughput of the proposed scheme can be expressed as

$$C_{ADB}(P_S, P_R) = \frac{1}{2 \min \left\{ \mathbb{E} \left[ \log_2 \left( 1 + P_S \min_{R_{SR} \in \mathcal{R}_1} \left\| h_{SR} \right\|^2 \right) \right], \mathbb{E} \left[ \log_2 \left( 1 + P_R \sum_{R_{SR} \in \mathcal{R}_2} \left\| h_{SR} \right\|^2 \right) \right] \right\}}{\frac{1}{2} \min \left\{ \left[ \log_2 \left( 1 + P_R \sum_{R_{SR} \in \mathcal{R}_1} \left\| h_{SR} \right\|^2 \right) \right], \mathbb{E} \left[ \log_2 \left( 1 + P_S \min_{R_{SR} \in \mathcal{R}_1} \left\| h_{SR} \right\|^2 \right) \right] \right\} +}.$$
Figure 5: The maximum achievable throughput of high/low SNR.

Figure 6: The maximum achievable throughput versus different grouping modes of the proposed scheme.
Proof. Considering each group of relays, we know that the achievable throughput of the data flow passing through group $R_1$ is given by 

$$C_{ADB_1} = \min \left\{ \lim_{N \to \infty} \frac{1}{N} \sum_{t_1 \in \mathcal{T}_1} \log_2 \left( 1 + P_S \min_{R_i \in \mathcal{R}_1} \left( \|h_{SR_i}(t_1)\|^2 \right) \right) \right\}$$

$$= \frac{1}{2} \min \left\{ \mathbb{E} \left[ \log_2 \left( 1 + P_R \sum_{R_i \in \mathcal{R}_1} \|h_{RD}(t_2)\|^2 \right) \right] \right\},$$

(11)

where $h_{SR_1}(t_1)$ and $h_{RD_1}(t_2)$ denote the channel coefficients of the $S - R_1$ and $R_1 - D$ links in the $t_1$-th and $t_2$-th time slots, respectively, and the last equality comes from the fact that each transmission mode occupies half of the whole transmission time slots, i.e., $|\mathcal{T}_1| = |\mathcal{T}_2| = N/2$. The computation of $C_{ADB_1}$ is similar to that of $C_{ADB}$. Therefore, the achievable throughput of ADB can be calculated as

$$C_{ADB}(P_S, P_R) = C_{ADB_1} + C_{ADB_2},$$

(12)

as shown in (10).

Remark 2. The mode switching frequency $Q$, i.e., the relays selected to receive data in every $Q$ consecutive time slots, does not change the throughput. It is obvious that as $Q$ increases, the queue length increases, and the average delay increases as well. Without loss of generality, we consider the case that $\mathcal{T}_1 = \{1, 3, 5 \cdots \}$ while $\mathcal{T}_2 = \{2, 4, 6 \cdots \}$ throughout this paper, i.e., the two different transmission modes alternate every time slot.
Denote
\[
C_{11} = \mathbb{E} \left[ \log_2 \left( 1 + P_R \min_{k \in \mathbb{F}_2} \left( \frac{1}{\|h_{SR}\|^2} \right) \right) \right],
\]
\[
C_{12} = \mathbb{E} \left[ \log_2 \left( 1 + P_R \left( \sum_{k \in \mathbb{F}_2} \|h_{SR,D}\|^2 \right) \right) \right],
\]
\[
C_{21} = \mathbb{E} \left[ \log_2 \left( 1 + P_R \min_{k \in \mathbb{F}_2} \left( \frac{1}{\|h_{SR}\|^2} \right) \right) \right],
\]
\[
C_{22} = \mathbb{E} \left[ \log_2 \left( 1 + P_R \left( \sum_{k \in \mathbb{F}_2} \|h_{SR,D}\|^2 \right) \right) \right].
\]

(13)

**Proposition 3.** Given $P_S$ and $P_R$, the approximate closed-form expressions for the achievable throughput of ADB in Rayleigh fading channels are given by
\[
C_{ADB}(P_S, P_R) = \frac{1}{2} \min(C_{11}, C_{22}) + \frac{1}{2} \min(C_{21}, C_{12})
\]
\[
= \begin{cases} 
\frac{1}{2} (C_{11} + C_{22}) & \text{if } C_{11} < C_{22} \text{ and } C_{21} < C_{12}, \\
\frac{1}{2} (C_{11} + C_{12}) & \text{if } C_{11} < C_{22} \text{ and } C_{21} > C_{12}, \\
\frac{1}{2} (C_{22} + C_{12}) & \text{if } C_{11} > C_{22} \text{ and } C_{21} < C_{12}, \\
\frac{1}{2} (C_{22} + C_{12}) & \text{otherwise,}
\end{cases}
\]

(14)

where
\[
C_{11} = \sum_{n_1+\cdots+n_M=M \atop n_1+n_2+\cdots+n_M=M} \left( \frac{M}{n_1,n_2,\ldots,n_M} \right) \cdot \left( \frac{1}{2\sigma_r^2} \right)^p \cdot q \cdot \ln 2
\]
\[
C_{22} = \sum_{k=0}^{\min(\max(L-M,0),N-1)} \frac{1}{k!(1/2\sigma_r^2)^k} \ln 2 \left( \frac{1}{2\sigma_r^2} \right)^k e^{-(L-M)/2\sigma_r^2} \cdot E_{1}(L-M/2\sigma_r^2).
\]
\[
C_{21} = \sum_{n_1+\cdots+n_M=M \atop n_1+n_2+\cdots+n_M=M} \left( \frac{M}{n_1,n_2,\ldots,n_M} \right) \cdot \left( \frac{1}{2\sigma_r^2} \right)^p \cdot q \cdot \ln 2
\]
\[
C_{12} = \sum_{k=0}^{\min(\max(L-M,0),N-1)} \frac{1}{k!(1/2\sigma_r^2)^k} \ln 2 \left( \frac{1}{2\sigma_r^2} \right)^k e^{-(L-M)/2\sigma_r^2} \cdot E_{1}(L-M/2\sigma_r^2).
\]

(15)

where the symbol $C$ is used for the analysis of the approximate closed-form expressions for the achievable throughput of the proposed ADB scheme in Rayleigh fading channels, and
\[
p = \sum_{i=0}^{N-1} i \cdot n_{i+1},
\]

(16)
\[
q = \prod_{i=0}^{N-1} (d_i)^{n_{i+1}},
\]

(17)

with $\left( n_1, n_2, \ldots, n_{N_r} \right) = M!n_1!n_2!\cdots n_{N_r}!$, and $E_1(x) = \int_x^\infty (e^{-t}/t) dt; x > 0$ is the exponential integral function.

**Proof.** Please see Appendix A.

Note that the values of $C_{11}, C_{12}, C_{21},$ and $C_{22}$ only depend on the transmit power values, the ADB parameter $M$, the total number of relays $L$, and the channel statistics. Therefore, given these parameters, we can compute the values of $C_{11}, C_{12}, C_{21},$ and $C_{22}$ offline and conduct the following design on the optimal power allocation in advance.

Given the total power constraint of the network denoted as SNR, we can allocate the total power to the source and relays to achieve the best performance. Note that the derived optimal values of the $P_S$ and $P_R$ can be viewed as an optimal configuration of the transmission power for the relay networks.

**ADB.** The receiving and transmitting relays for each time slot have been determined before the system starts its normal operation. The source transmits in every time slot, while the relays to enable each relay to be capable of being selected for transmission. Again, the source works for all time slots. So we should have $P_S + L/2P_R \leq SNR$.

**SFD-MMRS.** We allocate transmit power to the source and $L$ relays to enable each relay to be capable of being selected for transmission. Again, the source works for all time slots. Therefore, we should have $P_S + L/2P_R \leq SNR$.

**CRS.** We should allocate transmit energy to the source and $L$ relays, albeit the data transmission occupies two time slots. Therefore, we should have $1/2(P_S + L/2P_R) \leq SNR$.

**DF.** Each relay must decode the common message transmitted by the source node and beamform their transmissions to the destination, and obviously we need to allocate transmit energy to source and $L$ relays. It is also performed in two time slots, so we should have $1/2(P_S + L/2P_R) \leq SNR$.

Consider the achievable throughput in (14), once given the total power SNR, it is obvious that when $P_S$ is small, the throughput is limited by the source-relay hop. On the
other hand, when $P_R$ is small, the relay-destination hop will be the bottleneck of the system. Therefore, there is always an optimal power allocation that maximizes the achievable throughput.

**Definition 4.** The maximum achievable throughput of ADB is given by

$$C_{\text{max}} = \max_{P_S + L/2P_R \leq \text{SNR}} C_{\text{ADB}}(P_S, P_R).$$

Similarly, we can define the maximum achievable throughput for DF, CRS, and SFD-MMRS with the total power constraint of the network SNR.

### 4.3. High-SNR Regime

In this part, we perform the asymptotic analysis for the maximum achievable throughput of the proposed scheme in the high-SNR regime. We have the following result.

**Theorem 5.** Given the total power SNR, the asymptotic closed-form expression for the maximum achievable throughput of ADB in the high-SNR regime for Rayleigh fading channels is given by (19).

$$C_{\text{ADB}}^{\text{high}} = \frac{1}{2} \ln 2 \left[ \ln \left( \frac{2\text{SNR}}{e^{\sigma^2_M/\sigma^2_h} + L/2\sigma^2_h} \right) + \sum_{k=1}^{N_k} \frac{1}{k} \ln \left( \frac{2\text{SNR}}{e^{\sigma^2_M/\sigma^2_h} + L/2(1 - M)\sigma^2_h} \right) + \sum_{k=1}^{N_k(l-M)} \frac{1}{k} \ln 2^{y} \right],$$

(19)

where $y \approx 0.5772156649$, and

$$a_1 = \sum_{k=1}^{N_k} \frac{1}{k} - \sum_{n_i \geq 0, n_1 + n_2 + \cdots + n_N = M} \binom{M}{n_1, n_2, \cdots, n_N} (p-1)! \frac{\text{SNR}}{qM^p},$$

(20)

$$a_2 = \sum_{k=1}^{N_k(l-M)} \frac{1}{k} - \sum_{n_i \geq 0, n_1 + n_2 + \cdots + n_N = L-M} \binom{L-M}{n_1, n_2, \cdots, n_N} (p-1)! \frac{\text{SNR}}{q(L-M)^p}.$$
4.5. Single-Antenna Relays. In this part, we consider the single-antenna relay scenario as a special case. We use \( g_i \) and \( h_i \) for \( i = 1, \ldots, L \) to denote the channel coefficients of \( S \rightarrow R_i \) and \( R_i \rightarrow D \) links, respectively. The instantaneous received SNR at relay \( R_i \) is now given by \( \gamma_i = P_i |g_i|^2/N_0 \), and the instantaneous received SNR at the destination from relay \( R_i \) is given by \( \gamma_i = P_i |h_i|^2/N_0 \).

Similarly, given the transmit power levels \( P_S \) and \( P_R \), the achievable throughput of ADB can be expressed as

\[
C_{ADB}(P_S, P_R) = \frac{1}{2} \min \left\{ \mathbb{E} \left( \log_2 \left( \frac{1+P_S \min_{R_i \in \mathcal{R}} (|g_i|^2)}{1+P_R \sum_{R_i \in \mathcal{R}} |h_i|^2} \right) \right), \mathbb{E} \left( \log_2 \left( \frac{1+P_R \sum_{R_i \in \mathcal{R}} |h_i|^2}{1+P_S \min_{R_i \in \mathcal{R}} (|g_i|^2)} \right) \right) + \frac{1}{2} \min \left\{ \mathbb{E} \left( \log_2 \left( \frac{1+P_S \min_{R_i \in \mathcal{R}} (|g_i|^2)}{1+P_R \sum_{R_i \in \mathcal{R}} |h_i|^2} \right) \right), \mathbb{E} \left( \log_2 \left( \frac{1+P_R \sum_{R_i \in \mathcal{R}} |h_i|^2}{1+P_S \min_{R_i \in \mathcal{R}} (|g_i|^2)} \right) \right) \right\}.
\]

(25)

Proposition 8. Given \( P_S \) and \( P_R \), the approximate closed-form expressions for the achievable throughput of ADB in Rayleigh fading channels are given by

\[
C_{ADB} = \frac{1}{2} \min \left( C_{11}, C_{22} \right) + \frac{1}{2} \min \left( C_{21}, C_{12} \right)
\]

where

\[
C_{11} = \frac{e^{M/2\sigma^2}}{\ln 2} \frac{M}{2\sigma^2},
\]

\[
C_{22} = \frac{1}{\ln 2} \left\{ \frac{e^{1/2b_1P_{R}\sigma^2}}{2} \left( \frac{1}{P_{R}\sigma^2} \right) + 1 \right\}
\]

\[
C_{21} = \frac{1}{\ln 2} \left\{ \frac{e^{1/2b_1P_{R}\sigma^2}}{2} \left( \frac{1}{P_{R}\sigma^2} \right) + 1 \right\}
\]

(27)

4.4. Low-SNR Regime. In this part, we perform the asymptotic analysis for the maximum achievable throughput of ADB in low-SNR regime. We have the following result.

Theorem 7. Given the total power SNR, the approximate closed-form expression for the maximum achievable throughput of ADB in low-SNR regime for Rayleigh fading channels is given by (23), and the corresponding optimal power allocation is

\[
C_{11} = \frac{e^{M/2\sigma^2}}{\ln 2} \frac{M}{2\sigma^2},
\]

\[
C_{22} = \frac{1}{\ln 2} \left\{ \frac{e^{1/2b_1P_{R}\sigma^2}}{2} \left( \frac{1}{P_{R}\sigma^2} \right) + 1 \right\}
\]

\[
C_{21} = \frac{1}{\ln 2} \left\{ \frac{e^{1/2b_1P_{R}\sigma^2}}{2} \left( \frac{1}{P_{R}\sigma^2} \right) + 1 \right\}
\]

where

\[
b_1 = \frac{\sigma^2}{M}, \quad (2M-1)!! = (2M-1)(2M-3)\ldots 1,
\]

(28)

and \( (2M-1)!! = (2M-1)(2M-3)\ldots 1 \), and \( E_1(x) = \int_0^\infty (e^{-t})dt, t > 0 \) is the exponential integral function.

Proof. Please see Appendix D.

5. Numerical Results

In this section, we evaluate the proposed ADB scheme and compare it with that of CRS [3], SFD-MMRS [4], and DF [2]. We assume that \( \sigma^2 = \sigma^2_0 = 1 \), unless specified otherwise.

Figure 3 plots the achievable throughput versus \( P_S/P_R \). We assume SNR = 10 dB, \( N_R = 6 \), \( L = 4 \), and \( M = 2 \). We can find that the achievable throughput always has a peak value as \( P_S/P_R \) varies, which verifies that once given the total power SNR, there is always an optimal power allocation that maximizes the achievable throughput. We can see that the
proposed scheme achieves the largest maximal throughput. We also note that the analytical results obtained based on the derivation in (14) match the simulation results, which verifies the approximate closed-form expressions.

In Figure 4, we compare the maximum achievable throughput of the proposed ADB scheme with several existing schemes as SNR varies. We assume \( N_R = 6, L = 4, \) and \( M = 2 \). We can find that the proposed scheme achieves the best performance.

In Figure 5, we plot the maximum achievable throughput of the proposed ADB scheme in the high- and low-SNR regimes, respectively. We assume \( N_R = 6, L = 4, \) and \( M = 2 \). We note that the theoretical results obtained based on Theorems 5 and 7 match the simulation results in the high- and low-SNR regimes, respectively, which verifies the asymptotic closed-form expressions.

In Figure 6, we compare the maximum achievable throughput of the ADB scheme versus different \( M \), i.e., different grouping modes. We assume SNR = 10 dB, \( N_R = 6 \), and \( L = 6 \). It is interesting that, the symmetric allocation of relays achieves the best performance with the given setting, which verifies Corollary 6. This is generally because that balanced beamforming gain can be retained within each group.

In Figure 7, we plot the maximum achievable throughput of each scheme versus the number of antennas of each relay. We assume SNR = 10 dB, \( L = 4 \), and \( M = 2 \). We can clearly see that the maximum achievable throughput improves as \( N_R \) increases. And the proposed scheme achieves a significant improvement in achievable throughput. We also observe that as \( N_R \) increases, the superiority of the proposed scheme over other strategies in achievable throughput becomes more apparent.

In Figure 8, we plot the maximum achievable throughput of each scheme versus the number of relays for the single-antenna scenario. We assume SNR = 10 dB and \( M = L/2 \). We can find that the proposed scheme achieves the best performance in all cases. It is interesting that for a given SNR, the proposed scheme achieves the largest maximum throughput when \( L = 10 \), and the DF strategy achieves the best throughput performance when \( L = 6 \). This is generally due to the tradeoff between the reduction in the power allocated to relays and the increased beamforming gain as \( L \) increases considering the total power constraint.

Figure 9 plots the maximum achievable throughput versus the number of relays \( L \) for a fixed total number of antennas. We assume \( M = L/2 \), and the total number of antennas is fixed as \( N_f = 24 \). It is interesting that the maximum achievable throughput decreases as \( L \) increases. In other words, when the total number of antennas at the relays is constant, it is better to increase the number of antennas per relay rather than the number of single-antenna relays.

6. Conclusion

In this paper, we have proposed a novel transmission protocol named ADB for buffer-aided multiple relay systems, in which the relays are divided into two groups, with one group receiving the signals transmitted by the source node while at the same time, the other group beamforming the previously received data to the destination. The two groups are activated alternately in their transmitter and receiver modes to combat the HD loss. In this protocol, the relays used for reception and transmission are predetermined without the need of instantaneous channel state information (CSI). We have considered the multiantenna multirelay scenario. We have obtained the closed-form expressions of the achievable throughput for the proposed scheme. We have also obtained the asymptotic expressions for the maximum achievable throughput and corresponding optimal power allocation in the high- and low-SNR regimes, respectively. Our results corroborate that the ADB scheme achieves significant improvement over the benchmark schemes in terms of the maximum achievable throughput. In addition, we have found that the almost or exact symmetric allocation of relays may achieve the best performance in the high-SNR regime. Furthermore, for a given total number of antennas, the proposed scheme achieves the better throughput performance with the increased number of antennas per relay rather than the increased number of single-antenna relays.

Future directions include the study of relay grouping based on the proposed scheme considering the path loss. The successive opportunistic relay selection policy based on the assumption of full CSI could also be designed to further improve the throughput performance. In addition, the analysis of the interference between the relays could be involved, and more efficient ways of interference mitigation and exploitation can be examined. On top of that, when the buffer size is finite, packet-based transmission and Markov chain methods could be adopted, which is also a good direction to expand.

Appendix

A. Proof of Proposition 3

To compute the achievable throughput of ADB in (14), we need to find \( C_{11}, C_{12}, C_{21}, \) and \( C_{22} \).

**Computation of** \( C_{11} \). In this case, we denote \( z = \min_{i \in \{1, \ldots, M\}} t_i \), where \( t_i = \|h_{SR_i}\|^2 \). Therefore, to derive \( C_{11} \), we first compute the cumulative distribution function (CDF) of \( z \). Denote \( z = \min (\|h_{SR_1}\|^2, \|h_{SR_2}\|^2, \ldots, \|h_{SR_M}\|^2) \). \( t_i \) follows the Erlang distribution. The CDF of \( t_i \) is given by ([27], 17.2)

\[
F_{T_i}(t_i) = 1 - \sum_{r=0}^{N_f-1} \left( \frac{t_i/2\sigma_g^2}{r!} \right)^r e^{-t_i/2\sigma_g^2}. \tag{A.1}
\]

Then, the CDF of \( z \) can be calculated as

\[
F_z(z) = P\left( \min_{i \in \{1, \ldots, M\}} t_i \leq z \right) = 1 - P\left( \min_{i \in \{1, \ldots, M\}} t_i \geq z \right) = 1 - (1 - F_{T_i}(z))^M = 1 - \left( \sum_{r=0}^{\infty} \left( \frac{z/2\sigma_g^2}{r!} \right)^r e^{-z/2\sigma_g^2} \right)^M. \tag{A.2}
\]
Then, $C_{11}$ can be obtained as

$$C_{11} = \mathbb{E} \left[ \log_2 \left( 1 + P_S \min_{i \in \{1, \ldots, M\}} \| h_{SR} \|_2^2 \right) \right] = \mathbb{E} [\log_2 (1 + P_S z)]$$

$$= \int_0^\infty \log_2 (1 + P_S z) d( F(z) (z - 1))$$

$$= \sum_{n_1 \geq 0} \frac{M}{n_1, n_2, \ldots, n_N R} \cdot \left( 1/2 \sigma_g^2 \right) \sum_{n=1}^{N_R-1} \frac{z^{n-1} e^{-z}}{z + 1/PS}$$

$$= b \sum_{n_1 \geq 0} \frac{M}{n_1, n_2, \ldots, n_N R} \cdot \left( 1/2 \sigma_g^2 \right) \sum_{n=1}^{N_R-1} \frac{z^{n-1} e^{-z}}{z + 1/PS}$$

$$= \left[ \sum_{i=0}^{N_R-1} \frac{i^{n_1+1} e^{-i/PS}}{i!}, \frac{M}{2 \sigma_g^2} \right]$$

$$+ \sum_{r=1}^{N_R-1} (r-1)! \left( -1/PS \right) \sum_{i=0}^{N_R-1} i^{n_1+r} \cdot \left( \frac{M}{2 \sigma_g^2} \right)^r,$$

where polynomial theorem is used in equality (a) and [28], Eq. 3.353.5, is used to obtain equality (b). And $E_1(x) = \int_x^\infty (e^{-t}/t) dt, x > 0$ is the exponential integral function.

The computation of $C_{21}$ is similar to that of $C_{11}$ and is expressed as

$$C_{21} = \sum_{n_1 \geq 0} \frac{L - M}{n_1, n_2, \ldots, n_N R} \cdot \left( 1/2 \sigma_g^2 \right) \sum_{n=1}^{N_R-1} \frac{z^{n-1} e^{-z}}{z + 1/PS}$$

$$+ \sum_{r=1}^{N_R-1} (r-1)! \left( -1/PS \right) \sum_{i=0}^{N_R-1} i^{n_1+r} \cdot \left( \frac{L - M}{2 \sigma_g^2} \right)^r.$$

**Computation of $C_{22}$**. In this case, let $z = \sum_{i=1}^M z_i$, where $z_i = \| h_{SR, i} \|_2^2, i \in \{1, \ldots, M\}$. Denote $t_i = \| h_{SR, i} \|_2^2, i \in \{1, \ldots, M\}$. $t_i$ follows the Erlang distribution, and the probability density function (PDF) of which is given by

$$f_{T_i}(t_i) = \frac{(1/2 \sigma_g^2)^{N_R-1} e^{-t_i/2 \sigma_g^2}}{(N_R-1)!}.$$

Note that when the shape parameter of gamma distribution is integer, the gamma distribution coincides with the Erlang distribution. And it is well known that a Nakagami-m random variable (RV) is the square root of a gamma RV, so we can know that $s_i$ follows the Nakagami distribution, and the PDF of $s_i$ is correspondingly given by

$$f_{S_i}(s_i) = \frac{(1/2 \sigma_g^2)^{N_R-1} e^{-s_i/2 \sigma_g^2}}{(N_R-1)!}.$$

We denote $z_i \sim \mathcal{M} (s_i, m, \Omega)$, where $m = N_R$ and $\Omega = 2N_R \sigma_R^2$. So far we know that $z$ is a sum of $M$ i.i.d. Nakagami random variables (RVs). A relatively simple and widely used approximation for the sum PDF was given in [29], (82), (84), from which we have $z \sim \mathcal{M} (zM, m, \Omega)$, where $m = mM$ and $\Omega = M^2 \Omega$. Then, the approximate PDF of $z$ is given by

$$f_Z(z) = \frac{(1/2 \sigma_g^2)^{N_R} z^{2N_R-1} e^{-z/2 \sigma_g^2}}{(N_R-1)!}.$$

The CDF of $z$ is simply obtained by integrating the PDF in (A.7) with respect to $z$ and is given by

$$F_Z(z) = \int_0^z \frac{(1/2 \sigma_g^2)^{N_R} t^{2N_R-1} e^{-t/2 \sigma_g^2}}{(N_R-1)!} dt = \frac{1}{(2M \sigma_g^2)^{N_R}} \left( \frac{z}{N_R-1} \right)^{N_R-1}. (N_R-1)!$$

Then, $C_{22}$ can be computed as

$$C_{22} = \mathbb{E} \left[ \log_2 \left( 1 + P_R \sum_{i=1}^M \| h_{SR, i} \|_2^2 \right)^2 \right]$$

$$= \int_0^\infty \log_2 \left( 1 + P_R z^2 \right) f_Z(z) dz$$

$$= \int_0^\infty \log_2 \left( 1 + P_R z^2 \right) \left[ \left( 1/2 \sigma_g^2 \right)^{N_R-1} e^{-z/2 \sigma_g^2} \right]$$

$$+ \sum_{k=0}^{N_R-1} \frac{2k}{k! (1/2 \sigma_g^2)^k} \int_0^\infty e^{-z/2 \sigma_g^2} dz,$$

$$= \sum_{k=0}^{N_R-1} \frac{2k}{k! (1/2 \sigma_g^2)^k} \log_2 \left( 1 + P_R z^2 \right).$$
where [28], Eq. 3.353.5, is used to obtain the final equality.

The computation of $C_{12}$ is similar to that of $C_{22}$, and is written as

$$C_{12} = \sum_{k=0}^{N_{c}(L-M)-1} \frac{1}{k! (2(L-M) \sigma_n^2)^k} \ln 2 \left[ \frac{1}{P_R} \right]^k e^{\frac{1}{2P_R} (L-M) \sigma_n^2} E_1 \left( \frac{1}{2P_R} \sigma_n^2 \right) + \sum_{r=1}^{k} \left( \frac{1}{P_R} \right)^{k-r} \left( \frac{1}{2(L-M) \sigma_n^2} \right)^{r-1} \right].$$

$$= \left. \sum_{k=0}^{N_{c}(L-M)-1} \frac{1}{k! (2(L-M) \sigma_n^2)^k} \ln 2 \left[ \frac{1}{P_R} \right]^k e^{\frac{1}{2P_R} (L-M) \sigma_n^2} E_1 \left( \frac{1}{2P_R} \sigma_n^2 \right) \right|_{r=1}^{k}.$$  

Finally, $C_{ADB}$ is obtained by substituting (A.3), (A.4), (A.10), and (A.11) into (14).

**B. Proof of Theorem 5**

First, to compute the achievable throughput of the proposed ADB scheme in high-SNR regime, we need to find $C_{11}$, $C_{21}$, $C_{22}$, and $C_{12}$ in high-SNR regime, which is denoted as $C_{11}^{\text{high}}$, $C_{21}^{\text{high}}$, $C_{22}^{\text{high}}$, and $C_{12}^{\text{high}}$, respectively.

(a) **Computation of $C_{11}^{\text{high}}$.** As the total power $\text{SNR} \rightarrow \infty$, we know that $P_S \rightarrow \infty$, and $C_{11}^{\text{high}}$ can be obtained as

$$C_{11}^{\text{high}} = \lim_{P_S \rightarrow \infty} C_{11} = \lim_{P_S \rightarrow \infty} \sum_{n_1 \geq 0} \sum_{n_2 \geq 0} \cdots \sum_{n_M \geq 0} \left( \frac{M}{n_1, n_2, \ldots, n_M} \right) \left( \frac{1}{2\sigma_n^2} x \right)^{p} \sum_{q=2}^{\infty} \frac{(-x)^q}{k!} x > 0,$$  

where equality (c) is obtained from the fact that as $P_S \rightarrow \infty$, $e^{M(2P_S \sigma_n^2)} \rightarrow 1$, $-1/(P_S)^{p-q} \rightarrow 0$ as $r \neq p$ and $(-1/P_S)^{p-q} \rightarrow 1$ as $r = p$. (B.1) utilizes the fact that when $n_1 = M$, $p = q = 1$ according to (17). We first consider the term $P_S^{-\infty}$. Here, we use an approximation for the exponential integral function $E_1(x)$ [30]:

$$E_1(x) = \ln \left( \frac{1}{x} \right) - \gamma - \sum_{k=1}^{\infty} \frac{(-x)^k}{kk!}, x > 0.$$
where $\gamma = 0.5772156649$ is the Euler constant. Then, the value of $A$ as $P_S \rightarrow \infty$ can be calculated as

$$
\lim_{P_S \rightarrow \infty} A = \lim_{P_S \rightarrow \infty} \frac{E_1\left(\frac{M}{2P_S\sigma^2}\right)}{P_S^\gamma} = 0.
$$

Then, $C_{11}^{\text{high}}$ can be obtained by

$$
C_{11}^{\text{high}} = \lim_{P_S \rightarrow \infty} \frac{1}{\ln 2} E_1\left(\frac{M}{2P_S\sigma^2}\right) + \sum_{\substack{n_1, n_2, \ldots, n_q \geq 0,\cr n_1 + n_2 + \cdots + n_q = M}} \frac{M}{M^q \ln 2} (p-1)!.
$$

(b) Computation of $C_{21}^{\text{high}}$. The computation of $C_{21}^{\text{high}}$ is similar to that of $C_{11}^{\text{high}}$ and is obtained as

$$
C_{21}^{\text{high}} = \frac{1}{\ln 2} \left[ \log \left( \frac{2P_S\sigma^2}{M-L} \right) - \log \frac{L-M}{(L-M)^q} \right]
$$

(c) Computation of $C_{22}^{\text{high}}$. As the total power $\text{SNR} \rightarrow \infty$, we also have $P_R \rightarrow \infty$, and $C_{22}^{\text{high}}$ in high-SNR regime, which is denoted as $C_{22}^{\text{high}}$, can be obtained as

$$
C_{22}^{\text{high}} = \lim_{P_R \rightarrow \infty} C_{22} = \lim_{P_R \rightarrow \infty} \frac{N_{ae} - 1}{k} \log \left( \frac{M}{(1/2\sigma^2)^k} \right) + \sum_{k=0}^{N_{ae}-1} \frac{1}{k!} \left( \frac{1}{2\sigma^2} \right)^k
$$

(d) Computation of $C_{12}^{\text{high}}$. The computation of $C_{12}^{\text{high}}$ is similar to that of $C_{22}^{\text{high}}$ and is derived as

$$
C_{12}^{\text{high}} = \frac{1}{\ln 2} \left[ \log \left( \frac{2P_R\sigma^2}{M-L} \right) - \log \frac{L-M}{(L-M)^q} \right] + \sum_{k=0}^{N_{ae}-1} \frac{1}{k}.
$$

Considering (14), (18), (B.4), and (B.6), the maximal achievable throughput can be achieved when
\[
\begin{align*}
C_{11}^{\text{high}} = C_{22}^{\text{high}} & \quad \Rightarrow \quad P_S = \frac{\text{SNR}}{1 + \frac{L\sigma^2_g}{2e^{\alpha M}M \sigma_h^2}}, \\
P_R = \frac{\text{SNR}}{e^{\alpha M}M \sigma_h^2 + L/2},
\end{align*}
\]

(B.8)

where

\[
a_1 = \sum_{k=1}^{N_n M - 1} \frac{1}{L} - \sum_{k=0}^{n_1 + \cdots + n_{N_n} = M} \frac{M}{qM^p} (p-1)!,
\]

\[
a_2 = \sum_{k=1}^{N_n (L-M) - 1} \frac{1}{L} - \sum_{k=0}^{n_1 + \cdots + n_{N_n} = L-M} \frac{L-M}{q(L-M)^p} (p-1)!,
\]

(B.9)

Finally, given the total power SNR, the approximate closed-form expressions for the maximum achievable throughput of ADB in high-SNR regime, which is denoted as \(C_{\text{ADB}}^{\text{high}}\), can be obtained by substituting (B.8) into (B.4)-(B.7) and then (14). Specifically,

\[
C_{\text{ADB}}^{\text{high}} = \frac{1}{2} \ln 2 \left[ \ln \left( \frac{25\text{SNR}}{e^{\alpha (L-M)/\sigma_h^2 + L/2 (L-M) e^{\alpha M} M \sigma_h^2}} \right) + \sum_{k=1}^{N_n M - 1} \frac{1}{L} + \ln \left( \frac{25\text{SNR}}{e^{\alpha (L-M)/\sigma_h^2 + L/2 (L-M) e^{\alpha M} M \sigma_h^2}} \right) + \sum_{k=1}^{N_n (L-M) - 1} \frac{1}{L} - 2\gamma \right].
\]

(B.10)

C. Proof of Theorem 7

First, to compute the achievable throughput of the proposed ADB scheme in low-SNR regime, we need to find \(C_{11}, C_{21}, C_{22}, \) and \(C_{12}\) in low-SNR regime, which is denoted as \(C_{11}^{\text{low}}, C_{21}^{\text{low}}, C_{22}^{\text{low}}, \) and \(C_{12}^{\text{low}}\), respectively.

(a) Computation of \(C_{11}^{\text{low}}\). We first compute the \(C_{11}^{\text{low}}\). We denote \(z = \min_{i \in \{1, \ldots, M\}} \|h_{SRi}\|^2\), the CDF of which has been given in (A.2). As the total power SNR \(\rightarrow 0\), we have \(P_S \rightarrow 0\). Here, we use the approximation \(\log_2 (1 + x) \approx x \log_2 e\) for small \(x\) to get

\[
C_{11}^{\text{low}} = \mathbb{E} \left[ \log_2 (1 + P_S \min_{i \in \{1, \ldots, M\}} \|h_{SRi}\|^2) \right] = \mathbb{E} [\log_2 (1 + P_S z)] = \mathbb{E} [P_S \log_2 e] = P_S \log_2 e \mathbb{E} (z),
\]

(C.1)

where \(\mathbb{E} (z)\) can be computed as

\[
\mathbb{E} (z) = \int_{0}^{\infty} z f(z) dz = \int_{0}^{\infty} zdF(z) = -z \left( \int_{0}^{\infty} \left( \sum_{n=1}^{N_n} \left( \frac{z/2\sigma^2_g}{r^2} \right)^n \right) M e^{\alpha n} dz \right) + \int_{0}^{\infty} z \left( \sum_{n=1}^{N_n} \left( \frac{z/2\sigma^2_g}{r^2} \right)^n \right) M e^{-\alpha n} dz
\]

\[
= -z \left( \sum_{n=1}^{N_n} \frac{(z/2\sigma^2_g)^n}{n!} \right) + \int_{0}^{\infty} z \left( \sum_{n=1}^{N_n} \frac{(z/2\sigma^2_g)^n}{n!} \right) M e^{-\alpha n} dz
\]

\[
= \left( \sum_{n=1}^{N_n} \frac{1}{n!} \right) z e^{\alpha M} e^{-\alpha n}
\]

(C.2)

where polynomial theorem is used in equality (h) and [28], Eq. 3.31.3, is used to obtain equality (i). Then, \(C_{11}^{\text{low}}\) can be obtained by substituting (C.2) into (C.1) and is expressed as

\[
C_{11}^{\text{low}} = \sum_{n_1 \geq 0} \left( \frac{M}{n_1, n_2, \ldots, n_{N_n}} \right) 2P_S \sigma^2_g \log_2 e \frac{qM^{p+1}}{1 + \log_2 e}
\]

(n_1 + n_2 + \cdots + n_{N_n} = M)

(C.3)

(b) Computation of \(C_{21}^{\text{low}}\). The computation of \(C_{21}^{\text{low}}\) is similar to that of \(C_{11}^{\text{low}}\) and is given by

\[
C_{21}^{\text{low}} = \sum_{n_1 \geq 0} \left( \frac{L-M}{n_1, n_2, \ldots, n_{N_n}} \right) 2P_S \sigma^2_g \log_2 e \frac{q(L-M)^{p+1}}{1 + \log_2 e}
\]

(n_1 + n_2 + \cdots + n_{N_n} = L-M)

(C.4)

(c) Computation of \(C_{22}^{\text{low}}\). Next, we compute the \(C_{22}^{\text{low}}\). We denote \(z = \sum_{i=1}^{M} \|h_{GBi}\|^2\), the PDF of which has been given in (A.7). As the total power SNR \(\rightarrow 0\), we also have \(P_R \rightarrow 0\)
Similarly,
\[
C_{22}^{\text{low}} = \mathbb{E}[\log_2 (1 + P_R z^2)] = \mathbb{E}(P_R z^2 \log_2 e) = P_R \log_2 e (z^2),
\]
where \(E(z^2)\) can be calculated as
\[
E(z^2) = \int_0^\infty z^2 f(z) dz
= \int_0^\infty \frac{2 \left(1/(2\Sigma)^{N_N M} \right)^{N_R M} \sigma_M^{-2} z^2 e^{-z^2/(2\Sigma)^2}}{(N_R M - 1)!} dz = \frac{\left(1/(2\Sigma)^{N_N M} \right)^{N_R M}}{(N_R M - 1)!} \int_0^\infty \frac{\left(1/(2\Sigma)^{N_N M} \right)^{N_R M}}{N_M!} \left(\frac{1}{2\Sigma M} \right)^{-N_M - 1} = 2N_R M^2 \sigma_h^2.
\]
(C.6)

To compute the achievable throughput of ADB in (26), we need to approximate closed-form expressions for the maximum achievable throughput of ADB in low-SNR regime, which is denoted as \(C_{22}^{\text{low}}\), can be obtained by substituting (C.6) into (C.5), and as a result,
\[
C_{22}^{\text{low}} = 2P_R N_R M^2 \sigma_h^2 \log_2 e.
\]
(C.7)

(d) Computation of \(C_{12}^{\text{low}}\). The computation of \(C_{12}^{\text{low}}\) is similar to that of \(C_{22}^{\text{low}}\) and is obtained as
\[
C_{12}^{\text{low}} = 2P_R N_R (L-M)^2 \sigma_h^2 \log_2 e.
\]
(C.8)

Following the same reasoning in Appendix B, the maximal achievable throughput can be achieved when
\[
\left\{
\begin{array}{l}
C_{11}^{\text{low}} = C_{22}^{\text{low}} \\
P_s + \frac{L}{2} P_R = \text{SNR}
\end{array}ight.
\Rightarrow
\left\{
\begin{array}{l}
P_s = \frac{P}{\text{SNR}} = \frac{1 + L/2 \sum n_i \geq 0, n_1 + n_2 + \cdots + n_N = M}{M} \sigma_h^2 p/(\sigma_h^2 N_R q M^{p+3})
P_R = \frac{P_s}{\text{SNR}} = \frac{1/\sum n_i \geq 0, n_1 + n_2 + \cdots + n_N = M}{M} \sigma_h^2 p/(\sigma_h^2 N_R q M^{p+3} + L/2)
\end{array}
\right.
\]
(C.9)

Finally, given the total power SNR, the approximate closed-form expressions for the maximum achievable throughput of ADB in low-SNR regime, which is denoted
\[
C_{\text{ADB}}^{\text{low}} = \frac{\text{SNR} \log_2 e}{1/\sum n_i \geq 0, n_1 + n_2 + \cdots + n_N = M} \left(\frac{M}{n_1, n_2, \cdots, n_N} \right) \sigma_h^2 p/(\sigma_h^2 N_R q M^{p+3}) + L/2 N_R M^2 \sigma_h^2
\]
\[
= \frac{\text{SNR} \log_2 e}{1/\sum n_i \geq 0, n_1 + n_2 + \cdots + n_N = M} \left(\frac{M}{n_1, n_2, \cdots, n_N} \right) \sigma_h^2 p/(\sigma_h^2 N_R q M^{p+3} + L/2 N_R (L-M)^2 \sigma_h^2)
\]
(C.10)

\section*{D. Proof of Proposition 8}
To compute the achievable throughput of ADB in (26), we need to find \(C_{11}, C_{12}, C_{21},\) and \(C_{22}\).

Computation of \(C_{11}\). In this case, we denote \(z = \min (g_1^2, g_2^2, \cdots, g_M^2)\). Therefore, to derive \(C_{11}\), we first compute the probability density function (PDF) of \(z\). The cumulative distribution function (CDF) of \(z\) is given by
\[
F_z(z) = P(\min (g_1^2, g_2^2, \cdots, g_M^2) \leq z) = 1 - e^{-M/2\sigma_h^2 z}.
\]
(D.1)

where [28], Eq. 3.351.3, is used to obtain equality (j). Then, \(C_{11}^{\text{low}}\) can be computed as
\[
C_{11}^{\text{low}} = 2P_R N_R M^2 \sigma_h^2 \log_2 e.
\]
(C.7)

Take the derivative of (D.1), and the PDF of \(z\) can be computed as \(f_z(z) = M/2\sigma_h^2 e^{-M/2\sigma_h^2 z}\). Then, \(C_{11}\) can be obtained as
\[
C_{11} = \mathbb{E}[\log_2 (1 + P_s z)] = \int_0^\infty \log_2 (1 + P_s z) f_z(z) dz
= \frac{e^{M/2\sigma_h^2 P_s}}{\ln 2} E_1 \left(\frac{M}{2\sigma_h^2 P_s}\right).
\]
(D.2)
The computation of $C_{21}$ is similar to that of $C_{11}$ and is given by

$$C_{21} = \frac{e^{-\frac{M^2}{2M^2}}}{\ln 2} E_1 \left( \frac{L - M}{2M^2} \right).$$  \hfill (D.3)

**Computation of $C_{22}$.** In this case, let $z = \sum_{i=1}^{M} h_i$ be a sum of $M$ i.i.d. Rayleigh random variables (RVs). Note that the distribution of an arbitrary sum of Rayleigh RVs does not exist in closed-form. As a result, numerical evaluations and approximations must be used [31]. A relatively simple and widely used small argument approximation (SAA) for the sum PDF was derived in [32]. Then, the SAA to the PDF of $z$

$$f_{SAA}(t) = \frac{t^{(2M-1)} e^{-t/2b}}{2^{M-1} M! (M - 1)!},$$  \hfill (D.4)

where $b = \sigma^2 / M[(2M - 1)!!]^{1/M}$, $(2M - 1)!! = (2M - 1)(2M - 3) \cdots 1$ and $t = z / \sqrt{M}$ is the normalized argument. Integration of (D.4) yields a SAA to the CDF of a Rayleigh sum given by

$$F_{SAA}(t) = 1 - e^{-t^2/2b} \sum_{k=0}^{M-1} \left( \frac{t^2/2b}{k!} \right)^k.$$  \hfill (D.5)

Then, the $C_{22}$ can be computed as

$$C_{22} = \mathbb{E} \left[ \log_2 \left( 1 + P_R (h_1 + h_2 + \ldots + h_M)^2 \right) \right] \nonumber$$

$$= \int_0^{\infty} \log_2 \left( 1 + P_R \left( 1 + \sqrt{M} \right)^2 \right) f(t) dt \nonumber$$

$$= \frac{1}{\ln 2} \left\{ \frac{t^{1/2b} P_R M}{(2b)^k \cdot k!} E_1 \left( \frac{1}{2b P_R M} \right) \left( \sum_{k=1}^{M-1} \frac{(-1)^k (1/2b P_R M)^k}{(2b)^k \cdot k!} \right) + 1 \right\} \nonumber$$

$$+ \sum_{k=1}^{M-1} \frac{1}{(2b)^k \cdot k!} \sum_{s=1}^{k} (s-1)! \left( \frac{1}{P_R M} \right)^{k-1} \left( \frac{1}{2b} \right)^{s-1}. \nonumber$$  \hfill (D.6)

The computation of $C_{12}$ is similar to that of $C_{22}$ and is derived as

$$C_{12} = \frac{1}{\ln 2} \left\{ \frac{e^{1/2b P_R (L - M)}}{P_R} \left( \frac{1}{P_R M} \right) \right\} \left\{ \frac{1}{P_R M} \right\} \left( \sum_{k=1}^{L-M-1} \frac{(-1)^k (P_R M)^k}{(2b)^k \cdot k!} \right) + 1 \right\} \nonumber$$

$$+ \sum_{k=1}^{L-M-1} \frac{1}{(2b)^k \cdot k!} \sum_{s=1}^{k} (s-1)! \left( \frac{1}{P_R M} \right)^{k-1} \left( \frac{1}{2b} \right)^{s-1}. \nonumber$$  \hfill (D.7)

where $b = (\sigma^2 / M - (2(L - M) - 1)!!)^{1/(L-M)}$. Finally, $C_{ADB}$ is obtained by substituting (D.2), (D.3), (D.6), and (D.7) into (26).

**Data Availability**

This article does not cover data research. No data were used to support this study.

**Disclosure**

This paper has been presented in part at the 2020 International Conference on Computing, Networking and Communications, Big Island, Hawaii, Mar. 2020 [33]. An earlier version of this work has been presented as Preprint according to the following link: https://arxiv.org/abs/2005.07699.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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