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Research Article

Sparse Spatial Spectral Fitting with Nonuniform Noise Covariance Matrix Estimation Based on Semidefinite Optimization

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In general, the azimuth estimation in array signal processing is derived under the assumption of uniform white noise, whose covariance matrix is a scaled identity matrix. However, in practice, the noise can be nonuniform with an arbitrary unknown diagonal covariance matrix. In this paper, the estimation of the noise covariance matrix is formulated into a solution to the semidefinite optimization problem which can obtain a more accurate sensor noise covariance matrix. In the proposed algorithm, the estimated nonuniform noise is subtracted from the sample covariance matrix. The simulation results show that the proposed algorithm can significantly improve the performance of the sparse spectrum fitting algorithm in nonuniform noise case, while the classic SpSF algorithm is used under uniform white noise assumption. The water pool experiments show that there are indeed significant differences in the noise covariance of the sensors.

1. Introduction

Determining the direction-of-arrival (DOA) of incoming signals is an important research problem in array signal processing, which is widely used in radar, sonar, navigation, wireless communications, and acoustics [1–7]. Under the uniform white noise assumption, several high resolution DOA estimation approaches are well known to provide high accuracy and excellent asymptotic performance, such as multiple signal classification (MUSIC) [8] and estimation of signal parameters via rotational invariance technique (ESPRIT) [9]. According to the assumption, sensor noises are presumed to be a zero-mean Gaussian process with the covariance matrix $\sigma^2 I$, which is the unknown uniform noise variance, and I is the identity matrix [10].

The sparse spectrum fitting (SpSF) [11] algorithm is another high resolution DOA estimation algorithm, which is based on the sparse signal model of the array and compressive sensing theory. SpSF is formulated by applying L_1-norm penalization to the spatial sparse model of the sample covariance matrix. It is similar to Lasso-type algorithms [12–15] which utilize compressive sensing theory with L_1-norm replacing L_0-norm. However, the SpSF algorithm still assumes the sensors noise is uniform Gaussian white noise. In fact, the sensor noise may be nonuniform [16–18], spatially correlated [19–24], or block-correlated [25–28]. In particular, in the field of underwater acoustic signal processing, the noise of different hydrophones tends to be nonuniform due to inaccuracy in the manufacture of hydrophones.

In this paper, as usual, we still assume that the noise entering the sensor is Gaussian noise, but the noise variances of sensor are nonidentical across the array, and then we propose a new DOA estimation algorithm with sparse spatial spectral fitting based on nonuniform noise estimation (NN-SpSF). The NN-SpSF algorithm significantly improves

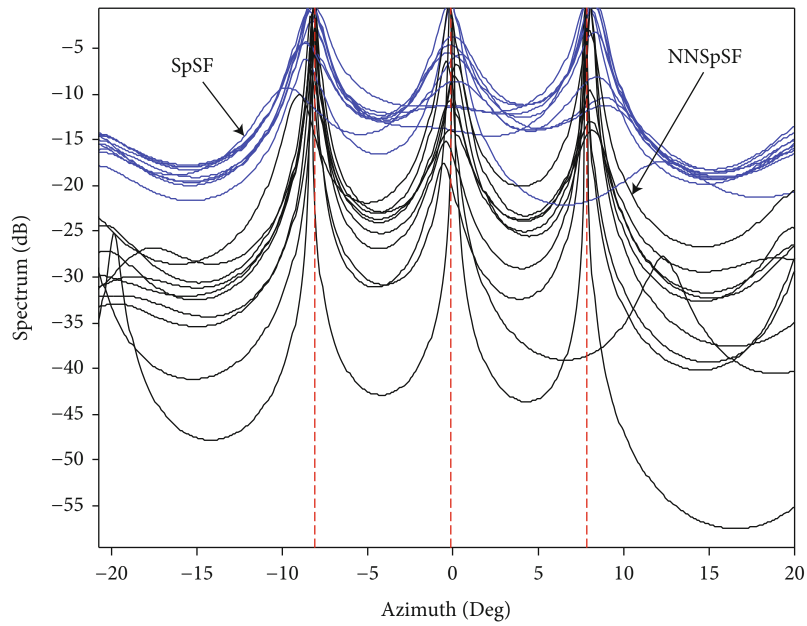

Figure 1: Comparisons of SpSF and NN-SpSF for DOA estimation.

the performance of DOA estimation under nonuniform noise compared with the SpSF algorithm.

The paper is organized as follows. The signal model is formulated in Section 2. The SpSF algorithm based on nonuniform noise estimation is developed in Section 3. Simulation results are presented in Section 4. The results of the water pool experiment are shown in Section 5. Conclusions are provided in Section 6.

2. Signal Model

Consider a uniform linear hydrophone array of M elements with its steering vector denoted by $\boldsymbol{a}(\theta)$. Suppose that $K(K < M)$ far-field narrow-band signals impinge on the array from the unknown DOAs $\theta = (\theta_1,\cdots,\theta_K)$, then the signal observed of the array at time t is given as

$$x(t) = A(\theta)s(t) + e(t), t = 1, \cdots, L, \tag{1}$$

where $A(\theta) = [a(\theta_1),\cdots,a(\theta_K)]$ is the steering matrix of the array, and $a_m(\theta) = [1, e^{j\varphi}\cdots,e^{j(M-1)\varphi}]^T$ is the steering vector [29]. In a uniform linear array (ULA) with half-wavelength interelement spacing, the mth entry of $a_m(\theta)$ is given by

$$\varphi = \frac{2\pi\Delta}{\lambda} \sin\theta, \tag{2}$$

where $\boldsymbol{s}(t) = [s_1(1),\cdots,s_K(n)]$ and $e(t)$ are the signal and noise vectors, respectively. They are assumed to be uncorrelated. L is the number of snapshots. Then, the array output covariance matrix can be expressed as

$$R = E\{x(t)x^{\mathrm{H}}(t)\} = \mathrm{APA}^{\mathrm{H}} + \mathrm{Q}, \tag{3}$$

where $E\{\cdot\}$ and $(\cdot)$ represent the mathematical expectation and Hermitian transpose, respectively. $P = E\{s(t)s^H(t)\}$ is the signal covariance matrix, and $Q = E\{e(t)e^H(t)\}$ is the noise covariance matrix. For uncorrelated sources, P is a diagonal matrix. In this paper, the sensor noise is considered to be nonuniform and can be modeled as a spatially and temporally uncorrelated zero-mean random process, and then Q is also a diagonal matrix having the form

$$Q = \mathrm{diag}\left\{\sigma_1^2,\cdots,\sigma_M^2\right\}, \tag{4}$$

where $\sigma_1^2, m = 1, \cdots, M$ are the sensor noise variances, and diag $\{\cdot\}$ denotes a diagonal matrix. In practical applications, the array covariance matrix is usually estimated as $\widehat{R} = 1/L \sum_{t=1}^{L} x(t)x^H(t)$.

3. The Proposed SpSF Algorithm with Nonuniform Noise Estimation (NN-SpSF)

The SpSF algorithm is formulated by applying l_1-norm penalization of fitting the source covariance model to the estimated spatial covariance, which applies the vectorization operation [30–32] to $\widehat{R}$, and it can obtain the following relation:

$$r = A_{N_\theta} p^{\circ} + \sigma^2 \mathrm{vec}(I), \tag{5}$$

where $r = \mathrm{vec}(\widehat{R})$, vec(), is the vectorization operation; $A_{N_\theta} = [a_v(\theta_1)a_v(\theta_2)\cdots a_v(\theta_{N_\theta})]$ is the array manifold matrix, in which $a_v(\theta) = \mathrm{vec}(a(\theta)a^{\mathrm{H}}(\theta))$, and N_θ is the number of search angles for θ. According to the Formula (5), the noise entering the array is assumed to be uniform Gaussian white noise. However, if the noise is nonuniform, (5) can be

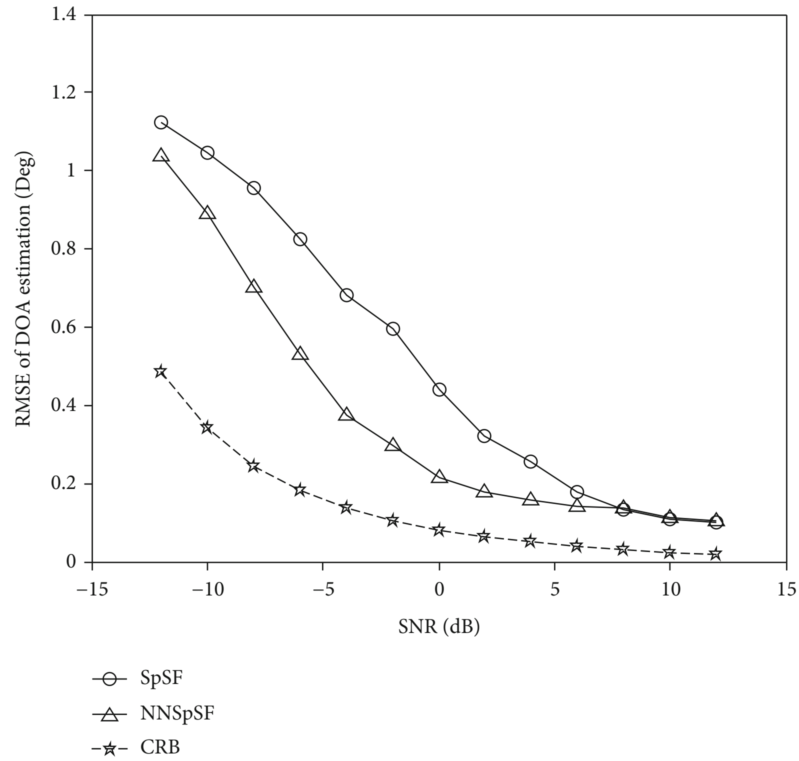

Figure 2: Comparison of the DOA estimation RMSEs versus SNR.

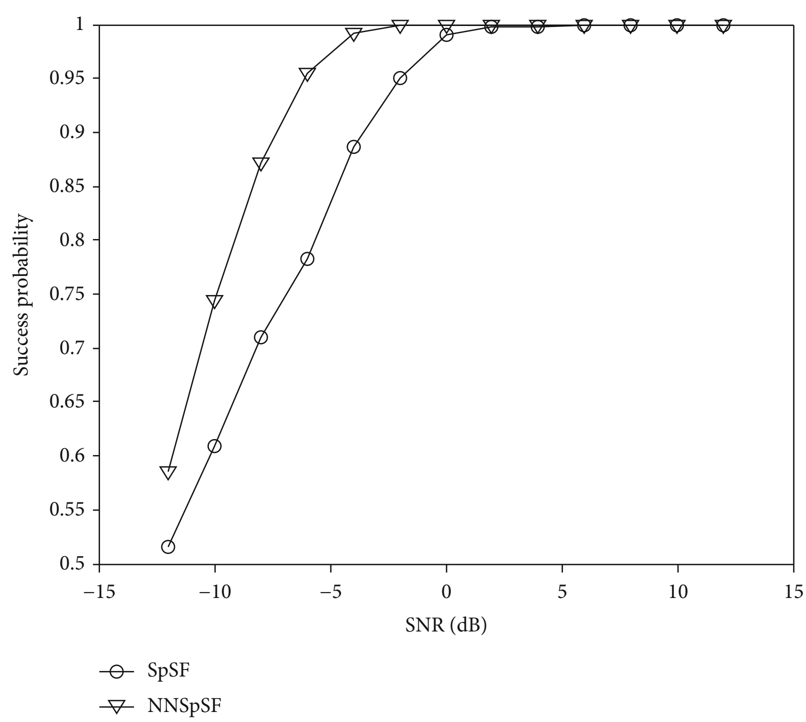

Figure 3: Comparison of the success probability of DOA estimation versus SNR.

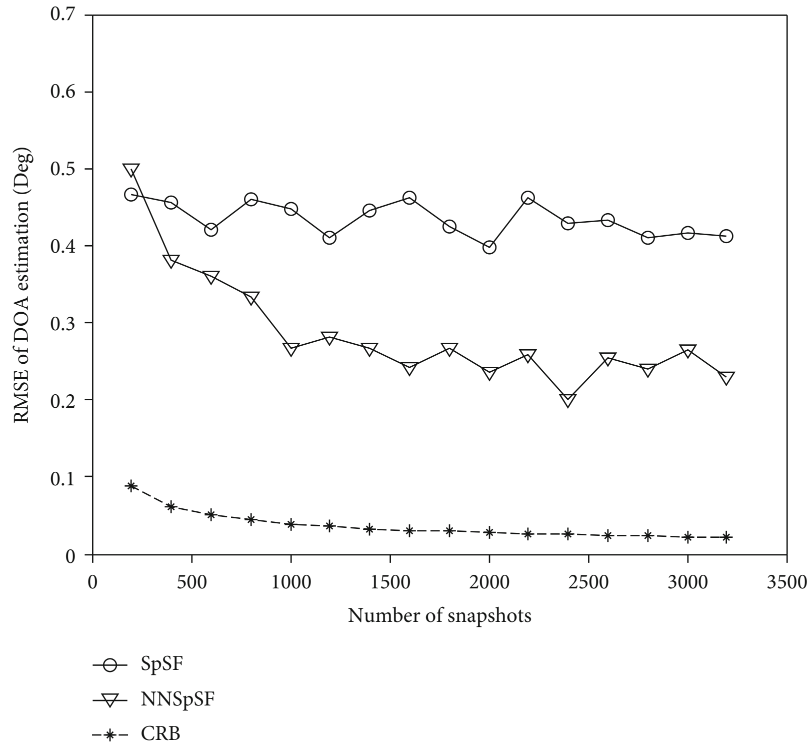

Figure 4: Comparison of the DOA estimation RMSEs versus the number of snapshots.

rewritten as

$$r = A_{N_\theta} p^{\circ} + \text{vec}(Q), \tag{6}$$

where Q is a diagonal matrix, but the elements are not equal; therefore, the bearing estimation performance of the SpSF algorithm will be significantly reduced. In the case of nonuniform Gaussian white noise, we need to remove the nonuniform noise information from Q by the following operations:

$$(Q\text{-}Q_N) \longrightarrow \lambda \cdot I. \tag{7}$$

To get the nonuniform diagonal matrix $Q_N = \text{diag}\ \{\sigma'^2_1, \cdots, \sigma'^2_M\}$, and $\sigma'^2_1 \neq \sigma'^2_2 \neq \cdots \neq \sigma'^2_M$, we can convert it to the following semipositive definite optimization problem, as shown in Formula (8):

$$\begin{gathered} \max \sum_{m=1}^{M} \sigma'^2_m, \\ s.t. \left|\widehat{R}\right| - Q_N \geq \eta \cdot I, \\ Q_N \geq 0, \\ \eta = \min \left(\text{diag}\left(\widehat{R}\right)\right). \end{gathered} \tag{8}$$

When Q_N is obtained, the DOA estimator of the SpSF algorithm in the nonuniform noise situation (written as NN-SpSF) can be given as

$$\begin{gathered} p^* = \arg \min_{p,\mu} \left\| \text{vec}\left(\widehat{R} - Q_N\right)\text{-}A_{N_\theta} p \right\|_2^2 + \mu \|p\|_1, \\ s.t. p_i \geq 0, i = 1, \cdots, N_\theta. \end{gathered} \tag{9}$$

Formula (8) and (9) can be solved with the convex optimization tool like CVX [33], which is used to solve the semipositive definite optimization problem. Here in formula (9), the variable μ takes the value of 0.8.

4. Simulations

In this section, a series of numerical experiment results under different conditions are conducted to validate the performance of the proposed NN-SpSF algorithm. The experiments are performed with a uniform linear array (ULA) with $M = 12$ sensors and half-wavelength space. Three equally powered independent narrowband signals impinge on the array from directions -8°, 0°, and 8° respectively. The noise is assumed spatially nonuniform and independent, which has the following covariance matrix:

$$Q = \text{diag}\ [10.2\ 5.6\ 8.5\ 11.2\ 7.8\ 9.5\ 8\ 4.8\ 9\ 6.1\ 7.2\ 2]. \tag{10}$$

The signal-to-noise ratio (SNR) is defined as

$$\text{SNR} = 10 \log 10 \left(\sigma_n^2 \frac{\sigma_s^2}{\sum_{i=1}^{M} \sigma_i^2} \right), \tag{11}$$

where σ_s^2 denotes the power of source signal.

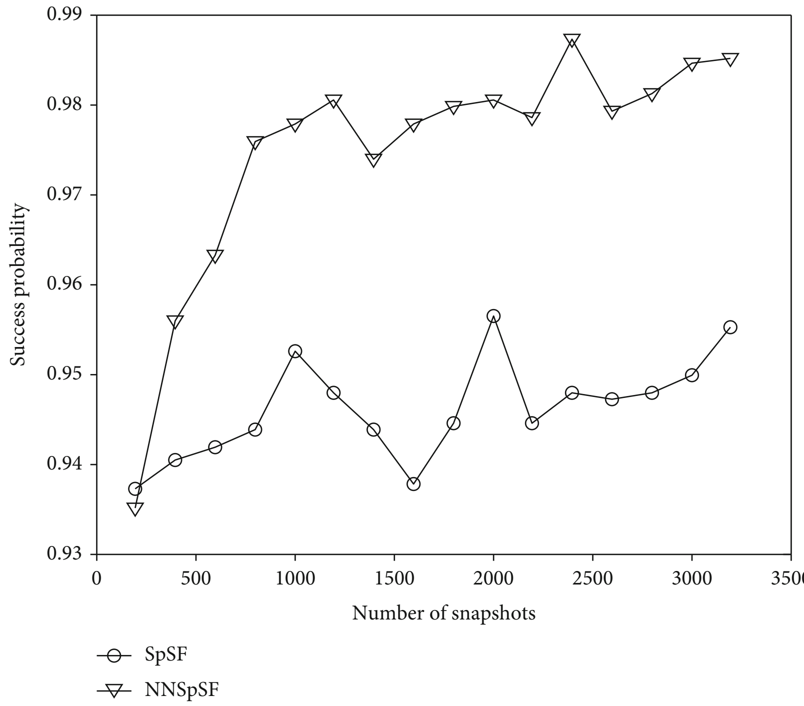

Figure 5: Success probability of DOA estimation versus the number of snapshots.

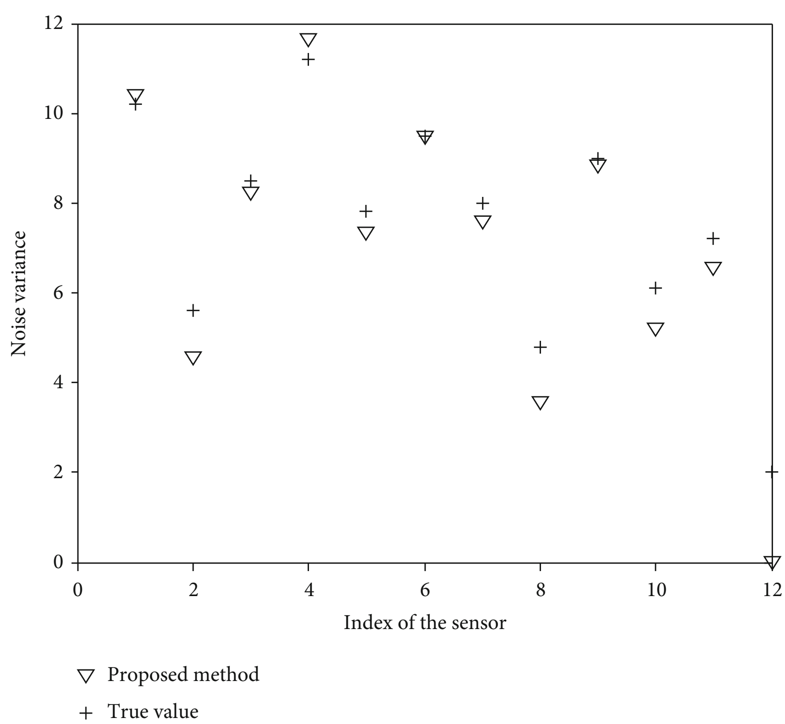

Figure 6: Averaged noise variances of sensors.

In the first simulation, the number of snapshots is 500, and 10 independent experiment runs for SNR = 0dB are performed. The comparison performance of the proposed NN-SpSF algorithm with the SpSF algorithm is shown in Figure 1.

As can be seen from Figure 1, the NN-SpSF algorithm has higher accuracy of azimuth estimation and lower sidelobe in nonuniform noise case.

Next, we set the number of snapshots to 800 and evaluate the performances of the proposed algorithms at different

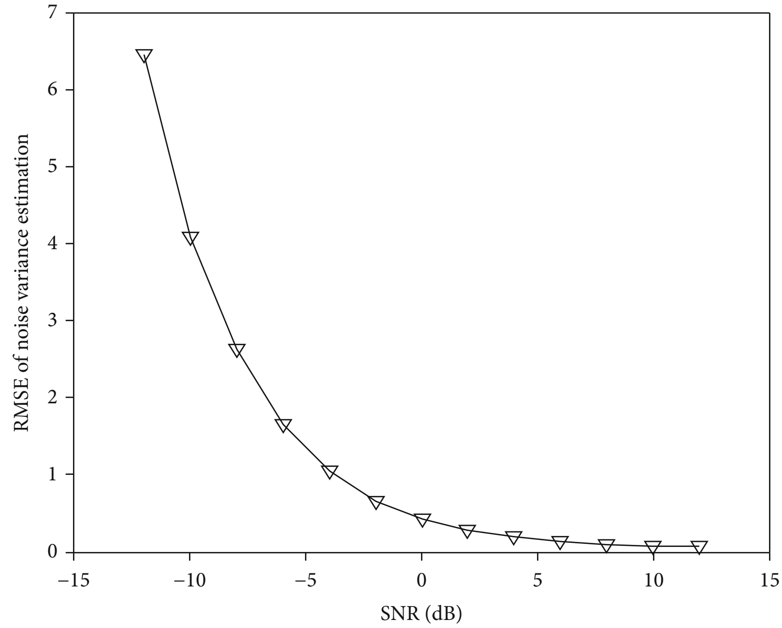

Figure 7: RMSE versus SNR for noise variance estimation.

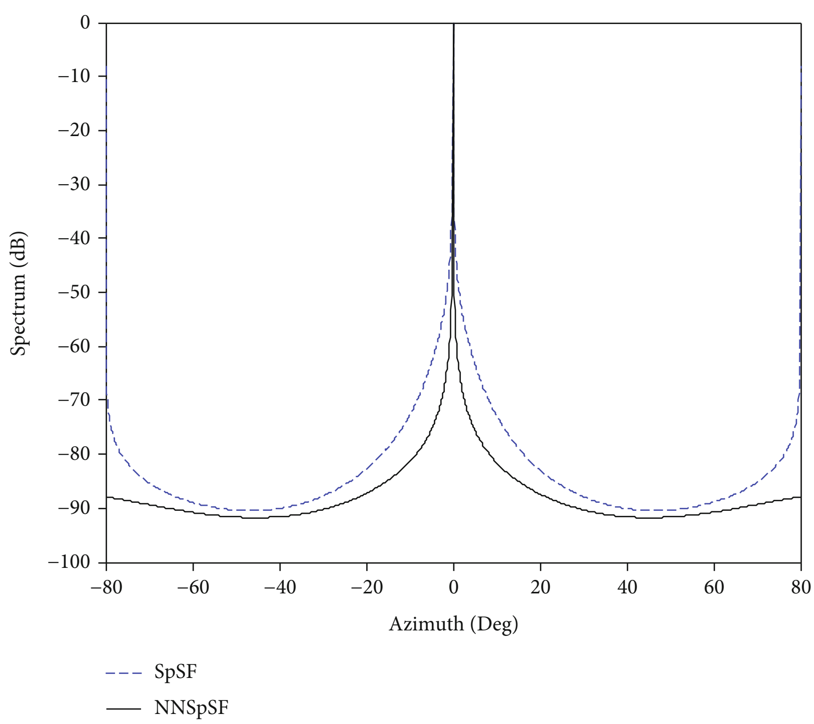

Figure 8: Comparison of SpSF and NN-SpSF algorithm for DOA estimation of pool experiment.

SNR levels. The root-mean-square error (RMSE) of the estimated DOA of the sources is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{K * N_m} \sum_{p=1}^{N_m} \sum_{k=1}^{K} \left(\widehat{\theta}_k(p) - \theta_k \right)^2}, \qquad (12)$$

where $\widehat{\theta}_k(p)$ is the estimate of θ_k for the pth Monte Carlo trial, K is the number of sources, N_m is the number of the Monte Carlo trials, and $N_m = 200$ in all the following simulations. The RMSE and success probability of DOA estimation versus SNR are shown in Figures 2 and 3. The SNR varies from -12 dB to 12 dB with 2 dB step size.

It can be seen from Figures 2 and 3 that the NN-SpSF algorithm has lower RMSE and higher success probability

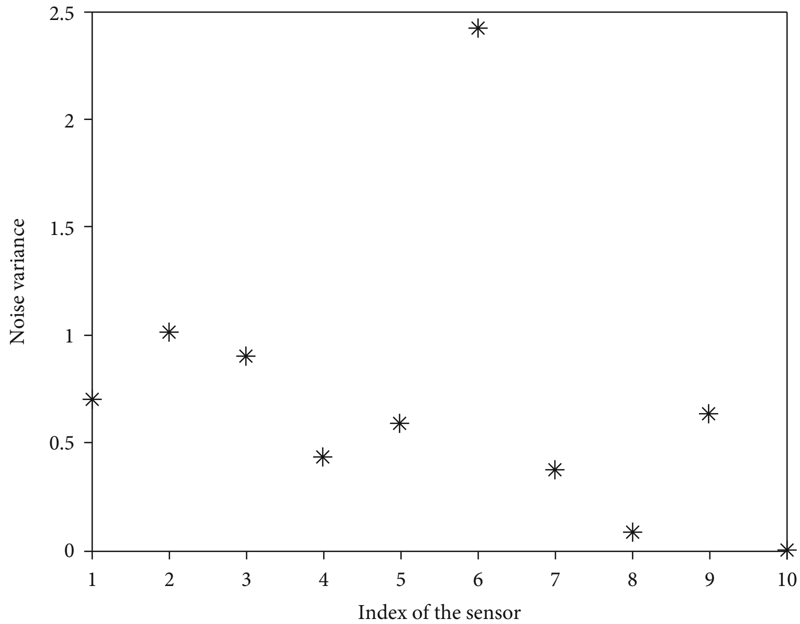

Figure 9: Noise variance estimation of the all sensors.

than the SpSF algorithms when the signal-to-noise ratio changes. Figures 4 and 5 show the RMSE and success probability of DOA estimation versus snapshots, which varies from 200 to 3200 with 200 step sizes.

It can be found that, as the snapshots increasing, the NN-SpSF algorithm has the similar performance with Figures 2 and 3. Besides the performance of DOA estimation, we also evaluate the performance of the proposed NN-SpSF and SpSF algorithm for noise variance estimation. Figure 6 depicts the estimated noise variances averaged from 500 Monte Carlo trials at SNR = −8 dB. Figure 7 shows the RMSE of noise variance estimation of all sensors versus SNR; the SNR varies from -12 dB to 12 dB with 2 dB step size.

It can be seen that the RMSE of noise variance decreases rapidly with the increase of the SNR, and the proposed algorithm can still estimate the nonuniform noise with a lower deviation at a lower SNR, such as the -8 dB case in Figure 6.

5. Water Pool Experiment

The algorithm in this paper was verified by pool experiments in an anechoic pool. The water pool is 20 m long, 8 m wide, and 7 m deep. The receiving array is a vertical uniform linear array with 10 array elements, and the first hydrophone was placed at a depth of 0.7 m. The transmitting transducer is 7 m away from the receiving array, and its transmitting signal is a CW pulse with a frequency of 3 kHz. The CW pulse signal has a length of 400 ms and a period of 1 s.

The results of the DOA estimation of the water pool experiment for the NN-SpSF algorithm is shown in Figure 8, and it can be observed that the NN-SpSF algorithm has lower sidelobe. The noise variance estimation of all sensors of the array is shown in Figure 9. In the actual situation, there are indeed some differences in the noise background of the sensors in the array due to manufacturing, installation, and other reasons.

6. Conclusion

In this paper, a new noise variance estimation algorithm of all sensors in an array in nonuniform noise for DOA estimation is proposed. The estimation of the noise covariance matrix is formulated into a solution to the semidefinite optimization problem which can obtain a more accurate sensor noise covariance matrix. Simulation results show that subtracting the estimated nonuniform noise from the sample covariance matrix can significantly improve the performance of the SpSF algorithm. The water pool experiments show that due to manufacturing and other reasons, nonuniform noise of the sensor does exist, but independent nonuniform noise in the sensor array is the simplest assumption. In future, we will study the noise covariance matrix estimation algorithm closer to the real situation to improve the performance of azimuth estimation.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declared that they have no conflicts of interest regarding this work.

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