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Research Article

Sensor Selection for Hybrid AOA-TOA Localization with Correlated Measurement Noise in Underwater Wireless Sensor Networks

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Underwater target localization is the most crucial part of the underwater wireless sensor network (UWSN). Due to limited communication range and energy constraints in underwater scenarios, only a subset of sensors can be selected to localize. This paper investigates the sensor selection schemes for hybrid angle-of-arrival (AOA) and time-of-arrival (TOA) localization in the underwater scenario. We first develop the Cramér-Rao lower bound (CRLB) for the hybrid AOA-TOA localization with correlated measurement noise model with Gaussian priors, and a Boolean vector is introduced to denote the selected sensors for hybrid measurement. Secondly, the sensor selection schemes are formulated as an optimization problem, and the optimality criterion is to minimize the trace of CRLB. The original nonconvex problem has been modified to the semidefinite problem program (SDP) by convex relaxation, and then, a randomization algorithm is chosen to advance the result of the SDP method. Finally, simulations verify that the proposed algorithm approaches the exhaustive search algorithm, and the effect of correlated measurement noise on the estimation performance in the hybrid localization system is proved.

1. Introduction

Target localization technology plays a significant role in marine target detection and tracking, marine environment monitoring, underwater vehicle navigation, etc. [1, 2]. However, owing to the limitations are high power consumption, severe propagation delay, and so on [3–6]; it is not feasible to active all sensors to localize or track the unknown target in the underwater wireless sensor network (UWSN). Thus, the problems of sensor selection for target localization in UWSN have been considered; the goal is to make a compromise solution between the localization estimation accuracy and the best subset of activated sensors [7, 8].

Several localization methods have been developed using different localization measurements, e.g., time-difference-of-arrival (TDOA) or time-of-arrival (TOA) [9], angle-of-arrival (AOA) [10], received signal strength (RSS) [11], and frequency difference of arrival (FDOA) [12]. The AOA-based and TOA-based localization are the most popularly used ones. The AOA-based localization can be easily obtained using triangulation approaches, but the estimation accuracy is lower than the TOA localization. A reduced-complexity algorithm based on a pseudo maximum likelihood (ML) estimation is presented in [13]; an AOA-based mechanism that associates the line-of-sight (LOS) over time for a given trial location was described. Recently, a novel AOA-based approximately unbiased estimation is derived by using semidefinite relaxation (SDR) in [14]. In contrast, the TOA can achieve better localization performance when the high-precision timing measurements are acquired [15]. In [16], a new algebraic localization method was derived based on a minimum number of localization measurements. On this basis, an optimal linear unbiased estimator is designed to calculate the final position estimation.

Extensive research has been conducted to investigate the sensor selection schemes in [17–21]. Generally, sensor selection schemes are transformed as an optimization issue by using different optimality criterion. Two frequently used optimization criteria are minimizing the trace of Cramér-Rao lower bound (CRLB) (A-optimality criterion) and maximizing the determinant of the Fisher information matrix (FIM) (D-optimality criterion). On this basis, the scheme of sensor selection is transformed into an integer programming (IP) scheme with an optimization variable. However, owing to the IP scheme is an NP-hard scheme. The direct way to acquire the best sensor subset is the exhaustive search algorithm, which has high computational complexity and does not apply to broad sensor networks. For these reasons, several suboptimal algorithms have been presented to deal with the sensor selection schemes.

The sensor selection scheme was formulated based on minimizing the log-determinant of the estimated error covariance matrix, and the convex relaxation was adopted to solve the problem in [17]. The authors proposed a sparsity-promoting method by minimizing the number of sensors to be selected with the limitation of the estimation performance in [18]. Thus, the original nonconvex optimization issue can be transformed to a sparse vector design scheme. The problem of sensor scheduling in the linear system with correlated measurement noise was presented in [19], which was transformed to minimize the trace of the inverse of the Bayesian FIM. Recently, the sensor selection issue for TDOA-based localization was formulated by minimizing the trace of CRLB and two independent Boolean vectors as the selected reference sensors and ordinary sensors in [20]. The convex relaxation methods are utilized to formulate the nonconvex problem as an SDP. The two suboptimal sensor selection algorithms were designed for DOA-based and TOA-based localization algorithms to minimize the trace of CRLB, which only used one Boolean vector as the optimization variable in [21]. Both of the nonconvex problems were relaxed as convex SDP.

In the literature mentioned above, sensor selection schemes usually utilize only one kind of measurement. The key to locating a target is obtaining sufficient measurement from the multiple sensors to improve estimation accuracy. Therefore, an intuitive method, knowns as hybrid measurements, has attracted considerable attention recently [22, 23]. Besides, since the underwater channel has the features of low communication channel bandwidth, serious multipath effect, and variable acoustic velocity [24], it is necessary to improve the localization performance using the hybrid localization method. Plenty of hybrid target location methods are implemented based on different combinations of mixed measurements, e.g., TDOA-RSS, AOA-TOA, and RSS-AOA [22, 23, 25–28]. The authors derived the CRLB for the hybrid method using both AOA and TOA estimates in [25]. Lately, a joint TOA and AOA estimator was proposed for UWB indoor ranging under LOS operating conditions [26]. In [27], the authors focus on the energy-efficient localization strategy using hybrid TOA/AOA measurements, and a joint resource allocation and an antenna selection framework were presented to minimize the estimation error. In [28], the authors extend the sensor selection method to a mixed TDOA and AOA localization scenario with the presence of sensor location error. However, the sensor selection schemes with correlated noise measurement were little considered in the above literature due to the FIM being a nonlinear function. Besides, the FIM becomes more complicated by using more types of measurements. Therefore, the sensor selection problems for the hybrid measurements with correlated noise in UWSN are nontrivial tasks. Moreover, the inaccurate sensor locations can also cause a considerable reduction of localization accuracy, which should be considered in the sensor selection schemes.

This paper investigates sensor selection strategies for hybrid AOA-TOA-based localization in underwater scenario. The key contributions are summarized as follows:

(1) The CRLB for the hybrid AOA-TOA cooperative localization measurement model is presented for Gaussian priors, and one Boolean vector is used for the expressions of sensor selection scheme with the hybrid AOA-TOA measurement under correlated measurement noise

(2) To mitigate the localization error caused by uncertainty in sensor locations, a calibration source with precisely known location is introduced. The sensor selection scheme is formulated by a nonconvex optimization problem based on minimizing the trace of CRLB, and the convex relaxation techniques are adopted to transform the original problem as SDP problem. Besides, a randomization algorithm is also approved to improve the result

The rest of this paper is organized as follows: In Section 2, the CRB for the hybrid AOA-TOA-based localization is derived, and sensor selection scheme is formulated. Section 3 investigates the method to reduce the sensor location error and then transform the original scheme to the nonconvex optimization schemes. The convex relaxation and a randomization algorithm are developed to solve the sensor selection problems in Section 4. The comprehensive simulation results are presented in Section 5. Finally, Section 6 is devoted to our conclusions and future research directions.

2. Problem Formulation

This section introduces the hybrid AOA-TOA measurement model and the CRLB for the hybrid AOA-TOA-based localization in UWSN, and the sensor selection scheme is introduced.

2.1. The Hybrid AOA-TOA Measurement Model. We consider a two-dimensional underwater scenario composed of multiple sensors with known locations and an unknown stationary target. Assuming that the unknown target follows a given prior probability density function (PDF) $\mathbf{p} \sim \mathcal{N}(\mathbf{p}_0, \mathbf{C}_0)$, where $\mathbf{p}_0$ and $\mathbf{C}_0$ are the mean and covariance matrix of $\mathbf{p}$. The unknown target location is $\mathbf{p} = (p_x, p_y)^T$, and the kth mutiple sensor is $\mathbf{s}_k = (s_{xk}, s_{yk})^T, k = 1, \cdots, N$. Each multiple sensors can achieve the hybrid AOA and TOA measurements. Consequently, the hybrid measurement can acquire an estimator for the unknown target. The calibration source is located at $\mathbf{c} = (c_x, c_y)^T$, which is utilized to correct the sensor location errors.

The kth sensor has the measurement model as follows:

$$\tilde{z}_k = z_k + \alpha_k = f_k(\mathbf{p}, \mathbf{s}_k) + \alpha_k, \tag{1}$$

and $f_k(\mathbf{p}, \mathbf{s}_k)$ denotes a nonlinear measurement model with $\mathbf{p}$ and $\mathbf{s}_k$. We assume that the measurement noise is α_k.

Stacking $\tilde{z}_k$ for $k = 1, \cdots, N$. The vector form of the above (1) is

$$\tilde{\mathbf{z}} = \mathbf{z} + \alpha, \tag{2}$$

with

$$\begin{aligned} \tilde{\mathbf{z}} &= [\tilde{z}_1, \cdots, \tilde{z}_N]^T, \\ \mathbf{z} = [z_1, \cdots, z_N]^T &= [f_1(\mathbf{p}, \mathbf{s}_1), \cdots, f_N(\mathbf{p}, \mathbf{s}_N)]^T, \\ \alpha &= [\alpha_1, \cdots, \alpha_N]^T, \end{aligned} \tag{3}$$

and we assume that α is white, Gaussian, zero-mean random vectors. Owing to the noise experienced is correlated among different sensors; the covariance matrix $\mathbf{R}_\alpha$ of α is a nondiagonal matrix.

Without loss of generality, different sensor types obtain different measurements and parameter expressions. Hence, we first introduce the AOA and TOA measurements, respectively.

The measurement model of AOA-based localization at the kth sensor is given [29]

$$z_k = f_k(\mathbf{p}, \mathbf{s}_k) = \tan^{-1} \frac{p_y - s_{yk}}{p_x - s_{xk}}, \tag{4}$$

and $\tan^{-1}$ denotes the 4-quadrant arctangent; and the noisy AOA measurement of the kth sensor is ${{\tilde{\theta }}_{k}}$.We assume AOA measurement noise vector $\beta \sim \mathcal{N}(0, \sigma_A^2 \mathbf{R}_A)$, with σ_A^2 denoting the noise power, while $\mathbf{\Sigma}_A = \sigma_A^2 \mathbf{R}_A$ represents the covariance matirx.

For the noisy circular-based TOA measurement of the kth sensor, we obtain [30]

$$\tilde{t}_k = t_k + l_k = \frac{\|\mathbf{p} - \mathbf{s}_k\|}{v} + l_k, \tag{5}$$

where t_k ignores the presence of distance errors, v is the signal velocity, and l_k denotes the measurement noise. Writing the range measurement equation in the (5) form gives

$$z_k = f_k(\mathbf{p}, \mathbf{s}_k) = \|\mathbf{p} - \mathbf{s}_k\|, \tag{6}$$

where $d_k = \|\mathbf{p} - \mathbf{s}_k\|$ denotes the distance of the target and sensor, and we also assume the TOA measurement noise vector $\chi \sim \mathcal{N}(0, \sigma_T^2 \mathbf{R}_T)$, with σ_T^2 denoting the noise, while $\mathbf{\Sigma}_T = \sigma_T^2 \mathbf{R}_T$ represents the covariance matirx.

Therefore, the hybrid AOA-TOA measurement noise vector can be expressed as

$$\zeta = [\beta, \chi], \tag{7}$$

the hybrid AOA-TOA measurement noise covariance matrix with $2N$ measurements is given by

$$\begin{aligned} \mathbf{\Sigma} = \mathbb{E}\left\{\zeta\zeta^T\right\} &= \operatorname{diag}\left\{\mathbb{E}\left\{\beta\beta^T\right\}, \mathbb{E}\left\{\chi\chi^T\right\}\right\} \\ &= \operatorname{diag}\left\{\mathbf{\Sigma}_A, \mathbf{\Sigma}_T\right\} = \operatorname{diag}\left\{\sigma_A^2 \mathbf{R}_A, \sigma_T^2 \mathbf{R}_T\right\}. \end{aligned} \tag{8}$$

The Jacobian matrices of the N sensor for the AOA measurement errors and the TOA measurement errors can be expressed as, respectively

$$\begin{aligned} \mathbf{J}_{\mathrm{AOA}} &= \left.\begin{bmatrix} \dfrac{\partial \tilde{\theta}_1}{\partial p_x} & \dfrac{\partial \tilde{\theta}_1}{\partial p_y} \\ \vdots & \vdots \\ \dfrac{\partial \tilde{\theta}_N}{\partial p_x} & \dfrac{\partial \tilde{\theta}_N}{\partial p_y} \end{bmatrix}\right|_{\mathbf{p}} = \begin{bmatrix} \dfrac{-\sin\theta_1}{d_1} & \dfrac{\cos\theta_1}{d_1} \\ \vdots & \vdots \\ \dfrac{-\sin\theta_N}{d_N} & \dfrac{\cos\theta_N}{d_N} \end{bmatrix}_{N\times 2}, \\ \mathbf{J}_{\mathrm{TOA}} &= \left.\begin{bmatrix} \dfrac{\partial \tilde{t}_1}{\partial p_x} & \dfrac{\partial \tilde{t}_1}{\partial p_y} \\ \vdots & \vdots \\ \dfrac{\partial \tilde{t}_N}{\partial p_x} & \dfrac{\partial \tilde{t}_N}{\partial p_y} \end{bmatrix}\right|_{\mathbf{p}} = \begin{bmatrix} 2\cos\theta_1 & 2\sin\theta_1 \\ \vdots & \vdots \\ 2\cos\theta_N & 2\sin\theta_N \end{bmatrix}_{N\times 2}. \end{aligned} \tag{9}$$

Thus, we can get the Jacobian matrix of the hybrid AOA-TOA measurements as

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_{\mathrm{AOA}} \\ \mathbf{J}_{\mathrm{TOA}} \end{bmatrix}_{2N\times 2}. \tag{10}$$

Using (8) and (10), the FIM for the hybrid AOA-TOA localization with Gaussian priors yields [31]

$$\mathbf{FIM} = \mathbf{C}_0^{-1} + \mathbf{J}^T \mathbf{\Sigma}^{-1} \mathbf{J}, \tag{11}$$

$$\mathbf{CRLB} = \mathbf{FIM}^{-1}. \tag{12}$$

2.2. Sensor Selection Problem. The goal of the sensor selection scheme is to select the best nonredundant set of sensors for localization tasks while satisfying some performance. We consider a localization problem, which chooses a specific subclass with M sensors of N ($N > M$) sensors to satisfy a range of demand. We assume N sensors can obtain N AOA and N TOA measurements in the UWSN. Consequently, two different measurements are obtained by the same sensor. A Boolean vector can be defined as

$$\mathbf{r} = \left[\mathbf{w}^T, \quad \mathbf{v}^T \right]^T, \tag{13}$$

$$\begin{aligned} \mathbf{w} &= [w_1, w_2, \cdots, w_N]^T, w_i \in \{0, 1\}, i = 1, 2, \cdots, N, \\ \mathbf{v} &= [v_1, v_2, \cdots, v_N]^T, v_j \in \{0, 1\}, i = 1, 2, \cdots, N, \end{aligned} \tag{14}$$

and ith element of $\mathbf{w}$ denotes if the ith sensor is selected or not for the AOA measurement, and the ith element of $\mathbf{v}$ denotes if the ith sensor is selected or not for the TOA measurement. It assumed that when the ith sensor is selected, both the AOA and TOA measurements from that sensor are jointly considered for the localization task.

More specifically, we define two sensing matrices $\mathbf{\Phi}_w$ ($\mathbf{\Phi}_w \in \mathbb{R}^{M\times N}$) and $\mathbf{\Phi}_v(\mathbf{\Phi}_v \in \mathbb{R}^{M\times N})$ [17], where $\mathbf{\Phi}_w$ is a submatrix of diag $(\mathbf{w})$(diag $(\mathbf{w}) \in \mathbb{R}^{N\times N}$) that only contains all rows corresponding to the selected sensors, diag $(\mathbf{w})$ is a diagonal matrix, and the diagonal elements are obtain by $\mathbf{w}$. The similarly definition is available for $\mathbf{\Phi}_v$ and $\mathbf{v}$. Note that the links of them are associated by

$$\begin{aligned} \mathbf{\Phi}_w \mathbf{\Phi}_w^T = \mathbf{I}_w, \mathbf{\Phi}_w^T \mathbf{\Phi}_w = \text{diag}\,(\mathbf{w}), \\ \mathbf{\Phi}_v \mathbf{\Phi}_v^T = \mathbf{I}_v, \mathbf{\Phi}_v^T \mathbf{\Phi}_v = \text{diag}\,(\mathbf{v}), \end{aligned} \tag{15}$$

thus

$$\mathbf{\Phi}_r \mathbf{\Phi}_r^T = \mathbf{I}_r, \mathbf{\Phi}_r^T \mathbf{\Phi}_r = \text{diag}\,(\mathbf{r}). \tag{16}$$

Based on the above definitions, the covariance matrix of the hybrid measurements for the selected sensor is given by

$$\mathbf{\Sigma}_r = E\left[\mathbf{\Phi}_r \zeta (\mathbf{\Phi}_r \zeta)^T\right] = \mathbf{\Phi}_r \mathbf{\Sigma} \mathbf{\Phi}_r^T. \tag{17}$$

With the above hybrid measurement model and the definitions, the FIM for selected sensors is expressed as

$$\mathbf{FIM}_r = \mathbf{C}_0^{-1} + \mathbf{J}^T \mathbf{\Phi}_r^T \mathbf{\Sigma}_r^{-1} \mathbf{\Phi}_r \mathbf{J}. \tag{18}$$

3. Sensor Selection Method for the Hybrid AOA-TOA-Based Localization

This section introduces a calibration source with a precisely known location to alleviate the localization performance degeneration caused by the sensor position errors. The sensor selection issue for hybrid AOA-TOA-based localization is transformed into an optimization issue. The optimality criterion chosen is to minimize the inverse of the FIM, which is also known as the A-optimality criterion. Other criteria are also available such as the D-optimality criterion (maximizing the determinant of the FIM or minimizing the volume of the localization error ellipsoid), which may be cause large estimation error with volume minimization in some cases, and the E-optimality criterion (minimizing the maximum eigenvalue of the CRLB matrix), and the A-optimality criterion is the most commonly used criteria for performance measurement, which is equivalent the estimation mean squared error (MSE).

3.1. The Method to Correct Sensor Location Errors. To mitigate the localization inaccuracy caused by sensor position error in the underwater scenario, a calibration source is introduced to correct the sensor location errors. Assuming that the calibration source location in the localization model is precisely known, the signal for sensor position compensating is sent to sensors through the calibration source. According to the received signal and the calibration source location, the sensors can use the arrival time to realize the correction of position error.

On the premise that the calibration source is completely synchronized with the sensor clock in the UWSN, the time delay of the sensors that received the signal is sent by the calibration source and can be expressed as

$$\tau_k = \frac{1}{v} \|\mathbf{s}_k - \mathbf{c} + \Delta \mathbf{s}_k\| + \varepsilon_k = \Delta\tau_k + \varepsilon_k, \tag{19}$$

where v represents the transmission speed of underwater acoustic signal and $\Delta \mathbf{s}_k$ denotes the position error contained in the kth sensor. ε_k denotes the measurement noise, which follows $\varepsilon_k \sim \mathcal{N}(0, \alpha_k^2)$. We assume that $\varepsilon = (\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_N)^T$, $\tau = (\tau_1, \tau_2, \cdots, \tau_N)\Delta\tau = (\Delta\tau_1, \Delta\tau_2, \cdots, \Delta\tau_N)$. When the noise variance of each sensor is different, $\mathbf{Q} = \text{cov}\,(\varepsilon\varepsilon^T) = \text{diag}\,(\alpha_1^2, \alpha_2^2, \cdots, \alpha_N^2)$, which is the covariance matrix of the time of arrival measurement error.

The likelihood function of sensor measurement can be expressed as

$$f(\tau|\mathbf{s}) = \frac{1}{(2\pi)^{N/2}\sqrt{\det \mathbf{Q}}} \exp\left[-\frac{1}{2}(\tau - \tau(\mathbf{s}))^T \mathbf{Q}^{-1} (\tau - \tau(\mathbf{s}))\right]. \tag{20}$$

The maximum likelihood estimation of target location is

$$\hat{\mathbf{s}}_\tau = \arg \min_{\mathbf{s}_\tau} f(\mathbf{s}_\tau), \tag{21}$$

where the $f(\mathbf{s}_\tau)$ is denoted as cost function.

$$f(\mathbf{s}_\tau) = (\tau - \tau(\mathbf{s}_\tau))^T \mathbf{Q}^{-1} (\tau - \tau(\mathbf{s}_\tau)). \tag{22}$$

It assumed that the variable estimation $\mathbf{s}_\tau$ has the intial estimation $\mathbf{s}_\tau^0$, and the kth estimation obtained by iteration is $\mathbf{s}_\tau^k$. We denote the residual error as

$$\mathbf{e}(\mathbf{s}_\tau) = \tau\left(\mathbf{s}_\tau^k\right) - \tau. \tag{23}$$

To compute the residual error, the first-order Taylor series approximation is used for $\mathbf{e}(\mathbf{s}_\tau)$ in $\mathbf{s}_\tau^k$. Therefore,

(23) can be rewritten as

$$\mathbf{e}(\mathbf{s}_\tau) \approx \mathbf{e}\left(\mathbf{s}_\tau^k\right) + \left.\frac{\partial \tau(\mathbf{s}_\tau)}{\partial \mathbf{s}_\tau}\right|_{\mathbf{s}_\tau=\mathbf{s}_\tau^k}\left(\mathbf{s}_\tau - \mathbf{s}_\tau^k\right) = \mathbf{e}\left(\mathbf{s}_\tau^k\right) + \mathbf{J}\left(\mathbf{s}_\tau^k\right)\Delta\mathbf{s}_\tau^k. \tag{24}$$

By substituting (24) into (22), we obtain

$$\begin{aligned} f(\mathbf{s}_\tau) &= (\tau - \tau(\mathbf{s}_\tau))^T\mathbf{Q}^{-1}(\tau - \tau(\mathbf{s}_\tau)) \\ &\approx \left(\mathbf{e}\left(\mathbf{s}_\tau^k\right) + \mathbf{J}\left(\mathbf{s}_\tau^k\right)\Delta\mathbf{s}_\tau^k\right)^T\mathbf{Q}^{-1}\left(\mathbf{e}\left(\mathbf{s}_\tau^k\right) + \mathbf{J}\left(\mathbf{s}_\tau^k\right)\Delta\mathbf{s}_\tau^k\right) \\ &= \mathbf{e}^T\left(\mathbf{s}_\tau^k\right)\mathbf{Q}^{-1}\mathbf{e}\left(\mathbf{s}_\tau^k\right) + 2\mathbf{e}^T\left(\mathbf{s}_\tau^k\right)\mathbf{Q}^{-1}\mathbf{J}\left(\mathbf{s}_\tau^k\right)\Delta\mathbf{s}_\tau^k + \Delta\mathbf{s}_\tau^k\mathbf{J}^T\left(\mathbf{s}_\tau^k\right)\mathbf{Q}^{-1}\mathbf{J}\left(\mathbf{s}_\tau^k\right)\Delta\mathbf{s}_\tau^k. \end{aligned} \tag{25}$$

We take the derivative of $\Delta\mathbf{s}_\tau^k$ to the above formula and make it zero to obtain

$$\Delta\mathbf{s}_\tau^k = -\left(\mathbf{J}^T\left(\mathbf{s}_\tau^k\right)\mathbf{Q}^{-1}\mathbf{J}\left(\mathbf{s}_\tau^k\right)\right)^{-1}\mathbf{J}^T\left(\mathbf{s}_\tau^k\right)\mathbf{Q}^{-1}\mathbf{e}\left(\mathbf{s}_\tau^k\right). \tag{26}$$

Thus, the $k+1$ estimation of the variable can be expressed as

$$\mathbf{s}_\tau^{k+1} = \mathbf{s}_\tau^k + \Delta\mathbf{s}_\tau^k. \tag{27}$$

Hence, we can adopt the above method to modify the sensor position error before sensor selection. For simplify, we still use $\mathbf{s}_k$ to represent the precise sensor position.

3.2. Sensor Selection for Correlated Noises. We shall investigate the sensor selection scheme for hybrid AOA-TOA-based localization with the correlated noises. Using [31], the noise covariance matrix can be decomposed as

$$\begin{aligned} \mathbf{\Sigma}_A &= \lambda_A\mathbf{I}_N + \mathbf{Z}_A, \\ \mathbf{\Sigma}_T &= \lambda_T\mathbf{I}_N + \mathbf{Z}_T, \end{aligned} \tag{28}$$

where the positive scalar is λ_A, λ_T is selected to make sure the matrix $\mathbf{Z}_A$, $\mathbf{Z}_T$ is all positive definite, and $\mathbf{I}$ denotes the identity matrix. The hybrid AOA-TOA measurement for the decomposition can be expressed as

$$\mathbf{\Sigma} = \mathbf{\Gamma}_N + \mathbf{Z}, \tag{29}$$

with $\mathbf{\Gamma} = \operatorname{diag}\{\lambda_A\mathbf{I}_N, \lambda_T\mathbf{I}_N\}$ and $\mathbf{Z} = \operatorname{diag}\{\mathbf{Z}_A, \mathbf{Z}_T\}$.

Using (29) in (17), we can obtain

$$\mathbf{\Sigma}_r = \mathbf{\Phi}_r(\mathbf{\Gamma}_N + \mathbf{Z})\mathbf{\Phi}_r^T = \mathbf{\Gamma}_r + \mathbf{\Phi}_r\mathbf{Z}\mathbf{\Phi}_r^T, \tag{30}$$

with $\mathbf{\Gamma}_r = \operatorname{diag}\{\lambda_A\mathbf{I}_M, \lambda_T\mathbf{I}_M\}$. Substituting (30) into (18), with the matrix lemma [32], we have

$$\begin{aligned} &= \mathbf{\Phi}_r^T\left(\mathbf{\Gamma}_r + \mathbf{\Phi}_r\mathbf{Z}\mathbf{\Phi}_r^T\right)^{-1}\mathbf{\Phi}_r \\ \mathbf{\Phi}_r^T\mathbf{\Sigma}_r^{-1}\mathbf{\Phi}_r &= \mathbf{Z}^{-1} - \mathbf{Z}^{-1}\left(\mathbf{Z}^{-1} + \mathbf{\Gamma}_r^{-1}\mathbf{\Phi}_r^T\mathbf{\Phi}_r\right)^{-1}\mathbf{Z}^{-1} \\ &= \mathbf{Z}^{-1} - \mathbf{Z}^{-1}\left(\mathbf{Z}^{-1} + \mathbf{\Gamma}_r^{-1}\operatorname{diag}(\mathbf{r})\right)^{-1}\mathbf{Z}^{-1}. \end{aligned} \tag{31}$$

Substituting (31) into (18), it derives

$$\mathbf{FIM}_r = \begin{matrix} \mathbf{C}_0^{-1} + \mathbf{J}^T\mathbf{Z}^{-1}\mathbf{J} \\ -\mathbf{J}^T\mathbf{Z}^{-1}\left(\mathbf{Z}^{-1} + \mathbf{\Gamma}_r^{-1}\operatorname{diag}(\mathbf{r})\right)^{-1}\mathbf{Z}^{-1}\mathbf{J}. \end{matrix} \tag{32}$$

As discussed above, the relationship between $\mathbf{FIM}_r$ and $\mathbf{r}$ is created absolutely by (32). We adopt the A-optimality criterion as the optimization objective, which is equivalent to minimize the trace of inverse of the FIM; thus, the sensor selection issue in the hybrid AOA-TOA-based localization is given by

$$\begin{aligned} \min_{\mathbf{r}} \quad & \operatorname{tr}\left(\mathbf{FIM}_r^{-1}\right) \\ \text{s.t.} \quad & 1^T\mathbf{r} = 2M \\ & \mathbf{r} \in \{0, 1\}^{2N} \end{aligned} \tag{33}$$

It is clear from the (33) is a nonconvex optimization scheme because of the last Boolean constraints. In the following sections, we propose employing the convex relaxation to approximately solve it.

4. Semidefinite Relaxation for Sensor Selection Problem

This section we analyze and present the method to settle the above nonconvex problem. The sensor selection scheme described in Section 3 is a nonconvex and NP-hard problem. We present a convex relaxation solution for the hybrid AOA-TOA sensor selection. What is more, a randomization algorithm is approved to advance the achievement of the SDP.

4.1. The SDP Method. To simplify the problem and facilitate theoretical analysis, we construe $\mathbf{A} = \mathbf{C}_0^{-1} + \mathbf{J}^T\mathbf{Z}^{-1}\mathbf{J}$ and $\mathbf{B} = \mathbf{Z}^{-1}\mathbf{J}$ in (32). Hence, the optimization issue in (33) is transformed as

$$\begin{aligned} \min_{\mathbf{r}} \quad & \operatorname{tr}(\mathbf{X}) \\ \text{s.t.} \quad & \mathbf{A} - \mathbf{B}^T\left(\mathbf{Z}^{-1} + \mathbf{\Gamma}_r^{-1}\operatorname{diag}(\mathbf{r})\right)^{-1}\mathbf{B}\mu\mathbf{X}^{-1} \\ & 1^T\mathbf{r} = 2M \\ & \mathbf{r} \in \{0, 1\}^{2N} \end{aligned} \tag{34}$$

and the $\mathbf{X} \in \mathbb{R}^{M\times M}$ in the above optimization issue is an auxiliary variable; the first constraint in (34) is given by [17]

$$\left(\mathbf{A} - \mathbf{B}^T\left(\mathbf{Z}^{-1} + \mathbf{\Gamma}_r^{-1}\operatorname{diag}(\mathbf{r})\right)^{-1}\mathbf{B}\right)'\mathbf{X}. \tag{35}$$

Here, we introduce the other one variable $\mathbf{Y} \in \mathbb{R}^{M\times M}$ and the inequality constraint in (35) can be equivalently transformed to

$$\mathbf{A} - \mathbf{Y}\mu\mathbf{X}^{-1}, \tag{36}$$

$$\mathbf{Y}\mu\mathbf{B}^T\left(\mathbf{Z}^{-1} + \mathbf{\Gamma}_r^{-1}\ \mathrm{diag}\ (\mathbf{r})\right)^{-1}\mathbf{B}. \tag{37}$$

Here, the $\mathbf{U} \le \mathbf{V}$(or $\mathbf{U} \ge \mathbf{V}$) shows that $\mathbf{V} - \mathbf{U}$ (or $\mathbf{U} - \mathbf{V}$) is the positive semidefinite matrix. Applying to the Schurs complement, the first constraint in (34) is transformed to the below linear matrix inequality (LMI)

$$\begin{bmatrix} \mathbf{A} - \mathbf{Y} & \mathbf{I} \\ \mathbf{I} & \mathbf{X} \end{bmatrix} \mu 0, \begin{bmatrix} \mathbf{Y} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{Z}^{-1} + \mathbf{\Gamma}_r^{-1}\ \mathrm{diag}\ (\mathbf{r}) \end{bmatrix} \mu 0, \tag{38}$$

Substituting (38) into (34), and the original optimization issue is rewritten as

$$\begin{aligned} &\min_{\mathbf{r},\mathbf{X},\mathbf{Y}} && \mathrm{tr}(\mathbf{X}) \\ &\text{s.t.} && \text{LMIs in } (38) \\ & && 1^T\mathbf{r} = 2M \\ & && \mathbf{r} \in \{0, 1\}^{2N} \end{aligned} \tag{39}$$

We can find that the above optimization problem is nonconvex owing to the Boolean selection vector $\mathbf{r}$. Thus, we utilize (16) and introduce an auxiliary variable $\mathbf{R} = \mathbf{r}\mathbf{r}^T$; the number of sensor selection and Boolean constraints in (39) is given by

$$\mathrm{tr}(\mathbf{R}) = M, \mathrm{diag}\ (\mathbf{R}) = \mathbf{r}, \tag{40}$$

and then, the $\mathbf{R} = \mathbf{r}\mathbf{r}^T$ can be relaxed to $\mathbf{R}\mu\mathbf{r}\mathbf{r}^T$; we reach the following optimization problem

$$\begin{aligned} &\min_{\mathbf{r},\mathbf{X},\mathbf{Y}} && \mathrm{tr}(\mathbf{X}) \\ &\text{s.t.} && \text{LMIs in } (38) \\ & && \mathrm{tr}(\mathbf{R}) = M \\ & && \mathrm{diag}\ (\mathbf{R}) = \mathbf{r} \\ & && \begin{bmatrix} \mathbf{R} & \mathbf{r} \\ \mathbf{r}^T & 1 \end{bmatrix} \mu 0 \end{aligned} \tag{41}$$

For the SDP (41) problem, the interior point algorithm is utilized to solve it quickly, and then, the fractional $\mathbf{r}$ can be obtained. Thus, the $\mathbf{w}$ and $\mathbf{v}$ also can be extracted from the $\mathbf{r}$ as defined before. The convenient method is to select the M largest sum of a fractional element w_k and v_k, and the corresponding index denotes the selected sensors.

We also can use a randomization algorithm to get a better solution, which consists of an iterative procedure. The aforementioned procedure is called "SDP," and the details of the randomization algorithm are shown in Algorithm 1, which is called "SDP with randomization."

5. Number Results

In this section, we first analyze the complexities of the proposed approach, and then, extensive simulations prove that the proposed approach can obtain high estimation performance for the hybrid TOA-AOA-based localization.

The commonly used method is the interior point method for the SDP solution, which the computational complexity is $\mathcal{O}(N^3)$ [33]. Furthermore, the SDP with the randomization algorithm should be considered. Due to the additional multiplication operations are needed for the randomization algorithm is$S\mathcal{O}_M$, and S denotes the number of random vectors. The exhaustive search algorithm itemizes all the possible sensor subsets with M from the total N sensors; thus, all possible sensor subsets are $N!/(N-M)!M!$, and we can choose the sensor subset with the minimum trace of CRLB. The exhaustive search algorithm that has high computational complexity is$\mathcal{O}(N!)$. It is noticeable that the proposed algorithm has lower computational complexity compared to the exhaustive algorithm.

In what follows, the proposed algorithm validity is demonstrated by simulation results. In the simulation experiment, let us assume that N sensors are randomly arranged in a region of size $1000\,\mathrm{m} \times 1000\,\mathrm{m}$. The prior PDF of $\mathbf{p}$ is given by $\mathbf{p} \sim \mathcal{N}(\mathbf{p}_0, \mathbf{C}_0)$, and $\mathbf{p}_0 = (0, 0)^T$, $\mathbf{C} = \mathrm{diag}\ (50, 50)$. The calibration source location is $\mathbf{c}_0 = (30,-20)^T$. The positive definite is $\mathbf{Z}_A$ and $\mathbf{Z}_T$, and we set $\lambda = 0.9\lambda_{\min}$, where $\lambda_{\min}$ is the minimum eigenvalue of $\mathbf{\Sigma}_A$ and $\mathbf{\Sigma}_T$. Two general approaches, the closest sensor algorithm and exhaustive search algorithm are recommended to contract with the proposed approach in this paper. The exhaustive algorithm can obtain the best result and is also used for comparison. In the closest sensor algorithm, the M shortest distance between the sensors and target of sensors is chosen. For simplicity, "Exhaustive search" is used to represent the exhaustive search algorithm, and "Closest sensors" is used to represent the M closest sensors. "SDP" and "SDP with randomization" are utilized to indicate our proposed SDP algorithm and SDP with Algorithm 1. Besides, we also use "All sensors" to represent the total activated sensors, and "Random selection" is utilized to represent randomly selected M sensors.

In the first experiment, we investigate the algorithm's performance to correct the sensor location errors using the calibration source. We assume that noise variance of the TOA measurement is $\sigma_1 = \cdots = \sigma_N = 1\,\mathrm{m}$, and the noise variance of the AOA measurement is $\gamma_1 = \cdots = \gamma_N = 1°$. The errors of the sensor position are within a 2-mile radius of the true position. Figure 1 depicts the localization accuracy with the different number of sensors. We observe that the $\mathrm{tr}(\mathrm{CRLB})$ of the corrected sensor position is close to tr(CRLB) of the ture sensors; the conclusion can be realized under the SDP solution and SDP with randomization algorithm. The effectiveness of the sensor position error correction method is proved, and the SDP with randomization obtains a high localization accuracy with the different number of selected sensors.

Next, we consider that the simulation scenarios are composed of $M = 5$ and $M = 10$ selected sensor, and the noise variance of AOA measurement remains constant while the noise variance of TOA measurement is a variable from $1\,\mathrm{m}^2$ to $10\,\mathrm{m}^2$. Figure 2 presents the $\mathrm{tr}(\mathrm{CRLB})$ increase as the noise variance growing of all these algorithms. However, the SDP with randomization algorithm always has better

Input: The fractional solution $\mathbf{w}$ and $\mathbf{v}$;
1: **for**$k = 1, 2, \cdots, N$**do**
2: Generate two random vectors: $\mathbf{w}_{\varsigma}^{k} \sim \mathcal{N}(\mathbf{w}^{O}, \mathbf{W}_{w} - \mathbf{w}^{o}\mathbf{w}^{oT})$, $\mathbf{v}_{\varsigma}^{k} \sim \mathcal{N}(\mathbf{v}^{O}, \mathbf{V}_{v} - \mathbf{V}^{o}\mathbf{V}^{oT})$.
3: set the largest M elements as 1 and the rest as 0 to generate two feasible vectors $(\mathbf{w}_{\varsigma}^{k})_{N}$ and $(\mathbf{v}_{\varsigma}^{k})_{N}$
4: Obtain the selected sensor index from $(\mathbf{w}_{\varsigma}^{k})_{N}$ and $(\mathbf{v}_{\varsigma}^{k})_{N}$, select the same sensor index of the two vectos as 1.
5: selsect the rest $(M - a)$ sensors from the rest of "1" sensors.
end for
6:Substitute each possible combination into objective fuction and choose vectors with the minimum value.
7: **Output:** the solved Boolean vector to choose with the minimum value

Algorithm 1: A randomization algorithm.

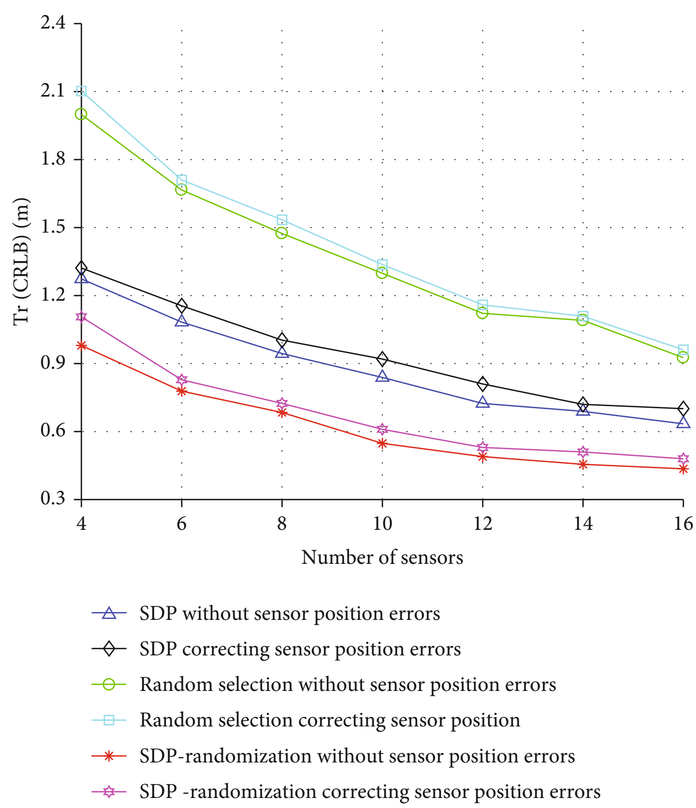

Figure 1: Comparison with different number of sensors.

localization accuracy and is close to the exhaustive search algorithm.

To further demonstrate the efficiency of the proposed method, we also consider the simulation scenarios in that the noise variance of TOA measurement remains constant. In contrast, the noise variance of AOA measurement varies from 1° to 10°. Other parameter settings remain unchanged as Figure 2.

We observe in Figure 3 that the tr(CRLB) does not increase significantly as γ^2 increases, which is consistent with the results in [34].

Furthermore, we consider the sensor selection scenario with each sensor having different noise variance while the number of the selected sensor varies from 4 to 16. Figure 4 plots the tr(CRLB) corresponding to these algorithms. It is observed from Figure 4 that the tr(CRLB) decreases with the selected sensor number from 4 to 16. The SDP with randomization algorithm yields a lower estimation error than other algorithms and almost achieves the exhaustive search algorithm.

Finally, the correlated measurement noise scenario is considered. The correlation parameter $\lambda_A = \lambda_T = \lambda$ varies from 0.1 to 0.9, and the remaining parameters of this simulation unchange as above. Figure 5 depicts the tr(CRLB) comparison of different number of selected sensors. and it can be observed that the tr(CRLB) curve of the SDP with randomization algorithm has the same variation tendency when the different selected sensors. That is the localization accuracy is raising as the correlation gets stronger, which is consistent with the results in [35]. Due to the strongly

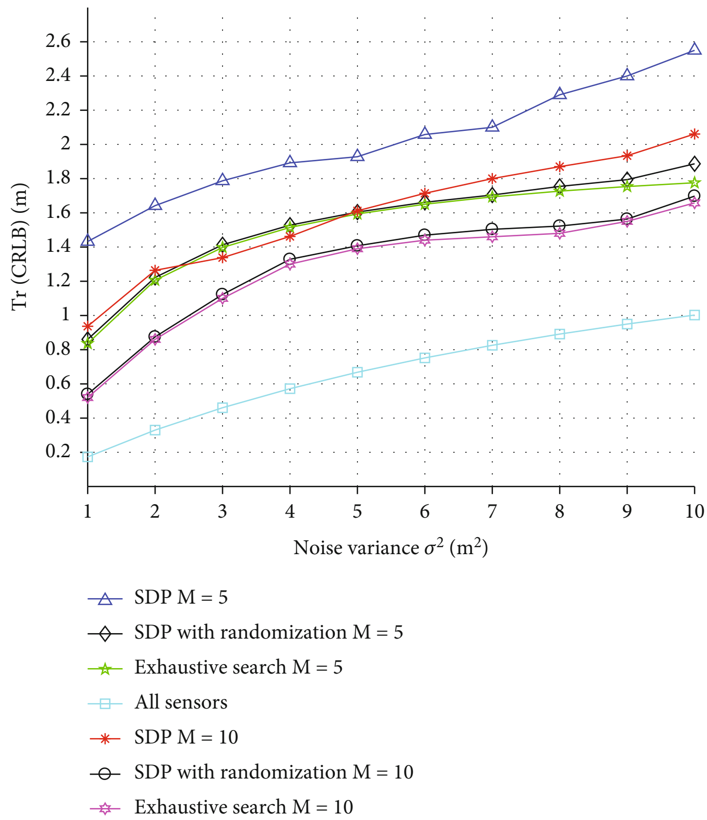

FIGURE 2: Comparison with different noise variance of TOA as $M = 5$ and $M = 10$.

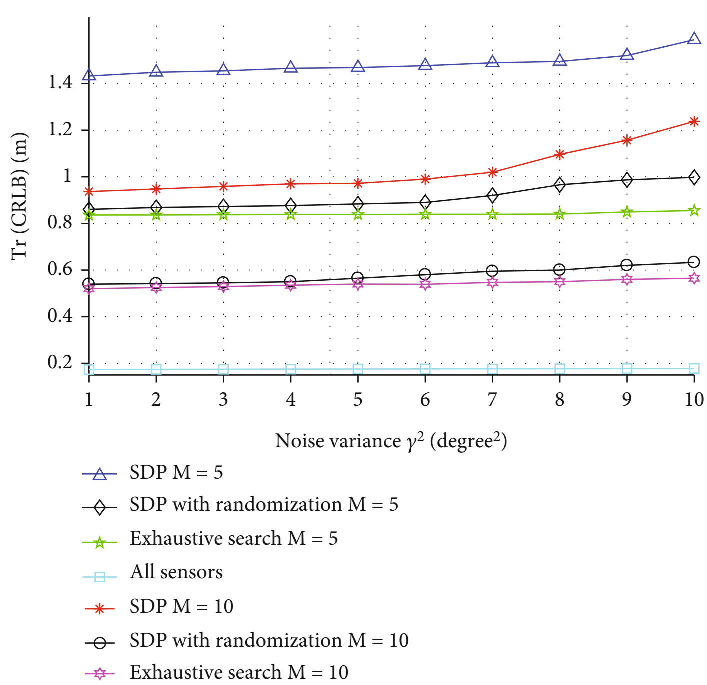

FIGURE 3: Comparison with different noise variance of AOA as $M = 5$ and $M = 10$.

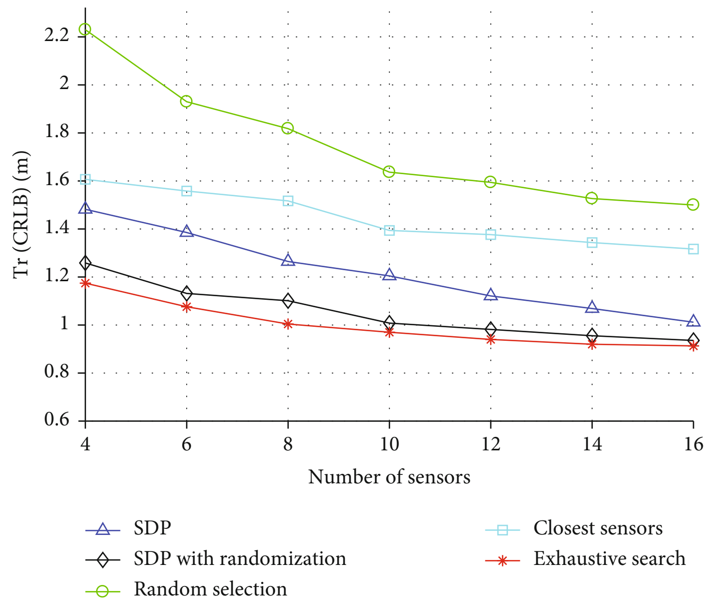

Figure 4: Comparison with different numbers of sensors and noise variance.

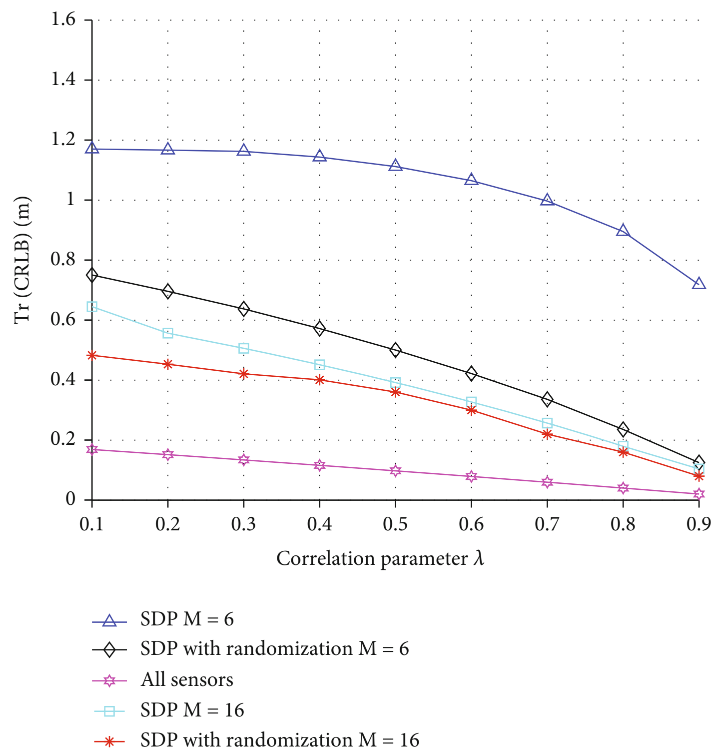

Figure 5: Comparison with different correlation parameter as $M = 6$, $M = 16$, and all sensors.

correlated noise, noise cancellation is achieved by subtracting one sensor from the rest. Furthermore, it is observed that SDP with randomization approach obtained better localization accuracy than the SDP solution.

6. Conclusion

This paper explores the sensor selection scheme for an uncertain target localization based on hybrid AOA-TOA measurements with the correlated noise in the underwater scenario. Considering that the original nonconvex optimization problem is formulated by minimizing the tr(CRLB), the optimization issue can be relaxed by convex relaxation and solved by the SDP method. The randomization algorithm is utilized to refine the results. Besides, a calibration source with a precise position is used to correct the sensor position. Simulation studies confirm that the superiority of the proposed algorithm over the existing algorithms; besides, the influence of the noise correlation on the sensor selection scheme is also discussed.

In the future work, we will investigate the sensor selection scheme for multiple underwater unknown targets with correlated noise, which will be formulated as a convex combination problem. We will develop the SDP solution for sensor selection for TDOA, AOA, and/or RSS. Furthermore, it is interesting to devise the sensor selection scheme with hybrid localization for target tracking in the underwater scenario. Sensor selection in the presence of non-line-of-sight propagation is also a challenging research topic.

Data Availability

The data used to support the finding of this study are included in the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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