


Research Article

Chebyshev-Gauss Approximation Analysis for Mobile Edge Computing-Aided IoT Networks

Fusheng Zhu,¹ Liming Chen,² Wen Zhou,³ Dan Deng,⁴ Yajuan Tang,⁵ Jun Liu ,⁶ Yuwei Zhang,⁶ Tao Cui,⁶ Lin Zhang,⁶ Jing Wang,⁶ and Sun Li⁷

¹Guangdong New Generation Communication and Network Innovative Institute (GDCNi), Guangzhou, China

²Electric Power Research Institute of CSG, Guangzhou, China

³Nanjing Forestry University, Nanjing, China

⁴University of Science and Technology of China, China

⁵Shantou University, Shantou, China

⁶Tsinghua University, Beijing, China

⁷Xi'an Jiaotong University, China

Correspondence should be addressed to Jun Liu; junliu.thu@ieee.org

Received 5 May 2022; Revised 25 May 2022; Accepted 30 May 2022; Published 18 July 2022

Academic Editor: Jun Li

Copyright © 2022 Fusheng Zhu et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Recently, mobile edge computing (MEC) has been widely applied into Internet of Things (IoT) networks, which has attracted a lot of attention from researchers. A critical challenge in the MEC-aided IoT networks is that the performance analysis is often complicated, where it is quite difficult for us to obtain some analytical or closed-form solution to the performance analysis, such as outage probability and bit error rate. This has been the bottleneck of the development of MEC-aided IoT networks. To address this challenge, we deeply investigate the Chebyshev-Gauss approximation method and derive the analytical solution to implement this powerful and useful approximation. We then give several examples to show the effectiveness of the Chebyshev-Gauss approximation in the performance analysis for the MEC-aided IoT systems. The results in this work can serve as an important reference and reveal some important inherent mechanisms for the MEC-aided IoT networks.

1. Introduction

Recently, a lot of wireless nodes cooperate together, to communicate and compute collaboratively, which form the Internet of Things (IoT) networks [1, 2]. In such a system, a lot of wireless nodes access the system spectrum, by using orthogonal or nonorthogonal multiple access schemes [3, 4]. These nodes can communicate and compute in a collaborative way, when facing some intensive calculating tasks. Besides the communication and calculation, the privacy protection also becomes a key research topic in the study of IoT networks [5, 6], where some privacy protection methods from the physical layer to the application layer should be incorporated into the system, in order to enhance the data communication privacy and

data calculation privacy, especially for some sensitive data such as medical data and financial data.

Some novel techniques have been proposed by researchers to promote the development of IoT networks, among which mobile edge computing (MEC) is a key technology [7, 8]. In the MEC-aided IoT networks, some edge nodes have some powerful ability to help calculate the intensive tasks from other nodes, which will be helpful in leading to a smaller delay and power consumption (PoCo). In this area, a lot of studies have been performed to utilize the communication resources as well as calculating resources in the MEC-aided IoT networks, through some conventional optimization methods such as convex optimization or some intelligent algorithms such as deep reinforcement learning (DRL) algorithms, in order to reduce the system delay and

PoCo [9]. This can help make the MEC-aided IoT networks fit the various applications [10–12].

Because of restricted regularity sources, cochannel interference has actually been unavoidable in the wireless communication systems. Cochannel interference has restricted the system efficiency seriously, as well as being the traffic jam of the wireless systems. The effect of cochannel interference on the system efficiency of wireless communications was thoroughly examined in the literary works. Some authors examined the dual-hop communicating systems in the existence of cochannel interference, as well as the system data rate and outage possibility. For the secure communicate systems in cochannel interference, the system efficiency might be examined through obtaining the capacity as well as asymptotic privacy outage possibility, whereby the impact of interfering energy on the system efficiency might be exposed.

A critical challenge in the MEC-aided IoT networks is that the performance analysis is often complicated, where it is quite difficult to obtain some analytical or closed-form solution to the performance analysis, such as outage probability and bit error rate. This has been the bottleneck of the development of MEC-aided IoT networks. To address this challenge, we deeply investigate the Chebyshev-Gauss approximation method and derive the analytical solution to implement this powerful and useful approximation. We then give several examples to show the effectiveness of the Chebyshev-Gauss approximation in the performance analysis for the MEC-aided IoT systems. The results in this work can serve as an important reference and reveal some important inherent mechanisms for the MEC-aided IoT networks.

2. Chebyshev-Gauss Quadrature

In numerical analysis, numerical integration is the method and theory of calculating the value of definite integration. We can use the Leibniz integral rule to calculate the definite integral through the original function. However, it is regularly difficult to calculate the original value of the function. There are few functions that can be expressed by elementary functions, and the integration of most integrable functions cannot be expressed by elementary functions or even analytical expressions. Therefore, in many cases, we can only use numerical integration to calculate the approximate value of the function.

At present, there are many algorithms for calculating definite integral. For instance, these algorithms mainly include the following:

- (i) Rectangle rule
- (ii) Trapezoidal rule
- (iii) Romberg's method
- (iv) Gauss quadrature

Among the above algorithms, the Gaussian quadrature rule with n -point is a quadrature rule constructed to yield an exact result for polynomials of degree $2n - 1$ or less by

a suitable choice of the nodes x_i and weights w_i for $i = 1, \dots, n$. Specifically, the Gauss quadrature includes three different forms:

- (1) Chebyshev-Gauss quadrature
- (2) Gauss-Hermite quadrature
- (3) Gauss-Jacobi quadrature

Now, we mainly discuss the Chebyshev-Gauss quadrature. Chebyshev-Gauss quadrature is an extension of Gaussian quadrature method, which is used to approximate the following two types of integral value. For the first kind, we can have

$$\int_{-1}^{+1} \frac{f(x)}{\sqrt{1-x^2}} dx \approx \sum_{i=1}^n w_i f(x_i), \quad (1)$$

where $x_i = \cos((2i-1)\pi/2n)$, the weight $w_i = \pi/n$, and the approximation error decreases with a larger number of item n .

On the contrary, for the second kind, we have

$$\int_{-1}^{+1} \sqrt{1-x^2} g(x) dx \approx \sum_{i=1}^n w_i g(x_i), \quad (2)$$

where $x_i = \cos(i\pi/(n+1))$, the weight $w_i = (\pi/(n+1)) \sin^2(i\pi/(n+1))$, and the approximation error decreases with a larger number of item n .

For a random function $f(x)$, its integral $\int_a^b f(x) dx$ can be approximated as

$$\int_a^b f(x) dx \approx \sum_{i=1}^n w_i f(x_i), \quad (3)$$

where w_i is the weight coefficient, and the approximation error decreases with a larger number of item n . Note that the accuracy can be improved by increasing the number of x_i or finding the right x_i . We can rewrite $f(x)$ as

$$f(x) = \rho(x)g(x). \quad (4)$$

Therefore, (3) can be approximated as

$$\int_a^b \rho(x)g(x) dx \approx \sum_{i=1}^n w_i g(x_i), \quad (5)$$

where $\rho(x)$ is the weight function and the approximation error decreases with a larger number of item n . We can use the Chebyshev-Gauss quadrature to approximate (5). The Chebyshev polynomials of the first kind are obtained

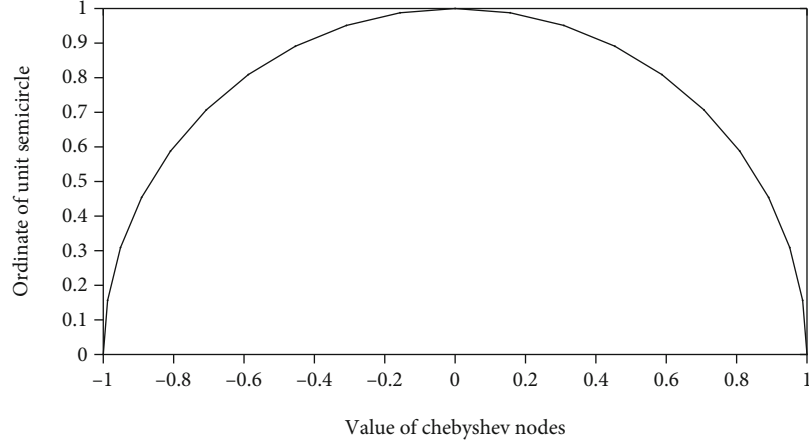


FIGURE 1: Chebyshev node on the unit semicircle.

from the following recurrence relation:

$$\begin{aligned}
 \cos 2\theta &= 2 \cos^2\theta - 1, \\
 \cos 3\theta &= 4 \cos^2\theta - 3 \cos \theta, \\
 \cos n\theta &= T_n(\cos \theta), \\
 T_n(x) &= \cos (n \cdot \arccos x).
 \end{aligned} \tag{6}$$

Let $T_n(x) = 0$, and the root of the equation can be obtained as

$$\begin{aligned}
 T_n(x) &= \cos (n \cdot \arccos x) = 0, \\
 n \cdot \arccos x &= \frac{\pi}{2} + k\pi = \frac{(2k+1)\pi}{2}, \\
 \arccos x &= \frac{(2k+1)\pi}{2n}, \\
 x_k &= \cos \left(\frac{(2k+1)\pi}{2n} \right),
 \end{aligned} \tag{7}$$

where $k=0, 1, \dots, n-1$, and x_k is the Chebyshev node, namely, the root of the Chebyshev polynomials of the first kind.

As shown in Figure 1, the Chebyshev node is equivalent to the x -axis coordinates of N equally spaced points on the unit semicircle. The Chebyshev-Gauss quadrature can obtain a relatively approximate solution only when the function $f(x)$ can be approximated by polynomials in the interval $[-1, 1]$. We use an affine transformation for nodes over an arbitrary interval $[a, b]$ as

$$x_k = \frac{1}{2}(a+b) + \frac{1}{2}(b-a) \cos \left(\frac{(2k-1)\pi}{2n} \right), \quad k = 1, 2, \dots, n. \tag{8}$$

For (3), let

$$x = \frac{1}{2}(a+b) + \frac{1}{2}(b-a)y, \tag{9}$$

and thus, we have

$$\int_a^b f(x) dx = \int_{-1}^1 \frac{1}{2}(b-a) \cdot f \left(\frac{1}{2}(a+b) + \frac{1}{2}(b-a)y \right) dy. \tag{10}$$

Let $y = \cos \theta$, where $\theta = ((2k+1)\pi/2n)$, $k=0, 1, \dots, n-1$, and then, we have

$$\begin{aligned}
 \int_{-1}^1 \frac{1}{2}(b-a) \cdot f \left(\frac{1}{2}(a+b) + \frac{1}{2}(b-a)y \right) dy \\
 = - \int_{\pi}^0 \frac{1}{2}(b-a) \sin \theta \cdot f \left(\frac{1}{2}(a+b) + \frac{1}{2}(b-a) \cos \theta \right) d\theta.
 \end{aligned} \tag{11}$$

We can further write

$$\int_a^b f(x) dx = \int_0^{\pi} \frac{1}{2}(b-a) \sin \theta \cdot f \left(\frac{1}{2}(a+b) + \frac{1}{2}(b-a) \cos \theta \right) d\theta. \tag{12}$$

According to the definition of definite integral, we can write $\int_a^b f(x) dx$ as

$$\begin{aligned}
 \int_a^b f(x) dx &= \sum_{k=0}^{n-1} \frac{1}{2}(b-a) \sin \left(\frac{(2k+1)\pi}{2n} \right) \frac{\pi}{n} \\
 &\cdot f \left(\frac{1}{2}(a+b) + \frac{1}{2}(b-a) \cos \left(\frac{(2k+1)\pi}{2n} \right) \right).
 \end{aligned} \tag{13}$$

By comparing with (5), we can obtain

$$x_i = \frac{1}{2}(a+b) + \frac{1}{2}(b-a) \cos \left(\frac{(2k+1)\pi}{2n} \right), \tag{14}$$

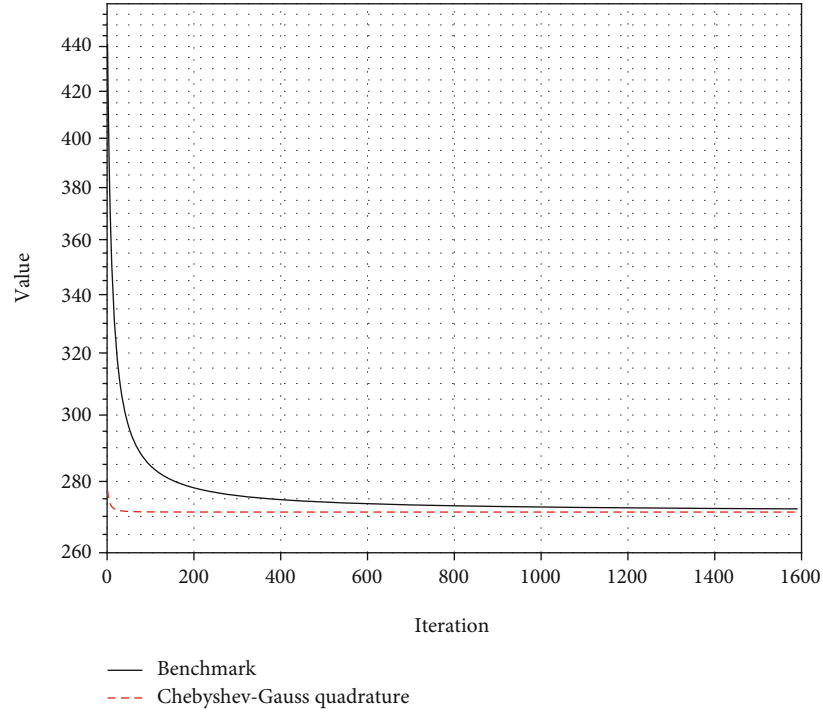


FIGURE 2: Comparison of the convergence effect of I_1 .

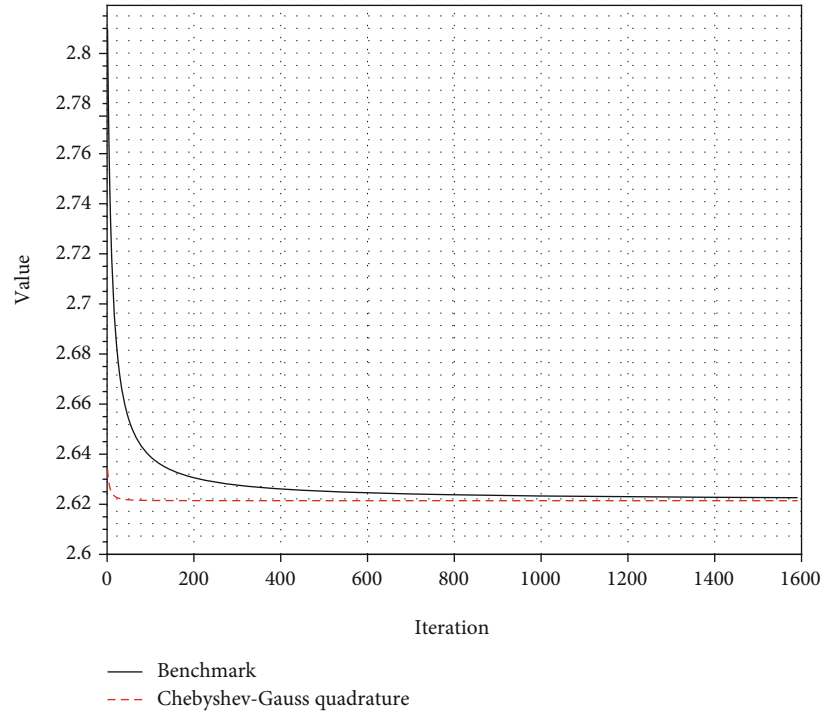


FIGURE 3: Comparison of the convergence effect of I_2 .

where $k = 1, 2, \dots, n - 1$, and

$$w_i = \frac{1}{2} (b - a) \sin \left(\frac{(2k + 1)\pi}{2n} \right) \frac{\pi}{n}, \quad (15)$$

3. Numerical and Simulation Results

In this part, we present some numerical and simulation results to verify the convergence effect of the conventional rectangle rule and the Chebyshev-Gauss quadrature. We compare the convergence effect of the following three

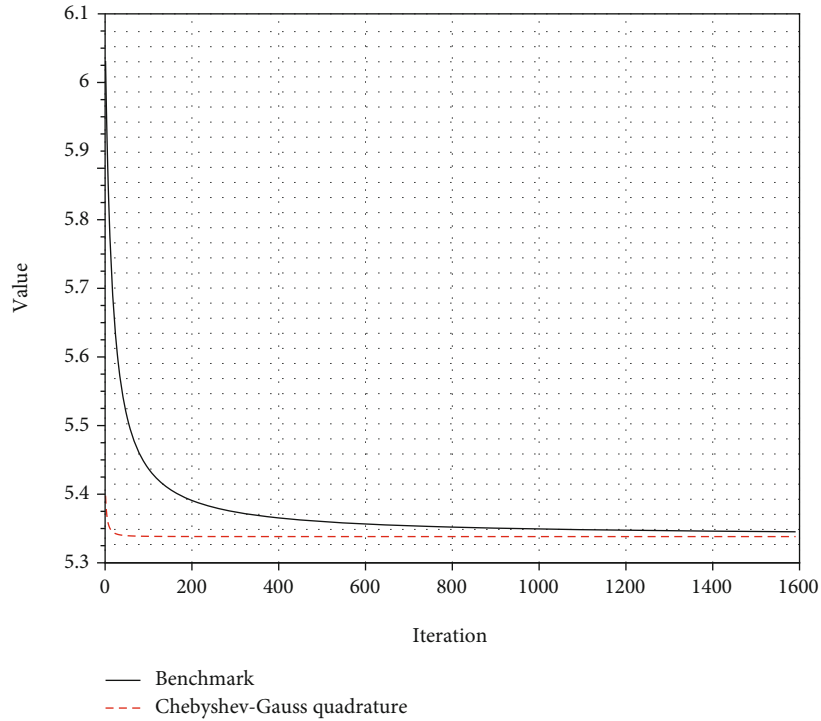


FIGURE 4: Comparison of the convergence effects of I_3 .

functions, in Figures 2–4 and Tables 1–3.

$$\begin{aligned}
 I_1 &= \int_1^3 e^{x^2-x^{1/2}} dx, \\
 I_2 &= \int_1^3 \log(e^x - x^2) dx, \\
 I_3 &= \int_1^3 \frac{x^{e^x-x}}{x^3} dx.
 \end{aligned} \tag{16}$$

Figure 2 shows the convergence effect of I_1 . We can find that the Chebyshev-Gauss quadrature and the benchmark method (namely the rectangle rule) both reach the convergent state after 1600 iterations, and the ultimate convergent values are 271.2296 and 272.1258, respectively. Similarly, as shown in Table 1, the Chebyshev-Gauss quadrature method approaches convergence with the convergent value of 271.2469, after about 200 iterations, which is much faster than the benchmark method which approaches the convergent state, after about 1200 iterations.

In Figure 3, the Chebyshev-Gauss quadrature approximation is performed on I_2 to demonstrate the advantage of faster convergence than the benchmark method. From Table 2, we can also find that the Chebyshev-Gauss quadrature method approaches the convergent state with the convergent value of 2.6207, after about 200 iterations, while the benchmark method approaches the convergent state, after about 1200 iterations. The associated ultimate convergent values are 2.6207 and 2.6218 after 1600 iterations, respectively.

TABLE 1: Approximation of I_1 versus the number of iterations.

Iteration	Benchmark method	Chebyshev-Gauss
1	441.1021	277.4677
200	278.1464	271.2469
400	247.7484	271.2329
600	273.5892	271.2310
800	273.0044	271.2303
1000	272.6519	271.2300
1200	272.4162	271.2298
1400	272.2475	271.2297
1600	272.1258	271.2296

TABLE 2: Approximation of I_2 versus the number of iterations.

Iteration	Benchmark method	Chebyshev-Gauss
1	2.8100	2.6329
200	2.6296	2.6207
400	2.6252	2.6207
600	2.6237	2.6207
800	2.6230	2.6207
1000	2.6225	2.6207
1200	2.6222	2.6207
1400	2.6220	2.6207
1600	2.6218	2.6207

Figure 4 illustrates the convergence of the function I_3 . It can be seen that the Chebyshev-Gauss quadrature method approaches the convergent state with the convergent value

TABLE 3: Approximation of I_3 versus the number of iterations.

Iteration	Benchmark method	Chebyshev-Gauss
1	6.026	5.3915
200	5.38493	5.3354
400	5.3608	5.3353
600	5.3525	5.3353
800	5.348	5.3353
1000	5.3457	5.3353
1200	5.3440	5.3353
1400	5.3428	5.3353
1600	5.3419	5.3353

of 5.3354 after about 200 iterations, while the benchmark method approaches the convergent state with the convergence value of 5.3428 after about 1400 iterations, as shown in Table 3. This also shows the advantage that the Chebyshev-Gauss quadrature method converges much faster than the benchmark method. The associated final convergent values are 5.3353 and 2.6218 after 1000 iterations, respectively.

4. Conclusions

In the MEC-aided IoT networks, a critical challenge was that the performance analysis was often complicated, where it was quite difficult to obtain some analytical or closed-form solution to the performance analysis, such as outage probability and bit error rate. To address this challenge, we deeply investigated the Chebyshev-Gauss approximation method and derived the analytical solution to implement this powerful and useful approximation. We then gave several examples to show the effectiveness of the Chebyshev-Gauss approximation in the performance analysis for the MEC-aided IoT systems. The results in this work could serve as an important reference and reveal some important inherent mechanisms for the MEC-aided IoT networks.

Data Availability

The data of this paper can be obtained through email to the authors.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this work.

Acknowledgments

The work in this paper was supported by the Key-Area Research and Development Program of Guangdong Province (No. 2019B090904014) and the National Natural Science Foundation of China (No. 62871349).

References

- [1] S. Arzykulov, A. Celik, G. Nauryzbayev, and A. M. Eltawil, "UAV-Assisted cooperative & cognitive NOMA: deployment, clustering, and resource allocation," *IEEE Transactions on Cognitive Communications and Networking*, vol. 8, no. 1, pp. 263–281, 2022.
- [2] Q. Tao, J. Wang, and C. Zhong, "Performance analysis of intelligent reflecting surface aided communication systems," *IEEE Communications Letters*, vol. 24, no. 11, pp. 2464–2468, 2020.
- [3] M. Liu, B. Li, Y. Chen et al., "Location parameter estimation of moving aerial target in space-air-ground-integrated networks-based IoV," *IEEE Internet of Things Journal*, vol. 9, no. 8, pp. 5696–5707, 2022.
- [4] B. Wang, F. Gao, S. Jin, H. Lin, and G. Y. Li, "Spatial- and frequency-wideband effects in millimeter-wave massive MIMO systems," *IEEE Transactions on Signal Processing*, vol. 66, no. 13, pp. 3393–3406, 2018.
- [5] X. Hu, C. Zhong, Y. Zhang, X. Chen, and Z. Zhang, "Location information aided multiple intelligent reflecting surface systems," *IEEE Transactions on Communications*, vol. 68, no. 12, pp. 7948–7962, 2020.
- [6] M. Liu, C. Liu, M. Li, Y. Chen, S. Zheng, and N. Zhao, "Intelligent passive detection of aerial target in space-air-ground integrated networks," *China Communications*, vol. 19, no. 1, pp. 52–63, 2022.
- [7] W. Zhou, L. Chen, S. Tang et al., "Offloading strategy with PSO for mobile edge computing based on cache mechanism," *Cluster Computing*, vol. 2021, no. 1, pp. 1–11, 2021.
- [8] J. Zhao, X. Sun, Q. Li, and X. Ma, "Edge caching and computation management for real-time internet of vehicles: an online and distributed approach," *IEEE Transactions on Intelligent Transportation Systems*, vol. 22, no. 4, pp. 2183–2197, 2021.
- [9] J. Zhao, Q. Li, Y. Gong, and K. Zhang, "Computation offloading and resource allocation for cloud assisted mobile edge computing in vehicular networks," *IEEE Transactions on Vehicular Technology*, vol. 68, no. 8, pp. 7944–7956, 2019.
- [10] Z. Zhu, S. Wan, P. Fan, and K. B. Letaief, "Federated multi-agent actor-critic learning for age sensitive mobile-edge computing," *IEEE Internet of Things Journal*, vol. 9, no. 2, pp. 1053–1067, 2022.
- [11] S. Premkumar and A. N. Sigappi, "ASIS edge computing model to determine the communication protocols for IoT based irrigation," *Journal of Mobile Multimedia*, vol. 18, no. 3, 2022.
- [12] F. Alqahtani, M. Al-Maitah, and O. A. Elshakankiry, "A proactive caching and offloading technique using machine learning for mobile edge computing users," *Computer Communications*, vol. 181, pp. 224–235, 2022.