Research Article

Graph Algorithm for considering Second-Order Effects of Rc Ring-Section Columns

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1. Introduction

Annular Section is widely used in engineering construction, because it has the advantages of reasonable force, material saving, beautiful shape, etc. With regard to the calculation of the annular section reinforcement of RC members, China’s Code for Design of Concrete Structures [1] (referred to as Code) uses transcendental equations to solve iteratively, but the manual calculation is extremely cumbersome. If we consider the second-order effect of members on this basis, we should first distinguish between the P-δ effect and the P-Δ effect. The two methods for calculating the magnification factor η of the second-order effect [2] are significantly different. The former is calculated by the ultimate curvature, while the latter is calculated by the magnification factor critical force. The former uses the ultimate curvature, while the latter applies the critical force. After the magnification factor η of the second-order effect is obtained, we need to substitute it into the transcendental equations of the annular section and solve it iteratively, indicating that it is unrealistic to solve the second-order effect of RC annular section column through the manual calculation.

In addition, at the section level, the annular section equation in the Code adopts the conversion coefficients (α and β) of the rectangular section’s equivalent diagram [3], at the bar level, the P-δ second-order effect magnification factor in the Code also adopts the rectangular section’s ultimate curvature equation. In other words, the rectangular equation is used at both levels, but unfortunately, this simplification deviates from the accurate calculation to a large extent.

In the face of the triple nonlinearity [4] of material, geometry and section width, this paper only makes the assumption that the second-order deformation curve is quadratic parabola at the geometric nonlinear level [5–10], while the rest are derived strictly according to the constitutive relation of the material and the actual size of the annular section. Compared with the Code, its calculation result is...
more accurate. In order to facilitate the manual calculation of reinforcement, we have drawn the calculation results into a nomogram.

2. Calculation Method and Equation Derivation

2.1. Constitutive Relationship. In this work, the complete constitutive relation of concrete and steel bar given in Code 6.2.1 is adopted, and its graphic expression is shown in Figure 1.

Regardless of the tensile strength of concrete, the mathematical expression of the concrete’s constitutive relation is:

$$\sigma_c = \begin{cases} 0 & \varepsilon_c \leq 0 \\ f_c \left[ 1 - \left( \frac{\varepsilon_c - \varepsilon_y}{\varepsilon_y} \right)^2 \right] & 0 < \varepsilon_c \leq 0.002 \\ f_c & 0.002 < \varepsilon_c \leq 0.0033 \end{cases}$$

(1)

Wherein, $\sigma_c$ represents the concrete’s compressive stress, $f_c$ represents the design value of the concrete’s axial compressive strength, $\varepsilon_c$ represents the concrete’s compressive strain, $\varepsilon_{cu}$ represents the ultimate compressive strain of normal section, and $\varepsilon_y$ represents the compressive strain when the compressive stress reaches $f_c$, which is 0.002. Only the concrete whose strength grade is lower than C50 is considered. According to the Code, when $\varepsilon_{cu}$ is greater than 0.0033, it is taken as 0.0033, and when it is under axial compression, $\varepsilon_{cu} = \varepsilon_y$.

The tensile or compression steel bar adopts the ideal elastic-plastic [11, 12] stress-strain relationship without considering the strengthening stage, the expression is:

$$\sigma_s = \begin{cases} E_s \varepsilon_s & 0 < \varepsilon_s < \varepsilon_y \\ f_y & \varepsilon_y \leq \varepsilon_s \leq 10\% \end{cases}$$

(2)

Wherein, $\sigma_s$ represents the tensile stress of steel bar, $f_y$ represents the design value of the steel bar’s tensile strength, $\varepsilon_s$ represents the tensile strain of steel bar, and $\varepsilon_y$ represents the yield strain of steel bar.

2.2. Strain Distribution. The calculation method at the section level is that the stress is determined by the known strain, and then the internal force is calculated. The key of this method is to determine the possible strain distribution of the cross section. According to the relevant provisions in the Code 6.2.1, taking the ultimate strain of concrete and steel bar as the boundary and ensuring that the section is in the ultimate strain state, then the strain area A-C represented by the shade in Figure 2 is constructed. The values of the three strain areas are as follows: Area A, $\varepsilon_s = 10\%$; $\varepsilon_{c1}$ ranges from 10% to -3.3%; Area B, $\varepsilon_{c2} = -3.3\%$, $\varepsilon_{c3}$ ranges from 10% to 0; Area C, $\varepsilon_c = -2\%$, $\varepsilon_{c1}$ ranges from -3.3% to -2%, that is, the area through which the right boundary of Area B rotates counterclockwise around K point.

Among them, the upper and lower edges of the section are concrete strain, which are $\varepsilon_{c1}$ and $\varepsilon_{c2}$, respectively, the upper and lower steel bar strain are $\varepsilon_{s1}$ and $\varepsilon_s$, respectively, and the intersection point between the ultimate compressive strain 2% of axial compression and the right boundary of Area B is $K$.

Using the plane section hypothesis, the relationship between curvature $\Phi$ and edge strain is obtained, and the dimensionless curvature curve is obtained by multiplying it by the section height $\phi$, see Eq. (3).

$$\Phi = \frac{\varepsilon_{c1} - \varepsilon_{c2}}{d} = \frac{\varepsilon_{c1} - \varepsilon_{s1}}{d - a} ; \phi = \Phi d$$

(3)

2.3. Calculation of Stress and Internal Force. The cross-sectional area of the annular section because of its own characteristics, which indicates that when calculating the internal force of the annular section, the compression area of the inner annulus (hollow) should be subtracted from the outer annulus compression area, as detailed in Figure 3.

The distribution of steel bars is discrete [13–15] and uniform. For the convenience of calculations, the steel bar area is spread out on the perimeter of the annulus where the steel bar is located, forming a steel bar annulus, that is:

$$\bar{a}_s = \frac{A_s}{2\pi r_s} = \frac{\omega r_s^2 f_y}{2 r_s f_y}$$

(4)

Wherein, $\bar{a}_s$ represents the linear reinforcement ratio of steel bar annulus. As represents the cross-sectional area of steel bar in annular section, and $\omega = A_{sfy}/\pi r_s^2$ represents the strength reinforcement ratio of dimensionless section.

After transformation, the annular section is divided into three annuli, $r_1$ is the radius, $r_2$ and $r_s$ are the outer and inner radii of the annulus, respectively, and $r_1$ is the radius of the steel bar annulus, as shown in Figure 3.

According to Figure 4, the height $x_c$ of the compression zone is calculated, as shown in Eq. (5).

$$x_c = \frac{\varepsilon_{c1}}{\varepsilon_{c1} - \varepsilon_{c2}} d$$

(5)

Wherein, $\varepsilon_{c1}$ and $\varepsilon_{c2}$ represent the strain at the upper edge and the lower edge of the annulus, respectively, and $d$ represents the diameter of the annulus.

After the $x_c$ is obtained, the outer and inner circular strain $\varepsilon_{ri}$ and $\varepsilon_{2ri}$ at the arbitrary microstrip of the annular section and the strain $\varepsilon_{ri}$ at any place of the steel bar annulus are determined, see Eq. (6).

$$\begin{align*}
\varepsilon_{ri} & = \frac{x_c - r_1 (1 - \cos \phi)}{x_c} \varepsilon_{c1} \\
\varepsilon_{2ri} & = \frac{x_c - r_2 (1 - \cos \phi)}{x_c} \varepsilon_{c1} \\
\varepsilon_{ri} & = \frac{x_c - (r_1 - r_s \cos \phi)}{x_c} \varepsilon_{c1}
\end{align*}$$

(6)

According to Eqs. (5) and (6), the strains in the concrete and steel bar annulus vary with the strain at the upper and lower edges of the section. By substituting Eq. (6) into Eqs.
annular section are calculated according to Eq. (8). From which the axial force and bending moment of the concrete parts, then the increment of each strip of the section. Assuming the section is divided into microstrip are obtained. In the calculation, the moment equation for calculating the area of the microstrip is:

\[
\tau = \frac{2r_1^2 \sin^2 \varphi}{\pi} \, d\varphi
\]

\[
\tau = 2r_2^2 \sin^2 \varphi \, d\varphi
\]

\[
\tau = 2\Delta r \, d\varphi = \omega r_1^2 f \, d\varphi / f_y
\]

Thus, the axial force and bending moment of each microstrip is obtained. In the calculation, the moment point of the bending moment is selected as the central axis of the section. Assuming the section is divided into \( k \) equal parts, then the increment of each strip’s center angle is \( d\varphi = \pi / k \), and the center angle of the \( k \)-th strip is \( \varphi_k = k \varphi \), from which the axial force and bending moment of the annular section are calculated according to Eq. (8).

\[
N = \sum_{i=1}^{k} (\sigma_1 \, dA_{1i} - \sigma_2 \, dA_{2i} + \sigma_3 \, dA_i)
\]

\[
M = \sum_{i=1}^{k} (\sigma_1 \, dA_{1i} \cos \varphi_i - \sigma_2 \, dA_{2i} \cos \varphi_i + \sigma_3 \, dA_i \cos \varphi_i)
\]

The equations for dimensionless axial force (axial compression ratio) \( n \), bending moment \( m \) and curvature ratio \( \phi \) are:

\[
n = N / (\pi r_1^2 f_c), \quad m = M / (\pi r_1^3 f_c), \quad \phi = \Phi d
\]

Wherein, \( r_1 \) represents the radius of the outer annulus, \( d \) represents the height of the section, and \( f_c \) represents the design value of the concrete’s compressive strength.

After dimensionless substitution, Eq. (8) is obtained as:

\[
\begin{align*}
\omega &= \frac{k \sum \sigma_1 \, dA_{1i} - \sigma_2 \, dA_{2i} + \sigma_3 \, dA_i}{A_f \, f_y} \\
\omega &= \frac{k \sum \sigma_1 \, dA_{1i} \cos \varphi_i - \sigma_2 \, dA_{2i} \cos \varphi_i + \sigma_3 \, dA_i \cos \varphi_i}{A_f \, f_y}
\end{align*}
\]

Equation (10) is the exact solution for the calculation of the bearing capacity of the annular section. It can be seen that after dimensionless, \( n \) and \( m \) have nothing to do with \( f_c \), which makes \( n-m \) more general, and C15 ~ C50 concrete can be used. Although Equation (10) is an exact solution, it is very difficult to obtain the exact value by hand calculation. The method described later will solve this problem.

2.4. Cross-Section Limit N-Mu Correlation. Calculation idea: assign different values to \( c \) to get the corresponding stress from the constitutive relationship, and then integrate the stress to get the internal force. It should be noted that the strain should be within the range of the possible strain distribution in Figure 2, and the fixed side should become a ultimate strain \((\varepsilon_y = -3.3\%\), or \(\varepsilon_y = 10\%\)), while the strain on the other side can change from small to large, that is, a series of different values should be assigned to \( \varepsilon_x \) and \( \varepsilon_y \), respectively. From Eq. (10), the value of \( n-m \) can be obtained, which is the limit value \( n_m \). Below, five values with equal interval between \( \omega = 0 \) and 2 are selected as examples, and then we get 5 groups of \( n_m \) arrays and draw 5 curves.

Obviously, the moment bearing capacity of the annular section \( m_n \) has a non-linear correlation with the axial compression ratio \( n \), but has a significant linear correlation with the strength reinforcement ratio \( \omega \). With the increase of the strength reinforcement ratio, the curve also increases at equal intervals. In addition, the size eccentric boundary of the annular section is no longer the maximum bending moment, that is, the annular section no longer has the obvious big and small eccentric demarcation point as the rectangular section.

In Figure 5, the maximum internal force of each curve is point \((0, n_{mu})\), and the corresponding strain is \(-2\%\), that is, when the strain of concrete and steel bar is \(-2\%\), the limit value of axial compression ratio \( n_{cu} \) could be obtained from Eq. (10), as shown in Eq. (11):

\[
n_{cu} = \begin{cases} 
\frac{r_2^2}{r_1^2} \omega - 1 & \varepsilon_y \leq 2\% \\
\frac{r_2^2}{r_1^2} \omega - 2\omega \varepsilon_y & \varepsilon_y > 2\%
\end{cases}
\]
Figure 5 shows the $\omega$-$n$-$m_u$ correlation in which $\omega$ is an isoline, that is, a fixed $\omega$ value, and a series of $n$-$m_u$ values are calculated from the Eq. (10). If we want to obtain the $n$-$m_u$ correlation in which $n$ is an isoline, we can fix the $n$-value and calculate a series of $m_u$-$\phi_u$ values from the Eq. (10). In order to coordinate with the following content, here $m_u$ is expressed by eccentricity $e_u$, that is, $2e_u/d = m_u/n$, and the correlation of $n-e_u/d-\omega$ is given in Figure 6.
3. Approximate Algorithm for Second-Order Effect of Annular Section Column

3.1. Second-Order Deformation and Second-Order Bending Moment. Taking a cantilever column subjected to eccentric pressure $N$ as the research object, see Figure 7. In the process of calculating the second-order deformation, this article assumes that the curvature distribution of the column is a quadratic parabola to avoid complicated numerical calculations [16–18]. In this way, the curvature of the whole column can be controlled by the curvature ($\Phi$) of a section at the foot of the column, so that the calculation of the member hierarchy can be transformed into the calculation of the section hierarchy. That is, as long as $\Phi$ is known, the second-order deformation $e_2$ can be determined. The total bending moment at the fixed end of the column foot is $M_{tot} = M_1 + M_2$, and the second-order bending moment [19–22] is $M_2 = N \cdot e_2$, then $e_2$ can be obtained by moment multiplication, see Eq. (12).

$$e_2 = \int \frac{M_{tot}}{EI} \cdot Mdz = \int \Phi Mdz = \frac{2}{3} \cdot \frac{l_0}{2} \cdot \Phi \cdot \frac{5}{8} \cdot \frac{l_0}{2} = \frac{5}{48} \Phi^2$$

(12)
The expression of the maximum $M_{tot}$ is:

$$M_{tot} = M_1 + M_2 = N\varepsilon_1 + N\varepsilon_2 = N \left( \varepsilon_1 + \frac{5}{48} \Phi_0^2 \right)$$  \hspace{1cm} (13)\]

For the generality of the calculated results, Eq. (13) is expressed as follows after dimensionless:

$$m_{tot} = n \cdot \frac{\varepsilon_1}{m_1} + n \frac{5}{24} \left( \frac{l_0}{d} \right)^2 \phi$$  \hspace{1cm} (14)\]

According to Eq. (14), the right end of the equation is known except for curvature $\phi$. The total bending moment is a function of a linear equation, and the independent variable is $\phi$. The next section analyzes and discusses how to determine the independent variable $\phi$.

### 3.2. Calculation of Limit Curvature $\phi_u$.

The curvature in the previous section is still to be determined. How should $\phi$ be chosen? Should it be elastic $\phi_e$, elastic-plastic $\phi_p$, or ultimate curvature $\phi_u$? In this regard, we first analyze it by calculating the ultimate curvature. When calculating the ultimate bending moment with Eq. (10), the combination of cross-section strain is that one side is ultimate strain and the other is variable strain, by substituting these combined values with ultimate strain into Eq. (3), the ultimate curvature $\phi_u$ is obtained, and the change of its value is shown by the solid line in Figure 8. Figure 8 provides the variation (dotted line) of the ultimate curvature $\phi_u$ in China’s Code, and the strain $\phi_u$ at the demarcation point of large and the small eccentricity is selected in the Code to calculate the critical curvature $\phi_{cr}$:

$$\phi_{cr} = \frac{(3.3 + \varepsilon_s)d}{d - a_s} \cdot 10^{-3}$$ \hspace{1cm} (15)\]

In the large eccentric compression zone, the ultimate curvature is taken as the constant curvature, that is, $\phi_u = \phi_{cr}$, and the curvature in the small eccentric compression area is variable curvature, that is, an inverse function ($\zeta_c = 0.5f_cA/N$) is used to reduce the constant curvature $\phi_{cr}$.

The biggest difference between the Code and the curvature curve in this paper is that the curvature curve of the Code still exists when the axis is under pressure, which is obviously incorrect, and the curvature should be zero. In addition, the Code applies a single curvature reduction curve to represent a large number of actual curves, there must be a large error. The part of the Code curve higher than the actual curve is unsafe, the lower part is safe, and the too large lower part is too conservative.

It should be added that two groups of ultimate strains ($\varepsilon_s = \varepsilon_{s0}, \varepsilon_s = \varepsilon_{s1}$) and ($\varepsilon_s = 10\%\varepsilon_{s0}, \varepsilon_s = \varepsilon_{s1}$) are selected to calculate the ultimate curvature $\phi_{cr}$. The strain of the former group of large (small) eccentric demarcation points is the same as that of the Code, corresponding to the stress shape of the slender column (curve I in Figure 8), while the latter group of strain is the maximum strain of the section, corresponding to the stress shape of the short column (curve II in Figure 8). The front part of curve I and curve II coincided and separated at $\phi_u = \phi_{cr}$, and then separated at J1 and J2, respectively. The critical curvature $\phi_{cr}$ of the Code is between curve I and curve II, which is larger and safer than that of slender column curve I, so it is reasonable.

According to the analysis, it is not advisable for the Code to use the reduced curvature of a single curve to reflect different strength reinforcement ratio in the range of small eccentricity, which is too simple and has a large error. However, it takes a certain value of constant curvature in the range of large eccentricity, which is slightly larger and safer than the curve I of the slender column. Accordingly, we only improve the ultimate curvature in the range of small eccentricity, that is, we use a straight line to replace the actual curve (see the dashed line in Figure 9) and the straight line Eq. (16):

$$\phi_u = \begin{cases} 
\frac{n_{cu} - n}{n_{cu} + 0.2} 6.6 \times 10^{-3} & n < -0.2 \\
6.6 \times 10^{-3} & n \geq -0.2 
\end{cases}$$ \hspace{1cm} (16)\]

According to Figure 9, the dashed line is closer to the solid line, and the smaller the strength reinforcement ratio, the better the fit. In addition, the dotted lines are all lower than the solid lines, which is biased towards safety.

### 3.3. Simplified Model.

After the ultimate curvature variable is determined by the Eq. (16), we can use the Eq. (14) to calculate the total bending moment of the column foot section caused by the load. However, the bending moment of this load can not increase indefinitely, and the maximum can only reach the bearing capacity of the section, that is, $m_{tot} = m_s$. In this way, the Eq. (14) can be rewritten into the Eq. (17):

$$m_s = n \cdot \frac{\varepsilon_1}{m_1} + n \frac{5}{24} \left( \frac{l_0}{d} \right)^2 \phi_u$$ \hspace{1cm} (17)\]

The $m_s$ in Eq. (17) is a known quantity that can be calculated (see Section 1.3, 1.4). In this case, we can change
either of the two parameters $n$ and $l_0$ to solve the other. Finally, we can get the correlation ($m-n-\lambda$) shown by the dotted line in Figure 10. Correspondingly, the solid curve is calculated by using the conjugate beam method [23–25] and considering the elastoplastic moment-curvature relationship of the cross section, and the curvature does not make any assumptions, so the calculated value is more accurate. We will not expand the details of its algorithm here. According to the comparison between the two, the correlation calculated by the approximate simplified curvature given by the Eq. (16) is in good agreement with the exact value, which indirectly verifies the rationality of the Eq. (16).

The ultimate strain used in Section 1.4 ($\varepsilon_{c1} = -3.3\%$, $\varepsilon_s = 10\%$) is changed to the ultimate strain used in the Code ($\varepsilon_{c1} = -3.3\%$, $\varepsilon_s = \varepsilon_y$) in Eq. (16). The Eq. (10) is used to calculate the $n-m_{cu}\phi_u$ correlation in which $n$ is the isoline again, that is, a series of $m_{cu}\phi_u$ values corresponding to the fixed $n$ value. The relationship among the three can be seen in Figure 11. In addition, the relative eccentricity ($e/d$) is used instead of dimensionless moment ($m$) as the ordinate in
Figure 11, which enables the distribution of the energy curve more dispersed and clearer.

If both ends of Eq. (17) are divided by 2 at the same time, it becomes the expression of relative eccentricity:

\[
\frac{\epsilon_x}{d} = \frac{e_1}{d} + \frac{e_2}{d} = \frac{e_1}{d} + \frac{5}{48} \left(\frac{l_0}{d}\right)^2 \phi_u \quad (18)
\]

Eq. (18) is a linear equation with \( \phi_u \) as its independent variable, if the line is drawn in Figure 11, the first-order term is the intercept on the longitudinal axis, and the coefficient of the second-order term is the slope. The position to which the right end of the line can extend is determined by the axial compression ratio \( \nu \). It is more convenient to calibrate the slope by adding a slenderness ratio vertical axis (\( l_0/d \)) to the right of Figure 11, that is, Eq. (19):

\[
\frac{\epsilon_x}{d} = \frac{5}{48} \left(\frac{l_0}{d}\right)^2 \phi_x \quad (19)
\]

\( \phi_x \) in Eq. (19) can be any value, which is only used to select the position of the vertical axis. After the \( \phi_x \) value is determined, each value assigned to the \( \epsilon_x \) results in a value corresponding to the \( l_0/d \), so that the vertical axis on the right side of Figure 11 (\( l_0/d \)) is calibrated. Now, we can find the corresponding \( n \) inside the cross section from the known external parameters (\( e_1/d, l_0/d \)) in Figure 11, and then get the corresponding strength reinforcement ratio \( \omega \) through the correlation of \( n\epsilon_x/d\omega \) in Figure 6, and finally the required steel bar area is obtained.
For convenient reference, Figures 11 and 6 can be arranged together (see Figure 12), so that it is very clear to calculate the cross-section reinforcement or check the bearing capacity considering the second-order effect. By making 4 auxiliary lines in the nomogram of this three-coordinate system, the reinforcement of the annular section column can be calculated by hand, avoiding the iterative solution of the transcendental equation.

The method and steps of using the diagram are illustrated with a red line in Figure 12:

1. Draw a straight line \(a\) from the known \(l_0/d\);
2. Draw a parallel line of line \(a\) to the \(e_1/d\) on the left axis;
3. Pass through the intersection of the straight line \(b\) and the isoline \(n\), as the horizontal line \(c\), to get the \(e_{tot}/d\);
4. Use the intersection of the horizontal line \(c\) and the isoline in the right figure as the plumb line, and finally get the strength reinforcement ratio \(\omega\).

4. Calculation Examples

We choose an RC annulus column as the calculation example, and use the methods proposed in this article and the Code to perform calculations to compare the calculation process and calculation results of the two.

The material and section parameters of the RC annulus column are: C30 concrete (\(f_c = 14.3 \text{ N/mm}^2\)), HRB400 (\(f_y = 360 \text{ N/mm}^2\)), the inner and outer radii of the annular section: \(r_1 = 300 \text{ mm}, r_2 = 210 \text{ mm}\), the steel bar annulus radius: \(r_s = 255 \text{ mm}\), the calculated length \(l_0 = 21 \text{ m}\), \(l_0/h = 35\), pressure \(N = -1000 \text{kN}\), bending moment \(N = -1000 \text{kN}\).

4.1. Using Graph Algorithm. According to the following 4 steps, the total bending moment and the required steel bar area are obtained:

1. \(l_0/d = 35\) (draw straight line \(a\));
2. \(e_1/d = (-M/N + e_a)/d = 0.533\) (draw a parallel line \(b\));
3. \(n = N/(\pi r_1^2 f_c) = 0.247\) (draw horizontal line \(c\) and plumb line \(d\)), get \(e_{tot}/d = 1.352\), then \(M_{tot} = -N e_{tot} = 811.2 \text{kN.m}\)
4. Find \(\omega = 1.08\)

The required steel bar area is \(A_s = \omega (\pi r_1^2 f_y)/f_y\) = 12124 mm\(^2\).

4.2. Adopting the Method Proposed by Code
(1) The magnification factor of bending moment $\eta_{ns}$ is calculated as:

$$\xi_c = \frac{0.5f_c A}{N} = \frac{0.5 \times 14.3 \times 144126}{1000 \times 10^3} = 1.031 > 1.0, \text{ take } 1.0.$$

$$\eta_{ns} = 1 + \frac{1}{1300(M_2/N + \varepsilon_n)/R_0} \left(\frac{l_p}{h}\right)^2 \xi_c$$

$$= 1 + \frac{555}{1300 \times (300 \times 10^3/1000 + 20)} \times 35^2 \times 1.0 = 2.634$$

(20)

(2) The second-order bending moments are:

$$C_m = 0.7 + 0.3 \frac{M_1}{M_2} = 1.0$$

$$M = C_m \eta_{ns} M_2 = 1.0 \times 2.634 \times 300 = 790.2 kN \cdot m$$

(21)

(3) Solve transcendental equations (Code E.0.3)

$$\begin{align*}
N & \leq \alpha \varepsilon_f A + (\alpha + \alpha_f) f_y A_s \\
N \varepsilon_l & \leq \alpha \xi_f A_r (r_1 + r_2) \sin \pi \alpha + f_y A_r r_1 \sin \pi \alpha + \sin \pi \alpha, \\
\alpha_f & = 1 - 1.5 \alpha \cdots \varepsilon_l = \varepsilon_0 - \varepsilon_u
\end{align*}$$

(22)

Article 6.2.6 of the Code stipulates that when the strength grade of concrete does not exceed C50, $\alpha_f$ is 1.0. Substituting the known conditions into the above formula and using MATLAB programming to solve iteratively, we get: $\alpha = 0.4138$, $\alpha_f = 0.3793$.

(4) Substitute $\alpha$ and $\alpha_f$ into the above transcendental equations, we can get $A_r = 11848 \text{ mm}^2$

4.3. Comparison. The calculation results of the two methods are compared: the ratio of bending moment is 811.2/790.2 = 1.027, the ratio of steel bar area is 12124/11848 = 1.023, and the deviation is less than 5%. The curvature calculation deviation of the method proposed by the Code is large, causing the result to be small and unsafe; the curvature of the graph algorithm is more consistent with the actual curvature distribution, so the calculation results are more safe. Many other calculations were compared, and it was concluded that the calculation results of the method proposed by Code were small, and the errors between the graph algorithm and the method proposed by Code were less than 5%, which was acceptable for engineering design.

5. Conclusion

(1) We use the approximate algorithm to simplify the RC annular section column from the section level and member level, and the calculated results are in good agreement with the exact solution.

(2) We derive and design a fast tool diagram for calculating the reinforcement of annular columns considering the second-order effect. Due to the use of dimensionless parameters in the derivation process, this diagram can be applied to all concrete strength grades of C15 – C50.

(3) Graph algorithm has the advantages of fewer steps and simple operation, avoiding iterative solution of the annular section’s transcendental equation of the toroidal section in the Code, and brings great convenience for design. Only three basic variables (axial compression ratio $n$, first-order eccentricity $e_l/d$, slenderness ratio $l_0/d$) are needed, and then four auxiliary lines are made in the graph to obtain the cross-section reinforcement area $A_s$.

(4) The reinforcement calculation of RC annular eccentrically loaded columns involves the nonlinearity of material, geometry and section width. In this paper, the triple nonlinear problem is solved by graphic method, which provides an idea and reference for the nonlinear problems of other types of RC columns. For example, circular section, T-shaped section, etc.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author does not have any possible conflicts of interest.

References


