

Research Article

Vandermonde-Based Unitary Precoding Method for Integer-Forcing Design

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An integer-forcing linear receiver has significantly better performance than conditional receivers for slow-fading channels because it directly recovers an integer linear combination of signals instead of decoding all signals. The performance in terms of achievable rate, outage probability, and error rate can be improved with a unitary matrix precoder imposed at each channel realization. In this paper, a new special unitary matrix precoding approach is proposed to reduce the computational complexity. Different from the parameterization technique with many parameters, the new method constructs a unitary Vandermonde matrix with only a single parameter. The optimal Vandermonde matrix is determined on the basis of the shortest vector of a lattice generated by the precoding matrix in which the single parameter is searched. Therefore, its complexity is reduced to a polynomial time, whereas the traditional unitary precoder has exponential complexity. Simulation results show that the proposed scheme can achieve the performance similar to the benchmark schemes but with much lower complexity. The scheme offers a good trade-off between performance and complexity.

1. Introduction

Compute-and-forward (CF) is a promising new technique of physical-layer network coding for wireless relay networks [1–3]. CF exploits nested lattices and enables relays to decode the integer linear combination of transmitted messages by using the noisy linear combinations provided by the channel. The integer linear combination transforms into a linear combination of the information messages by modulo operation over the same finite field. In this regard, CF strategy makes use of interferences to obtain significantly higher rates between users in a network. The CF technique can be utilized in the massive MIMO system and is envisioned to support the demands of fifth-generation (5G) and beyond mobile system [4, 5].

Inspired by the idea of the CF strategy, a new linear integer-forcing (IF) receiver technique was subsequently proposed to harness the intrapair interference for multiple-in multiple-out (MIMO) architectures [6]. As any integer

linear combination of lattice codewords is itself a lattice codeword, the IF technique may recover linear integer combinations of messages for further original message detection. The IF strategy can achieve higher sum rate with low complexity by using well-structured codes and a good approximation of the channel's real coefficients by utilizing rational numbers. The practical approach for the IF receiver and the application in the MIMO multiuser system is discussed in [7–9]. Beyond that, MIMO relay multiuser systems equipped with IF precoding technique can achieve an outstanding performance [10].

When the knowledge of channel state information is available at the transmitter, the transmitter uses a precoding scheme to encode information symbols prior to transmission to increase reliability and overcome channel fluctuations. In [11], Sakzad and Viterbo proposed unitary precoded integer-forcing (UPIF) scheme in which the precoding matrices are designed as unitary matrices. The full diversity gain could be obtained along with full rate transmission. The performance

was also not dependent on the minimum distance of received constellations. In high-order modulation schemes, the UPIF also performs excellently.

Motivated by UPIF, an orthogonal precoding scheme for IF linear receivers was proposed by [12]. The orthogonal precoding could outperform its unitary counterpart. Then, a steepest gradient algorithm was proposed to find the “good” orthogonal precoder matrices. However, the optimal precoder matrix of a unitary or orthogonal was difficult to identify with the unitary or orthogonal constraint and lattice distance minimization problem. Furthermore, multiple parameters with respect to unitary matrix or orthogonal matrix are required to determine in a high-order MIMO system, leading to high computational complexity.

In this paper, we propose a low-complexity suboptimal precoding scheme for IF linear receivers. The unitary precoder matrix is designed as a Vandermonde matrix that only contains a single parameter associated with an angle based on a complex cyclotomic number field. Remarkably, owing to the orthogonal structure stipulated for the Vandermonde matrix, finding the optimal Vandermonde matrix can be transformed when searching the optimal parameter. Moreover, the problem can be solved in polynomial time; it is faster than the one in [11] whose computational complexity is exponential in the number of MIMO dimension. More importantly, the proposed method is suitable for any MIMO dimension. Compared with the unitary matrix precoder, the performance of the proposed scheme is near to parameterization technique in high-order MIMO, as validated by simulation results.

This paper is organized as follows. Section 2 describes the system model and the unitary precoded IF scheme. Section 3 presents a novel precoding strategy. Section 4 performs some numerical simulations to validate the usefulness of the proposed method. Finally, some concluding remarks are presented in Section 5.

1.1. Notations. The set \mathbb{Z} , \mathbb{R} , and \mathbb{C} denote the ring of integers, the field of real number, and the field of complex number, respectively. $\mathbb{Z}[i] = \mathbb{Z} + i\mathbb{Z}$ denotes the ring of Gaussian integers. The superscripts $(\cdot)^T$ and $(\cdot)^H$ represent the matrix transpose and Hermitian transpose operations, respectively. mod stands for the modulo operation imposed on a matrix or a vector over a lattice. The notation $\|\cdot\|$ denotes the Euclidean norm of a vector. \mathbf{I} is an identity matrix.

2. System Model

We consider a quasistatic flat-fading MIMO system comprising M transmit antennas and M receive antennas, i.e., an $M \times M$ point-to-point MIMO link. Let $\mathbf{H} \in \mathbb{C}^{M \times M}$ denote the complex valued channel matrix, the elements of which are independent and identically distributed (i.i.d.) with a zero-mean unit-variance complex Gaussian distribution. We assume that the transmissions are organized into bursts of L symbol slots ($L \gg 1$), where H is constant during the burst but changes randomly from one burst to the next (quasistatic channel). By using singular value decomposition (SVD), the channel matrix \mathbf{H} can be decomposed into $\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^H$, where \mathbf{U} and \mathbf{V} are uni-

tary matrices, such that $\mathbf{U}^H\mathbf{U} = \mathbf{V}^H\mathbf{V} = \mathbf{I}$, and \mathbf{D} is a diagonal matrix with the entries $d_1, d_2, \dots, d_M \in \mathbb{R}$ arranged in descending order on the diagonal.

Let \mathbf{w}_i denote the information vectors to be transmitted at the i -th antenna of the transmitter. \mathbf{w}_i is drawn independently and uniformly over a prime-size finite field F_p^L , where p is prime. A fine lattice, also called a coding lattice, is denoted by $\Lambda = \mathbf{G}\mathbb{Z}^L$, where $\mathbf{G} \in \mathbb{R}^{M \times L}$ is a full-rank generator matrix. Specifically, if we take $\mathbf{G} = \mathbf{I}$, then we recover Λ to a \mathbb{Z}^L lattice. A coarse lattice, also called a shaping lattice, is denoted by $\Lambda_c = p\Lambda$. The corresponding nested lattice can be represented as $\mathcal{C} = \Lambda \cap \mathcal{V}_{\Lambda_c}$. The encoder at the i -th antenna uses a bijective map ϕ to map the message \mathbf{w}_i over the finite field to the nested lattice code $\mathbf{s}_i = \phi(\mathbf{w}_i) = [\mathbf{G}\mathbf{w}_i] \text{ mod } \Lambda$. Each codeword is subject to the power constraint, i.e., $(1/L)\mathbb{E}\|\mathbf{s}_i\|^2 = \rho$. We also assume that $\mathbf{S} = [\mathbf{s}_1, \dots, \mathbf{s}_M]^T \in \mathbb{C}^{M \times L}$. With the precoding matrix \mathbf{U} and the precoding unitary matrix \mathbf{P} , the matrix $\mathbf{X} = \mathbf{VPS}$ is the matrix of transmitted vectors to be sent through the channel. The received signal at the receiver can be expressed as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}, \quad (1)$$

where \mathbf{Z} is an additive Gaussian noise in which each term is modeled as an i.i.d. zero-mean Gaussian random variable with a unit variance. Let $\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_M]^T$.

Then, the unitary matrix \mathbf{U}^H is imposed on the received signal \mathbf{Y} , thus yielding

$$\mathbf{Y}' = \mathbf{D}\mathbf{P}\mathbf{X} + \mathbf{Z}', \quad (2)$$

where $\mathbf{Z}' = \mathbf{U}^H\mathbf{Z}$ and the entries of \mathbf{Z}' also follow the i.i.d. Gaussian random distribution with zero mean and unit variance.

Subsequently, the receiver projects \mathbf{Y}' with an equalization matrix $\mathbf{B} \in \mathbb{R}^{M \times M}$ and then feeds it into the modulo function with lattice Λ_c . The effective received vector can then be expressed as

$$\begin{aligned} \mathbf{Y}_{\text{eff}} &= [\mathbf{B}\mathbf{Y}'] \text{ mod } \Lambda_c = [\mathbf{B}\mathbf{D}\mathbf{P}\mathbf{X} + \mathbf{B}\mathbf{Z}'] \text{ mod } \Lambda_c \\ &= [\mathbf{A}\mathbf{X} + (\mathbf{B}\mathbf{D}\mathbf{P} - \mathbf{A})\mathbf{X} + \mathbf{B}\mathbf{Z}'] \text{ mod } \Lambda_c \\ &= [\mathbf{W} + \mathbf{Z}_{\text{eff}}] \text{ mod } \Lambda_c, \end{aligned} \quad (3)$$

where $\mathbf{A} \in \mathbb{Z}[i]^{M \times M}$ is a full-rank target Gaussian-integer-valued matrix.

$$\mathbf{W} = [\mathbf{A}\mathbf{X}] \text{ mod } \Lambda_c \quad (4)$$

is a matrix with each row being a codeword in \mathcal{C} owing to the linearity of the code.

$$\mathbf{Z}_{\text{eff}} = [(\mathbf{B}\mathbf{D}\mathbf{P} - \mathbf{A})\mathbf{X} + \mathbf{B}\mathbf{Z}'] \text{ mod } \Lambda_c \quad (5)$$

is an additive noise statistically independent of \mathbf{W} , and \mathbf{B} is chosen to approximate the resulting MIMO channel DP

with the invertible Gaussian integer matrix \mathbf{A} . The full rank of matrices \mathbf{G} and \mathbf{A} provides sufficient conditions for recovering the original messages via the quantization of \mathbf{Y}_{eff} , i.e.,

$$[\tilde{\mathbf{w}}_1, \tilde{\mathbf{w}}_2, \dots, \tilde{\mathbf{w}}_M]^T = \mathbf{G}^{-1} \mathbf{A}^{-1} \mathcal{Q}_\Lambda(\mathbf{Y}_{\text{eff}}), \quad (6)$$

where $\tilde{\mathbf{w}}_i, i = 1, \dots, M$ is the estimate of $\mathbf{w}_i, i = 1, \dots, M$. If we take $\mathbf{G} = \mathbf{I}$, then an integer lattice is involved and can construct a regular constellation (e.g., PAM and QAM). Finding the optimal integer matrix \mathbf{A} is a successive shortest vector problem in the IF architecture [13]. The traditional search framework, including the Lenstra, Lenstra, and Lovász (LLL) algorithm [14], the greedy algorithm [15], and the sphere decoding algorithm [16], can be employed in our works. Besides, an optimal algorithm recently proposed in [13] can be used specially for IF systems.

Let $\mathbf{y}_{\text{eff},m}^T, \mathbf{a}_m^T$, and \mathbf{b}_m^T be the m -th rows of $\mathbf{Y}_{\text{eff}}, \mathbf{A}$, and \mathbf{B} , respectively. According to Eq. (5), the variance of $\mathbf{z}_{\text{eff},m}^T$ can be written as

$$\sigma_{\text{eff},m}^2 = \rho \left\| \mathbf{b}_m^T \mathbf{D} \mathbf{P} - \mathbf{a}_m^T \right\|^2 + \left\| \mathbf{b}_m^T \right\|^2. \quad (7)$$

\mathbf{b}_m^T can be optimized as $\mathbf{b}_m^T = P_s \mathbf{a}_m^T (\mathbf{D} \mathbf{P}^T (\mathbf{I} + P_s \mathbf{D} \mathbf{P} (\mathbf{D} \mathbf{P})^T)^{-1})$ by minimizing of the effective noise variance $\sigma_{\text{eff},m}^2$ [17], thus yielding

$$\sigma_{\text{eff},m}^2 = \rho \mathbf{a}_m^T \mathbf{P}^T (\mathbf{I} + \rho \mathbf{D}^T \mathbf{D})^{-1} \mathbf{P} \mathbf{a}_m. \quad (8)$$

Finding the optimal unitary precoder matrix is a hard problem due to the unitary constraint, multiple variances, and lattice minimum distance problem. These issues are addressed in the sequel.

3. Proposed Approach

The precoding matrix \mathbf{P} imposed on the channel gain should be designed to enable the Gaussian integer matrix \mathbf{A} to fully approximate the channel coefficient without having to amplify the effective noise. The performance in terms of achievable rate and error rate should be optimized. In this section, we focus on the problem of choosing the unitary matrix \mathbf{P} to maximize the achievable sum rate while ensuring low computational complexity.

Here, $\mathbf{I} + \rho \mathbf{D}^T \mathbf{D}$ is a positive definite matrix. Thus, its Cholesky decomposition is a factorization of the form $(\mathbf{I} + \rho \mathbf{D}^T \mathbf{D})^{-1} = \mathbf{L} \mathbf{L}^H$. Assuming $\mathbf{L}_P = \mathbf{P}^H \mathbf{L}$, we rewrite $\sigma_{\text{eff},m}^2$ as

$$\sigma_{\text{eff},m}^2 = \rho \mathbf{a}_m^H \mathbf{L}_P \mathbf{L}_P^H \mathbf{a}_m = \rho \left\| \mathbf{L}_P \mathbf{a}_m^H \right\|^2. \quad (9)$$

The performance in terms of SNR is decided by the worst subchannel. Hence, \mathbf{P} is chosen to minimize $d_M(\Lambda(\mathbf{L}_P))$, which is the largest successive minimum of the lattice $\Lambda(\mathbf{L}_P)$.

$$\mathbf{P}_{\text{opt}} = \arg \min_{\mathbf{P} \in \mathcal{U}_M} d_M^2(\Lambda(\mathbf{L}_P)), \quad (10)$$

where \mathcal{U}_M is the unitary group of all $M \times M$ unitary matrices. This minimization problem can be transformed into a maximization of the minima of the lattice $\Lambda(\mathbf{L}_P^{-1})$ [12], i.e.,

$$\mathbf{P}_{\text{opt}} = \arg \max_{\mathbf{P} \in \mathcal{U}_M} \min_{\mathbf{v} \in \mathbb{Z}[i]^M \setminus \mathbf{0}} \left\| \mathbf{L}^{-1} \mathbf{P} \mathbf{v} \right\|^2, \quad (11)$$

to simplify the solution for the maximization.

The optimal solution for problem in Eq. (11) seems difficult and combinatorial due to the unitary constraint and the distance minimization of the lattice. A parameterization technique was previously proposed in [11]. For example, for the 2×2 MIMO system, \mathbf{P} can be expressed with a parameter angle α to satisfy the orthogonal condition, i.e., $\mathbf{P} =$

$$\begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}. \quad \text{With this simple parameterization, the}$$

exhaustive search can be conducted with only one parameter $\alpha \in [0, \pi/4]$ by using the fine steps via the Gauss reduction algorithm. However, this approach is prohibitively hard to implement for a higher-dimensional MIMO system because the number of parameters can reach $M(M-1)/2$. At each iteration, the minimum distance of $\Lambda(\mathbf{L}_P^{-1})$ is required to be checked, leading to an exponential computational complexity with respect to the number of parameters.

The unitary matrix constraint is a major obstacle in finding the optimal \mathbf{P} . To overcome this challenge, we propose a special matrix as the precoding matrix of the lattice. The precoding matrix can be parameterized using only a single angle α for any dimension as follows:

$$\mathbf{P}(\alpha) = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 & \theta \cdot \theta_1 & \dots & (\theta \cdot \theta_1)^{M-1} \\ 1 & \theta \cdot \theta_2 & \dots & (\theta \cdot \theta_2)^{M-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \theta \cdot \theta_M & \dots & (\theta \cdot \theta_M)^{M-1} \end{bmatrix}, \quad (12)$$

where $\theta = \exp(i\alpha)$ is the rotation variation, and $\theta_k = \exp(2k\pi i/M), k = 1, 2, \dots, M$ is assumed to be the complex roots of the minimal polynomial $x^M - 1$ according to the algebraic number theory introduced in [18]. Here, the parameterized precoding matrix is a Vandermonde matrix and unitary, i.e., $\mathbf{P}(\alpha) \mathbf{P}^H(\alpha) = \mathbf{I}$. We are interested in choosing the optimal α , such that the minimum distance of lattice $\Lambda(\mathbf{L}_{P(\alpha)})$ can be maximized. Moreover, the Vandermonde structure of $\mathbf{P}(\alpha)$ can help to reduce the matrix-vector multiplication complexity to $\mathcal{O}(n \log n)$ flops in a similar manner as the complexity reduction of the fast Fourier transform.

The effect of the multiplication of the lattice points is periodic in α with the period $[0, 2\pi/M]$. Therefore, performing a search algorithm over a period is desirable. The corresponding function of minimum distance of the lattice $\Lambda(\mathbf{L}_{P(\alpha)})$ is denoted by $d(\Lambda(\mathbf{L}_{P(\alpha)}))$, and it has reflective symmetry in period $[0, 2\pi/M]$. The characteristic of the period and symmetry originates from the structure of the roots of a unit circle. Figure 1 shows the periodic and symmetrical behavior of the

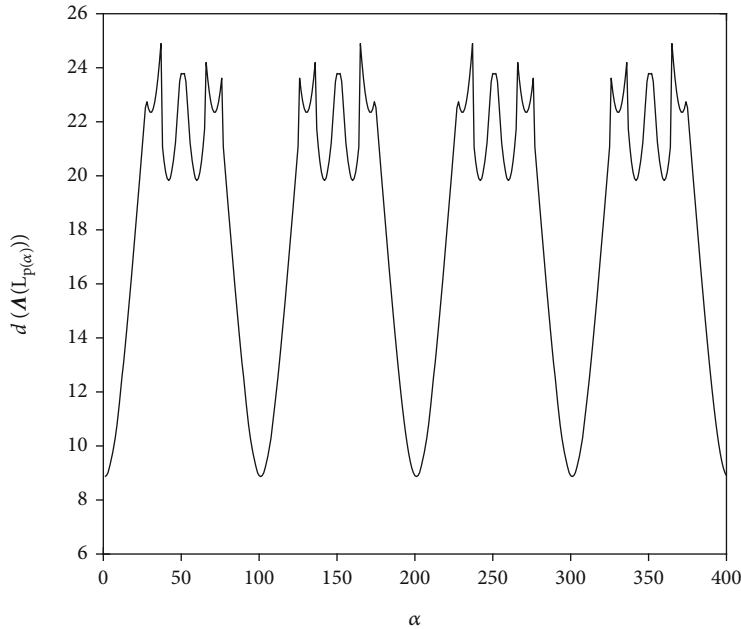


FIGURE 1: Periodic and symmetrical behavior of $d^2(\Lambda(\mathbf{L}_{P(\alpha)}))$ based on the variation of θ in a 2×2 complex MIMO.

function curve of $d^2(\Lambda(\mathbf{L}_{P(\alpha)}))$. Subsequently, the search interval of θ is further halved into $[0, \pi/M]$, which can also save half of the computational complexity.

The exhaustive search operation is performed by discretizing the space of $[0, \pi/M]$ into n samples with a fine step size. At each step, the minimum distance of the resulting lattice $\Lambda(\mathbf{L}_{P(\alpha)})$ is evaluated while the unitary matrix $\mathbf{P}(\alpha)$ is constructed with each sample of α . We choose the complex LLL reduction algorithm [19] to find the successive minima of a lattice with a complexity of $\mathcal{O}(M^4 \log M)$. Therefore, the overall complexity of the proposed technique is $\mathcal{O}(nM^4 \log M)$. In comparison, the parameterization approach introduced in [11] used at least $M(M-1)/2$ parameters to construct the unitary matrix. Each parameter has a separate search space of $[0, 2\pi]$. As a result, the overall complexity of the parameterization approach is $\mathcal{O}(n^{M(M-1)/2} M^4 \log M)$ with the same LLL algorithm. Thus, the proposed algorithm outperforms the parameterization technique in terms of computational complexity.

4. Results and Discussion

In this section, we present numerical results as a way of verifying the performance of our proposed Vandermonde matrix precoding scheme in terms of achievable sum rate and symbol error rate. The channel matrix is generated randomly from one burst to the next with i.i.d elements $H_{i,j} \sim \mathcal{CN}(0, 1)$, in which the real and imaginary parts are independent and have zero mean and equal variance. The transmitted symbols are modulated with 4-QAM, and the corresponding finite ring is $\mathbb{Z}_2 = \{0, 1\}$. The finite constellation \mathcal{S} represents the set of coset representatives of $\mathbb{Z}[i]/p\mathbb{Z}[i]$. Here, the symbols of \mathcal{S} should be transformed into scaled and shifted versions with a reduced average transmit power.

The theoretical exhaustive search based on the design criterion given in Eq. (11) is included to construct the Vandermonde matrix with single parameter. The step size for our brute force search for the cases is 0.001 radians. For the purpose of ensuring low complexity, the LLL algorithm is equipped with the capability of finding the minima of the lattice $\Lambda(\mathbf{L}_P^{-1})$ and the suboptimal matrix \mathbf{A} . For the sake of comparison, the unitary matrix scheme proposed in [11] and the orthogonal matrix scheme proposed in [12] are shown.

First, we compare the minimum distance of the resulting dual lattices of the Vandermonde matrix precoder and unitary matrix precoder. Figure 2 shows the average of $d(\Lambda(\mathbf{L}_P^{-1}))$ in 2×2 MIMO. Findings indicate that the Vandermonde precoder is always better than the orthogonal matrix precoder and is nearer to the unitary matrix precoder. This finding can be attributed to the unitary precoder that searches over all groups of the unitary matrix while the Vandermonde matrix precoder searches the part of the unitary matrix group. Figure 3 shows the performance in terms of average sum rate. For the sake of comparison, the curve of the conditional SVD precoding method is also shown. As can be clearly seen from Figure 3, the unitary matrix precoder and Vandermonde matrix precoder have nearly the same curve, but the orthogonal matrix precoder has a lower performance. In contrast, the SVD precoder does not outperform the above three precoders.

Then, we compare the performance in terms of symbol error rate (SER) for the 2×2 and 4×4 MIMO channels over 4-QAM constellations. The SVD scheme clearly cannot achieve the desired SER performance when SNR is large. Subsequently, we focus on the precoder schemes. As shown in Figures 4 and 5, among the three precoders, the orthogonal precoder always exhibits the best SER performance, and the Vandermonde matrix precoder scheme yields a performance that is inferior to unitary precoder. However, the performance distinction

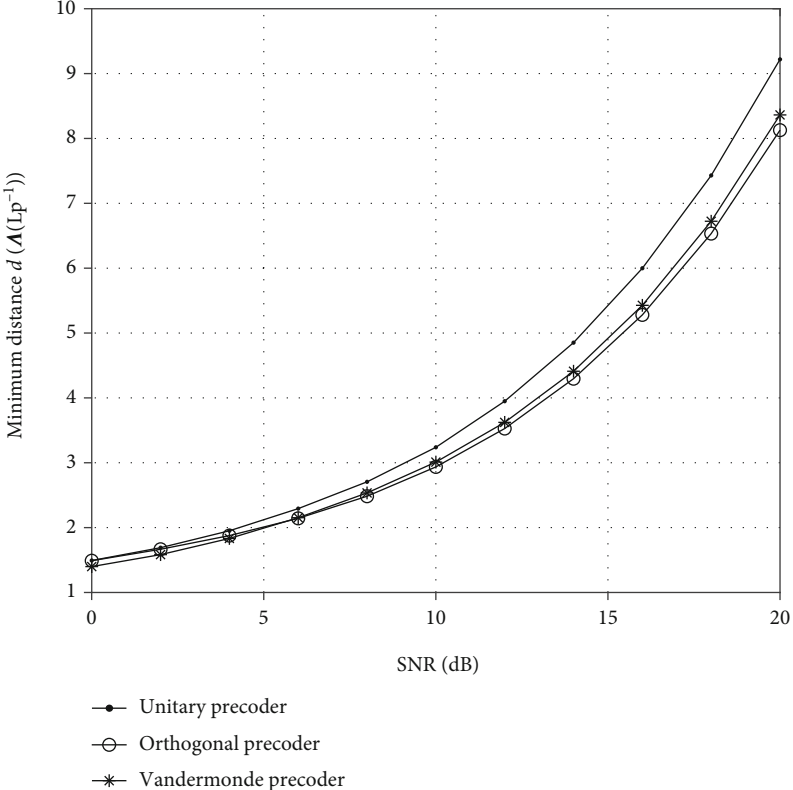


FIGURE 2: Comparison of average minimum distances of $\Lambda(L_p^{-1})$ of three schemes in a 2×2 complex MIMO.

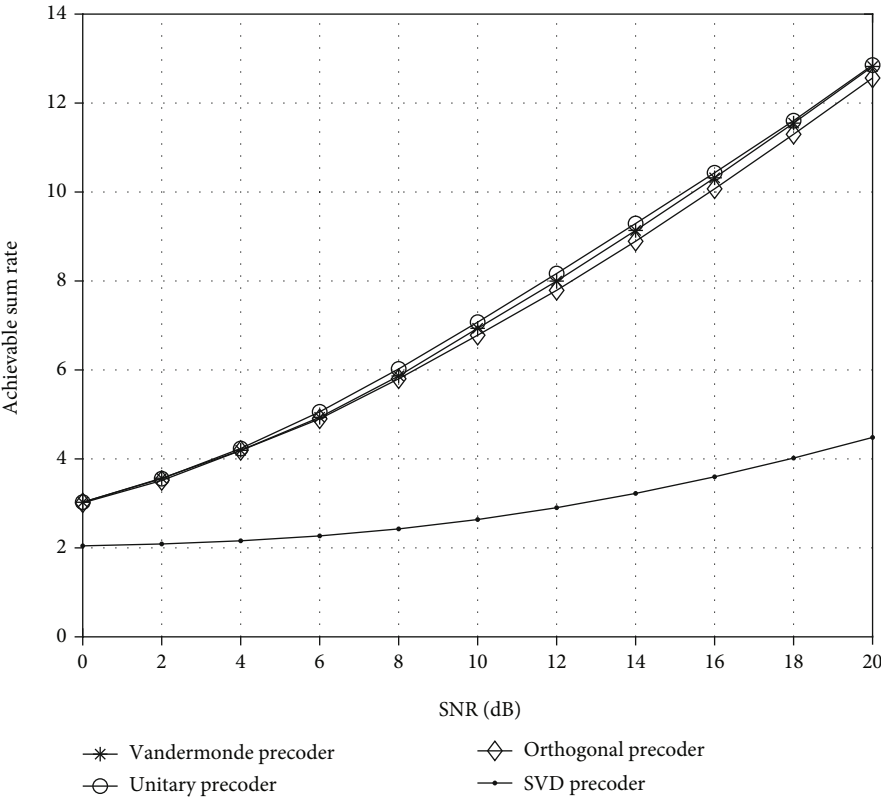


FIGURE 3: Comparison of achievable sum rate of four schemes in a 2×2 complex MIMO.

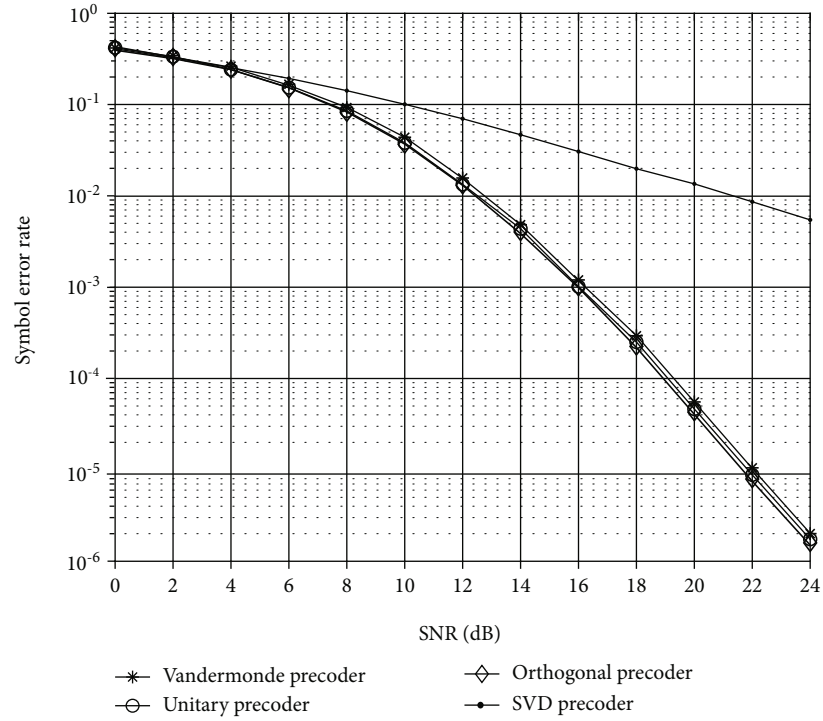


FIGURE 4: Comparison of symbol error rate of Vandermonde precoder, unitary precoder, orthogonal precoder, and SVD precoder in a 2×2 complex MIMO.

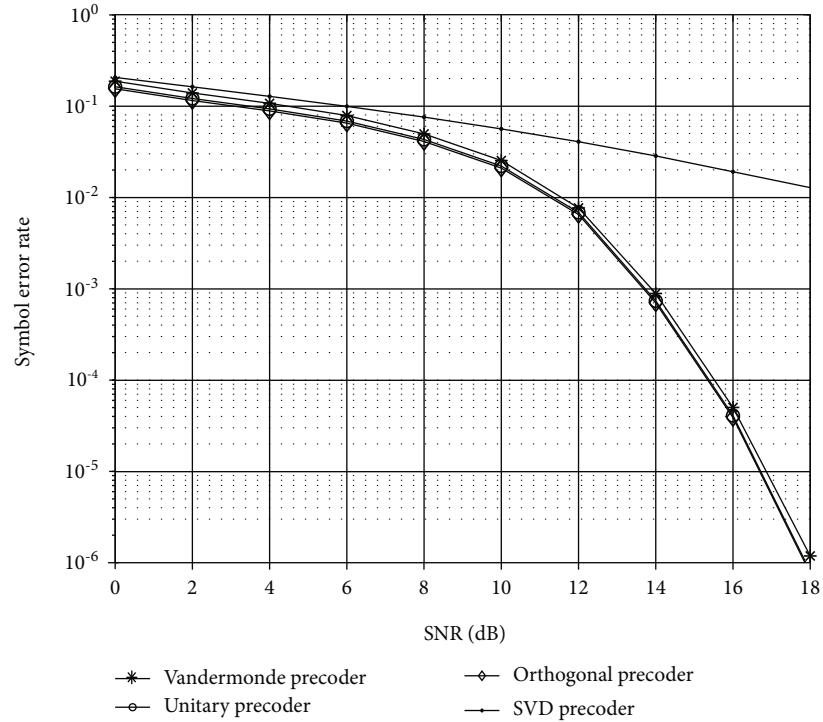


FIGURE 5: Comparison of symbol error rate of Vandermonde precoder, unitary precoder, orthogonal precoder, and SVD precoder in a 4×4 complex MIMO.

among the three precoders is only slight. In the 2×2 scenario, in this study, the orthogonal precoder only needs to deal with a single parameter, but in the 4×4 scenario, six parameters need to be determined. The unitary precoder needs three parameters in the 2×2 scenario and 12 parameters in the 4×4 scenario. The exponentially increasing complexity is intolerable. Meanwhile, the Vandermonde precoder can reach a good trade-off between performance and complexity.

5. Conclusions

In this study, we propose the Vandermonde matrix precoder with a low polynomial complexity to suboptimally alternate the unitary matrix precoding in MIMO IF systems. An SVD decomposition approach is employed to transform the channel into a diagonal matrix. It is of paramount importance to design precoder matrices to adapt to each channel realization. However, the optimal precoder matrix from the unitary group is a hard problem requiring an exhaustive search of multiple parameters. To overcome this shortcoming, we choose the Vandermonde matrix requiring only a single parameter. The corresponding complexity is reduced to a polynomial time, and the loss of SER performance is only slight. The simulation results verify the good trade-off between performance and complexity yielded by using our proposed scheme.

Data Availability

The data used to support the findings of this study are included in the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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