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Research Article

Fast and Accurate Approach for DOA Estimation of Coherent Signals

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An algorithm for estimation of direction of arrival (DOA) based on QR decomposition is proposed for coherent signals in this paper. When coherent is present, the rank of the signal covariance matrix is generally less than the signal number, making the estimation of the signal or noise subspace inaccurate. Therefore, we need to eliminate the spatial covariance matrix rank loss. According to the idea of matrix reconstruction, we try to construct three different data matrices, one is the signal covariance matrix, the other is the eigenvector reconstruction matrix of the signal covariance matrix, and the other is reconstructed matrix with the addition of spatial smoothing technology. Based on the resulting data matrix, whereafter, QR decomposition with the iteratively weighted least squares (IWLS) as solver is proposed to reduce the computational complexity of DOA estimation. Compared with other existing algorithms, the simulation results show that our method has high accuracy and great increase in the computational efficiency.

1. Introduction

In recent years, people have invested a lot of energy in researching high-resolution technology to estimate the angle of arrival of sources on linear arrays in wireless communications, including astronomy, radar, smart grid, and sonar [1–7]. Algorithms like MUSIC [8] and ESPRIT [9] are based on subspace decomposition to solve uncorrelated signals with good performance. And another classic subspace algorithm propagation operator (propagator method, PM) [10] used a series of linear operations to obtain a noise subspace orthogonal to the steering vector. The disadvantage is that the angle estimation accuracy is relatively poor under low signal-to-noise ratio (SNR). The prerequisite of these algorithms is incoherent signals. However, as the space environment becomes more and more complex, the signals received by the array are often mixed with coherent signals. These methods will encounter difficulties, such as multi-path propagation signals and co-frequency interference signals.

In response to this problem, scholars have proposed a series of algorithms, such as forward spatial smooth (FOSS) algorithm [11] splits the array into sub-arrays, and then superimposes each other to eliminate the coherence of the signal. But FOSS algorithm from the structure of the covariance matrix from the above point of view, forward smoothing does not make full use of all the information of the sample covariance matrix. In order to eliminate this shortcoming, while increasing the information utilization, Pillai and Kwon [12] proposed forward-backward spatial smoothing (FBSS), ignoring the cross-correlation information at both ends of the anti-diagonal line. Next, an improved spatial smoothing technique is proposed in [13] to use the statistics of the entire sample covariance matrix as much as possible. Matrix reconstruction technology [14, 15] is another type of method used to solve the problem of coherent signal sources, [15] proposed an eigenvector method (EVM), by selecting the largest feature value corresponding eigenvectors to construct a full-rank forward and backward matrix. [16, 17] proposed a singular value decomposition (SVD) algorithm, while [18, 19] achieved decoherence by constructing the data receiving matrix into a Toeplitz matrix.

Most of the above methods or methods developed on this basis to combat the DOA estimation problem of coherent sources are at the expense of array aperture, that is, the degree of freedom of the array will be reduced. This makes the performance of the subspace algorithm unable to reach the Cramer-Rao lower bound even under the condition of high SNR. In order to remedy the above problems, a method based on principal-singular-vector utilization for modal analysis (PUMA) [20–22] was proposed, but when the incident signal is completely coherent, the performance deteriorates severely. The algorithm goal for DOA estimation is able to provide reliable azimuth information in a limited sample and low signal-to-noise ratio environment. Many existing subspace algorithms have severe thresholding effects under such conditions.

A QR-IWLS method was proposed in [23] by applying QR decomposition to the parameter estimation of two-dimensional complex-valued sinusoidal signals under additive white Gaussian noise with the use of the rank and linear prediction properties of the noise-free data matrix; then, an iteratively weighted least squares (IWLS) algorithm is used to estimate the linear prediction coefficients. On the basis, [24] proposed a fast rank revealing technique for frequency estimation of complex sinusoids in a noisy environment, and a two-stage QR decomposition frequency estimation method using weighted least squares (WLS) as the solver.

Inspired by the literatures [23, 24], in this paper, we propose a fast and low-complexity DOA estimation method based on subspace decomposition. First, the method uses the steering vector to linearly express and constructs an equivalent covariance matrix under Gaussian noise based on the covariance matrix of the received information of the data. Second, the proposed method uses QR decomposition to replace the computationally intensive EVD/SVD, and uses the first row of the R matrix to reconstruct the equivalent covariance matrix Y again. In addition, we also try to use forward and backward method to construct the data matrix.

The rest of the paper is organized as follows. In the Section of System Model, the signal model for DOA estimation of coherent signal is given. The algorithm is proposed in the Section of Proposed Approach. In the Section of Simulation Results, we perform the simulation to evaluate the accuracy and efficiency of the proposed approach. Finally, conclusions are drawn in the Section of Conclusion.

2. System Model

In this work, we consider P far-field narrow-band sources from directions $\theta_i (i = 1, 2, \cdots, P)$ impinging on a uniform linear array (ULA) having $M (M > P)$ isotropic sensors with halfwavelength interspacing as shown in Figure 1 [18].

The first L signals are fully coherent, while the other $P - L$ signals are uncorrelated and independent of the first ones. We take $s_1(t)$ as a reference, and the lth coherent signal is:

$$s_l(t) = \beta_l s_1(t) = \rho_l e^{j\Delta\phi_l} s_1(t), l = 1, \cdots, L, \tag{1}$$

where $\beta_l = \rho_l e^{j\Delta\phi_l}$, ρ_l is the amplitude fading factor, and $\Delta\phi_l$ denotes the phase difference of $s_l(t)$ related to $s_1(t)$. Let the sensor 1 be the reference, and the mth array output at time t is:

$$\begin{aligned} x_m(t) &= \sum_{i=1}^{P} s_i(t) e^{(-j2\pi md \sin\theta_i)/\lambda} + n_m(t) \\ &= s_1(t) \sum_{i=1}^{L} \beta_i e^{(-j2\pi md \sin\theta_i)/\lambda} \\ &\quad + \sum_{i=L+1}^{P} s_i(t) e^{(-j2\pi md \sin\theta_i)/\lambda} + n_m(t), \end{aligned} \tag{2}$$

where $s_i(t)$ and $n_m(t)$ are the complex envelop of the ith signal and the additive white noise at the mth element $(m = 1, \cdots, M)$. $d = \lambda/2$ is the interspacing, and λ is the carrier wavelength. The final array received signal vector is:

$$x(t) = [x_1(t), \cdots, x_M(t)]^T = \mathbf{A}(\theta) s(t) + \mathbf{n}(t), \tag{3}$$

where $(t) = [s_1(t), \cdots, s_P(t)]^T$ denotes the source vector. $\boldsymbol{n}(t) = [n_1(t), \cdots, n_M(t)]^T$ is the complex Gaussian noise vector. $\boldsymbol{A}(\theta) = [\mathrm{a}(\theta_1), \cdots, \mathrm{a}(\theta_P)]$ is the array steering matrix with $a(\theta_i) = \left[e^{-j2\pi d \sin\theta_i/\lambda}, e^{-j2\pi 2d \sin\theta_i/\lambda}, \cdots, e^{-j2\pi Md \sin\theta_i/\lambda}\right]^T$ $(\cdot)^T$ is the transpose.

3. Proposed Approach

In the above received signal vector $x(t)$, when the number of snapshots is N, one can evaluate the output covariance matrix:

$$\mathbf{R}_{xx} = \frac{1}{N} \sum_{t=1}^{N} x(t) x^H(t), \tag{4}$$

where $(\cdot)^H$ is the conjugate transpose. We can perform the EVD/SVD on the covariance matrix $\mathbf{R}_{xx}$ to obtain the signal subspace and use MUSIC or other subspace-based method to estimate the DOA. However, the sources are completely coherent, namely, $L = P$. The rank of sources covariance matrix is one. After the EVD of $\mathbf{R}_{xx}$, the dimension of the signal subspace is less than the rank of the array manifold $\mathbf{A}(\theta)$, which leads to the consequence that the steering vector is no longer orthogonal to the noise subspace and makes failure of subspace algorithm. According to [25], we assume the noise is temporal white and the noise covariance matrix denoted by $\mathbf{R}_n$, is full-rank. As the steering vectors span the signal subspace, so we have linear representation form as:

$$\mathbf{R}_n \mathbf{e}_k = \sum_{n=1}^{P} \alpha_k(n) a(\theta_n), \tag{5}$$

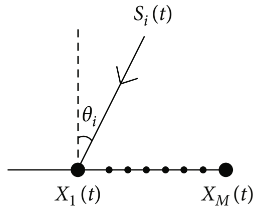

FIGURE 1: Received signal model of uniform linear array.

where $\mathbf{e}_k (1 \le k \le K)$ is an eigenvector of the received signal covariance matrix (the first K eigenvectors corresponding to first eigenvalues in decreasing order), and $\alpha_k(n)$ is a linear combination factor. When the noise covariance is an identity matrix, (5) is simplified as:

$$\mathbf{e}_k = \sum_{n=1}^{P} \alpha_k(n)\mathbf{a}(\theta_n). \quad (6)$$

For completely coherent case, namely, $K = 1$, the above equation is reduced to:

$$\mathbf{e}_1 = \sum_{n=1}^{P} \alpha_1(n)\mathbf{a}(\theta_n). \quad (7)$$

It indicates that the largest eigenvalue contains all the signal information. So the vector $\mathbf{e}_1$ can be used to reconstruct an equivalence covariance matrix $\boldsymbol{Y}$, constructed as:

$$\boldsymbol{Y} = \begin{bmatrix} e_{1,1} & e_{1,2} & \cdots & e_{1,g} \\ e_{1,2} & e_{1,3} & \cdots & e_{1,g+1} \\ \vdots & \vdots & \ddots & \vdots \\ e_{1,m} & e_{1,m+1} & \cdots & e_{1,M} \end{bmatrix}, \quad (8)$$

where $e_{1,m}$ is the element of $\mathbf{e}_1$, $m = M - g + 1$, $m > P$, $g > P$.

Here, QR decomposition is used instead of EVD/SVD to reduce the complexity of calculation. Then, the first row of the matrix $\boldsymbol{R}$ is used to construct a equivalence covariance matrix $\boldsymbol{Y}$, and $e_{1,m}$ is the elements of $\mathbf{r}_1$, $\boldsymbol{R} = [\mathbf{r}_1, \cdots, \mathbf{r}_M]^T$.

Finally, we apply the QR decomposition again to the constructed matrix Y and estimate the DOA with the use of the QR-IWLS method [23].

On the other hand, as we know, the performance of the forward and backward method [12] is much better than that only using the constructed $\boldsymbol{Y}$, which can also be used to solve the coherent situation. So in this paper, we also try this similar way; the backward matrix is:

$$\mathbf{Y}_b = \mathbf{J}_m \boldsymbol{Y}^* \boldsymbol{J}_q, \quad (9)$$

where both of $\mathbf{J}_m$ and $\mathbf{J}_q$ are anti-diagonal eye matrices, and $(\cdot)^*$ denotes the conjugate operation, then we obtain the equivalence covariance matrix:

$$\mathbf{Y}_r = \frac{1}{2}(\mathbf{Y} + \mathbf{Y}_b). \quad (10)$$

Then, we also perform the QR factorization on $\mathbf{Y}_r$ to obtain the signal subspace. And next the QR-IWLS method is used to estimate the DOA.

For the above analysis, the QR-IWLS algorithm steps are summarized as follows:

Step 1: Calculate the covariance matrix $\mathbf{R}_{xx}$ of the array received signal data by (4).

Step 2: Perform QR decomposition on $\mathbf{R}_{xx}$, take the eigenvector $\mathbf{e}_k$ corresponding to the largest eigenvalue.

Step 3: Reconstruction the matrix $\boldsymbol{Y}$, $\mathbf{Y}_r$ by (8), (10).

Step 4: Apply the QR decomposition to the $\mathbf{R}_{xx}$, $\boldsymbol{Y}$, $\boldsymbol{Y}_r$, respectively, to obtain the signal subspace.

Step 5: Use QR-IWLS to estimate DOAs of signal.

To sum up, we construct three different data matrices: covariance matrix (QR-IWLS-matrix), eigenvector reconstruction matrix using signal covariance (QR-IWLS-rematrix), and reconstruction matrix by applying smoothing technique (QR-IWLS-fbmatrix); then, we use QR-IWLS method to estimate DOA. The simulation experiments are carried out under different SNR, different snapshots, and different array numbers, as shown in Figures 2–4. The results show that as the SNR increases, QR-IWLS-rematrix is better than the other two methods, and QR-IWLS-fbmatrix performs poorly. So in the next section, we choose QR-IWLS-matrix and QR-IWLS-rematrix for experiments.

4. Simulation Results

In the simulation part, computer simulations are conducted to evaluate the performance of the QR-IWLS method in white Gaussian noise, compared with MUSIC [17] and PUMA [22] algorithms. In the experiment, we consider a ULA composed of different numbers of isotropic sensors and the element spacing is equal to half a wavelength, if not mentioned, two narrow-band far-field signal sources with angles of 5° and 75°. Furthermore, all results are averaged after 200 independent Monte Carlo experiments. The mean square error (MSE) is used to evaluate and analyze the performance of the algorithm, which is computed by $\mathrm{MSE} = 1/200\sum_{i=1}^{G}[(\theta_i - \widehat{\theta}_i)^2]$, where θ_i is the actual value, and $\widehat{\theta}_i$ is the measured value, respectively. The simulation is performed on the MATLAB R2019b on a computer with 16 GB of RAM and the 64-bit Windows 11 operating system.

In our first test, the number of snapshots N is set to 512, and the number of sensors M is 18. We investigate the MSE of the proposed method versus the SNR, as shown in Figure 5. The proposed method achieves DOA estimation performance better than the MUSIC method and QR-IWLS-matrix method, and has comparable performance with the PUMA algorithm.

In the second test, we set the SNR to 10 dB and the number of sensors M to 18. We compare the performance of each method in the case of different snapshots. As seen in Figure 6, with the number of snapshots increases, the performance of MUSIC method is severely damaged due to the

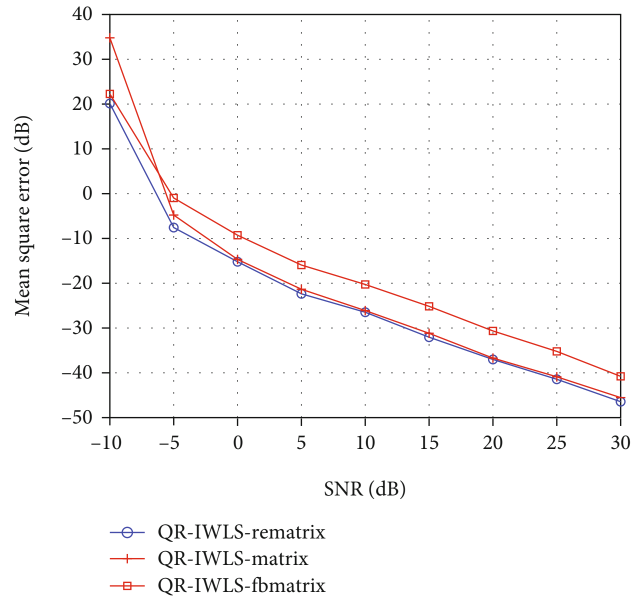

Figure 2: MSE versus SNR for three different data matrices.

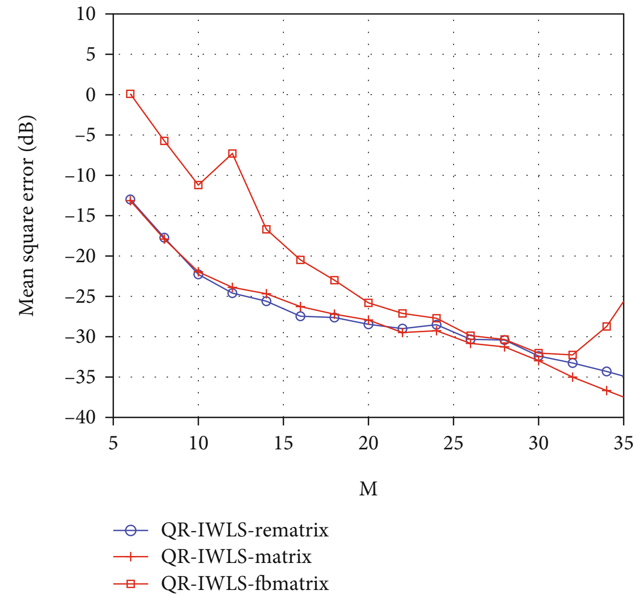

Figure 4: MSE versus array number for three different data matrices.

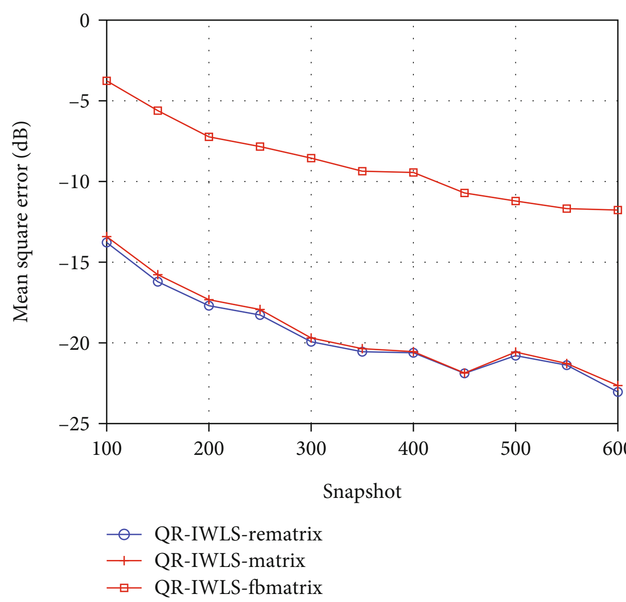

Figure 3: MSE versus snapshots for three different data matrices.

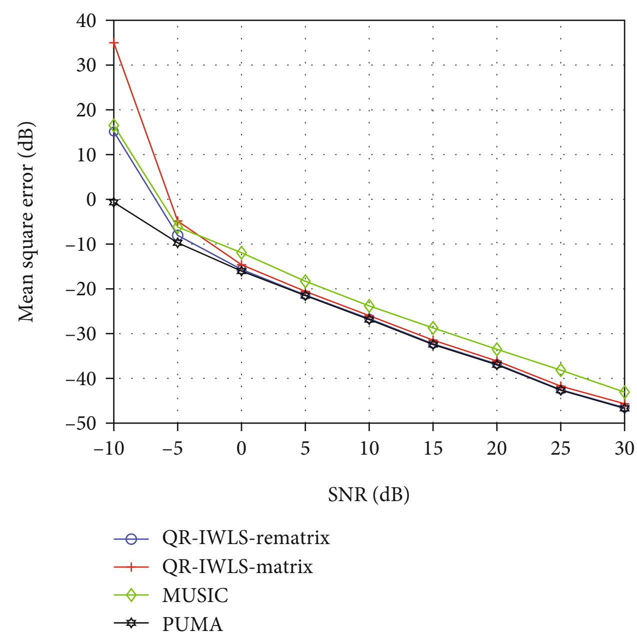

Figure 5: MSE versus SNR.

coherent of signals. The figure also confirms that for the QR-IWLS-matrix method and the QR-IWLS-rematrix method, the error is much smaller than other methods. Among them, the QR-IWLS-rematrix method has the best performance.

In the third experiment, the MSE versus the number of sensors is investigated. The number of sensors M is varied from 5 to 35, the SNR is 10 dB, and the number of snapshots is 512. As shown in Figure 7, the proposed method achieves comparable performance with the PUMA method and it is superior to the MUSIC algorithms.

Next, we investigate the relationship between success probability and SNR; the results are plotted in Figure 8, where the ratio of the number of successful runs to the total number of independent runs is calculated as the probability of success. We assumed that the number of DOAs is 2, when $\max |\theta_i - \widehat{\theta}_i| \leq |\theta_2 - \theta_1|/2$. It is concluded that all methods achieve 100% success at SNR ≥ 5 dB, and our QR-IWLS-rematrix method and the PUMA method achieve the highest resolution probability.

Finally, the computational complexity of the algorithm is compared, where the computer runtime is shown in Figure 9 and the complexity analysis is shown in Table 1. For

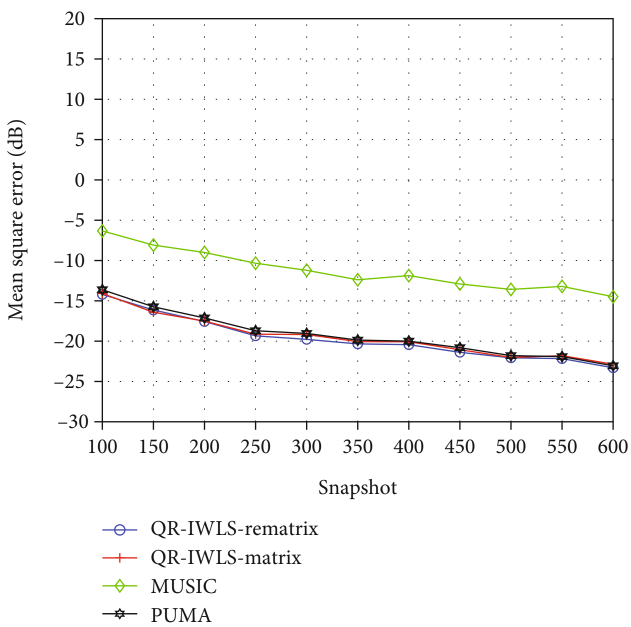

Figure 6: MSE versus snapshots number.

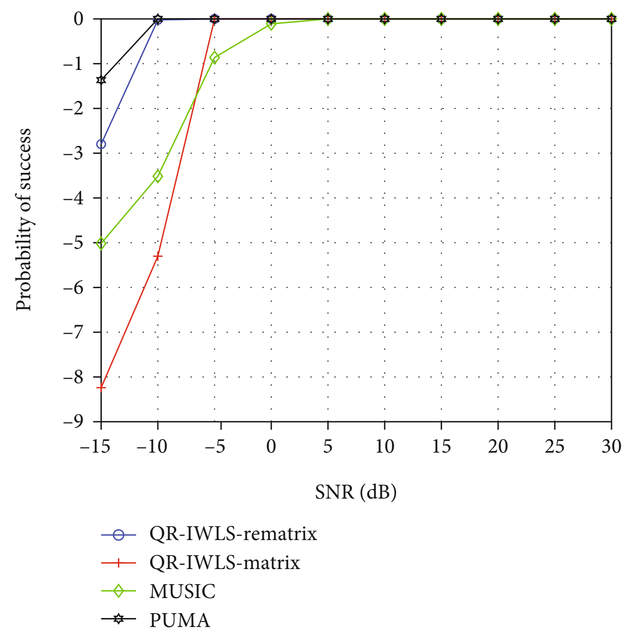

Figure 8: Probability of success.

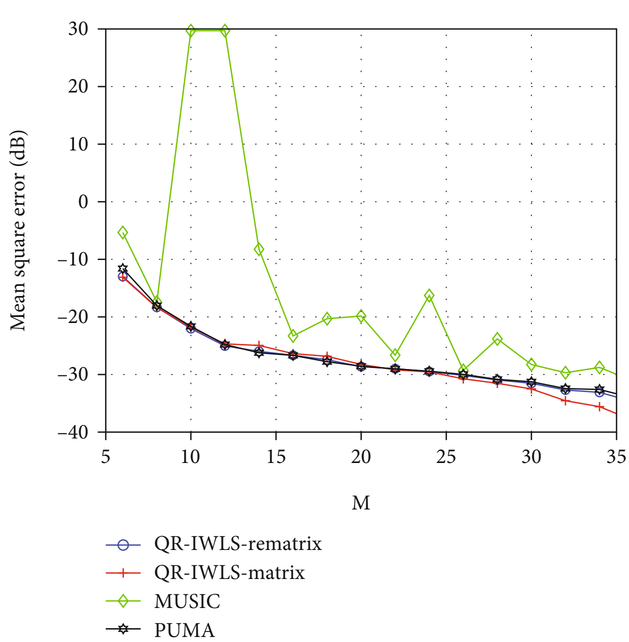

Figure 7: MSE versus array number.

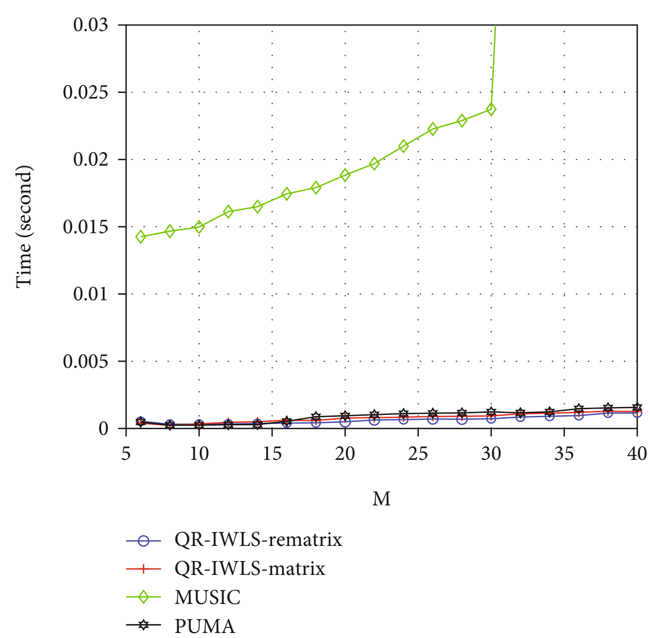

Figure 9: Time comparison versus array number.

simplicity, we only consider the main complexity of each method. Both simulation results and the complexity analysis demonstrate that the QR-IWLS-rematrix method has the lowest computational complexity among all methods, that is, under the conditions of corresponding SNR and sufficiently large number of sensors, the proposed method has a smaller MSE with less complexity compared with other mentioned methods.

In the above experiments we did, the setting parameters are large. To further investigate the performances of our proposed approaches, we simulate another experiment, that is, turn down the parameter setting. Fix $\theta_1 = 5^\circ$, and ranging θ_2 from 10° to 80° with $M = 10$ elements. As shown in Figure 10, our proposed algorithms have better results than the other two algorithms, and the performance of MUSIC algorithm is unstable and even fails when θ_2 is larger than 60°. Thus, our algorithm is persuasive and effective.

Table 1: The computational complexity comparison.

Algorithm	Computational complexity
QR-IWLS-rematrix	$O(-M^2N - MN + M^3)$
QR-IWLS-matrix	$O(-M^2N + M^3)$
PUMA	$O(M^2N + M^3)$
MUSIC	$O(M^2N + 2MN + M^3)$

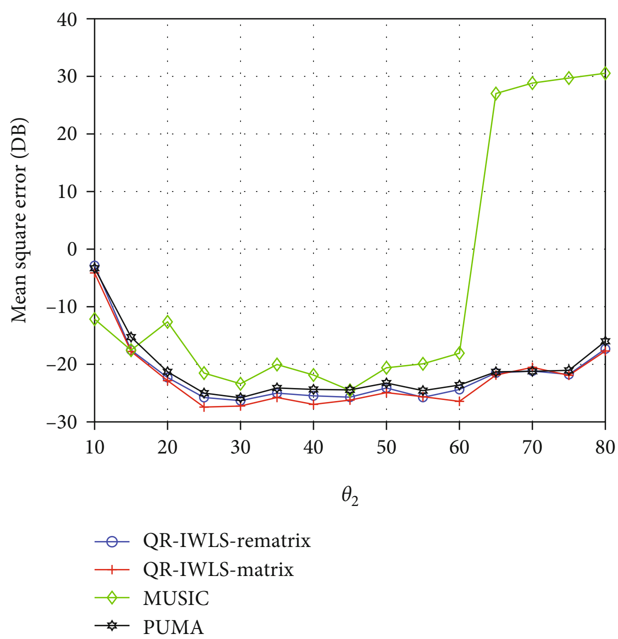

Figure 10: Fix θ_1 and change θ_2 with 10 elements.

5. Conclusion

In this paper, we proposed a QR decomposition-based method to handle DOA estimation of coherent signals. The coherent of the received signal is deduced by the reconstructing of a data matrix, whose elements come from the eigenvalues of a covariance matrix. The DOA is then estimated with the use of QR decomposition and IWLS-based method. The simulation results verify that the proposed approach has a comparable performance and less complexity compared with other advanced methods.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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