WILEY | Hindawi

Research Article

A Novel Optimal Strategy for Communication System in the Maritime Industry Based on Game Theory

Weijie Li [1] **, Qin Qi** [2,3,4] **and Yifang Liu** [5]

[1]*Institute of Biothermal Science and Technology, University of Shanghai for Science and Technology, Shanghai 200093, China*
[2]*School of Management, Xiamen University, Xiamen 361005, China*
[3]*Department of Publishing and Dissemination, Shanghai Publishing and Printing College, Shanghai 200093, China*
[4]*Shenzhen Value Online Information Technology Co., Ltd, Shenzhen 518000, China*
[5]*College of Science, University of Shanghai for Science and Technology, Shanghai 200093, China*

Correspondence should be addressed to Qin Qi; qiqin_jxsh@163.com

In this paper, we employ convex optimization and the saddle point equation to find the two-player optimal payoff in iterated rock-paper-scissors game. We also describe the equivalent of payoff written in a two-person non-zero-sum matrix in the hypothetical game system, which provides a possible way to make quantitative analyses. In addition, we use the interior-point methods to simulate rock-paper-scissors game and our numerical results verify that our hypothesis of the payoff equation, $\max_Y \min_X Y^T A X = \min_X \max_Y Y^T A X$, can still work very well even if A changes with the payoff g, but it is never affected by other factors.

1. Introduction

It is challenging for human beings to make optimal decisions in noncooperative strategic interactions [1]. The finest approach for scissors of rock paper is to really behave allegedly. This means that each choice is played around a third of the time and a player cannot imagine what is next. Rock-paper-scissors (RPS for short) game widely used to study competitive phenomena in society and biology, especially species diversity and pattern formation [2–7], offers a new way. As is known to all, the concept of Nash equilibrium (NE), developed under the assumption that the players are sufficiently rational to ensure that they can accurately learn the strategies of the competing players and to optimize their own strategy accordingly, plays a fundamental role in both classic game theory and evolutionary game theory [1, 8–13]. Furthermore, much effort has been devoted to investigating RPS game using a variety of models, such as the "rock-paper-scissors dynamics model" [14, 15], scale-free memory model [16], and cyclic dominance model [17–19]. Note that rock play is a dominant strategy for both players (i.e., the best

choice of rock, whenever your opponent plays! So, the balance for this game is unique: both players always choose rock).

Convex optimization is a reliable and efficient optimization method, which can obtain the global optimal solution precisely by selecting the appropriate algorithm. Convex optimization, frequently used to solve optimization problems due to its strong scalability and wide applications which include electronic technology [20–23], software engineering [24–27], and machine learning [28], can be fully applied to the game theory and sheds light on the optimal strategy in iterated RPS game. When we check the payment table for rock, paper, and scissors, we see that such a balance does not exist. There is no option where the selections for the two players are the best answer for the other player. So, there is no true Nash strategy balances.

In this paper, we employ convex optimization (CVX for short) and the saddle point equation to optimize two players' payoff in iterated RPS game, which has a considerable amount to offer in both theoretical researches and practical applications. Make a move that either gives you

a win or a stalemate that ensures you will not lose. You can suppose for instance that your opponent will not play it three times in a row, if they toss out scissors twice. Either rock or paper they will play. In order to verify our hypothesis of the study, we assign different values to the convex optimization model and then draw graphs with simulation software to find the pertinent rule, which shows the feasibility of the convex optimization method. Rock scissoring (also known as "rock" or ro-sham-bo) is a hand game usually played between two people, in which each participant produces a single one of three forms with an extended hand (articles of rock, "rocks" are sometimes known by various orders). The hand game normally is played in between two people. The forms "rock," "paper," and "scissors" are "clocked fist" (a fist with the index finger and middle finger extended, forming a V). "Scissors" are the same as the two-fingered V sign (also denoting "victory" or "peace"). It is not held vertically but is directed horizontally.

The marginal contribution of our study is that we introduce the two-player non-zero-sum matrix $A = [1, 0, g\,; g, 1, 0\,; 0, g, 1]$ to describe the payoff of players quantitatively as $X^T A Y$ and $Y^T A X$; we simulate the system with Newton algorithm. We use the saddle point equation $\max_Y \min_X Y^T A X = \min_X \max_Y Y^T A X$ to calculate how players can obtain the maximal payoff and find out that the EPRs of player X and player Y increase with the payoff g. This method takes into account the opponent's previous movements, to decide whether the opponent wants to choose one move over another. In essence, each rock, paper, and scissor have a "score." The rating of the opponent's move is increased after every move. If the rock paper scissors is a game that is bad, then they can only play in the devil's sons, and of course, the frenzied fans are spectators. Therefore, the concept that the game is bad is not even taken into account or that the game is not bad.

2. Model

For the sake of simplicity, we take a two-person non-zero-sum RPS game model as an example to study the noncooperative game system. In this game, each player plays innumerable rounds of the game and can only choose one action among R, P, and S in each round, as shown in Figure 1. The payoff $g\,(g > 1)$ is defined as the only parameter of the winning action in this game [1] (see Figure 2), and rational players just make decisions according to the value of g. Two players get a unit payoff when they choose the same action. Furthermore, player X will win with payoff g while player Y is going to get zero payoff when player X beats player Y, and vice versa.

During competitions, players often plan three gestures before the tournament begins. Some tourney players utilize methods to mislead or fool other players into an illegal move, which leads to a loss. One such approach is to call the name of one move in order to misdirect and mislead the other.

3. Results

3.1. Convex Optimization Equation. The expected payoff per round (EPR) W_X of player X and the EPR W_Y of player Y are as follows:

$$W_X = X^T A Y,$$
$$W_Y = Y^T A X, \tag{1}$$

where

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & g \\ g & 1 & 0 \\ 0 & g & 1 \end{pmatrix}, \tag{2}$$
$$X^T = (X_R, X_P, X_S), Y^T = (Y_R, Y_P, Y_S).$$

X_R, X_P, and X_S denote the payoff probabilities of player X, and Y_R, Y_P, and Y_S denote the payoff probabilities of player Y.

Compared with the traditional payoff function, our payoff equation is strictly convex. The convex optimization problem is described by (3) and (4).

$$X_R + X_P + X_S = 1,$$
$$Y_R + Y_P + Y_S = 1, \tag{3}$$

$$X_R > 0, X_P > 0, X_S > 0,$$
$$Y_R > 0, Y_P > 0, Y_S > 0. \tag{4}$$

Let $B_1^T = (1, 0, 0)$, $B_2^T = (0, 1, 0)$, and $B_1^T = (0, 0, 1)$. With the incidence matrices, we can rewrite problems (3) and (4) as (5) and (6).

$$\sum_i B_i^T X = 1, \quad i = 1, 2, 3,$$
$$\sum_j B_j^T Y = 1, \quad j = 1, 2, 3, \tag{5}$$

$$B_i^T X \geq 0, \quad i = 1, 2, 3,$$
$$B_j^T Y > 0, \quad j = 1, 2, 3, \tag{6}$$

Here, player X selects a strategy $i \in \{1, 2, 3\}$, while player Y selects a strategy $j \in \{1, 2, 3\}$. As (1)–(4) above are convex, all their optima are global optima. Two adversaries randomly throw out motions in the game rock paper scissors, and each wins, loses, or draws with equal probability. It must be a game of sheer luck, not competence—and certainly, if everybody could be perfectly alleged, nobody could take the lead on anybody else.

In this paper, the system we study is a two-person non-zero-sum game, so it can be written as $Y^T A X + X^T A Y \neq 0$. In order to study it quantitatively, we suppose that player X makes his decision first and player Y acts according to player X's decision later. In an object larger than another,
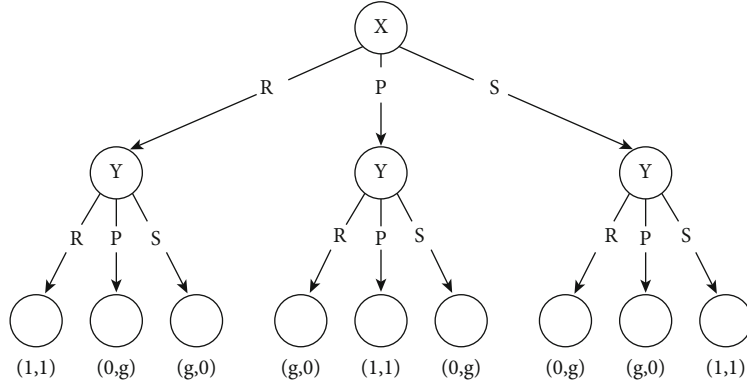
FIGURE 1: The payoff tree: each player (player X or player Y) has three possible actions: rock (R) beats scissors (S), S beats paper (P), and P in turn beats R. As it is doubtful that the adversary will play rock again, shears are unsurpassable. In the event of paper, scissors win; it is a tie, if the adversary chooses scissors. RPS players are mentally classified as winners and losers. Next time, a losing player will more likely switch to another throw.

FIGURE 2: The payoff matrix: each element of the payoff matrix is from row player to column player. It has just two potential outcomes in a simultaneous zero-sum game: one player draw, a win, and a loss for the other. A player who decides to play rock is going to beat another player who has selected scissors ("rock smashes scissors" or "blunt scissors" at times) but is going to lose to the player who has picked paper ("paper covers rock") ("scissors cuts paper"). The game is tied, usually played to break the tie quickly, if both players decide to play the same form. The game type was created in China and spread through increased contact with East Asia, and various varieties of signs were developed throughout time. A genuinely random opponent is not feasible to acquire an edge. However, it is possible to gain an important advantage by using the psychological flaws of intrinsically nonrandom adversaries. Actually, people tend to be nonrandom players. As a result, competition for algorithms playing rock paper scissors was held.

a paper which covers a rock still makes sense. That is why the paper beats the rock; only because the rock is not harmed does it invisibly render the rock unnecessary to the rest of the world. For rational players X and Y, player X wants to minimize W_Y, while player Y wants to maximize W_Y. Similarly, player X wants to maximize W_X, while player Y wants to minimize W_X. Game theory just renders it ineffective as an instrument to analyse the occurrences of the real world with the highly problematic assumptions on "rationality," equilibrium solutions, information, and knowledge.

Player X's best defense is to use X to minimize $Y^T A X$.

$$\min_X Y^T A X, \tag{7}$$

while player Y should choose Y to maximize one of the payoffs of the system.

$$\max_Y \left(\min_X Y^T A X \right), \tag{8}$$

$$Y^T A X = (Y_R, Y_P, Y_S) \begin{pmatrix} 1 & 0 & g \\ g & 1 & 0 \\ 0 & g & 1 \end{pmatrix} \begin{pmatrix} X_R \\ X_P \\ X_S \end{pmatrix}$$

$$= (Y_R + gY_P, Y_P + gY_S, Y_S + gY_R) \begin{pmatrix} X_R \\ X_P \\ X_S \end{pmatrix}$$

$$= Y_R X_R + Y_P X_P + Y_S X_S + g(Y_R X_S + Y_P X_R + Y_S X_P)$$

$$= (Y_R + gY_P)X_R + (Y_P + gY_S)X_P + (Y_S + gY_R)X_S. \tag{9}$$

We define $\chi = Y_R + gY_P$, $\xi = Y_P + gY_S$, and $\mu = Y_S + gY_R$, where χ, ξ, μ denote the values of the expected payoff (gain), respectively.

When $\chi < \xi, \mu$ and player X wants to minimize $Y^T A X$, then he should maximize X_R and its coefficient χ. That is to say, when $X_R = 1$, $X_P = 0$, and $X_S = 0$, $Y^T A X$ is the minimal payoff. From the inner minimization in (8), we have $e_1^T = (X_R, X_P, X_S) = (1, 0, 0)$. The other two cases ($\xi < \chi, \mu ; \mu < \chi, \xi$) proceed similarly. Two-person games are the simplest form of competing situations. These games have only two players; it is dubbed zero-sum games, as one player wins the other player lose.
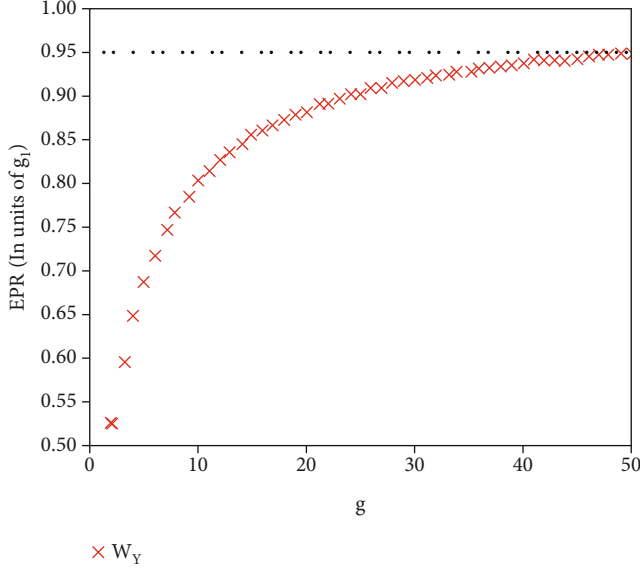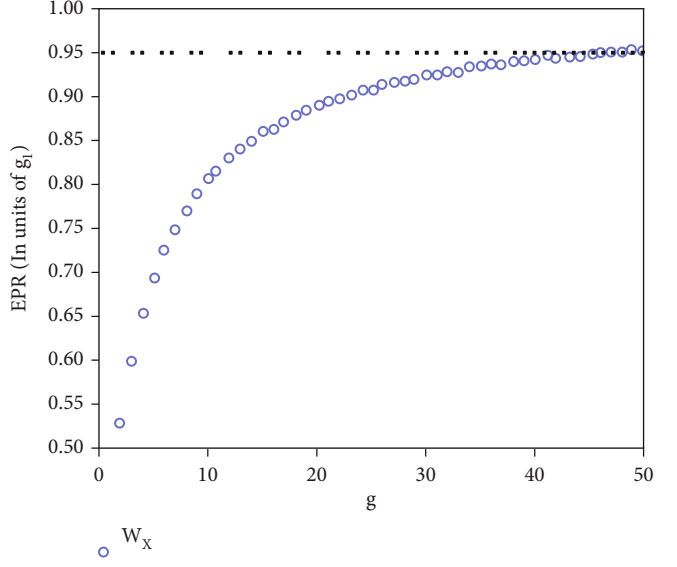
(a) The optimal value of EPR of player $X(W_X^*)$, when $g\epsilon 0$-50

(b) The optimal value of EPR of player $Y(W_Y^*)$, when $g\epsilon 0$-50

FIGURE 3: The optimal value of EPR of player X and player Y.

For $\max\limits_{Y}(\min\limits_{X} Y^T AX)$, inner optimization can be described as follows:

$$\min_{X} Y^T AX = Y^T A e_i, \quad i = 1, 2, 3, \tag{10}$$

where e_i denotes the vector that is all zeros except for one in the ith position, that is, deterministic strategy i. These optimization expressions are to be compared with the following standard form of a convex optimization function: the most difficult aspect of making decisions, according to a study presented at the annual conference of the academy of management this month, is not finding the proper answer; it has the fortitude to really act on that information.

$$\begin{cases} \min f_0(x) \\ \text{subject to } f_i(x) \leq 0, \quad i = 1, \cdots m \\ h_i(x) = 0, \quad i = 1, \cdots, p \end{cases} \tag{11}$$

In this paper, we introduce a scalar variable α representing the value of the inner minimization:

$$\begin{cases} \max \alpha \\ \text{subject to } \alpha \leq Y^T A e_i, \quad i = 1, 2, 3 \\ \sum_j B_j^T Y = 1, \quad j = 1, 2, 3 \\ B_j^T \geq 0, \quad j = 1, 2, 3 \end{cases} \tag{12}$$

Writing in matrix notation

$$\begin{cases} \max \alpha \\ \text{subject to } \alpha \mathbf{I} - AY \leq \mathbf{0} \\ \mathbf{I}^T Y \equiv 1 \\ B^T Y \geq \mathbf{0}, \end{cases} \tag{13}$$

where $\alpha = \min\limits_{X} Y^T AX$, $\mathbf{I}^T = (1, 1, 1)$, $\mathbf{O}^T = (0, 0, 0)$, and

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{14}$$

The symbol $\geq$ denotes being greater or equal to, the symbol $\leq$ denotes being less or equal to, and the symbol $\equiv$ denotes being equivalent to. Thus, we have obtained the standard convex optimization equation of our model.

3.2. The Saddle Point Equation. The primal problem (11) is convex with convex payoff mainly decided by player X.

$$\begin{cases} \min Y^T AX \\ \text{subject to } \sum_i B_i^T X = 1, \quad i = 1, 2, 3 \\ -B_i^T X \leq 0, \quad i = 1, 2, 3 \end{cases} \tag{15}$$

The Lagrangian is

$$L(X, \lambda, v) = Y^T AX - \lambda_i B_i^T X + \sum_{i=1}^{3} V_i \left(B_i^T X - 1 \right). \quad (16)$$

Thus, the dual function is

$$g(\lambda, v) = \min_X L(X, \lambda, v) = \min \ Y^T AX - \lambda_i B_i^T X + \sum_{i=1}^{3} V_i \left(B_i^T X - 1 \right). \quad (17)$$

For each pair (λ, v) with $\lambda \geq 0$, the Lagrange dual function gives us a lower bound on the optimal value p^* of the optimization problem (15). You are calm; scissors shows. You must be very careful to cut an object or open a box with scissors if you wish to cut it. Scissors shows that you are crafty and are awaiting a chance. You have just made a lovely dinner for scissors if you believed you could suffocate rock by throwing paper. Thus, we have a lower bound that depends on some parameters λ and v. Problem (15) is translated into the optimization problem:

$$\begin{cases} \max - \sum_{i=1}^{3} V_i, & i = 1, 2, 3 \\ \text{subject to } \lambda \geq 0 \end{cases} \quad (18)$$

This is called the Lagrange dual problem corresponding to problem (15). The Slater condition [28] says that strong duality between (17) and (18) holds if the quadratic inequality constraints are strictly feasible; i.e., if there exists an X with $-B_i^T X < 0, \sum_i B_i^T X = 1$, $i = 1, 2, 3$. Strong duality between (17) and (18) will be proven in the next section.

Now, suppose the order of play is reversed; player Y chooses Y_R, Y_P, Y_S first, and then, player X chooses X_R, X_P, X_S. Following a similar argument, if the players follow the optimal strategy, player X should choose X to minimize $\max_Y Y^T AX$, which results in a payoff of $\min_X \max_Y Y^T AX$ which is equivalent to

$$\begin{cases} \min \ \beta \\ \text{subject to } \beta \mathbf{I} - AX \geq \mathbf{0} \\ \mathbf{I}^T X \equiv 1 \\ B^T X \geq \mathbf{0}, \end{cases} \quad (19)$$

where Equations (13) and (19) are standard convex optimization payoff functions of players X and Y. Note that player Y's problem is dual to player X's in game theory. Many powerful algorithms have developed as a result of competitions for programming rock paper scissors, the heuristic compilation of techniques by Iocaine Powder, for instance, who was the winner of the First International RoShamBo Programing Competition in 1999. It also contains six metastrategies for each method it deploys, which defeat the opponent in second, third, and second guessing and so on.

The previous hypothesis is based on the sequence of how player X and player Y make decisions. Now, player X and player Y are making decisions at the same time. In comparison with Equations (13) and (19), we accordingly have

$$\max_Y \min_X Y^T AX = \min_X \max_Y Y^T AX. \quad (20)$$

We call Equation (20) the saddle point equation. It is a description of the saddle point at which players get the maximal payoff. Equation (13) gives the left part of Equation (20), whereas (19) provides the right part of Equation (20).

Similarly, looking at the payoff of player X, we have another saddle point equation

$$\max_X \min_Y X^T AY = \min_Y \max_X X^T AY. \quad (21)$$

The above equation is also a description of the saddle point equation at which players can achieve the maximal payoff of the system we study. Equations (13) and (20) represent the left part and the right part of Equation (20), respectively. Based on this, we denote the left and right parts of Equation (21) as

$$\begin{cases} \max \ \delta \\ \text{subject to } \delta \mathbf{I} - AX \leq \mathbf{0} \\ \mathbf{I}^T X \equiv 1 \\ B^T X \geq \mathbf{0} \end{cases} \quad (22)$$

$$\begin{cases} \min \ \gamma \\ \text{subject to } \gamma \mathbf{I} - AY \geq \mathbf{0} \\ \mathbf{I}^T Y \equiv 1 \\ B^T Y \geq \mathbf{0} \end{cases} \quad (23)$$

We use strong duality theorem to prove Equation (20). The same principle can also be used to prove Equation (21).

To prove it, first, it is noted that

$$\min_X L(X, \lambda, v) = \min \left\{ Y^T AX - \lambda_i B_i^T X + \sum_{i=1}^{3} V_i \left(B_i^T X - 1 \right) \right\}. \quad (24)$$

This means that we can write the optimal value of the primal payoff as

$$d^* = \max_{\lambda \geq 0} \min_X L(X, \lambda, v). \quad (25)$$

We also have

$$p^* = \min_X \max_{\lambda \geq 0} L(X, \lambda, v) \quad (26)$$

by the definition of the dual function. Thus, strong duality can be described as the equality $d^* = p^*$. $Y^T AX$ satisfies the strong max-min property or the saddle point property [29].

3.3. Optimization Algorithm

3.3. Optimization Algorithm. With linear equality and inequality constraints reduced to a sequence of linear constraint problems, the optimization problem in standard form is as follows:

$$
\begin{cases}
\min f_0(x) \\
\text{subject to } f_i(x) \leq 0, & i = 1, \cdots\cdots, m \\
h_i(x) = 0, & i = 1, \cdots\cdots, p
\end{cases}
\tag{27}
$$

We have

$$
\begin{cases}
f_i(\tilde{x}) \leq 0, & i = 1, \cdots\cdots m, \\
h_i(\tilde{x}) = 0, & i = 1, \cdots\cdots, p, \\
\tilde{\lambda} \geq 0, & i = 1, \cdots\cdots, m, \\
\tilde{\lambda}_i f_i(\tilde{x}) = 0, & i = 1, \cdots\cdots m, \\
\nabla f_0(\tilde{x}) + \sum_{i=1}^{m} \tilde{\lambda}_i \nabla f_i(\tilde{x}) + \sum_{i=1}^{p} \tilde{v}_i \nabla h_i(\tilde{x}) = 0,
\end{cases}
\tag{28}
$$

which are called the Karush-Kuhn-Tucker (KKT) conditions [29].

Because problem (15) is convex, the KKT conditions are also sufficient for the points to be primal and dual optimal. Thus, we have

$$
\begin{cases}
-B_i^T \tilde{X} \leq 0, & i = 1, 2, 3, \\
\sum_{i=1}^{3} B_i^T \tilde{X} - 1 = 0, & i = 1, 2, 3, \\
\tilde{\lambda}_i \geq 0, & i = 1, 2, 3, \\
-\tilde{\lambda}_i B_i^T \tilde{X} = 0, & i = 1, 2, 3, \\
Y^T A + \sum_{i=1}^{3} \tilde{\lambda}_i \left(-B_i^T\right) + \sum_{i=1}^{3} \tilde{v}_i B_i^T = 0.
\end{cases}
\tag{29}
$$

Then, $X^{\cdot}$ and $(\lambda^{\cdot}, v^{\cdot})$ are primal and dual optimal, with zero duality gap. Interior-point methods solve problem (15) by applying Newton's method to a sequence of equality constrained problems.

First, we translate problem (15) into an optimization problem, which states that the inequality constraints are implicit in the objective

$$
\begin{cases}
\min \left\{ Y^T A X + \sum_{i=1}^{3} I_-\left(-B_i^T X\right) \right\}, & i = 1, 2, 3 \\
\text{subject to } B_i^T X - 1 = 0,
\end{cases}
\tag{30}
$$

where $I_- : \mathbf{R} \longrightarrow \mathbf{R}$ is the indicator function for the nonpositive reals

$$
I_-(z) = \begin{cases}
0 & z \leq 0, \\
\infty & z > 0,
\end{cases}
\tag{31}
$$

and I_0 is the indicator function of $\{0\}$.

Then, we define the function

$$
\hat{I}_-(z) = -\left(\frac{1}{t}\right) \log(-z),
$$
$$
\text{dom } \hat{I}_- = R_{++}
\tag{32}
$$

to approximate the indicator function by using the barrier method, where $t > 0$ is a parameter that determines the accuracy of the approximation. The function $\hat{I}_-$ is convex and nondecreasing and takes on the value ∞ for $z > 0$. $\hat{I}_-$ is differentiable and closed and increases to ∞ as z increases to 0.

Subsequently, we replace I_- with $\hat{I}_-$ in (30).

$$
\begin{cases}
\min \left\{ Y^T A X + \sum_{i=1}^{3} -(1/t) \log\left(B_i^T X\right) \right\}, & i = 1, 2, 3 \\
\text{subject to } B_i^T X - 1 = 0
\end{cases}
\tag{33}
$$

The objective function here is convex, since $-(1/t) \log(-Z)$ is convex, increasing in z, and differentiable [29]. We obtain the function

$$
\varnothing(X) = \sum_{i=1}^{3} -\left(\frac{1}{t}\right) \log\left(B_i^T X\right), \quad i = 1, 2, 3,
\tag{34}
$$

with $\text{dom}\varnothing = \{X \in R^n | -B_i^T X < 0, i = 1, 2, 3\}$, which is the logarithmic barrier for problem (15). Finally, we obtain the gradient and Hessian of the logarithmic barrier function $\varnothing$:

$$
\begin{cases}
\nabla\varnothing(X) = \sum_{i=1}^{3} \frac{1}{B_i^T X} \nabla\left(-B_i^T X\right), & i = 1, 2, 3, \\
\nabla^2\varnothing(X) = \sum_{i} \frac{1}{-B_i^T X} \nabla\left(-B_i^T X\right) \nabla\left(-B_i^T X\right)^T + \sum_{i=1}^{3} \frac{1}{\left(B_i^T X\right)^2} \nabla^2\left(-B_i^T X\right), & i = 1, 2, 3.
\end{cases}
\tag{35}
$$

3.4. Verifying Hypothesis

3.4. Verifying Hypothesis. The hypothesis in our study is that the saddle point equation $\max_Y \min_X Y^T A X = \min_X \max_Y Y^T A X$ can still work very well even if A changes with g, but it is never affected by other factors. Assigning a specific value to A in simulation software, such as $A = A_1 = [1, 0, 2.7, 1; 2.7, 1, 0; 0, 2.7, 1]$, we can calculate the left part of the saddle point equation $\max_Y \min_X Y^T A X$ easily using the CVX software package. With our specified values, the calculation result of the left part of the saddle point equation is $\max_Y \min_X Y^T A X = +1.23333$. The calculation result of the right part is $\min_X \max_Y Y^T A X = +1.2333$. Assigning another specific

value to A in simulation software, such as $A = A_2 = [1, 0, 4.1; 4.1, 1, 0; 0, 4.1, 1]$, the calculation result of the left part of the saddle point equation is $\max_Y \min_X Y^T A X = +1.7$. The calculation result of the right part is $\min_X \max_Y Y^T A X = +1.7$. It means that no matter $A = A_1$ or $A = A_2$, the left part of the saddle point equation stays the same with the right part. As indicated in Figure 3, suppose $g_1 = (g + 5.1)/3$, the optimal value of EPR of player $X(W_X^*)$ equals to that of player $Y(W_Y^*)$, which increases with g and reaching towards 0.95. Thus, we believe numerical results are consistent with our theoretical hypothesis.

4. Conclusion

We have demonstrated in this paper how players can obtain the optimal payoff in two-player iterated RPS game with the convex optimization method and saddle point equation and verify the hypothesis of the payoff equation with interior methods by using simulation software. Hence, it can be concluded that convex optimization is a feasible method to maximize the payoff in two-player iterated RPS game regardless of other factors. Furthermore, the research method is operational and the results of our study can be extended to game systems like social cycling, species competition, election, and economical issues and provide insight into further related quantitative research.

Rock paper scissors (RPS) is not only a game popular with children but also a basic and classic model system for studying the mechanism of decision-making in noncooperative strategic interactions in depth. The RPS is a topic of increasing interest and significance for it helps improving our understanding on many complex competition issues (species divergence, price cycling, human decision-making, rationality and cooperation, and so on). It is also a starting point to enter into the interdisciplinary field between statistical physics and game theory [9].

As stated in Bi and Zhou's paper [8], cooperation in a finite-population RPS game system with more than two players may be much more difficult and complex to achieve than the case of only two players; we only provide the simplest model to probe into the optimal strategy in iterated RPS game in the paper; much more complicated related research needs being carried out in the future.

Data Availability

The figures used to support the findings of this study are included in the article.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

W.J.L. was responsible for conceptualization. Y.F.L. was responsible for the software. Q.Q. and Y.F.L. were responsible for writing the original draft. Q.Q. was responsible for writing, review, and editing. All authors read and agreed to the published version of the manuscript.

References

[1] Z. Wang, B. Xu, and H. J. Zhou, "Social cycling and conditional responses in the rock-paper-scissors game," *Scientific Reports*, vol. 4, no. 7, pp. 1–7, 2014.

[2] J. M. Biernaskie, A. Gardner, and S. A. West, "Multicoloured greenbeards, bacteriocin diversity and the rock-paper-scissors game," *Journal of Evolutionary Biology.*, vol. 26, no. 10, pp. 2081–2094, 2014.

[3] R. A. Laird, "Population interaction structure and the coexistence of bacterial strains playing 'rock–paper–scissors,'" *Oikos*, vol. 123, no. 4, pp. 472–480, 2014.

[4] S. Loertscher, "Rock-scissors-paper and evolutionarily stable strategies," *Economics Letters.*, vol. 118, no. 3, pp. 473-474, 2013.

[5] Q. He, M. Mobilia, and U. C. Täuber, "Spatial rock-paper-scissors models with inhomogeneous reaction rates," *Physical Review E Statistical Nonlinear & Soft Matter Physics.*, vol. 82, no. 1, pp. 1–11, 2010.

[6] B. Sinervo, B. Heulin, Y. Surget-Groba et al., "Models of density-dependent genic selection and a new rock-paper-scissors social system," *American Naturalist.*, vol. 170, no. 5, pp. 663–680, 2007.

[7] B. Kerr, M. A. Riley, M. W. Feldman, and B. J. M. Bohannan, "Local dispersal promotes biodiversity in a real-life game of rock-paper- scissors," *Nature*, vol. 418, no. 6894, pp. 171–174, 2002.

[8] Z. Bi and H. J. Zhou, "Optimal cooperation-trap strategies for the iterated rock-paper-scissors game," *PLoS One*, vol. 9, no. 10, pp. 1–6, 2014.

[9] H. J. Zhou, "The rock-paper-scissors game," *Contemporary Physics.*, vol. 57, no. 2, pp. 151–163, 2016.

[10] E. Bahel, "Rock-paper-scissors and cycle-based games," *Economics Letters.*, vol. 115, no. 3, pp. 401–403, 2012.

[11] A. V. D. Nouweland, "Rock-paper-scissors a new and elegant proof," *Department of Economics-Working Papers Series.*, vol. 86, no. 43, pp. 178–184, 2007.

[12] E. Bahel and H. Haller, "Cycles with undistinguished actions and extended rock-paper-scissors games," *Economics Letters.*, vol. 120, no. 3, pp. 588–591, 2013.

[13] P. W. Goldberg, "A survey of PPAD-completeness for computing Nash equilibria," *Surveys in Combinatorics*, vol. 3, pp. 1–32, 2011.

[14] E. Wesson and R. Rand, "Hopf bifurcations in delayed rock-paper-scissors replicator dynamics," *Dynamic Games & Applications.*, vol. 6, no. 1, pp. 1–18, 2016.

[15] D. Semmann, H. J. Krambeck, and M. Milinski, "Volunteering leads to rock-paper-scissors dynamics in a public goods game," *Nature*, vol. 425, no. 6956, pp. 390–393, 2003.

[16] I. Lubashevsky and S. Kanemoto, "Scale-free memory model for multiagent reinforcement learning. Mean field approximation and rock-paper-scissors dynamics," *European Physical Journal B.*, vol. 76, no. 1, pp. 69–85, 2010.

[17] C. M. Postlethwaite and A. M. Rucklidge, "Spirals and heteroclinic cycles in a spatially extended rock-paper-scissors model

of cyclic dominance," *Europhysics Letters*, vol. 17, no. 9, article 48006, 2016.

[18] G. Verma, K. Chan, and A. Swami, "Zealotry promotes coexistence in the rock-paper-scissors model of cyclic *dominance*," *Physics*, vol. 92, no. 5, article 052807, 2015.

[19] A. Szolnoki and M. Perc, "Zealots tame oscillations in the spatial rock-paper-scissors game," *Physical Review E.*, vol. 93, no. 6-1, article 062307, 2016.

[20] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: a unified framework for convex optimization," *IEEE Transactions on Signal Processing.*, vol. 51, no. 9, pp. 2381–2401, 2003.

[21] A. B. Gershman, N. D. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming," *Signal Processing Magazine IEEE.*, vol. 27, no. 3, pp. 62–75, 2010.

[22] S. Joshi and S. Boyd, "Sensor selection via convex optimization," *IEEE Transactions on Signal Processing.*, vol. 57, no. 2, pp. 451–462, 2009.

[23] F. Altenbach, S. Corroy, G. Böcherer, and R. Mathar, "Strategies for distributed sensor selection using convex optimization," in *2012 IEEE Global Communications Conference (GLOBECOM)*, pp. 2367–2372, Anaheim, CA, USA, December 2012.

[24] J. Mattingley and S. Boyd, "CVXGEN: a code generator for embedded convex optimization," *Optimization and Engineering*, vol. 13, no. 1, pp. 1–27, 2012.

[25] J. Mattingley and S. Boyd, "Real-time convex optimization in signal processing," *IEEE Signal Processing Magazine.*, vol. 27, no. 3, pp. 50–61, 2010.

[26] S. P. Boyd, "Real-time embedded convex optimization," *Ifac Proceedings Volumes.*, vol. 42, no. 11, p. 9, 2009.

[27] T. Wang, R. Jobredeaux, M. Pantel, P. L. Garoche, E. Feron, and D. Henrion, "Credible autocoding of convex optimization algorithms," *Optimization and Engineering*, vol. 17, no. 4, pp. 781–812, 2016.

[28] Y. Ploengpit and T. Phienthrakul, "Rock-paper-scissors with Myo Armband pose detection," in *2016 International Computer Science and Engineering Conference (ICSEC)*, pp. 1–5, Chiang Mai, Thailand, December 2016.

[29] V. Boyd and L. Vandenberghe, "Convex optimization," *IEEE Transactions on Automatic Control*, vol. 51, no. 11, pp. 1859–1859, 2006.