

Retraction

Retracted: Finite-Time Attitude Cooperative Control of Multiple Unmanned Aerial Vehicles via Fast Nonsingular Terminal Sliding Mode Control

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

- (1) Discrepancies in scope
- (2) Discrepancies in the description of the research reported
- (3) Discrepancies between the availability of data and the research described
- (4) Inappropriate citations
- (5) Incoherent, meaningless and/or irrelevant content included in the article
- (6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

- [1] Q. Han, X. Tuo, Y. Tang, and P. He, "Finite-Time Attitude Cooperative Control of Multiple Unmanned Aerial Vehicles via Fast Nonsingular Terminal Sliding Mode Control," *Wireless Communications and Mobile Computing*, vol. 2022, Article ID 4324626, 11 pages, 2022.

Research Article

Finite-Time Attitude Cooperative Control of Multiple Unmanned Aerial Vehicles via Fast Nonsingular Terminal Sliding Mode Control

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This paper discusses the attitude cooperative control of multiple unmanned aerial vehicle systems (MUAVs) with unknown dynamics and external disturbances. Distributed fast nonsingular terminal sliding mode control (FNTSMC) is used with quaternion-description dynamical systems. The dynamics and external disturbances are changed into lumped disturbances by formula transformation. A robust nonlinear disturbance observer is proposed to estimate the lumped disturbances in finite time. Then, combining the fast terminal sliding mode control, distributed FNTSMC controllers are designed under the directed topology. Based on the Lyapunov stability theory and graph theory, convergence stability of the nonlinear systems is strictly proved, and the tracking errors between the leader and the followers approach to a small residual set. Finally, the simulation example is presented to illustrate the effectiveness and advantage of the proposed controllers.

1. Introduction

In recent years, formation flight control (FFC) for multiple unmanned aerial vehicle systems has attracted more and more attention owing to its various applications in both military and civilian areas [1–4]. Attitude cooperative is required to complete the formation flight mission. However, attitude dynamics of MUAVs are highly nonlinear, such as inertia uncertainties and external disturbances. How to develop a robust attitude control law is challenging work. Sliding mode control (SMC) has been widely used in attitude control for unmanned aerial vehicles (UAVs) [5]. Because of its robustness and insensitivity to disturbances and uncertainties, \mathcal{H}^∞ controller was proposed for unmanned helicopters with uncertain models [6]. Although most of the existing robust control methods provide sufficient stability and reliability, they can only guarantee the asymptotic convergence as time goes into infinity.

Finite-time control (FTC) is more desirable for attitude synchronization of spacecraft or some other flight vehicles due to its rapid maneuverability highly demanded in some real-time actions [7]. Besides high precision performance and fast convergence rate, finite-time control can also provide better disturbance rejection properties and better robustness against uncertainties, and it has been gradually implemented in spacecraft attitude stability and tracking control [7, 8]. Terminal sliding mode control (TSMC) has been proposed, which can significantly improve the transient performance substantially in finite time [9]. Adaptive sliding mode control method was proposed for quadrotors to handle the parametric uncertainties [10]. However, disadvantages also exist in TSMC. One of the disadvantages is the singularity problem, and the other is that TSMC has slower convergence to the equilibrium point than the traditional SMC when the system state is far away from the equilibrium point. Hence, a nonsingular TSMC (NTSMC)

scheme was proposed, which has been used in multiple agent systems while maintaining the advantages of the conventional TSMC [11], the NTSMC can eliminate the singularity problem, and fast TSMC (FTSMC) was proposed to provide fast convergence rate for the slow convergence rate problem [12]. To overcome the two disadvantages of the SMC, fast nonsingular TSMC (FNTSMC) has been used for attitude synchronization of spacecrafts [13]. Recently, FNTSMC combined with event-triggered scheme was designed for distributed attitude coordination with hyperbolic tangent function for multiple agent systems [14]. However, inertia uncertainties were not considered, which is not practical in dynamical environments in the mentioned work [13]. Based on the robust decentralized attitude control, finite-time attitude synchronization mechanism was proposed for flying vehicles; however, the precise knowledge of the inertial matrix should be known, which is not practical because of its time-varying [15–18]. Neural network (NN) and fuzzy logic systems (FLSs) can approximate any smooth functions over a compact set to arbitrary accuracy [19]. Based on Chebyshev NN and TSMC, the attitude tracking problem of spacecrafts is studied, and a switching is designed between the adaptive NN controller and robust controller [20], finite-time formation control for multiple helicopters was proposed [21], and the disturbances were estimated by the NN technique without considering the uncertain parameters. However, the mentioned works [20, 21] estimated the disturbances by NN and FLS only effective on some compact sets.

In this paper, the FNTSMC method is used for the attitude cooperative control for MUAVs, considering the uncertain inertia part and external disturbances. It is significantly noted how to deal with lumped disturbances in cooperative attitude control. However, adaptive control or SMC scheme cannot reconstruct information about the lumped disturbances. Nonlinear disturbances observer (NDO) can eliminate the problem and reconstruct the lumped disturbances without an additional sensor [22]. The NDO technique has attracted much attention and has been widely used in unmanned aerial vehicle (UAV) attitude control and mobile robots [21, 23]. A composite disturbance observer-based control (DOC) and TSMC were proposed for a class of spacecraft formation [24]. A composite control based on NDO was proposed to compensate the disturbances through feedforward [25]. The NDO technique combined with an event-triggered scheme has been used in multiple agent systems [26]. Recently, the observer schemes for different objects have been investigated [21, 27–32]. A nonlinear internal model-based observer is proposed to estimate the nonlinear signals with parametric uncertainty. The NDO-based controller is proposed to compensate the disturbances for the MUAVs or single quadrotor [29–31]. A distributed antidisturbance method combining with Nussbaum function, adaptive NNs, and disturbances observer for attitude tracking of MUAVs was proposed [31]; however, the abovementioned works used the NDO not considering the finite-time property. In order to obtain high precision performance and fast convergence rate, the NDO scheme was proposed, which could obtain the lumped disturbances information in finite time [32]. Decentralized FNTSMC combined with terminal sliding mode disturbance

observer was proposed for quaternion-description attitude synchronization of multiple rigid spacecrafts [33]. However, these works have their own limitations. The authors only consider the attitude cooperative problem under the undirected communication topology [33]. However, cooperative control for attitude cooperative of MUAVs under directed communication topology is more challenging.

Motivated by the aforementioned analysis, this paper aims to consider a more interesting cooperative attitude control problem for MUAVs with inertia uncertainties and external disturbances. The contributions of this paper are summarized as follows:

- (i) Compared with the attitude cooperative under undirected topology [33], our analysis is based on the directed topology, and it is more challenging and practical
- (ii) A robust NDO is proposed to estimate the lumped disturbances, which can obtain the information of the lumped disturbances in finite time
- (iii) Based on the directed communication topology, combining the NDO and FNTSMC, distributed cooperative attitude controller for MUAVs is proposed, which can guarantee that the tracking errors converge to the regions containing the origin in finite time

The rest of this paper is organized as follows. In Section 2, preliminaries of graph theory, quaternion-based attitude dynamics of MUAVs, and some useful lemmas are given. Section 3 gives main results. The effectiveness of the controller is illustrated by a numerical simulation example in Section 4. Finally, the conclusion is given in Section 5.

Notation. For convenience, the following notations are used throughout this paper. All matrices are assumed to have compatible dimensions. R^n and $R^{n \times m}$ denote the n -dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. I_n is a $n \times n$ dimensional identity matrix. A^T denotes the transpose of vector or matrix A , $\text{tr}(\cdot)$ is the trace of a given matrix, \otimes stands for Kronecker product, and $|\cdot|$ refers to the absolute value. $\|\cdot\|$ refers to the Euclidean vector norm or the induced matrix 2-norm, and $\|B\| = (\text{tr}(B^T B))^{1/2}$. The superscript ‘ T ’ denotes matrix transposition. I_n denotes the $n \times n$ identity matrix, and $I_n \in R^n$ ($0_n \in R^n$) is a column vector with each entry being $I(0)$. $\lambda_{\max}(M)$ be the largest eigenvalue of matrix M . $\text{diag}(\cdot)$ represents the diagonal matrix, and $\text{sgn}(\cdot)$ denotes the sign function. $\text{sig}^r(\cdot) = \text{sgn}(\cdot)|\cdot|^r$.

2. Preliminaries and Problem Formulation

In this section, basic algebraic graph theory and notations are introduced. Then, the system description and attitude tracking problems of MUAVs are described.

2.1. Communication Algebraic Graph Theory. Graph theory is a useful tool to solve the cooperative problems among the MUAVs. In this paper, we consider the directed communication topology. A weighted directed graph is denoted by

$$\mathcal{E} = (\nu, \xi, \mathcal{E}), \quad (1)$$

where $\nu = \{1, 2, \dots, n\}$ stands for the single UAV and $\xi = \nu \times \nu$ denotes the edges set. If there exists an edge between two nodes, the two nodes are called adjacent. $A = (a_{ij})_{N \times N} \in \mathbb{R}^{n \times n}$ is weighted matrix which is symmetric, the symmetric adjacency matrix A is defined as $a_{ii} = 0$, and $a_{ij} = a_{ji} \geq 0$ means that agent ν_i and ν_j are connected by an edge. The Laplacian matrix corresponding to the weighted undirected graph is defined as $L = [l_{ij}]_{n \times n}$, which has the property $L = D - A$, where $D = \text{diag}\{d_1, d_2, \dots, d_n\}$ is the out-degree of the i th node $d_i = \sum_{j=1}^n a_{ij}$.

In this paper, we consider the MUAVs composed of one leader and N followers. Without loss of generality, let agent 0 be the leader, and the N followers are labeled as i ($i = 1, 2, \dots, n$). The communication topology of MUAVs between the followers is characterized by a digraph \mathcal{E} with the Laplacian matrix L . The communication between the followers and the leader is represented by a diagonal matrix $B = \text{diag}\{b_1, b_2, \dots, b_n\}$. If the follower agent i has access to the information of leader 0, then $b_i > 0$; otherwise, $b_i = 0$. Besides, the communication topology between followers and the leader is characterized by \mathcal{E} . Obviously, the graph $\overline{\mathcal{E}}$ is the subgraph of \mathcal{E} .

Assumption 1. The augmented digraph $\overline{\mathcal{E}}$ contains a directed spanning tree.

Lemma 1 (see [34]). *If the digraph $\overline{\mathcal{E}}$ has a directed spanning tree, then the matrix $L + B$ is invertible.*

In this paper, we consider that the directed communication topology is firmly connected. If there at least one UAV can receive the information from the leader, which implies that $b_i \neq 0$, then $L + B$ is dominant and irreducible diagonal matrix. By Lemma 1, we can obtain that $L + B$ is invertible, and the matrix $(L + B) \otimes I_3$ is invertible.

2.2. Dynamic Model of MUAVs. Compared with the rotation matrix for attitude description, the quaternion-based attitude description has more advantages because of providing simplified design and analysis of control systems. Hence, the attitude dynamics equations of the i th UAV in terms of quaternion-based are given as follows:

$$\begin{cases} \dot{Q}_i = \frac{1}{2}T(Q_i)\omega_i, \\ J_i\dot{\omega}_i = u_i - S(\omega_i)J_i\omega_i + d_i, \end{cases} \quad (2)$$

where $Q_i = [q_i \ q_{i0}]^T$ represents the attitude of the i th UAV, $q_i \in \mathbb{R}^3, q_{i0} \in \mathbb{R}, Q_i \in \mathbb{R}^4, |Q_i| = 1$, and $\omega_i \in \mathbb{R}^3$ is the angular velocity. J_i is the symmetric positive definite constant moment of the inertial matrix of the i th UAV; u_i represents the control torque of the rotor thrust for each UAV; $T(Q_i)$ is given by $T(Q_i) = \begin{pmatrix} q_{i0}I_3 + S(q_i) \\ -q_i^T \end{pmatrix}$, and

$$S(\omega_i) = \begin{pmatrix} 0 & -\omega_{i3} & \omega_{i2} \\ \omega_{i3} & 0 & -\omega_{i1} \\ -\omega_{i2} & \omega_{i1} & 0 \end{pmatrix}; R(Q_i) \text{ denotes the rotation}$$

matrix associated to $Q_i = [q_i \ q_{i0}]^T$ that brings the inertial frame into the body frame, and $R(Q_i)$ is given as $R(Q_i) = (2q_{i0}^2 - 1)I_3 + 2q_i q_i^T - 2q_{i0}S(q_i)$; d_i denotes the external disturbances.

The multiplication between two unit quaternions is given by

$$Q_1 \odot Q_2 = \left(\frac{q_{10}q_2 + q_{20}q_1 + S(q_1)q_2}{q_{10}q_{20} - q_1^T q_2} \right), \quad (3)$$

where $Q_1 = [q_1 \ q_{10}]^T$ and $Q_2 = [q_2 \ q_{20}]^T$.

In this paper, the control objective for the MUAVs is that the followers can track the desired attitudes of the leader. Based on the mentioned relevant knowledge of multiplication, the tracking errors of the i th UAV can be formulated as

$$\begin{cases} \tilde{Q}_{i,d} = Q_i^{-1} \odot Q_d, \\ \tilde{\omega}_i = \omega_i - R(\tilde{Q}_{i,d})\omega_d, \end{cases} \quad (4)$$

where $Q_d \triangleq [q_d, \eta_d]^T$ and ω_d are the desired attitude and desired angular velocity. $\tilde{Q}_{i,d}$ and $\tilde{\omega}_i$ are the attitude tracking error and angular velocity error, respectively.

Based on equations (2)–(4), the attitude error systems can be obtained:

$$\begin{aligned} J_i\dot{\tilde{\omega}}_i &= -S(\omega_i) \cdot J_i\omega_i + u_i \\ &+ J_i \cdot [S(\tilde{\omega}_i) \cdot R(\tilde{Q}_{i,d}) \cdot \omega_d - R(\tilde{Q}_{i,d}) \cdot \dot{\omega}_d] + d_i, \end{aligned} \quad (5)$$

$$\dot{\tilde{Q}}_{i,d} = \frac{1}{2}T(\tilde{Q}_{i,d}) \cdot \tilde{\omega}_i. \quad (6)$$

Assumption 2. The desired angular velocity ω_d and its derivatives are bounded and satisfied. The tracking error $\tilde{\omega}_i$ and its derivatives are bounded.

Assumption 3. The inertial matrix J^* is known and non-singular. ΔJ denotes the uncertainties satisfying $\|\Delta J\| \leq J_\theta$ with J_θ as a positive constant.

Lemma 2 (see [35]). *For the system given by $\dot{x} = f(x), f(0) = 0, x \in \mathbb{R}^n$. There exists a function $V(x) \in C^1$ defined on a neighborhood of the origin such that $V(x)$ is positive definite; $\dot{V}(x) + c_1 V^\eta(x) + c_2 V(x) \leq 0$, where $\eta \in (0, 1), c_1, c_2 > 0$.*

Then, the origin is locally finite-time stable, and the finite time T depending on the initial state $x(0)$ is

$$T(x(0)) \leq \frac{\ln\left[\left(c_2 V^{1-\eta}(x(0)) + c_1\right)/c_1\right]}{c_2(1-\eta)}. \quad (7)$$

2.3. Control Objective. In the presence of inertia uncertainties and external disturbances, we aim at designing FNTSMC such that the closed-loop systems (5) and (6) can reach the sliding mode surface (SMS) in finite time. Then, $\tilde{\omega}_i$ and \tilde{q}_i can converge to a residual set in finite time, respectively.

TSMC can effectively achieve high performance of finite-time stability and high robustness in the presence of model dynamics and environmental disturbances. However, it has some drawbacks, such as chattering and singularity. The modified finite-time nonsingular sliding mode surface for the MUAVs is designed to avoid the chattering and singularity problems, and the sliding mode surface is as follows:

$$s_i = b_i \cdot J_i^* (\tilde{\omega}_i + k_1 \cdot \tilde{q}_i + k_2 \cdot h_i \cdot (\tilde{q}_i)) + \sum_{j=1, j \neq i}^n a_{ij} \cdot [(J_i^* \tilde{\omega}_i - J_j^* \tilde{\omega}_j) + (k_1 \cdot J_i^* \cdot \tilde{q}_i - k_1 \cdot J_j^* \cdot \tilde{q}_j) + (k_2 \cdot J_i^* \cdot h_i \cdot (\tilde{q}_i) - k_2 \cdot J_j^* \cdot h_j \cdot (\tilde{q}_j))], \quad (8)$$

where $h_i(\tilde{q}_i) = [h_{i1}(\tilde{q}_{i1})h_{i2}(\tilde{q}_{i2})h_{i3}(\tilde{q}_{i3})]^T$ is a function and k_1 and k_2 are positive constants.

$$h_{ij}(\tilde{q}_{ij}) = \begin{cases} |\tilde{q}_{ij}|^r \cdot \text{sgn}(\tilde{q}_{ij}), & \text{if } \bar{s}_{ij} = 0 \text{ or } \bar{s}_{ij} \neq 0, |\tilde{q}_{ij}| > g, \\ \delta_1 \cdot \tilde{q}_{ij} + \delta_2 \cdot |\tilde{q}_{ij}|^2 \cdot \text{sgn}(\tilde{q}_{ij}), & \text{if } \bar{s}_{ij} \neq 0, |\tilde{q}_{ij}| \leq g, \end{cases} \quad (9)$$

where $i = 1, 2, \dots, n, j = 1, 2, 3, \bar{s}_i = \bar{s}_{ij, i=1,2,3} = [\bar{s}_{i1}, \bar{s}_{i2}, \bar{s}_{i3}]^T, \bar{s}_i = \tilde{\omega}_i + \delta_1 \cdot \tilde{q}_i + \delta_2 \cdot |\tilde{q}_i|^r \cdot \text{sgn}(\tilde{q}_i), \delta_1 = (2-r) \cdot g^{r-1}, \delta_2 = (r-1) \cdot g^{r-2}, r = r_1/r_2. r_1$ and r_2 are positive odd

$$\Delta M_i = -S(\omega_i) \cdot \Delta J_i \cdot \omega_i + \Delta J_i \cdot [S(\tilde{\omega}_i) \cdot R(\tilde{Q}_{i,d}) \cdot \omega_d - R(\tilde{Q}_{i,d}) \cdot \dot{\omega}_d] + d_i + \Delta J_i \cdot k_1 \cdot \left(\frac{1}{2}\tilde{q}_i + \tilde{q}_{i0} \cdot I_3\right) \cdot \tilde{\omega}_i + \Delta J_i \cdot k_2 \cdot \dot{h}(\tilde{q}_i). \quad (13)$$

In view of equation (11), the attitude cooperative problem has become the regulation problem, which is to design FNTSMC to compensate the certain dynamics x_i and ΔM_i , and then the variable states satisfy $\dot{s} \leq 0$.

Lemma 3 (see [36]). *Consider the error system equations (9)–(11) for sliding surface $\delta_i = \tilde{\omega}_i + c_1 q_i + c_2 q_i^r$, where $0 < r < 1, c_1 > 0, c_2 > 0$, for $i = 1, 2, \dots, n$. If $\delta_i = 0$, then $\tilde{\omega}_i = 0, q_{i0} = 1$ and $q_i = 0$ can be reached in finite time, respectively.*

Remark 1. Based on Lemma 3 and sliding mode variable structure theorem, the states reach the sliding mode surface $s = [(L+B) \otimes I_3] \cdot J^* \cdot [s^*] = 0$ with the controller to be designed. Based on the definition of matrix $B, [(L+B) \otimes I_3] \cdot J^*$ has full rank, and we can deduce that $s^* = 0$.

integers; $r \in (0, 1), g$ is a small positive constant. Based on the knowledge of graph theory, the parameter a_{ij} is the weighted value, which is the control signal of the attitude cooperative between the i th and j th UAV to keep the formation behavior. b_i is the weighted value denoting the information change between the i th UAV and the leader for tracking the leader's attitude.

Equations (8) can be rewritten by the Kronecker product.

$$s = [(L+B) \otimes I_3] \cdot J^* \cdot [s^*], \quad (10)$$

where $s^* = \tilde{\omega} + k_1 \tilde{q} + k_2 h(\tilde{q}), s = [s_1, s_2, \dots, s_n]^T, s^* = [s_1^*, s_2^*, \dots, s_n^*]^T, \tilde{\omega} = [\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_n]^T, J^* = \text{diag}\{J_1^*, J_2^*, \dots, J_n^*\}, \tilde{q} = [\tilde{q}_1, \tilde{q}_2, \dots, \tilde{q}_n]^T, \text{ and } h(\tilde{q}) = [h(\tilde{q}_1), h(\tilde{q}_2), \dots, h(\tilde{q}_n)]^T.$

Using equations (8) and (9), we can obtain the derivative of equation (10) as follows:

$$J_i^* \dot{s}_i^* = x_i + u_i + \Delta M_i, \quad (11)$$

where x_i is the certain dynamics of the MUAVs:

$$x_i = -S(\omega_i) \cdot J_i^* \cdot \omega_i + J_i^* \cdot [S(\tilde{\omega}_i) \cdot R(\tilde{Q}_{i,d}) \cdot \omega_d - R(\tilde{Q}_{i,d}) \cdot \dot{\omega}_d] + J_i^* \cdot k_1 \cdot \left(\frac{1}{2}\tilde{q}_i + \tilde{q}_{i0} \cdot I_3\right) \cdot \tilde{\omega}_i + J_i^* \cdot k_2 \cdot \dot{h}(\tilde{q}_i), \quad (12)$$

where J_i^* and ΔJ_i are the nominal inertial matrix and uncertain inertial matrix of the i th UAV, and the ΔM_i denotes the lumped disturbances.

3. Main Results

3.1. Nonlinear Distributed Disturbance Observer. In practical environments, disturbances cannot avoid in the dynamics of the MUAVs. In [22], distributed traditional observer has been designed for the multiple agent systems, which is widely used to reconstruct the external disturbances. However, the disturbance cannot compensate in finite time. Inspired by reference [32], a nonlinear disturbance observer is to be designed to compensate the lumped disturbances in equation (11), and we denote $\Delta \tilde{M}_i$ as the output of the observer. The NDO is designed as

$$\Delta \tilde{M}_i = \phi_i + P_i, \quad (14)$$

$$P_i = n_1 \cdot J_i^* \cdot \tilde{\omega}_i + n_2 \cdot \int_0^t \text{sig}^{n_m}(\Delta \tilde{M}_i) d\tau + n_3 \cdot \int_0^t \text{sgn}(\Delta \tilde{M}_i) d\tau, \quad (15)$$

$$\dot{\phi}_i = -n_1 \cdot (x_i + u_i + \Delta\widehat{M}_i), \quad (16)$$

where n_1, n_2, n_3 and n_m are positive constants and $\Delta\widehat{M}_i = \Delta M_i - \Delta\widetilde{M}_i$ is the estimate error of ΔM_i .

Theorem 1. Under Assumption 1 and Assumption 2, consider the closed-loop systems (15), the designed NDO (18) guarantees that the estimation of lumped disturbances $\Delta\widehat{M}_i$ converges to the neighborhoods of ΔM_i in finite time.

Proof. Construct the following Lyapunov function candidate for the lumped disturbances:

$$V_{\Delta M} = \frac{1}{2} \cdot \Delta\widehat{M}_i^T \cdot \Delta\widehat{M}_i. \quad (17)$$

Taking the derivative of $V_{\Delta M}$, $\dot{V}_{\Delta M} = \Delta\widehat{M}_i^T \cdot (\Delta\dot{\widehat{M}}_i)$.

Applying equation (15), we can obtain the following inequality:

$$\begin{aligned} \dot{V}_{\Delta M} &= \Delta\widehat{M}_i^T \cdot [\Delta\dot{M}_i - n_1 \cdot \Delta\widetilde{M}_i - n_2 \cdot (\Delta\widetilde{M}_i)^{n_m} \\ &\quad \cdot \text{sgn}(\Delta\widetilde{M}_i) - n_3 \cdot \text{sgn}(\Delta\widetilde{M}_i)]. \end{aligned} \quad (18)$$

Under Assumption 3, the lumped disturbances is bounded, $\|\Delta M_i\| \leq \rho$, where ρ is a positive constant.

$$\begin{aligned} \dot{V}_{\Delta M} &\leq \sqrt{2} \cdot (n_3 - \rho) \cdot V_{\Delta M}^{1/2} - 2 \cdot n_1 \cdot V_{\Delta M} \\ &\quad - 2^{(n_m+1/2)} \cdot n_2 \cdot V_{\Delta M}^{(n_m+1/2)}. \end{aligned} \quad (19)$$

By Lemma 3, the estimation error $\Delta\widehat{M}_i$ will converge to the residual set within a finite time. The residual set of $\Delta\widehat{M}_i$ is $\Delta\widehat{M}_i \leq \sqrt{2} \cdot ((\rho - n_3)/2)^{(n_m+1/2)} \cdot n_2^{1/n_m}$. \square

Remark 2. The parameter n_1 determines the convergence speed of the lumped disturbances. Hence, a trade-off between stability and convergence speed should be considered. Furthermore, we will consider the upper bound of the lumped disturbances and first derivatives unknown in future work.

3.2. Observer-Based FNTSMC Design. Based on the NDO and FTNSMC technique, a decentralized cooperative controller for MUAUVs is proposed, which can achieve a fast finite-time convergence result without singularity in the control input.

Theorem 2. Consider the attitude dynamics (2)–(4) and suppose that Assumption 2 holds. With the proposed NDO (14)–(16), the decentralized finite-time control law (20) guarantees that the tracking errors \tilde{q}_i and $\tilde{\omega}_i$ converge to the regions in finite time, respectively.

The attitude coordinated control law is proposed as

$$\begin{aligned} U &= -X - \Delta\widehat{M} + [(L+B) \otimes I_3]^{-1} \times [-Z \\ &\quad \cdot S - \beta \cdot \text{sgn}(S) - \alpha \cdot \text{sgn}(S) \cdot |S|^r], \end{aligned} \quad (20)$$

where $\Delta\widehat{M} = [\Delta\widehat{M}_1, \Delta\widehat{M}_2, \dots, \Delta\widehat{M}_n]$, $X = [x_1, x_2, \dots, x_n]$, $X = -S(\omega_i) \cdot J_i^* \cdot \omega_i + J_i^* \cdot [S(\tilde{\omega}_i) \cdot R(\tilde{Q}_{i,d}) \cdot \omega_d - R(\tilde{Q}_{i,d})$

$\dot{\omega}_d] + J_i^* \cdot k_1 \cdot ((1/2) \cdot \tilde{q}_i + \tilde{q}_{i0} \cdot I_3) \cdot \tilde{\omega}_i + J_i^* \cdot k_2 \cdot \dot{h}(\tilde{q}_i)$, $S = [s_1, s_2, \dots, s_n]$, $\beta = \text{diag}\{\beta_1, \beta_2, \dots, \beta_n\}$, $\alpha = \text{diag}\{\alpha_1, \alpha_2, \dots, \alpha_n\}$, and $Z = \text{diag}\{z_1, z_2, \dots, z_n\}$. $z_i, \beta_i, \alpha_i \in R^{3 \times 3}$ are positive definite matrices, $r \in (0, 1)$. If β_i is selected as $\beta_i > \varepsilon_0$, where β_i is the minimum eigenvalue of β_i , ε_i is the absolute value of estimation error, $\varepsilon_i = \|(L+B) \otimes I_3\| \cdot \Delta\widetilde{M}_i$, $\varepsilon_0 \geq \varepsilon_m$, and $\varepsilon_m = \max\{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n\}$.

Proof. In the presence of the inertia uncertain matrices and external disturbances, we define the candidate Lyapunov function as follows:

$$V_S = \frac{1}{2} S^T \cdot S. \quad (21)$$

Using equations (10) and (11), taking the derivative of V_S ,

$$\begin{aligned} \dot{V}_S &= S^T \cdot \dot{S} \\ &= S^T \cdot [(L+B) \otimes I_3] \cdot [X + \Delta M + U] \\ &= S^T \cdot [(L+B) \otimes I_3 \cdot \Delta\widetilde{M} - Z \cdot S \\ &\quad - \beta_i \cdot \text{sgn}(S) - \alpha \cdot \text{sig}^r(S)] \\ &\leq -S^T \cdot Z \cdot S - S^T \cdot \alpha \cdot \text{sig}^r(S) \\ &\leq -\lambda_{\min}(Z) \cdot S^T \cdot S - \lambda_{\min}(\alpha) \cdot \|S\|^{r+1} \\ &\leq -\mu_1 \cdot V_S - \mu_2 \cdot V_S^{(r+1)/2}, \end{aligned} \quad (22)$$

where $\mu_1 = 2 \cdot \lambda_{\min}(Z)$ and $\mu_2 = 2^{((r+1)/2)} \cdot \lambda_{\min}(\alpha)$. From Lemma 2, we can conclude that the sliding mode surface S converges to its origin in finite time:

$$T_r \leq \frac{2}{\mu_1(1-r)} \ln \frac{\mu_1 \cdot V_{S(0)}^{(1-r)/2} + \mu_2}{\mu_2}, \quad (23)$$

where $V_{S(0)}$ is the initial value of V_S .

When the tracking error system is on the sliding mode manifold $s_i = 0$, three cases should be considered as follows:

Case 1. If $\tilde{s}_{ij} = 0$, $i = 1, 2, \dots, n$ and $j = 1, 2, 3$, it means that

$$\tilde{\omega}_{ij} + \delta_1 \cdot \tilde{q}_{ij} + \delta_2 \cdot \text{sig}^r(\tilde{q}_{ij}) = 0. \quad (24)$$

By Lemma 3, we can obtain $\tilde{\omega} \rightarrow 0$ and $\tilde{q} \rightarrow 0$, and the closed-loop system can achieve finite-time stability.

Case 2. If $\tilde{s}_{ij} \neq 0$ and $|\tilde{q}_{ij}| > g$, then we can obtain $s_{ij} \neq 0$; obviously, that would not occur.

Case 3. If $\tilde{s}_{ij} \neq 0$ and $|\tilde{q}_{ij}| \leq g$, then we can obtain

$$\tilde{\omega}_{ij} + k_1 \cdot \tilde{q}_{ij} + k_2 (\delta_1 \cdot \tilde{q}_{ij} + \delta_2 \cdot \text{sig}^r(\tilde{q}_{ij})) = 0, \quad (25)$$

$\tilde{\omega}_{ij}$ will converge to the residual set

$$|\tilde{\omega}_{ij}| \leq k_1 |\tilde{q}_{ij}| + k_2 |\delta_1 \cdot \tilde{q}_{ij} + \delta_2 \cdot \text{sig}^r(\tilde{q}_{ij})|, \quad (26)$$

in finite time. \square

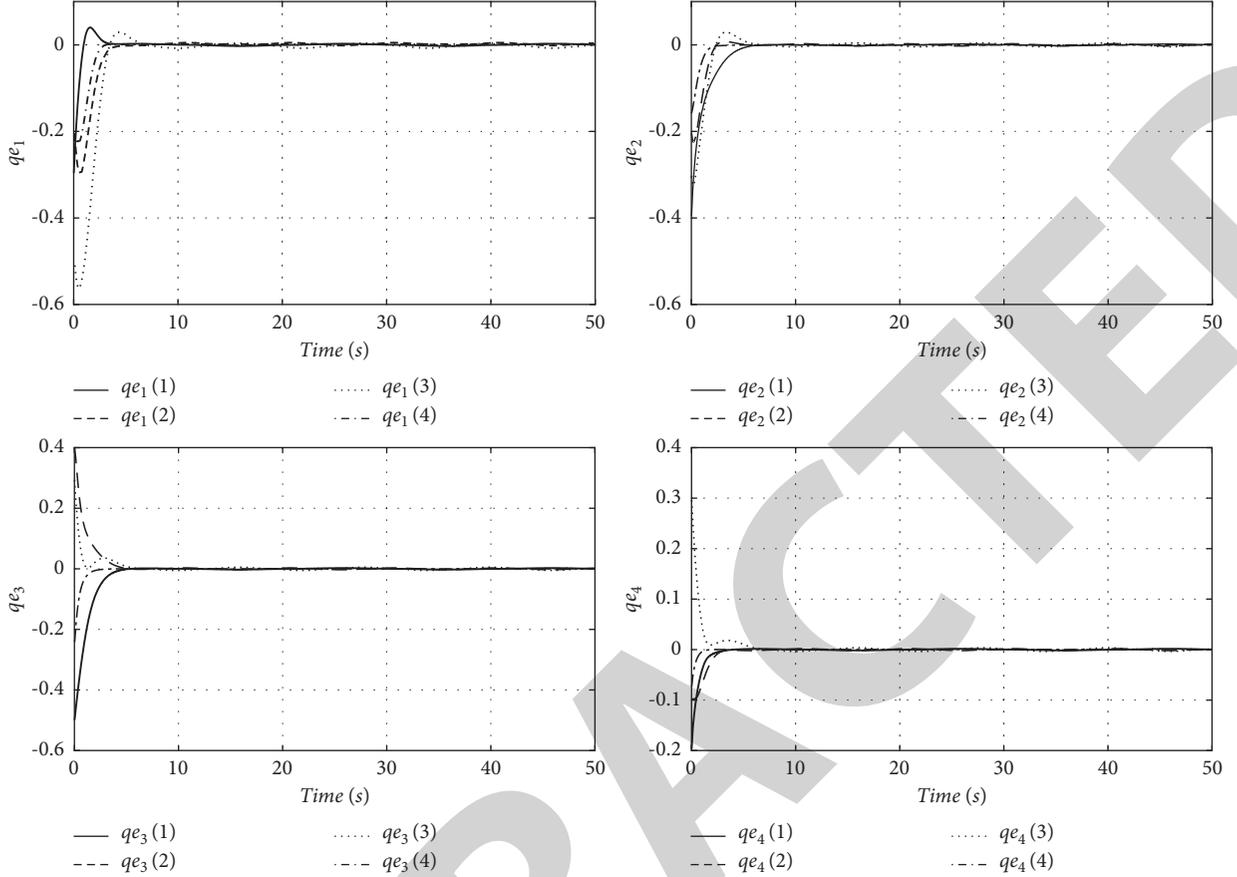


FIGURE 1: Quaternion errors of each UAV.

Remark 3. Attitude synchronization is the most important in the formation of the MUAVs. In this paper, we only consider the attitude cooperative problem, the position tracking problem is more challenging, and we will develop the work in the future.

Remark 4. The parameter β_i in the control law in designed control law is to compensate the estimation error and gives the robustness of the control law. The robustness parameter β_i can also be modified by the adaptive scheme. If we remove the parameters α_i , then the control law can guarantee that the system is asymptotically stable but have less precision performance compared with the finite-time control law.

Remark 5. This work considers the parameter uncertain and external disturbances case under directed communication topology. It is noted that in the case of the undirected communication topology, $(L+B) \otimes I_3$ does not satisfy Lemma 1, and then the item $[(L+B) \otimes I_3]^{-1}$ cannot be used in the control law. In [33], we note that the item $[(L+B) \otimes I_3]^{-1}$ can be eliminated by constructing the Lyapunov function containing $(L+B) \otimes I_3$.

Remark 6. It is noted that the item $Z \cdot S$ is the feedback control which could make that all the state signals of the MUAVs are uniformly ultimately bounded (UUB), and the term $\alpha \cdot \text{sgn}(S) \cdot |S|^r$ is the nonlinear feedback to make the system achieve the finite-time stability.

4. Numerical Example

In this section, numerical simulations demonstrate the effectiveness of the proposed observer-based FNTSMC scheme. Consider a group of UAV modeled with equations (2) and (3). The corresponding weighted Laplacian matrix is given as

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}^T. \quad (27)$$

The initial states of the UAV are arbitrary. The parameters of the MUAVs are as follows: the inertial matrix

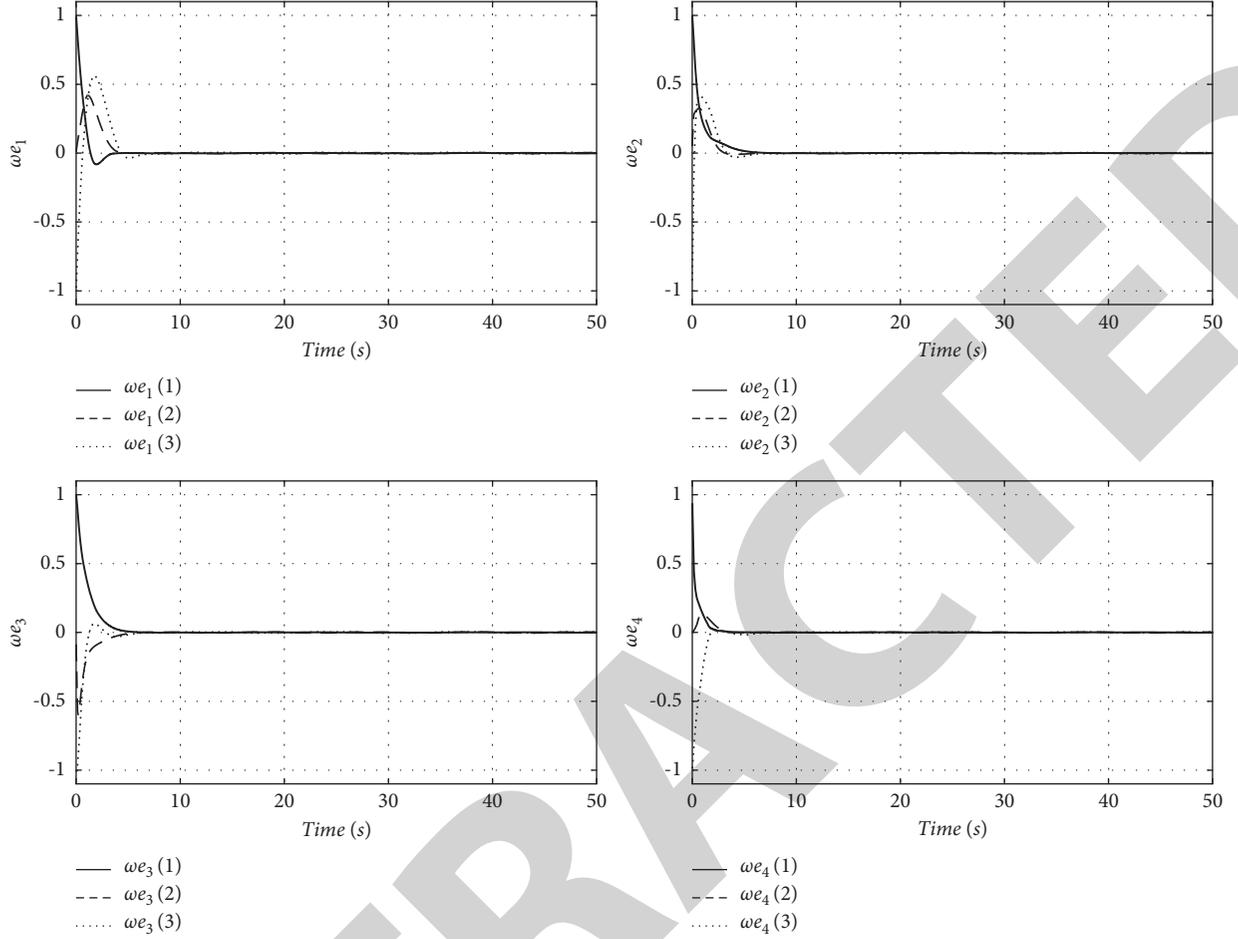


FIGURE 2: Angular velocity errors of each UAV.

$$\begin{aligned}
 J_1 &= \begin{bmatrix} 20 & 2 & 0.5 \\ 2 & 22 & 0.9 \\ 0.5 & 0.9 & 30 \end{bmatrix}, \\
 J_2 &= \begin{bmatrix} 22 & 1 & 0.9 \\ 1 & 25 & 0.3 \\ 1.5 & 0.7 & 25 \end{bmatrix}, \\
 J_3 &= \begin{bmatrix} 23 & 0.5 & 1 \\ 1 & 21 & 1.5 \\ 1 & 1.5 & 22 \end{bmatrix}, \\
 J_4 &= \begin{bmatrix} 25 & 0.5 & 1.5 \\ 0.9 & 28 & 0.5 \\ 0.5 & 0.7 & 25 \end{bmatrix},
 \end{aligned} \tag{28}$$

the nominal inertial matrix $J_1^* = J_2^* = J_3^* = J_4^* = \text{diag}\{20, 20, 20\}$ ($\text{kg} \cdot \text{m}^2$), $Q_d(t) = [0 \ 0 \ 0 \ 1]^T$, and $\omega_d(t) = [0 \ 0 \ 0]^T$, and the parameter of $k_1 = 1$, $k_2 = 0.3$, $\beta = \text{diag}\{0.2, 0.2, 0.2\}$, $n_1 = 20$, $n_2 = n_3 = 0.5$, $n_m = 0.4$, $r = 0.6$, $z_1 = z_2 = z_3 = z_4 = 2$, $\alpha = \text{diag}\{2, 2, 2\}$, $d_1 = [0.5 \sin(\pi t/10), -0.6 \cos(\pi t/5), 0.8 \sin(\pi t/6)]^T$, $d_2 = [0.5 \cos(\pi t/10), -0.6 \cos(\pi t/5), -0.8 \sin(\pi t/6)]^T$, and

$d_3 = [0.5 \sin(\pi t/10), -0.6 \cos(\pi t/5), 0.8 \cos(\pi t/6)]^T$, and $d_4 = [0.5 \cos(\pi t/10), -0.6 \cos(\pi t/5), 0.8 \sin(\pi t/6)]^T$.

The quaternion \bar{q}_i ($i = 1, 2, 3, 4$) and angular velocity errors $\tilde{\omega}_i$ ($i = 1, 2, 3, 4$) of each UAV are shown in Figures 1 and 2, respectively. From Figures 1 and 2, we can see the quaternion errors and angular velocity errors converge in finite time, respectively.

For convenience, let RFC represent the designed controller [36]; the response for the RFC and the FNTSMC is shown in Figures 3 and 4. Based on the proposed FNTSMC, the attitude of the MUAVs converges to equilibrium point smoothly with a settling time in less than 10 s. The results show that FNTSMC has faster convergence rate and higher control performance than the RFC controller, indicating that FNTSMC has a better disturbances rejection than the RFC controller.

Figure 5 shows the curves of the lumped disturbances. It is clear that reconstruction of the lumped disturbances can be obtained. The lumped disturbances contain the disturbances and uncertainty. The errors between the estimation and the real values converge to zero not only asymptotically but also achieve finite-time stability. The precise estimation errors showed the high performance of the decentralized finite-time NDO. In the finite-time disturbance observer

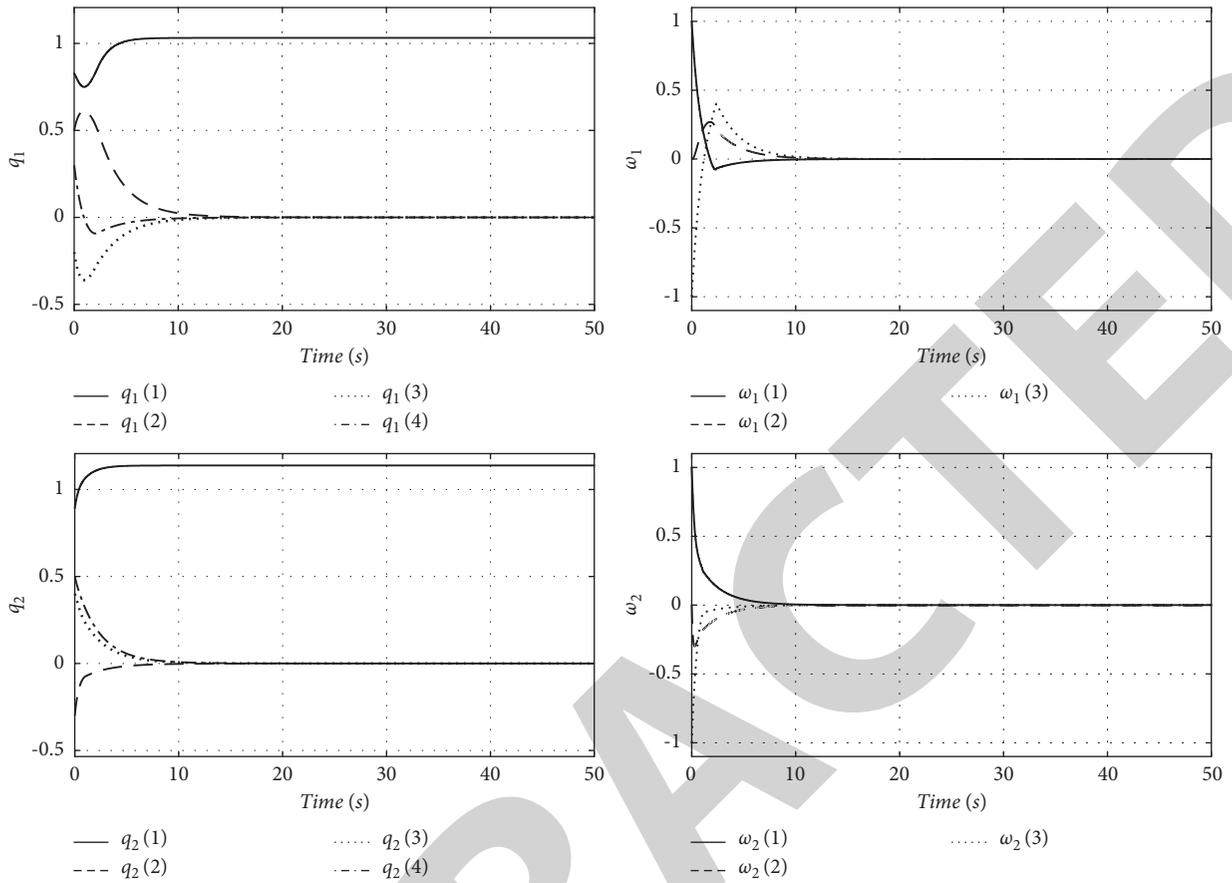


FIGURE 3: The attitude and angular velocity of each UAV based on RFC.

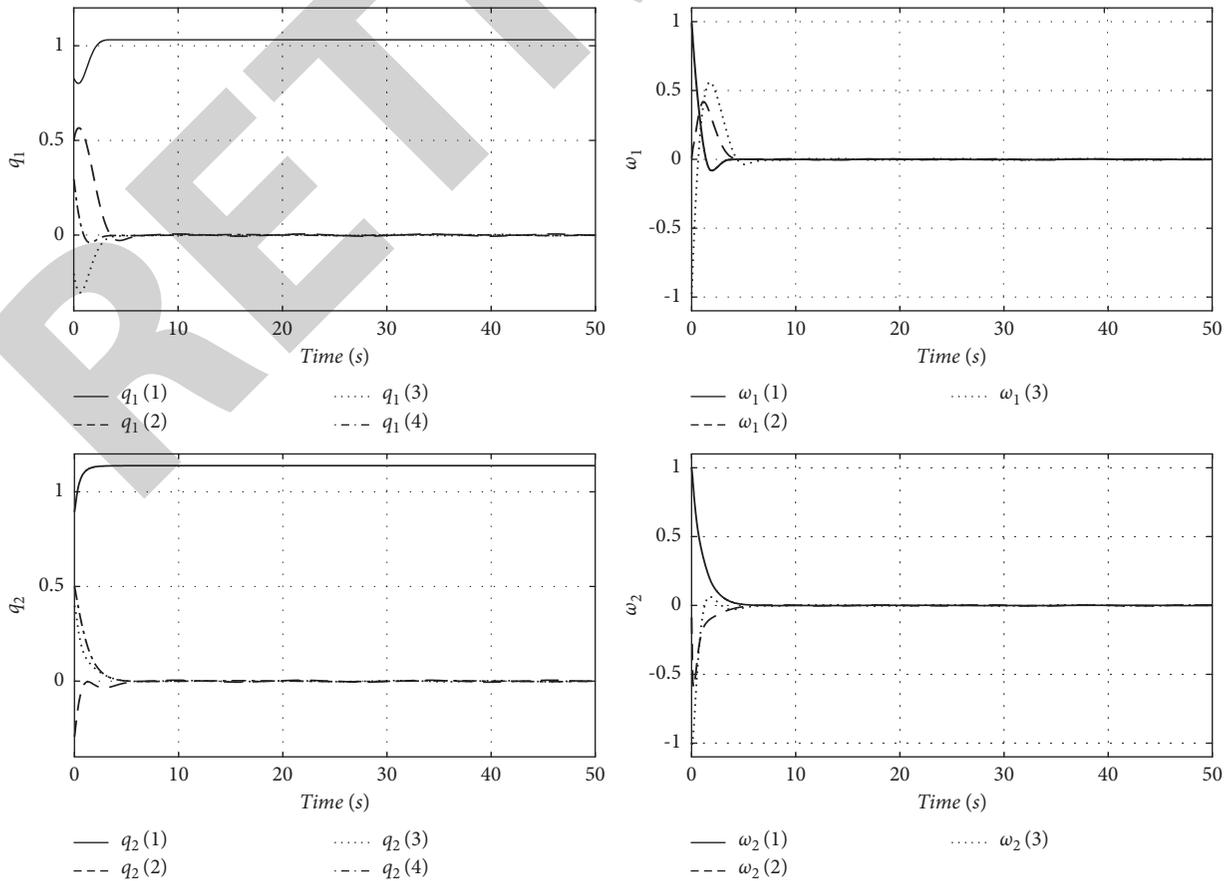


FIGURE 4: The attitude and angular velocity of each UAV based on FNTSMC.

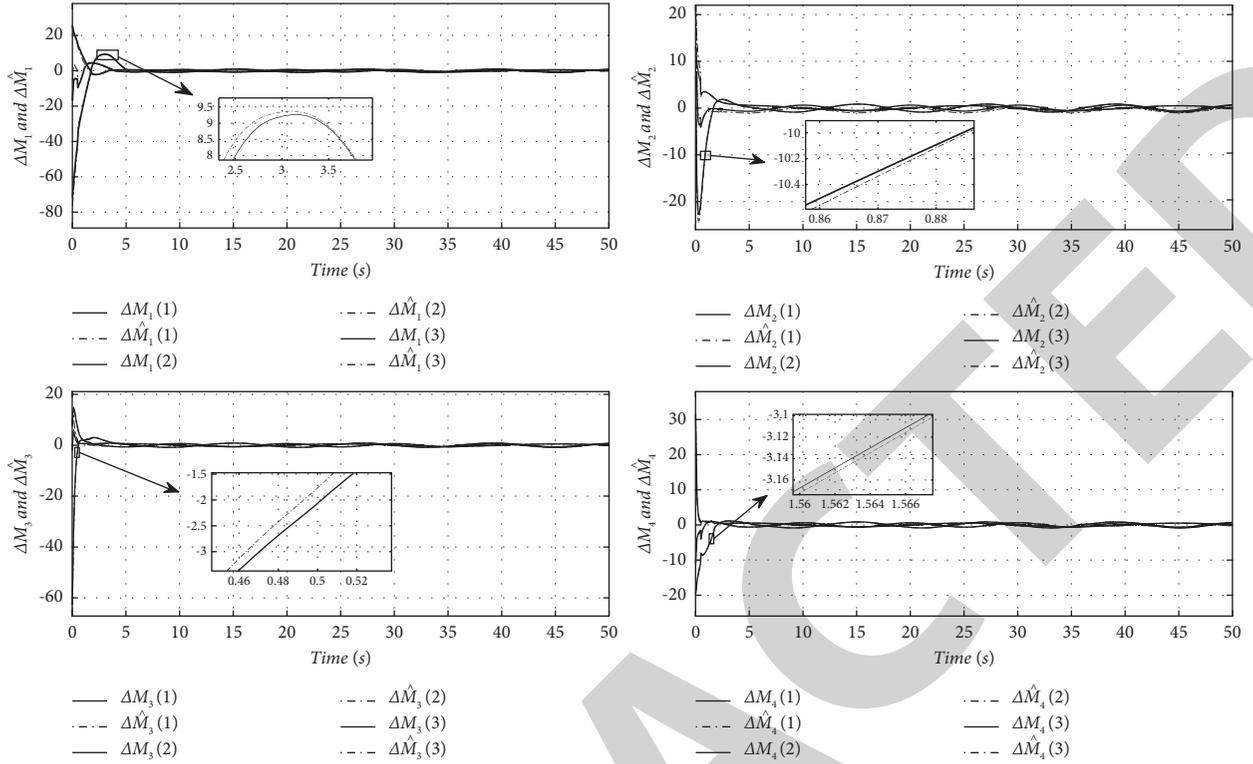
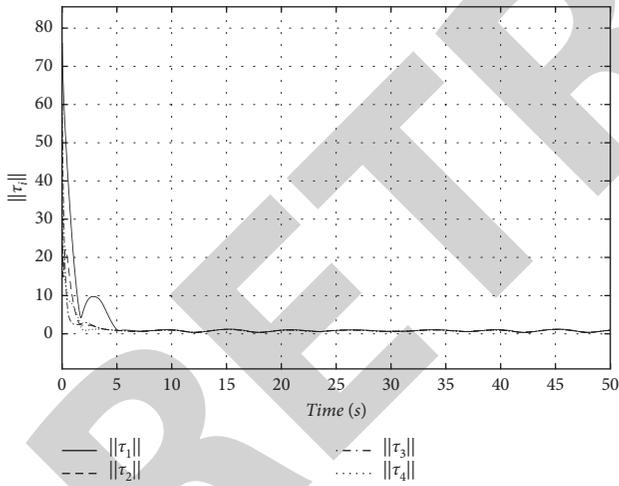

 FIGURE 5: Actual observer output ΔM and estimated observer output $\hat{\Delta M}$.


FIGURE 6: Control torques of each UAV.

technique, the parameter n_1 determines the convergence rate of observer errors of lumped disturbances. Based on the feedforward compensation, the switch gain is greatly reduced and the chattering is effectively reduced. It is noted that the finite-time NDO is given in equations (14)–(16); if we remove the term $n_2 \cdot \int_0^t \text{sig}^\gamma(\Delta \hat{M}_i) d\tau$, then we can get the similar observer proposed in [25]. Figure 6 shows the control torques of each UAV. It is observed from the above Figures 5 and 6 that the finite-time attitude cooperative controllers for MUAVs have high control performance.

5. Conclusion

This paper has solved the cooperative problem of multiple unmanned vehicles composed of one leader and many followers under directed topology. A distributed finite-time nonlinear disturbance observer has been proposed to estimate the lumped disturbances containing the inertia uncertainties and external disturbances. Based on the nonlinear observer, decentralized finite-time terminal sliding mode controller has been proposed such that each UAV can track the desired attitude and angular velocity. Future work includes extending the results to the case of communication delays and no angular velocity.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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