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Research Article

Portfolio Real Option Based on Trigeminal Tree Model in Sustainable Utilization of Exploration Asset

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Uncertainty of crude oil market causes the value of oil and gas exploration assets to fluctuate upward, remain unchanged, and fluctuate downward. So exploration project investment is usually chartered by multistages. This brings many challenges to investment decisions regarding the sustainable utilization of exploration assets. To avoid the investment risk of exploration projects, a portfolio option method based on the trigeminal tree model was proposed. By analysing the real options involved in oil and gas exploration assets, the investment process is divided into European options and American options. The pricing model of portfolio options is constructed by the trigeminal tree method, and sequential compound diagram and compound option diagram are built. Volatility is predicted based on the GARCH model, and then, the trinomial tree of the underlying asset value, evolution chart, sequential compound option chart, and portfolio option chart is established, respectively. Finally, the project option value is calculated. The application shows that the ROV of oil and gas exploration project of company A is 1.39 million US$ (USMM$), and the project value is 279.87 USMM$. Compared with the binary tree model, the model is superior to the traditional option pricing method in the application effect, which provides a scientific basis for investment decision-making.

1. Introduction

Real options are derived from financial options; therefore, their classification can refer to real financial options. According to the different execution times of option contracts, real options are divided into American and European options. After Stewart Myers introduced the real option theory into the investment field, the option theory and method have made great progress in solving the problem of option pricing and project risk [1–3] and have been extended to the fields of land development, mine operation, and mining investment [4–10]. For example, Acciaro applied the real option model to calculate the correlation between LNG investment cost and fuel price and suggested that decision-makers should adopt advanced technology to keep LNG price at a certain level [11]. However, most of the current research results continue the premise and assumption of financial option pricing method; it is unreasonable and undesirable. Some scholars have begun to focus on more complex compound option pricing model, but the application of compound option pricing in oil investment has not been reported publicly.

Due to the uncertainty of oil and gas market, oil and gas exploration project investment is usually chartered by multistages. In order to reduce uncertain market risks, decision-makers must formulate flexible investment strategies to deal with complex and changeable markets [12]. According to the stages of oil and gas projects and the mutual influence of each stage, the compound options are involved in the decision-making of oil and gas assets and can be divided into two types: sequential options and compound options. The oil and gas exploration projects witness the common situations: when the oil market presents optimistic, the enterprise will expand oil and gas production and carry out the expansion period to obtain more profits; if the oil market turns down, the decision-maker can sell some shares to reduce the loss as much as possible. At this time, there are contraction options [13–18]. If the market continues to depress, the decision-maker can sell the assets and exercise the abandonment option. There are three options at three points in time; that is to say, at same time, the decision-maker has the right to choose a variety of options [19], which constitutes a compound option. The paper innovates means to establish the pricing model of portfolio options by the trigeminal tree method.

2. Theory and Methodology

At present, the research on trigeminal tree option pricing model is generally limited to single option, but there are three forms of oil and gas asset value in global crude oil market: upward fluctuation, constant fluctuation, and downward fluctuation, so investment decision-making is more complex. To solve this problem, the trinomial tree model extends from single option to compound option, which includes time series option and compound option.

2.1. Single Option Model in Investment Decision. $S(t)$ represents the underlying asset value of time t change. In order to construct the trigeminal tree model, the continuous variable $S(t)$ needs to be discretized into equal intervals of N time intervals $\Delta t = T/\mathrm{N}$. The change of the asset value of the internal standard in a certain time interval is shown in the following Figures 1 and 2, and the option value changes are as follows in the chart.

According to the definition of mathematical expectation,

$$E(S) = P_u S_u + P_m S + P_d S_d, \tag{1}$$

$$E\left(S^2\right) = P_u (S_u)^2 + P_m (S)^2 + P_d (S_d)^2, \tag{2}$$

$$E\left(S^3\right) = P_u (S_u)^3 + P_m (S)^3 + P_d (S_d)^3. \tag{3}$$

According to the hypothesis, the change of the value of the underlying asset follows the geometric Brownian motion, and the expected rate of return of the underlying asset u is the risk-free rate of return r. Therefore,

$$E(S) = Se^{r\Delta t}. \tag{4}$$

According to Ito's lemma, the differential equations of S^2 and S^3 are obtained.

$$\frac{dS^2}{S^2} = \left(2r + \sigma^2\right) d_t + 2\sigma d_z, \tag{5}$$

$$\frac{dS^3}{S^3} = \left(3r + 3\sigma^2\right) d_t + 3\sigma d_z. \tag{6}$$

Formulas (2) and (3) are discretized, and the following results are obtained:

$$\frac{\Delta S^2}{S^2} = \left(2r + \sigma^2\right) d_t + 2\sigma \Delta_z, \tag{7}$$

$$\frac{\Delta S^3}{S^3} = \left(3r + \sigma^3\right) d_t + 3\sigma \Delta_z. \tag{8}$$

The integral solution can be obtained:

$$E\left(S^2\right) = S^2 e^{\left(2r+\sigma^2\right)\Delta t}, \tag{9}$$

$$E\left(S^3\right) = S^3 e^{\left(3r+3\sigma^2\right)\Delta t}. \tag{10}$$

Formulas (1)–(4), (9), and (10) in parallel can be immediately derived the following equations:

$$P_u + P_m + P_d = 1, \tag{11}$$

$$ud = 1, \tag{12}$$

$$e^{r\Delta t} = uP_u + P_m + dP_d, \tag{13}$$

$$e^{\left(2r+\sigma^2\right)\Delta t} = u^2 P_u + P_m + d^2 P_d, \tag{14}$$

$$e^{\left(3r+3\sigma^2\right)\Delta t} = u^3 P_u + P_m + d^3 P_d. \tag{15}$$

The equations are solved and simplified by Taylor's expansion formula, and the calculation formulas of rising factor u, falling factor d, and risk neutral probabilities P_u, P_m, and P_d are obtained as follows:

$$u = e^{\sigma\sqrt{3\Delta t}} = 1 + \sigma\sqrt{3\Delta t} + \frac{3\sigma^2 \Delta t}{2} + o\left(\sqrt{\Delta t}\right), \tag{16}$$

$$d = e^{-\sigma\sqrt{3\Delta t}} = 1 - \sigma\sqrt{3\Delta t} + \frac{3\sigma^2 \Delta t}{2} + o\left(\sqrt{\Delta t}\right), \tag{17}$$

$$P_u = \frac{1}{6} + \left(r - \frac{\sigma^2}{2}\right)\sqrt{\frac{\Delta t}{12\sigma^2}} + o\left(\sqrt{\Delta t}\right), \tag{18}$$

$$P_m = \frac{2}{3} + o\left(\sqrt{\Delta t}\right), \tag{19}$$

$$P_d = \frac{1}{6} - \left(r - \frac{\sigma^2}{2}\right)\sqrt{\frac{\Delta t}{12\sigma^2}} + o\left(\sqrt{\Delta t}\right). \tag{20}$$

2.2. Sequential Compound Option Model in Investment Decision. Sequential option is a kind of real option due to the stage of investment. It is assumed that an investment project can be divided into two stages. The first stage is the initial investment stage with a period of 1 year and the cost requires C1; the second stage is the expansion investment stage, with a period of 2 years and the cost is C2. It is known that the logarithmic current value of the project's net income is zero, and there is no risk in the future. According to the above parameters, use formulas (16)–(20); the ascending factor u, the descending factor D, and the risk neutral probabilities P_u, P_m, and P_d can be derived from formulas (1)~(5). This is the basis of the sequential option pricing model.

First of all, establish the value of the underlying assets trigeminal tree (Figure 3).

Then, calculate value of the second stage, so as to establish the net asset value chart of long-term options (Figure 4).

The determination of nodes in the net asset value chart needs to use the back steeping technology, which is obtained from the end in turn.

The value of the terminal node is determined and took $F(S_0u^3, 3)$ as an example.

$$F(s_0u^3) = \max\left(u^3s_0 - c_2, 0\right). \tag{21}$$

Refer to this rule to determine the value of the last column node. The intermediate node is selected by comparing the value of exercise option and maintenance option and took$F(S_0u^3, 2)$ as an example:

$$\begin{aligned} F(S_0u^2, 2) = \max\{u^2S_0 - C_2, e^{-r\Delta t}P_u \cdot F(S_0u^3, 3) \\ + P_m \cdot F(S_0u^2, 3) + P_d \cdot F\}, \end{aligned} \tag{22}$$

where C_2 is the option execution cost, $e^{-r\Delta t}\lceil P_u \cdot F(S_0u^3, 3) + P_m \cdot F(S_0u^2, 3) + P_d \cdot F(S_0u, 3)\rceil$ is the value when the option continues to be hold in the node, $u^2S_0 - C_2$ is the return on the exercise of the option, and the larger one is selected to determine the remaining node values.

Finally, on the basis of trigeminal tree with the long-term option net asset value, the option valuation chart is established, and the total option value is composed of the above option and the shorter option in the first stage. Then, the trigeminal tree of sequential option valuation is obtained to get the trigeminal tree of compound option valuation (Figure 5).

The analysis of the trigeminal tree of sequential option valuation depends on the one of net asset value of long-term option. $V(S_0u, 1)$ is taken as an example to determine the value of terminal nodes:

$$V(S_0u, 1) = \max\left(F(S_0u, 1) - C_1, 0\right), \tag{23}$$

According to this rule, the end node value is determined.

The option value of the initial node is derived from the later node.

$$V(S_0, 1) = e - r\Delta t[Pu \cdot V(S_0u, 1)]. \tag{24}$$

If there is an intermediate node, the rule can be still used: max (the value of holding the option, the income from exercising the option) to determine the option valuation at all times, and finally obtain the option value at the initial time.

Finally, establish the joint option valuation chart of sequential conforming to the option (Figure 6), and the value of the sequential option is $V(S_0, 0)$.

2.3. Portfolio Option Model. Portfolio option in oil and gas investment decision-making is a kind of compound option due to the multiple choices of investment schemes. If the oil company adopts strategic option strategy to reduce risks, in any time in the next three years, the decision-makers can choose to reduce it to a fraction of the previous business scale according to the market, so as to save cost a. Or expand the business scale several times at the cost of B. Or give up the current project and harvest the residual value C. The present value of the expected income of the oil company is S_0, the implied volatility of logarithm yield of expected return cash flow is σ, and the nonrisk interest rate of the next five years is r. It can be seen from the case that the company holds a portfolio option including expansion option, curtailment option, and abandonment option in the next three years and still uses the trigeminal tree method to construct the pricing model of portfolio option.

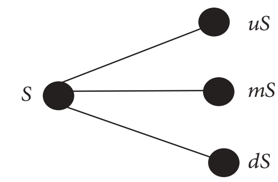


Figure 1: The change of underlying assets.

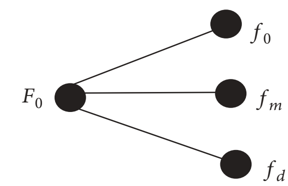


Figure 2: The change of option values.

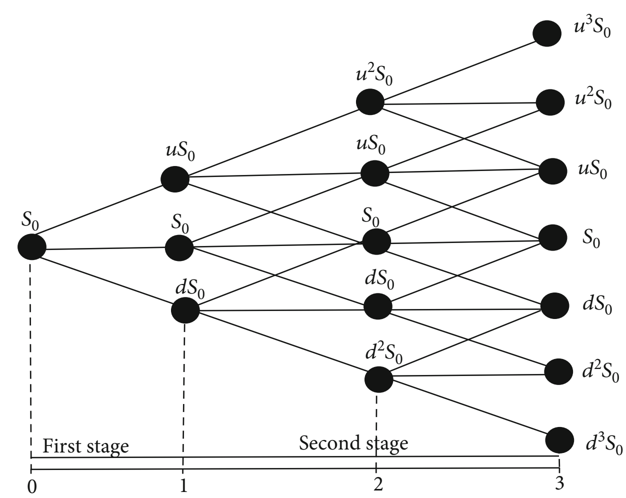


Figure 3: The trinomial tree of multistage compound of underlying asset.

Firstly, according to the known parameters, Taylor's expansion formulas (1)–(5) are applied to calculate the rising factor, decline factor, and risk neutral probability, and the compound option pricing model could be established. The expansion factor b is known, as well the expansion cost B, the contraction factors a, the reduced balance A, and the abandoned residual value C. Based on the above data, a trigeminal tree of the underlying asset value is established (Figure 7).

Secondly, establish the trigeminal tree of portfolio option valuation.

Decision rules of the end node are given and taken $F(S_0u^2, 3)$ as an example.

$$F(S_0u^2, 3) = \max\{b \cdot S_0u^2 - B, a \cdot S_0u^2s\}, \tag{25}$$

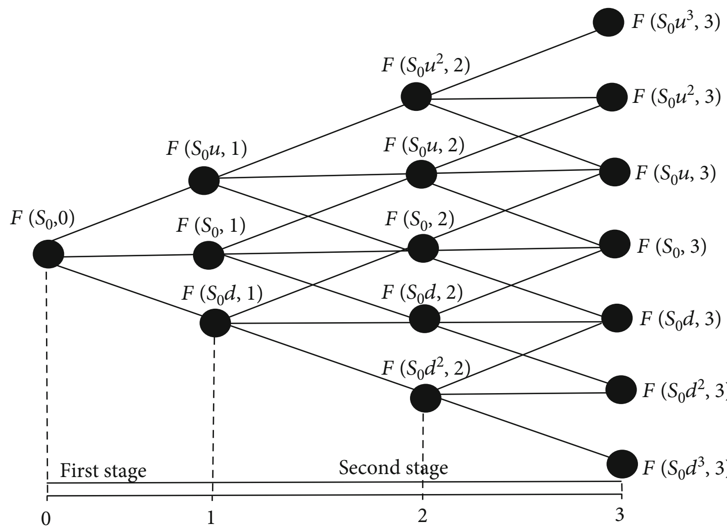


Figure 4: The trinomial tree of net value of underlying asset.

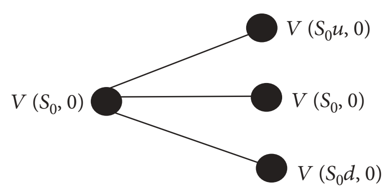


Figure 5: Trinomial tree of sequential options valuation.

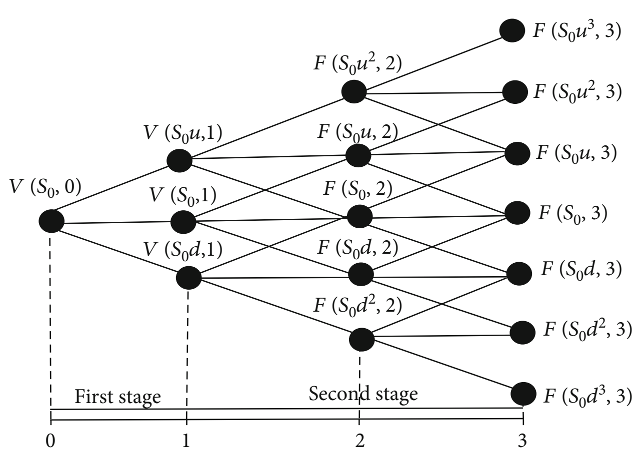


Figure 6: The trinomial tree of sequential compound options.

where $b \cdot S_0 u^2 - B$ represents the expansion value and $a \cdot S_0 u^2 + A$, C, S_0 are the reduced value. C is the value of exercising abandonment options, $S_0 u^2$ is the value of maintaining the status quo, and the maximum can be selected as the end node value.

Decision rules of intermediate node value are given and taken $F(S_0u, 1)$ as an example.

$$F(S_0u, 1) = \max \{b \cdot S_0 u - B, a \cdot S_0 u + A, C, e - r\Delta t \left[P_u \cdot F\left(S_0 u^2, 2\right) + Pm \cdot F(S_0 u, 2) + P_d \cdot F(S_0, 2)\right]\}, \quad (26)$$

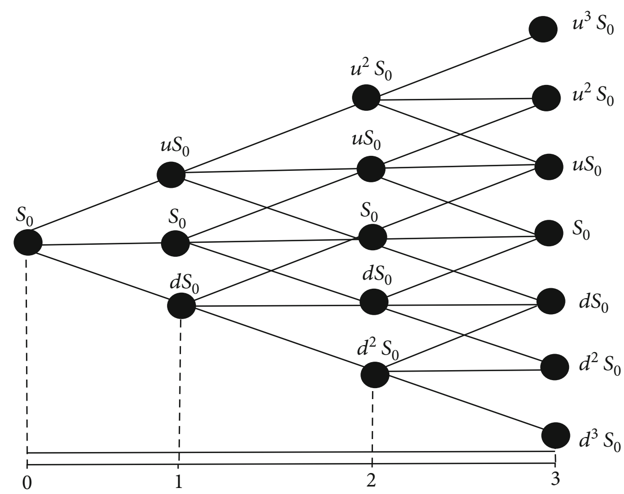


Figure 7: The trinomial tree of underlying asset.

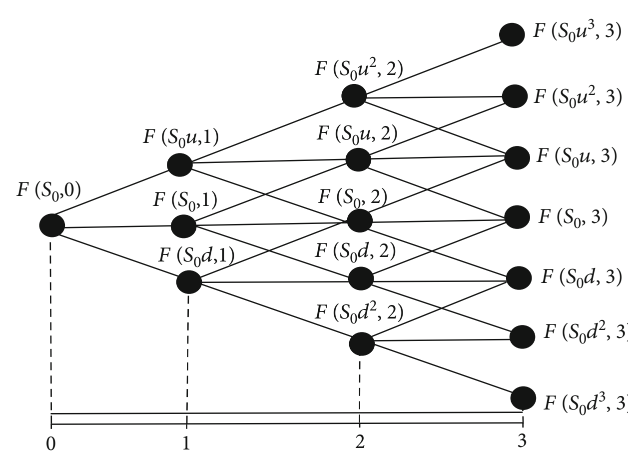


Figure 8: The trinomial tree of combined compound options.

Table 1: Net cash flow of company A in the next ten years.

Year	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
NCF/USMM$	-240	34.4	73.4	229.4	104.6	43.5	-4.6	34.3	115	156.6	73.4

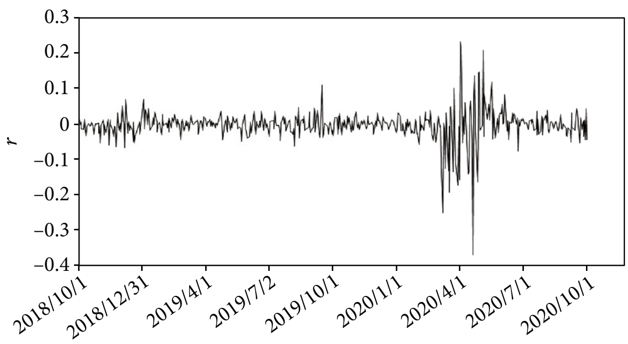


Figure 9: The time series of logarithmic rate of return.

where $e^{-r\Delta t}\lceil P_u \cdot F(S_0u2, 2) + P_m \cdot F(S_0u, 2) + P_d \cdot F(S_0, 2)\rceil$ represents the value of the point to maintain the status quo. The value of each node is determined in turn by the back stepping technique, and finally, trigeminal tree valuation chart of the portfolio option is obtained (Figure 8).

Finally, the price of the portfolio option is $F(S_0, 0) - S_0$.

Brent oil is one of the most widely traded crude on the futures market, accounting for about 65 percent of global crude volume. The expansion option described in the paper is a European option with a term of two years. Therefore, the data period is set to two years, namely, September 30, 2018, to 2020.10.1, which meets the research needs.

2.4. Option Volatility Estimation. The reason why the trigeminal tree model shows a trend of up and down changes is that the underlying asset value has volatility. The parameter of two volatility σ, which represents the change of asset value [20], is the key factor affecting the value of options. In the option pricing model, the accuracy of σ is a key to evaluate the value of investment projects accurately. In the study of real options, four kinds of volatility are mainly involved: implied volatility, actual volatility, historical volatility, and forecast volatility. Forecast volatility is commonly used in the study of option value calculation. Generally speaking, forecasting volatility is different from historical volatility; it comes from the understanding of historical volatility and people's prediction and judgment of the actual situation.

In the actual trading process, the historical volatility is often used as the basic data for forecasting, and then based on the continuous correction of quantitative data, the forecast volatility is finally determined. In this paper, EViews software is applied to establish GARCH (1, 1) model to predict volatility, the formula is shown in (27).

$$\sigma^2 = \alpha_0 + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{27}$$

where ε_{t-1}^2 is the lag term of residual square and σ_{t-1}^2 is the lag term of conditional variance. The parameters α and β in the model are nonnegative, and α_0 is a positive number.

3. Case Study

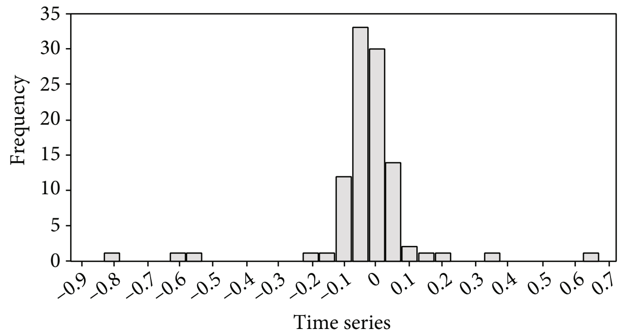
3.1. Case Background. At present, the oil company A has an oil and gas exploration block abroad, covering an area of 360 km^2. Three oil fields have been discovered with 230 million tons of oil reserves. In 2019, the crude oil production reached 3.5 million tons, and there are 5 prospective traps. Undiscovered oil resources are estimated to be 90 million tons. At the beginning of 2020, due to the outbreak of 2019-nCoV, the price of Brent crude oil dropped to below 40 US$/bbl. The profit of company A fell sharply, and it lost 240 USMM$ in 2020. In order to ensure the sustainable development, decision-makers plan to increase the crude oil production to 5.5 million tons in 2022 when the oil price rises to 60 $/bbl. The total exploration wells, development wells, and surface construction related to the increase of production will cost 55 USMM$. If the operation benefit is not optimistic after the production increase, the company will choose to farm out 50% of the block shares in the next three years of production rising to obtain 140 million US dollars of income, or all exploration blocks to obtain 280 million US dollars. Assuming that the production period of the block is 10 years, the remaining value D of the block is $6 million. According to the characteristics of the oil and gas industry, the nonrisk interest rate in the next five years is determined as 4.5%.

3.2. Option Analysis. The overall investment plan of the above case contains both sequential options and compound options, and the whole investment process can be divided into two stages: the first stage is two-year expansion option, which can be regarded as European rising option; the second stage is the compound option with three valid years, which composed of contraction option and abandonment option and can be regarded as American option. The execution of the first stage expansion option is the basis of the second stage compound option. Therefore, the two-stage expansion option and the portfolio option constitute the time series portfolio option.

Company A forecasts the cash flow and income in the next 10 years according to its own operation and management situation and the expectation of future income (Table 1).

According to the net cash flow (NCF) method, company A discounts the income of the next 10 years to the initial node. NPV will be as follows:

$$\text{NCF} = \sum_{i=1}^{10} \frac{ai}{(1+R)^t}, \tag{28}$$



Series:	R
Sample:	10/01/2018-09/30/2020
Observations	104
Mean	−0.004737
Median	−0.000740
Maximum	0.676375
Minimum	−0.805116
Std. dev.	0.146216
Skewness	−1.442369
Kurtosis	17.99977
Jarque-Bera	1011.031
Probability	0.000000

Figure 10: The time series of logarithmic rate of return.

where R is the discount (10%) and D is the residual value of the block.

In NCF method, NCF represents the whole value of exploration investment project, but in option pricing method, NCF is only the value at the starting point in trigeminal tree model of underlying assets, and the final value of exploration investment project is the sum of NCF and option value of project.

3.3. Estimation of Volatility. Only the impact of global oil prices on expected revenue is considered in the paper, and other factors are out of consideration in the process of production and operation currently. Using GARCH (1BI) and considering the influence of historical data, the volatility of the freight rate within the validity period of the option is predicted as the value of σ in the option pricing.

3.4. Date Processing. Brent oil price is selected as the sample data, and the oil price index series $\{L\}$ is established in EViews. The premise of using GARCH model to predict volatility is that the sample data is stationary, the volatility of crude oil market is large, and the sequence $\{L\}$ is obviously not stationary.

Therefore, the logarithmic difference of oil price is processed, that is,

$$r_t = 1nL_{t-1}. \tag{29}$$

On this basis, the logarithmic rate of return $\{r\}$ of the oil price index is constructed, and the time series of the logarithmic rate of return is obtained (Figure 9).

It can be observed that under the influence of the new crown epidemic, the fluctuation of logarithmic yield of oil price is mainly concentrated in the first half of 2020, especially in May 2020, and its fluctuation is quite large, and the fluctuation in the rest of the time is relatively stable.

By using the numerical analysis function of EViews, the frequency analysis and statistics of the sample data are carried out, and the column statistical chart of logarithmic rate of return is obtained (Figure 10).

It can be seen from Figure 10 that the Jarque-Bera test value of sequence $\{r\}$ is 1011.031, the corresponding p value is 0, the skewness is -1.442367, and the peak value is 17.99977. The sample data shows an obvious trend of left thick tail, so the rejection sequence is subject to the original hypothesis of normal distribution.

Null hypothesis: R has a unit root
Exogenous: constant
Lag legnth:1 (automatic-based on SIC, maxiag=18)

		1-statistic	Prob.*
Augmented Dickey-Fuller test statistic		−7.391646	0.0000
Test critical values:	1% level	−3.495677	
	5% level	−2.890037	
	10% level	−2.582041	

*Mackinnon (1995) one-sided p values.

Augmented Dickey-Fuller test equation
Dependent variable: D (R)
Method: Least squares
Date: 06/29/2020 Time: 17:38
Sample (adjusted): 10/01/2018-9/30/2020
Included ovservations: 102 after adjustments

Variable	Coefficient	Std. error	1-statistic	Prob.*
R (-1)	−1.020184	0.138019	−7.391646	0.0000
D (R-1)	0.082367	0.100446	0.820014	0.4142
C	−0.005156	0.014715	−0.350555	0.7267

R^2	0.473920	Mean dependent var	0.000777
Adjusted R^2 c	0.463292	S.D. dependent var	0.202529
S.E. of regression	0.148374	Akaike into criterion	−0.949195
Sum squared resid	2.179460	Schwarz criterion	−0.871990
Log likelihood	51.40895	Hannon-Quinn criter	−0.917932
F-statistic	44.59209	Durbin-Watson star	2.014837
Prob (F-statistic)	0.000000		

Figure 11: Adf unit root test.

3.5. Stationarity Test. In order to investigate the stationarity, the unit root test of the sequence $\{r\}$ is carried out (Figure 11).

As can be seen from Figure 11, the statistical value of t is -73.91646 and the corresponding p value is close to 0, indicating that the sequence $\{r\}$ is stable.

3.6. Autocorrelation and Partial Autocorrelation Test. In order to judge the degree of correlation between sample data and analyse the time series characteristics of data fluctuations, the Correlogram module is used to test the autocorrelation and partial autocorrelation of oil price return series (Figure 12).

As can be seen, the autocorrelation coefficient and partial autocorrelation coefficient of the sequence are close to 0, and the corresponding p values are greater than 0.05. Therefore, it accepts the original hypothesis that $\{r\}$ has no significant correlation at 5% significance level.

Correlogram of R

Date: 06/29/2020 Time: 17:38
Sample (adjusted): 10/01/2018-9/30/2020
Included observations: 104

Autocorrelation	Partial correlation		AC	PAC	Q-stat	Prob.
		1	0.056	0.056	0.3544	0.552
		2	−0.078	−0.082	1.0196	0.601
		3	−0.129	−0.121	2.8417	0.417
		4	−0.055	−0.048	3.177	0.529
		5	−0.098	−0.115	4.2546	0.513
		6	−0.053	−0.07	4.5736	0.6
		7	−0.0064	−0.095	5.0403	0.655
		8	−0.003	−0.042	5.0412	0.753
		9	−0.008	−0.053	5.0484	0.83
		10	0.04	−0.004	5.2339	0.875
		11	0.069	0.034	5.7937	0.887
		12	−0.007	−0.04	5.7991	0.926
		13	−0.046	−0.049	6.0551	0.944
		14	0.073	0.077	6.7034	0.946
		15	−0.078	−0.099	7.4487	0.944
		16	−0.11	−0.104	8.9608	0.915
		17	0.022	0.033	9.0233	0.94
		18	0.027	−0.018	9.1178	0.957

Figure 12: The Correlogram result of series r.

Correlogram of Y

Autocorrelation	Partial correlation		AC	PAC	Q-stat	Prob.
		1	−0.042	−0.042	0.1873	0.665
		2	0.440	0.439	21.135	0.000
		3	0.003	0.040	21.136	0.000
		4	−0.035	−0.283	21.274	0.000
		5	0.011	−0.021	21.288	0.001
		6	−0.039	0.142	21.462	0.002
		7	−0.049	−0.057	21.733	0.003
		8	−0.056	−0.120	22.089	0.005
		9	−0.052	0.003	22.403	0.008
		10	−0.040	0.074	22.592	0.012
		11	−0.024	−0.022	22.661	0.020
		12	−0.037	−0.091	22.828	0.029
		13	−0.003	0.015	22.829	0.044
		14	−0.024	0.045	22.900	0.052
		15	−0.037	−0.077	23.072	0.083
		16	−0.012	−0.054	23.088	0.111
		17	−0.041	0.024	23.300	0.140
		18	−0.032	−0.013	23.435	0.174
		19	−0.023	−0.051	23.505	0.216
		20	−0.041	−0.031	23.731	0.254
		21	−0.022	0.007	23.797	0.303
		22	−0.036	−0.025	23.971	0.349
		23	−0.037	−0.050	24.153	0.395

Figure 13: The Correlogram result of residual square series.

Dependent variable: D (R)
Method: ML-ARCH (Marquardt)-normal distribution
Date: 06/29/2020 Time: 17:38
Sample (adjusted): 10/01/2018-9/30/2020
Included ovservations: 102 after adjustments
Convergence achieved after 128 iterations
Presample variance: backcast (parameter=0.7)
GARCH=C(2)+C(3)×RESID(−1)^2+C(4)×GARCH(−1)

Variable	Coefficient	Std. error	1-statistic	Prob.*
C	0.008708	0.017161	0.507733	0.6116
Variance equation				
C	0.007935	0.001554	5.105348	0.0000
RESID(−1)^2	0.539366	0.235677	2.288595	0.0221
Garch(−1)	0.309374	0.137388	2.251831	0.0243

R^2	−0.008537 Mean dependent var	−0.004737
Adjusted R^2 c	−0.008537 S.D. dependent var	0.145216
S.E. of regression	0.146839 Akaike into criterion	−1.136296
Sum squared resid	2.220864 Schwarz criterion	−1.024589
Log likelihood	62.56739 Hannon-Quinn criter	−1.085091
Durbin-Watson stat	1.86567	

Figure 14: The result of GARCH (1, 1) regression.

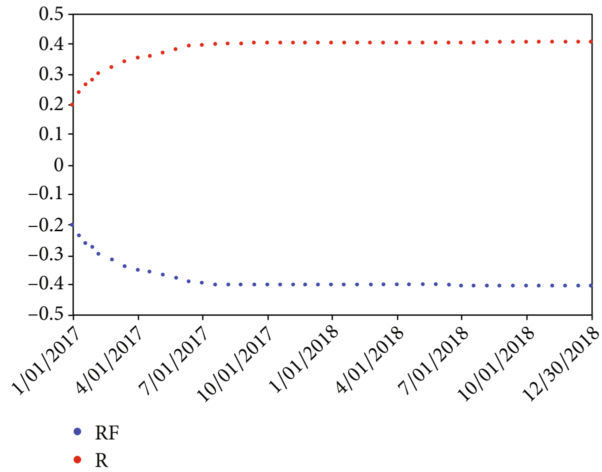


Figure 15: Historical data chart of variance.

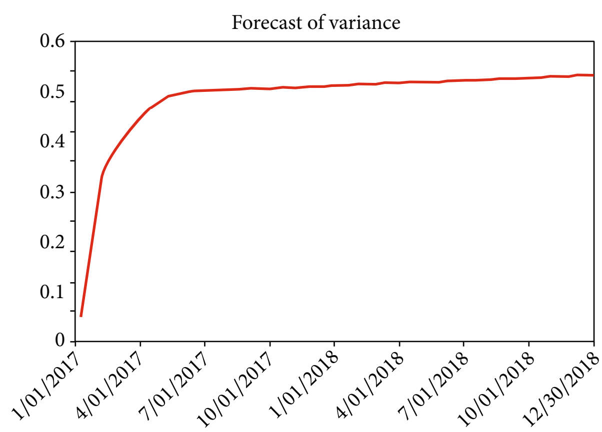


Figure 16: Forecasting chart of variance.

3.7. ARCH Effection Test. The heteroscedasticity is tested by the square correlation diagram of the residual [21], the logarithmic return series of oil price is dehomogenized, the residual sequence is obtained, a new residual sequence $\{r\}$ is established, and the Correlogram module is applied to verify the autocorrelation of the residual series (Figure 13).

According to the observation of p value, there is autocorrelation in the square sequence of residual error, so there is ARCH effection, and the GARCH model can be established.

3.8. Establishment of GARCH (1, 1) Model. Using the equation function of EViews, the calculation results of GARCH (1, 1) model are obtained (Figure 14).

There are three parameters of α_0 and α and β estimated by GARCH (1, 1), which have passed t-test at the significance level of 5%. According to formula (30), $\alpha_0 = 0.007935$, $\alpha = 0.539368$, $\beta = 0.309374$, and the available volatility is forecast as follows:

$$\sigma^2 = a_0 + a\varepsilon_t^2 + \beta\sigma_{t-1}^2 = 0.007935 + 0.539368\varepsilon_t^2. \quad (30)$$

3.9. Volatility Calculation. On the basis of the influence of historical data, the volatility of selling oil price during

Table 2: Forecasting sheet of variance.

Forecast	Actual	Forecast sample Start	Forecast sample End	Adjusted sample Start	Adjusted sample End	Root mean squared error	Mean absolute error	Mean abs. percent error	Theil inequality coefficient	Bias proportion	Variance proportion	Covariance proportion
RF	R	1/01/2017	12/30/2018	1/08/2017	12/30/2018	0.146132	0.071640	108.7168	0.947079	0.008455	NA	NA

Table 3: Parameters of compound options.

Parameters	S0	T	σ	μ	d	P_u	P_m	P_d	a	b	A	B	C
Value	278.47	5	2.8%	1.484	0.674	0.191	0.667	0.142	0.5	1.2	140	55	280
Unit	USMM$	Year									USMM$	USMM$	USMM$

the period of option validity is predicted as the value of σ in option pricing. The model is mainly bulit by the econometric software EViews. The two curves in Figure 15 represent the confidence level of the rate of return R, which is 95% (Figure 15). In the interval, with the increase of forecast periods, the confidence zone widens slightly. The lower curve shows the prediction results of conditional variance (Figure 16 and Table 2). As can be seen, the predicted value slowly increases and finally converges to 0.052, so the volatility is 0.2280.

According to formulas (1)–(6), the parameters for building the model are shown in Table 3.

3.10. Calculation of Option Value

3.10.1. Modelling the Trinomial Tree of the Underlying Asset Value. In this case, the effective period of the total compound option is 5 years. Therefore, there are 6 nodes in 0-5. According to the rising factor μ and the falling factor D, the trinomial tree of the underlying asset value is established (Figure 17).

3.10.2. Modelling the Evolution Chart of Underlying Assets. Due to the existence of the expansion period in the second year, the value of the underlying asset increases by 1.2 times of the previous one. Therefore, the value of the underlying asset of node 2 is changed to 1.2 times of the corresponding node in Figure 4, and the subsequent nodes are pushed forward in turn to build the evolution chart of the underlying asset (Figure 18).

3.10.3. Modelling the Trigeminal Tree of Portfolio Option Valuation. The compound option consists of expansion one in the first stage and portfolio one in the second stage. Firstly, the trigeminal tree of option valuation is built based on the second stage of portfolio options. In the last column, a decision rule is introduced: max (the balance of exercise

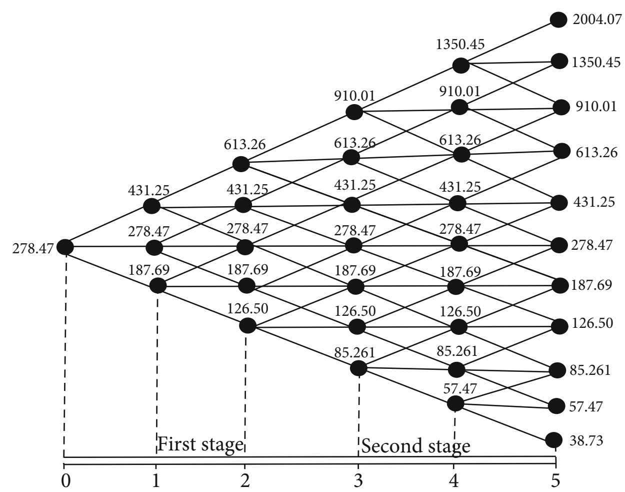


Figure 17: The trinomial tree of underlying asset value.

contraction options -1/2 of the underlying asset value of the corresponding node, the balance of the exercise and abandonment option—the underlying asset value of the corresponding node, 0), so as to determine the end node value of the trigeminal tree of portfolio option valuation. Then, the decision rule is introduced in nodes 2, 3, and 4: max (balance of exercise contraction options -1/2 of the underlying asset value of the corresponding node, the balance of the exercise and abandonment option—the value of the underlying asset of the corresponding node and the value of the option to continue to be held), where the value of the continued holding option is the discount value of the expected option return at the current time node according to the non-risk interest rate.

Taking $F(S_0u^4, 4)$ as an example, the decision rules of this point are as follows:

$$F(S_0u4, 4) = \max\left\{A - \frac{1}{2}u^4S_0, C - u^4S_0, e^{-r\Delta t}\left[P_u \cdot F\left(S_0u^5, 5\right) + Pm \cdot F\left(S_0u^4, 5\right) + Pm \cdot F\left(S_0u^4, 5\right) + P_d \cdot F\left(S_0u^3, 5\right)\right]\right\}, \quad (31)$$

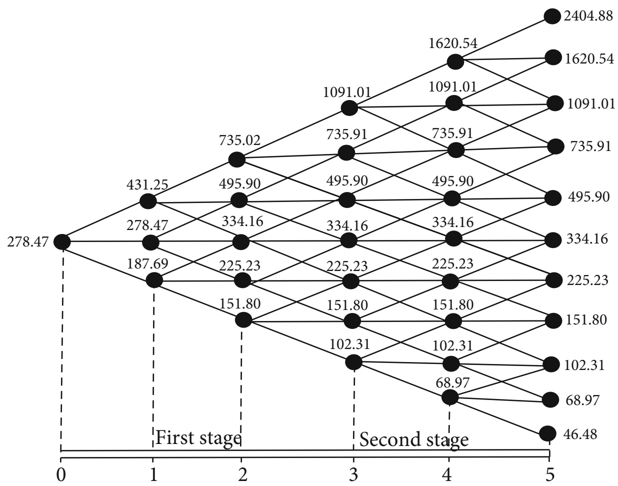


Figure 18: Evolution chart of underlying assets.

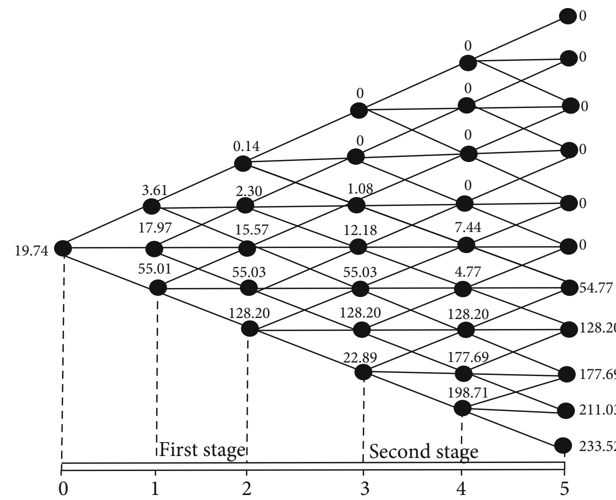


Figure 19: Trigeminal tree of portfolio option valuation.

where $e^{-r\Delta t}[P_u \cdot F(S_0u^5, 5) + P_m \cdot F(S_0u^4, 5) + P_d \cdot F(S_0u^3, 5)]$ is the discount value of the future income of the options held at note $F(S_0u^4, 4)$, so as to determine the node value and establish the valuation chart of portfolio options (Figure 19).

3.10.4. Modelling Trigeminal Tree Valuation Chart of Sequential Options. Company A holds expansion options in the first stage, so it is necessary to construct the valuation chart of overall conforming options based on the trigeminal tree valuation chart of portfolio options and the impact of expansion options. For nodes 0, 1, and 2, the decision rule max (the value of the exercise expansion option, 0) is used at the end node to determine the valuation of the last column. Since the expansion options of the company belong to European options, the value of the intermediate node does not need to be compared, and the value of the maintenance option can be calculated directly. Given calculation formula (11) of intermediate node value, option value is determined by $V(S_0u, 1)$.

$$V(S_0u, 1) = e^{-r\Delta t}\left[P_u \cdot V\left(S_0u^2, 2\right) + \right]. \quad (32)$$

Finally, the option value of the initial node, that is, the value of the overall compound option$V(S_0, 0)$, is derived from backward technique of trigeminal tree (Figure 20).

Through trigeminal tree pricing model, the value of the compound option ROV is 1.39 USMM$, and the value of oil and gas exploration project is NCF + ROV =279.87 USMM$.

4. Comparison and Discussion

In order to further clarify the advantages and disadvantages of trigeminal tree model, according to the parameters in Table 3, the binary tree method is applied to recalculate the assets, and the results are compared and analysed.

The first step is to construct the underlying asset grid and use table (Table 4) instead of binary tree. According to the given parameters, the underlying asset grid is obtained by backward technique of trigeminal tree.

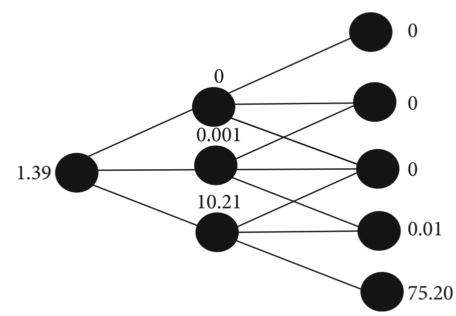


Figure 20: Trigeminal tree valuation chart of sequential options.

The second step is to construct the evolution grid of the underlying assets, as shown in Table 5.

The third step is to construct the compound option value grid, as shown in Table 6.

The compound option value V of the asset is 2.74 USMM$, that is, the Expected Net Cash Flow ENCF is as follows: ENCF = NCF + OV = 278.47 + 2.74 = 281.21 USMM$.

However, the asset value obtained by the trigeminal tree pricing model is 279.87 USMM$, which is lower than that of the binary tree model. This is because the value of the underlying asset remains unchanged by the binomial tree model, which is a kind of waiting option and does not change over time, so the option value is overestimated. This change is fully considered in the trigeminal tree model, and the value of the asset falls with the continuous forward discount. Therefore, the accuracy of the trigeminal tree model is higher than that of the binary tree model.

However, the trigeminal tree option pricing model is based on the known accurate future income and investment cost. In the process of estimating the option value, it is difficult to predict the exact value of income and cost. Because of the volatility of investment outlook, the range of the rise and fall in the underlying asset value itself has certain uncertainty. Therefore, although the trigeminal tree model considers random value variance, it ignores the uncertainty of the underlying asset itself.

Table 4: Underlying asset sheet of compound options.

Grid series	0	1	2	3	4	5
Asset value/USMM$	278.47	352.54	446.32	565.04	715.34	905.62
	—	219.96	278.47	352.54	446.32	565.04
	—	—	173.75	219.96	278.47	352.54
	—	—	—	137.24	173.75	219.96
	—	—	—	—	108.41	137.24
	—	—	—	—	—	85.63

Table 5: Evolution grid of the underlying asset.

Grid series	0	1	2	3	4	5
Asset value/USMM$	278.47	352.54	535.58	678.05	858.41	1086.75
	—	219.96	334.17	423.06	535.59	678.05
	—	—	208.50	263.96	334.17	423.06
	—	—	—	164.69	208.50	263.96
	—	—	—	—	130.09	164.70
	—	—	—	—	—	102.76

Table 6: Compound option value sheet.

Grid series	0	1	2	3	4	5
Asset value/USMM$	2.74	0	0	0	0	0
	—	6.20	0	3.13	0	0
	—	—	14.043	35.22	7.08	0
	—	—	—	115.31	71.50	16.04
	—	—	—	—	149.91	115.30
	—	—	—	—	—	177.24

In order to effectively define the above uncertainties, it is necessary to introduce the fuzzy theory into the real option investment decision-making model of oil and gas exploration assets in the future research and establish a trigeminal tree model based on fuzzy theory and real options, so as to improve the shortcomings of the classic trigeminal tree model.

5. Conclusions

(1) The paper constructs the trigeminal tree option pricing model. Compared with the binary tree model, it has faster convergence speed and higher calculation accuracy because of allowing more possibilities within the time interval. By analysing the possibility that the underlying assets remain unchanged, it further clarifies the existence of investment extension option, which has better applicability in calculating the value of oil and gas exploration assets

(2) The GARCH (1, 1) model is built to predict the volatility based on the historical data to improve the accuracy of the parameters

(3) Due to many factors such as resources and oil prices, the value of frontier exploration assets and low to medium exploration assets is highly uncertain. The trigeminal tree model proposed in this paper can be widely used in the value evaluation of the above assets, effectively improve forecasting precision, and provide scientific basis for investment decision-making

Data Availability

All data, models, and code generated or used during the study have been submitted in the paper.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to affect the work reported in this paper.

Authors' Contributions

Yiping Wu supervised the study; Jianjun Wang supervised the study and was responsible for the analysis; Qing Wang was responsible for the experiment; Qian Li was responsible for the analysis; Haowu Li was responsible for the data analysis; Ningning Zhang was responsible for the results; Qingchao Cao was responsible for the experiment.

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