Research Article

On the Performance of the Unified NOMA Framework for Satellite-Terrestrial Networks with Hardware Impairments

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A unified nonorthogonal multiple access (NOMA) framework for satellite-terrestrial network is investigated over Shadowed-Rician fading channels, which can be applied to both power domain NOMA (PD-NOMA) and code domain NOMA (CD-NOMA). Under the realistic assumption of hardware impairments (HIs) and imperfect successive interference cancellation (SIC), the achievable outage probability of the paired users is analyzed first, and the closed-form expressions of exact and asymptotic outage probability are derived. Next, the ergodic sum rate is investigated in different levels of HIs and residual interference. In addition, the energy efficiency in the delay-tolerant transmission mode is studied to characterize the performance of the satellite-terrestrial network. Finally, the Monte Carlo simulation results demonstrate the accuracy of theoretical derivation and showcase the effects of a wide range of satellite channel parameters and the realistic assumption on outage probability, ergodic rate, and energy efficiency.

1. Introduction

The satellite-terrestrial network has been envisioned as one of the research focuses to provide global service to users in future wireless communication networks [1]. A number of recently published studies have demonstrated its superiority over traditional satellite communications. Nonorthogonal multiple access (NOMA) is an effective approach to provide massive connectivity and is considered a new paradigm for the multiple access technology [2–4]. Therefore, introducing NOMA technology into the satellite-terrestrial network can improve spectrum efficiency and resource utilization [5, 6].

The application of NOMA technology to satellite communications is a promising way to further expand the integration of space and earth. The NOMA-based hybrid satellite-terrestrial relay system was investigated, in which the outage probability, asymptotic outage probability, and achievable rate were derived in [7]. In order to analyze the multilayer heterogeneous satellite network, the authors in [8] proposed a NOMA-enabled two-layer geostationary earth orbit/low earth orbit (LEO) satellite network, where the ergodic capacity of the proposed system was investigated in the case of coexistence of frequencies. In [9], a dedicated half-duplex (HD) terrestrial decode-and-forward (DF) relay was used to assist a user with weak channel condition to complete communications with the satellite. Moreover, in [10], the authors extended the work of [9] to investigate an amplify-and-forward multiantenna relay scenario and performed the performance analyses for fixed gain and variable gain relaying protocols, in which the exact and asymptotic outage probability expressions were obtained. In order to guarantee the transmission delay and transmission rate for satellite industrial Internet of things, the authors in [11] proposed the satellite-terrestrial integrated industrial Internet of
things and optimized the power distribution ratio of terrestrial users, thus achieving the purpose of reducing transmission cost. Considering the various delay quality of service (QoS) requirements for satellite Internet of things (S-IoT) applications, the authors in [12] investigated the delay-limited performance of NOMA-based downlink S-IoT networks by analyzing the effective capacity and proved the effects of different power distribution schemes on the proposed networks. In [13], the authors investigated the NOMA-assisted integrated satellite terrestrial networks with the partial relay selection scheme under imperfect successive interference cancellation (SIC), which can improve the spectral efficiency and expand the transmission coverage area. In [14], uplink and downlink NOMA were considered in intersatellite communication networks, where the optimum control methods were employed to delay suppression, reduce bit error rate, and increase throughput. The authors in [15] adopted an unmanned aerial vehicle using the DF protocol to assist satellite communicated with the terrestrial NOMA users; specifically, the outage probability and system throughput in delay-limited transmission mode were deduced to validate the superiority of the proposed system. In order to maximize the system capacity and energy efficiency, the NOMA architecture-enabled space-terrestrial satellite network was investigated, in which the satellite and terrestrial base stations provided services in a collaborative manner for users, and the NOMA scheme was only implemented for terrestrial networks [16]. More recently, a great deal of works have focused on the performance of hybrid satellite-terrestrial networks in cognitive radio (CR) scenarios [17–20]. In [17], the NOMA-based cognitive hybrid satellite-terrestrial overlay networks were proposed, in which the outage probability performance was investigated. One of the assumptions was that the power distribution scheme of NOMA users was determined by instantaneous channel conditions. The partial secondary network selection schemes were analyzed for NOMA-based overlay cognitive integrated satellite-terrestrial relay networks in [18], which proved the superiority of the proposed scheme. In [19], the authors derived the closed-form expression of outage probability for the primary satellite network, and the secondary transmitter was regarded as a relay to assist the primary transmission. Furthermore, the authors of [20] investigated the performance of the secondary network in terms of outage probability, sum throughput, and ergodic rate in which the nearby NOMA user operated in full-duplex mode and adopted DF relay protocol to improve the performance of the remote NOMA user.

Unfortunately, for some practical reasons such as quantization error, phase noise, in-phase/quadrature imbalance, and limitations on transceiver performance, the transmission nodes are vulnerable to hardware impairments (HIs) [21, 22]. However, the HIs from different sources have a certain randomness, resulting in these impairments which cannot be eliminated satisfactorily [23]. In [24–26], the authors proposed a realistic scenario in which the effects of HIs and imperfect channel state information (CSI) on the performance of simultaneous wireless information and power transfer NOMA relaying network were considered. The authors in [27] studied the outage performance of the NOMA-based integrated satellite-terrestrial relay network in the presence of HIs. In [28], the authors derived a closed-from expression for the outage probability in the NOMA-based satellite communication network considering HIs and channel estimation errors. The authors of [29] investigated the impacts of HIs on the secrecy performance of the NOMA-based integrated satellite multiple-terrestrial relay network.

However, existing NOMA schemes can be classified into two categories: power domain NOMA (PD-NOMA) and code domain power NOMA (CD-NOMA) according to the characteristics of resource mapping and spread spectrum mode. This division method is not conducive to the unification of the NOMA standard. Therefore, in order to facilitate the unification of NOMA technical solution, NOMA is divided into single-carrier NOMA and multicarrier NOMA according to the number of resource element (RE) or subcarrier occupied by users. CD-NOMA is a typical multicarrier NOMA, which uses a sparse matrix to map the data flow of multiple users to multiple REs or subcarriers. PD-NOMA can be regarded as a single carrier NOMA to realize user reuse by power adjustment. Therefore, from this perspective, PD-NOMA can be considered a special case of CD-NOMA, and CD-NOMA can be regarded as a special extension of PD-NOMA. While the aforementioned works have provided us with a better understanding of the application of PD-NOMA technology in satellite-terrestrial networks, the analysis for the unified NOMA framework-based satellite-terrestrial networks is far from well understood, which inspires the construction of this paper. In addition, from the perspective of practical application, it is of great significance to investigate the effects of HIs and imperfect SIC on satellite-terrestrial networks. In this paper, we propose a unified NOMA framework for satellite-terrestrial networks and analyze the performance of the proposed system with HIs and imperfect SIC under Shadowed-Rician fading channels. The essential contributions of this paper are summarized as follows:

(1) We propose a unified NOMA technology framework for satellite-terrestrial networks, which integrates two design patterns, PD-NOMA and CD-NOMA. A sparse matrix is applied to map the data stream of K users to N REs, and the unified NOMA framework can be transformed into PD-NOMA scheme \((N = 1)\) and CD-NOMA scheme \((N > 1)\) according to the different numbers of RE occupied by users.

(2) Considering two imperfections, HIs and imperfect SIC, we investigate the outage performance of the proposed system and derive the closed-form expressions of exact outage probability for the paired NOMA users. More specifically, the asymptotic outage probability is provided to get more insights into the high SNR region. Additionally, the impacts of a wide range of satellite parameters on network performance are studied.

(3) We study the ergodic rate of the unified NOMA framework for the satellite-terrestrial network, and the effects of HIs and residual interference on ergodic rate are discussed. In addition, the energy...
efficiency in the delay-tolerant transmission mode is investigated to characterize the performance of the proposed system.

(4) Numerical results validate the theoretical analysis and the advantages of the satellite-terrestrial network adopting the NOMA scheme. The HIs and residual interference have significant negative effects on the outage performance, ergodic sum rate, and energy efficiency. Moreover, a pair of terrestrial users is able to achieve better system performance as the number of REs increases.

The remainder of this article is organized as follows. In Section 2, the unified NOMA framework for satellite-terrestrial network is presented and the instantaneous SINR is derived. Section 3 provides the exact and asymptotic outage performance for a pair of terrestrial NOMA users. Then, in Section 4, the ergodic sum rate of satellite-terrestrial NOMA networks is analyzed. Section 5 obtains the numerical Monte Carlo results to verify the theoretical analysis. This paper is summarized in Section 6, and the mathematical proof is shown in Appendix.

The main mathematical symbols used in this paper are shown as follows: $F_X(\cdot)$ and $f_X(\cdot)$ denote the cumulative distribution function (CDF) and the probability density function (PDF) of a random variable $X$, respectively; $E\{\cdot\}$ is the expectation operator. $\|\cdot\|^2_2$ denotes the Euclidean two norms of a vector; $\odot$ is the dot product of two matrix elements.

## 2. System Model

Consider a unified downlink NOMA scenario for the satellite-terrestrial network, where the satellite (S) transmits the superimposed signals to the $K$ terrestrial users directly or with the aid of the DF relay (R), as depicted in Figure 1. In particular, S maps the data flows of $K$ users into $N$ time/frequency REs by using a sparse spread spectrum with AWGN) with zero mean and average power of $N_0$. The superimposed signals to the $K$ users are disturbed by additive white Gaussian noise (AWGN) or with the aid of the DF relay ($K$). By applying the SIC principle, the instantaneous received signal to interference and noise ratio (SINR) of $D_k$ is worse than that of $D_p$ due to the influence of shadow fading. In addition, due to the propagation characteristics and large transmission delay of the satellite channel, it is difficult to obtain perfect CSI for the satellite channel, which has been studied in [6, 32]. However, the main aim of this paper is to investigate the performance of the unified NOMA framework for satellite-terrestrial networks with HIs and imperfect SIC. Therefore, perfect CSI is assumed in this paper, and the results of this paper will be the basis for future research on imperfect CSI.

Two phases are used throughout the transmission process. In the first phase, $S$ sends the superposed information $X = g_s\sqrt{\rho_s} u_s P'_{s} x_s + g_q \sqrt{\rho_q} u_q P'_{q} x_q$ to $R$ and terrestrial users, where the destination user’s data flow is spread over a column of $G_{N \times K}$. In addition, $P_s$ is the transmit power at $S$, $\xi = \{r, p, q\}$, $x_u$ denotes the message transmitted to $D_u$, with unit power, and $u = \{p, q\}$. $\rho_s$ and $\rho_q$ are the power allocation coefficients for $D_s$ and $D_q$ that satisfy $a_p + a_q = 1$ and $a_p > a_q$ to guarantee the users’ fairness. $G_s(\phi_q) = G_s(J_1(u_t)/2 u_t + 36 J_1(u_t)/u_t^2)$ denotes the beam gain of node $\xi$, $G_\xi$ is the antenna gain of node $\xi$, and $\psi_\xi$ denotes the included angle between $\xi$ and beam center in relation to the satellite [32]. $u_t = 2.07123\sin \psi_\xi/\sin \phi_{3dB}$, $\phi_{3dB}$ denotes the 3 dB angle, and $J_1(\cdot)$ is the first-class-modified Bessel function with order 1. Moreover, $\chi_\xi = (\ell / 4\pi f_c d_\xi)^2$ is the free space loss between the satellite and node $\xi$, where $\ell, f_c$, and $d_\xi$ denote the speed of light, operating frequency, and the distance between the satellite and node $\xi$, respectively. Accordingly, the received signals at $R$ and terrestrial users can be given by

$$y_k^r = w_k h_k^r \odot \left(X + g_{\xi} \eta_{\xi}^r\right) + w_k n_k^r,$$

where $n_k^r \sim C.A(0, N_0 I_N)$ denotes the AWGN vector for the link between the satellite and node $\xi$ and $I_N$ is an identity matrix of size $N$. $\eta_{\xi}^r \sim C.A(0, k_{\xi}^2 \rho_s P_s)$ is the HIs between S and node $\xi$, where $k_{\xi}^2$ is the level of HIs [29, 32], $g_{\eta} = [g_{1\xi} g_{2\xi} \cdots g_{N\xi}]^T$, and $g_{\eta}$ denotes the HI indication. Since maximal ratio combiner (MRC) is adopted at $D_k$ over $N$ REs, we have $w_k = (h_k^r)^H/\|h_k^r\|^2$, where $w_k$ represents the receive weight vector. Assuming $\rho_{\xi}, \|h_k^r\|^2 > \rho_{\xi}, \|h_k^r\|^2$, and applying the SIC principle, the instantaneous received signal to interference and noise ratio (SINR) of $R$ decoding $x_q$ can be given by

$$I_{R, x_q}^r = \frac{\|h_k^r \odot g_{\xi}\|^2_2 a_p \rho_s \gamma}{\|h_k^r \odot g_{p}\|^2_2 a_p \rho_s \gamma + \|h_k^r \odot g_{q}\|^2_2 k_{\xi}^2 \rho_s \gamma + 1},$$

where

$$I_{R, x_q} = \frac{\|h_k^r \odot g_{\xi}\|^2_2 a_p \rho_s \gamma}{\|h_k^r \odot g_{p}\|^2_2 a_p \rho_s \gamma + \|h_k^r \odot g_{q}\|^2_2 k_{\xi}^2 \rho_s \gamma + 1}.$$
where \( y = P_y / N_0 \). By applying the SIC scheme, \( R \) begins decoding \( x_p \) after decoding \( x_q \). However, taking into consideration the influence of imperfect SIC, the instantaneous received SINR of \( R \) decoding \( x_p \) can be represented as

\[
\Gamma_{R,x_q}^1 = \frac{\| h_p^r \otimes g_{|p} \|^2}{\| h_p^r \otimes g_{|q} \|^2 + \| h_p^q \otimes g_{|q} \|^2 + 1},
\]

(3)

where \( \kappa \) represents the level of residual interference from the previous SIC process and \( 0 \leq \kappa \leq 1 \) (it is worth noting that we use the imperfect SIC model in [33, 34], assuming that \( \kappa \) is a constant. However, \( \kappa \) obeys the Gaussian distribution in some complicated cases [35, 36], which is beyond the scope of our research, but we will consider it in future work).

Similar to (2) and (3), we can express the instantaneous received SINRs via the \( S-D_p \) link and \( S-D_q \) link at respective nodes as

\[
\Gamma_{D_p,x_q}^1 = \frac{\| h_p^r \otimes g_{|p} \|^2}{\| h_p^r \otimes g_{|q} \|^2 + \| h_p^q \otimes g_{|q} \|^2 + 1},
\]

(4)

\[
\Gamma_{D_q,x_q}^1 = \frac{\| h_q^r \otimes g_{|q} \|^2}{\| h_q^r \otimes g_{|p} \|^2 + \| h_q^p \otimes g_{|p} \|^2 + 1},
\]

(5)

\[
\Gamma_{D_q,x_q}^1 = \frac{\| h_p^r \otimes g_{|q} \|^2}{\| h_p^r \otimes g_{|q} \|^2 + \| h_p^q \otimes g_{|q} \|^2 + 1},
\]

(6)

where \( \kappa_1 \) represents the level of residual interference from \( x_q \) and \( 0 \leq \kappa_1 \leq 1 \).

In the second phase, \( R \) transmits the superimposed signal \( Y = g_p^r \sqrt{P_R} b_p P_X + g_q^r \sqrt{P_R} b_q P_X \) to \( D_p \) and \( D_q \) with power \( P_R \) and power allocation coefficient \( b_u \), where \( b_p + b_q = 1 \), \( \Pi_u = 1/d_u^2 \), \( d_u \) denotes the distances from \( R \) to \( D_u \), and \( \Delta \) is the pathloss exponent. By spreading the terrestrial user’s data stream over a column of the matrix \( G_{N \times K} \), the received signal at \( D_u \) is represented as

\[
y_u' = w_u h_u' \otimes (Y + g_n t_n') + w_u n_u',
\]

(7)

where \( w_u = (h_u')^H ||h_u'|| \), the distortion noise from \( R \) to \( D_u \) is denoted as \( t_n' \sim \mathcal{C} \mathcal{N}(0,k_u'^2\Pi_u P_R) \), and \( n_u' \sim \mathcal{C} \mathcal{N}(0,N_0 \kappa') \) is the AWGN at \( D_u' \).

Assume that \( d_q > d_p \) and \( y = P_y / N_0 \), the instantaneous SINR of \( D_q \) decoding its own message can be given by

\[
\Gamma_{D_q}^2 = \frac{\| h_q^r \otimes g_{|q} \|^2}{\| h_q^r \otimes g_{|p} \|^2 + \| h_q^p \otimes g_{|p} \|^2 + 1}.
\]

(8)

Following the SIC scheme, \( D_p \) decodes \( x_q \) firstly with the received SINR as

\[
\Gamma_{D_p}^2 = \frac{\| h_p^r \otimes g_{|p} \|^2}{\| h_p^r \otimes g_{|q} \|^2 + \| h_p^q \otimes g_{|q} \|^2 + 1}.
\]

(9)

Subsequently, by subtracting the interfering term associated with \( x_q \) from \( y_p \), the received SINR of \( D_p \) decoding its own message can be given by
\[ T_{D_p,xy}^2 = \left\| h_p^y \otimes g_p \right\|^2_{2} b_p^2 \Pi_y^2 + \left\| h_p^x \otimes g_p \right\|^2_{2} \kappa_x b_x \Pi_x^2 Y + 1, \quad (10) \]

where \( \kappa_x \) represents the level of residual interference from the preceding SIC cycle and \( 0 \leq \kappa_x \leq 1 \).

3. Channel Statistical Properties

Assuming that the channel link from satellite to the \( n \)-th RE of terrestrial node \( \xi \) is modeled as Shadowed-Rician fading distributions, the PDF and CDF of \( |h_{n\xi}|^2 \) are given by [32]

\[ f_{|h_{n\xi}|^2}(x) = \alpha_q \sum_{i=0}^{m-1} \xi^{(i)} x^{(\beta_i - \delta_i)} e^{-x}, \quad (11) \]

\[ F_{|h_{n\xi}|^2}(x) = 1 - \alpha_q \sum_{i=0}^{m-1} i! \xi^{(i)} (\beta_i - \delta_i)^{-(i+1)-j} x^{(\beta_i - \delta_i)}, \quad (12) \]

with \( \alpha_q = (2b_q m_q)^{m_q}/(2b_q (2b_q m_q + \Omega_q)^{m_q}) \), \( \beta_q = 1/(2b_q) \), and \( \delta_q = \Omega_p/(2b_q (2b_q m_q + \Omega_q)) \), where \( 2b_q, m_q, \) and \( \Omega_q \) denote the average power of the multipath component, the shape parameter of Nakagami-\( m \), and the average power of the line-of-sight component, respectively. \( \xi(i) = (-1)^i (1 - m_q) \delta_q (i!)^2 \) and \( (\cdot) \) denotes the Pochhammer symbol ([37], p. xiii). Additionally, the terrestrial link from relay to the \( n \)-th RE of node \( u \) undergoes identically distributed (i.i.d.) Rayleigh fading distribution.

4. Outage Probability Analysis

4.1. Exact Outage Probability

The outage event is defined as the instantaneous SINR is inferior to the predefined threshold \( \phi_u \), where \( \phi_u = 2^{2R_u} - 1 \) and \( R_u \) is the predefined rate for \( D_p \). Assume that the terrestrial users process the data received from \( S \) and \( R \) nodes by selection combining. Thus, the outage probability of \( D_p \) can be formulated as

\[ P_{out,p} = \left[ 1 - P_{s}(G_{k,xy}^{|\phi_q|,\phi_p} > \phi_q, \Gamma_{k,xy}^{|\phi_q|,\phi_p} > \phi_p) \right] \left[ 1 - P_{r}(\Gamma_{k,xy}^{|\phi_q|,\phi_p} > \phi_p) \right], \quad (16) \]

\[ P_{out,p} = \left[ 1 - e^{-N_1 \sum_{i=0}^{N-1} B(i, i^{(i)} \pi, \Theta_k) \sum_{k=0}^{N} \xi_k^i e^{(\beta_k - \delta_k) k^2}} \right] \left[ 1 - e^{-N_1 \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \sum_{k=0}^{N} \xi_k^{i+j} e^{(\beta_k - \delta_k) k^2}} \right], \quad (17) \]

where \( \xi_1 = \max (\psi_1, \psi_2), \psi_1 = \phi_q / \rho_q \gamma (a_q - \phi_q (a_p + k_p^2)), \psi_2 = \phi_q / \rho_q \gamma (a_p - \phi_q (a_p + k_p^2)), \xi_2 = \max (\psi_3, \psi_4), \psi_3 = \phi_q / \rho_q \gamma (a_p - \phi_q (a_p + k_p^2)), \psi_4 = \phi_q / \rho_q \gamma (a_p - \phi_q (a_p + k_p^2)), \xi_3 = \max (\psi_3, \psi_4), \xi_4 = \phi_q / \rho_q \gamma (a_p - \phi_q (a_p + k_p^2)), \xi_5 = \max (\psi_3, \psi_4), \xi_6 = \phi_q / \rho_q \gamma (a_p - \phi_q (a_p + k_p^2)), \phi_u \) is the threshold for \( D_p \).

Proof. See Appendix B.
Corollary 3. The outage probability expression for $D_p$ with $N = 1$ is given by

\[
P_{\text{out},p} = \left[ 1 - a_p \sum_{i=0}^{m_p-1} \xi(i) \sum_{j=0}^{N_p-1} B(i_{R,j}, \Theta) \right] \frac{\lambda_1 e^{-\beta_j - \delta_j}}{(\beta_j - \delta_j)^{\frac{z}{\Delta_p}}} e^{\frac{z}{\Delta_p}}.
\]

Proof. By substituting (2)–(5), (9), and (10) into (16) and then by some algebraic application, we can obtain the outage probability expression for $D_p$ by means of (12). The proof is completed.

Next, we focus attention on calculating the outage probability of $D_q$. $D_q$ falls into outage if it cannot decode its own signal at both phases or $R$ cannot decode $D_q$’s signal in the first phase. Hence, the outage probability of $D_q$ can be formulated as

\[
P_{\text{out},q} = P_r \left[ \Gamma_{R,x_q}^1 < \phi_q \right] \left[ 1 - P_r \left( \Gamma_{R,x_q}^1 > \phi_q, \Gamma_{D_q,y_q}^2 > \phi_q \right) \right].
\]

(19)

Theorem 4. The expression for the outage probability of $D_q$ for $N > 1$ is given by

\[
P_{\text{out},q}^{\text{CD}} = \left[ 1 - a_q \sum_{i=0}^{m_q-1} \cdots \sum_{j=0}^{N_q-1} \prod_{k=0}^{N_q-1} B(i_{R,k}, \Theta_{k}) \sum_{j=0}^{N_q-1} \xi(j) \right] \frac{\lambda_2 e^{-\beta_j - \delta_j}}{(\beta_j - \delta_j)^{\frac{z}{\Delta_q}}} e^{\frac{z}{\Delta_q}}
\]

\[
\times \left[ 1 - a_q \sum_{i=0}^{m_q-1} \cdots \sum_{j=0}^{N_q-1} \prod_{k=0}^{N_q-1} N \zeta(i_j) \right]
\]

\[
\times \prod_{l=1}^{t=1} N - 1 B(i_{k',1}, \Theta_{k'}) \sum_{\mu=0}^{\infty} \frac{\lambda_2 e^{-\beta_j - \delta_j} \lambda_{\mu}}{(\beta_j - \delta_j)^{\frac{z}{\Delta_q} + \mu + 1}},
\]

(20)

where $\lambda_1 = \phi_q / r \gamma(a_q - (a_p + k_{p,q}^2) \phi_q)$, $\lambda_2 = \phi_q / r \gamma(a_q - (a_p + k_{p,q}^2) \phi_q)$, and $\lambda_3 = \phi_q / r \gamma(b_q - (b_p + k_{p,q}^2) \phi_q)$. The above variables need to satisfy the conditions that $b_q > (b_p + k_{p,q}^2) \phi_q$ and $a_q > (a_p + k_{p,q}^2) \phi_q$. The approximate expression of $f_{Y_k}$ for $Y_k$ can be expressed as

\[
f_{Y_k}(y) = \frac{a_k^N}{(N - 1)!} y^{N - 1} + o(y^{N - 1}), \quad N > 1,
\]

(22)

where $o(y^{N - 1})$ is a higher-order infinitesimal of $y$. Based on (22), the approximate expression CDF of $Y_k$ for $N > 1$ at the high SNR regime can be expressed as

\[
F_{Y_k}(y) = \frac{a_k^N}{N!} y^{N} + o(y^{N}), \quad N > 1.
\]

(23)

In addition, we can easily acquire the approximate expression CDF of $Y_k$ for $N = 1$ at the high SNR regime which can be expressed as [32]

\[
F_{Y_k}(y) = a_k y, \quad N = 1.
\]

(24)

In the following, the approximate expression CDF of $Z_u$ will be derived. By applying the Taylor series expansion, $e^z$ can be expanded as $e^z = \sum_{k=0}^{\infty} z^k / k!$. Thereby, $\sum_{k=0}^{\infty} z^k / k!$ in (25) can be replaced by $Y = e^z - \sum_{k=0}^{N-1} z^k / k!$. By substituting $Y$ into (25), when $z \to 0$, the approximate expression CDF of $Z_u$ for $N > 1$ can be expressed as

\[
F_{Z_u}(z) = \frac{1}{N!} z^N, \quad N > 1.
\]

(25)

For the special case of $N = 1$, when $z \to 0$, $F_{Z_u}(z)$ with the approximation of $e^z = 1 - z$ can be expressed as

\[
F_{Z_u}(z) = \frac{z^N}{N!}, \quad N = 1,
\]

(26)

where $\bar{\Omega}_q$ denotes the channel mean power.
By substituting (23)–(26) into (B.1) and (C.1), respectively, we can get the asymptotic outage probability of $D_p$ and $D_q$ at the high SNR regime as follows:

$$p_{\text{out}, p}^\infty = \begin{cases} \frac{\alpha_p^N \epsilon_p^N}{1 - \left(1 - \frac{\alpha_p^N \epsilon_p^N}{\alpha} \right) \left(1 - \epsilon_p^N \frac{N}{N^2} \right)} & , N > 1, \\ \alpha_p \epsilon_p \left[1 - \left(1 - \alpha_p \epsilon_p \right) \frac{\epsilon_p}{\epsilon_q} \right] & , N = 1, \end{cases}$$

$$p_{\text{out}, q}^\infty = \begin{cases} \frac{\alpha_q^N \lambda_q^N}{1 - \left(1 - \frac{\alpha_q^N \lambda_q^N}{\alpha} \right) \left(1 - \lambda_q^N \frac{N}{N^2} \right)} & , N > 1, \\ \alpha_q \lambda_q \left[1 - (1 - \alpha_q \lambda_q) \left(1 - \lambda_q \frac{N}{N^2} \right) \right] & , N = 1, \end{cases}$$

respectively, where $\alpha = (N - 1)!$.

According to the conclusions of (27) and (28), the diversity orders of $D_p$ and $D_q$ in CD-NOMA ($N > 1$) and PD-NOMA ($N = 1$) are both $3N/3$.

5. Ergodic Sum Rate and Energy Efficiency

5.1. Ergodic Rate. The ergodic sum rate is an important indicator of system performance, which depends on the conditions of the channel links and the decoding process at all the terrestrial nodes. Hence, in this section, the ergodic sum rate of the proposed system is investigated.

The achievable rate of $D_p$ and $D_q$ can be given by

$$R_p = \frac{1}{2} \log_b \left(1 + \min \left\{ \Gamma_1^{2} \right\} \right),$$

$$R_q = \frac{1}{2} \log_b \left(1 + \min \left\{ \Gamma_2^{1}, \Gamma_3^{1}, \Gamma_4^{1}, \Gamma_5^{1}, \Gamma_6^{1}, \Gamma_7^{1}, \Gamma_8^{1} \right\} \right),$$

respectively.

Thus, the ergodic rate of $D_p$ can be written as

$$R_p^E = \mathbb{E} \left\{ \frac{1}{2} \log_b \left(1 + \min \left\{ \Gamma_1^{2} \right\} \right) \right\}. \tag{31}$$

Since it is difficult to obtain the exact expression of the ergodic sum rate, we derive the approximate ergodic sum rate.

**Theorem 6.** The approximate expression for the ergodic rate of $D_p$ for $N > 1$ is given by

$$R_p^{CD} = \frac{1}{2} \log_b \left(1 + \min \left\{ \frac{X_1 a_p \rho_p}{X_1 \rho_p \gamma (\kappa_1 a_q + k_{\gamma}^2) + 1}, \frac{X_2 a_p \rho_p}{X_2 \rho_p \gamma (\kappa_1 a_q + k_{\gamma}^2) + 1}, \frac{b_p \Pi_p \gamma N}{\Pi_p \gamma (\kappa_2 b_q + k_{\gamma}^2) + 1} \right\} \right), \tag{32}$$

where $X_1 = \alpha_p^N \sum_{i=0}^{m_{\rho} - 1} \cdots \sum_{i_0=0}^{m_{\rho} - 1} \prod_{i=1}^{N-1} \hat{B}(i, \rho, \Theta, \Xi + 1)! (\beta_p - \delta_p)^{i-2}$ and $X_2 = \alpha_p^N \sum_{i=0}^{m_{\rho} - 1} \cdots \sum_{i_0=0}^{m_{\rho} - 1} \prod_{i=1}^{N-1} \hat{B}(i, \rho, \Theta, \Xi + 1)! (\beta_p - \delta_p)^{i-2}$.

**Proof.** See Appendix D.

**Corollary 7.** The approximate expression for the ergodic rate of $D_p$ for $N = 1$ is given by

$$R_p^{PD} = \frac{1}{2} \log_b \left(1 + \min \left\{ \frac{\hat{X}_1 a_p \rho_p}{\hat{X}_1 \rho_p \gamma (\kappa_1 a_q + k_{\gamma}^2) + 1}, \frac{\hat{X}_2 a_p \rho_p}{\hat{X}_2 \rho_p \gamma (\kappa_1 a_q + k_{\gamma}^2) + 1}, \frac{b_p \Pi_p \gamma}{\Pi_p \gamma (\kappa_2 b_q + k_{\gamma}^2) + 1} \right\} \right), \tag{33}$$

where $\hat{X}_1 = \alpha_p \sum_{i=0}^{m_{\rho} - 1} \hat{B}(i, \rho, \Theta, \Xi + 1)! (\beta_p - \delta_p)^{i-2}$ and $\hat{X}_2 = \alpha_p \sum_{i=0}^{m_{\rho} - 1} \hat{B}(i, \rho, \Theta, \Xi + 1)! (\beta_p - \delta_p)^{i-2}$.

In what follows, the ergodic rate of $D_q$ can be written as

$$R_q^{E} = \mathbb{E} \left\{ \frac{1}{2} \log_b \left(1 + \min \left\{ \Gamma_1^{2} \right\} \right) \right\}. \tag{34}$$

By substituting (2), (4), (6), (8), and (9) into (34), after some mathematical operations, the approximate expression for the ergodic rate of $D_q$ for $N > 1$ can be formulated as

$$R_q^{CD} = \frac{1}{2} \log_b (1 + \min \left\{ A_1, A_2 \right\}). \tag{35}$$
In (35), $A_1$ and $A_2$ can be expressed as

$$A_1 = \min \left\{ \frac{X_1a_p, p, y}{X_1 p, y(a_p + k^2_p) + 1}, \frac{X_2a_p, p, y}{X_2 p, y(a_p + k^2_p) + 1}, \frac{X_3a_p, p, y}{X_3 p, y(a_p + k^2_p) + 1} \right\},$$

$$A_2 = \min \left\{ \frac{b_q \Pi_p y N}{\Pi_p y b_p + k^2_p N + 1}, \frac{b_q \Pi_q y N}{\Pi_q y b_p + k^2_p N + 1} \right\},$$

(36)

where $X_1 = a_q^\alpha_q \sum_{i=0}^{m_q-1} \cdots \sum_{j=0}^{m_q-1} \sum_{l=0}^{\nu} \zeta(i) \zeta(j) \zeta(l) B(i, j, l, \Theta_{q}),$

$$X_2 = b_q \Pi_p y N,$$

$$X_3 = a_q^\alpha_q \sum_{i=0}^{m_q-1} \cdots \sum_{j=0}^{m_q-1} \sum_{l=0}^{\nu} \zeta(i) \zeta(j) \zeta(l) B(i, j, l, \Theta_{q}).$$

(37)

Due to the proof process being similar to (32), we will not elaborate on it.

For the special case with $N = 1$, the approximate expression for the ergodic rate of $D_q$ is calculated as

$$R^\text{PD}_q = \frac{1}{2} \log_2 \left( 1 + \min \{ A_1, A_2 \} \right).$$

(38)

$	ilde{A}_1$ and $	ilde{A}_2$ in (38) can be expressed as

$$\tilde{A}_1 = \min \left\{ \frac{X_1 a_p, p, y}{X_1 p, y(a_p + k^2_p) + 1}, \frac{X_2 a_p, p, y}{X_2 p, y(a_p + k^2_p) + 1}, \frac{X_3 a_p, p, y}{X_3 p, y(a_p + k^2_p) + 1} \right\},$$

(39)

respectively, where $X_3 = a_q^\alpha_q \sum_{i=0}^{m_q-1} \cdots \sum_{j=0}^{m_q-1} \sum_{l=0}^{\nu} \zeta(i) \zeta(j) \zeta(l) B(i, j, l, \Theta_{q}).$

5.2. Energy Efficiency. Energy efficiency is another important index to evaluate the system performance. According to the definition in [39], energy efficiency can be expressed as

$$\eta_{\text{EE}} = \frac{R_{\text{sum}}}{\nu P_c + P_t},$$

(41)

where $P_c$ is the fixed energy consumed by the transceiver and $\nu$ denotes the efficiency of the power amplifier. $R_{\text{sum}}$ is the ergodic sum rate, which can be obtained from (32), (33), (35), and (38).

6. Numerical Results

In this section, we provide numerical results to verify the analysis result and further evaluate the performance of the satellite-terrestrial networks. Assume that the type of satellite is low-orbit earth satellite, satellite links obey the Shadowed-Rician fading distribution, and channel parameters are shown in Table 1 [29, 40]. In this simulations, we set that the power allocation coefficients for $D_p$ and $D_q$ are $a_p = b_p = 0.25$ and $a_q = b_q = 0.75$, the distances from R to terrestrial users are $d_p = 0.5$ and $d_q = 1$, and the pathloss exponent of the terrestrial link is $\Delta = 2$. In addition, let the terrestrial channel mean power $\Omega_d = 1$. The target rates are $R_p = 1$ bit/s/Hz and $R_q = 0.5$ bit/s/Hz [32], respectively. The Monte Carlo simulation parameters are depicted in Table 2 [5].

Figure 2 illustrates the effects of the transmit SNR on the outage probability with different numbers of REs, where the satellite links undergo FHS. In this example, we set $\kappa = \kappa_r = \kappa_1 = \kappa_2$ and $k^2 = k^2_1 = k^2_2$. For comparison purposes, we choose the PD-NOMA case ($N = 1$) as the benchmark. It can be seen that the calculated result curves perfectly match well with the Monte Carlo simulation curves, and the asymptotic outage probabilities are approximated with the calculated results in the medium-high SNR region. The outage probabilities of $D_p$ and $D_q$ decrease as the value of transmit SNR increases. Additionally, the outage performance improves significantly when the number of RE increases from $N = 1$ to $N = 2$ and $N = 3$. The reason for this phenomenon is that CD-NOMA ($N > 1$) has a higher diversity order than PD-NOMA, which is consistent with the analysis of (27) and (28). Thus, this important observation also indicates that the diversity gains of the paired NOMA users are closely related to the number of REs.

Figure 3 shows the impacts of the transmit SNR and HIs on the outage probabilities of paired users. In this simulation, we set $\kappa^2 = \{0.0, 0.01, 0.02, 0.03\}$, $\kappa = 0$, and $N = 2$. We can observe that the exact analytical and asymptotic results are well aligned to the Monte Carlo results. From the outage probability versus SNR curves, it can also be seen that the outage performances for $D_p$ and $D_q$ increase as the transmit SNR increases. In particular, different levels of HIs are set to be from 0.01 to 0.03. As can be observed from the figure, the outage performances of paired NOMA users are becoming worse with the value of $k^2$ increasing from 0 to 0.03. Another important observation is that the outage probability of $D_p$ is more sensitive to HIs than that of $D_q$. 

Table 1: The table of Shadowed-Rician fading channel parameters.

<table>
<thead>
<tr>
<th>Shadowing condition</th>
<th>$b_k$</th>
<th>$m_k$</th>
<th>$\Omega_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequent heavy shadowing (FHS)</td>
<td>0.063</td>
<td>2</td>
<td>0.0005</td>
</tr>
<tr>
<td>Average shadowing (AS)</td>
<td>0.251</td>
<td>5</td>
<td>0.279</td>
</tr>
<tr>
<td>Infrequent light shadowing (ILS)</td>
<td>0.158</td>
<td>10</td>
<td>1.29</td>
</tr>
</tbody>
</table>

Table 2: The table of simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency</td>
<td>1 GHz</td>
</tr>
<tr>
<td>Satellite antenna gain</td>
<td>24.3 dBi</td>
</tr>
<tr>
<td>User antenna gain</td>
<td>3.5 dBi</td>
</tr>
<tr>
<td>3 dB angle</td>
<td>0.4°</td>
</tr>
<tr>
<td>The initial angle between the beam center and user</td>
<td>0.1°</td>
</tr>
<tr>
<td>The distance from the satellite to the user</td>
<td>1000 km</td>
</tr>
</tbody>
</table>
Figure 4 depicts the outage probabilities of paired NOMA users versus the imperfect SIC and transmit SNR in three different satellite shadowing conditions. For comparison, the outage probabilities of paired users experiencing FHS fading are selected as the benchmark. It is observed that the outage performance for both NOMA users improves as the degree of shadow fading experienced by satellite links decreases. The reason for this phenomenon is that the paired users can achieve a better outage performance when they undergo a better shadowing scenario. Additionally, the effect of residual interference from the SIC process on the outage probability is focused, where satellite links are subject to FHS. In this simulation, we set $\kappa = \{0, 0.01, 0.02, 0.03\}$ and $\kappa = 0$. Apparently, the outage performance of the $D_p$ will be significantly degraded if the level of residual interference $\kappa$ increases by 0.03 from zero. Therefore, it is necessary to develop a reasonable SIC scheme in a practical scenario.
Figure 5 reveals the outage probability of the considered system against the transmit SNR and power allocation factor $a_p$ under the FHS scenario. We assume that the system performance is analyzed under ideal conditions ($\kappa = 0$) and set the transmit SNR at $S$ and $R$ to be 30 dB. As can be seen from Figure 5, the outage probability decreases for $D_p$ and...
increases for $D_q$ as the value of $a_q$ increases. However, the outage performance of $D_p$ starts to decline once $a_p$ goes beyond 0.4. This phenomenon indicates that the increase in $a_p$, the corresponding power allocation factor $a_q = 1 - a_p$, will decrease and will affect the $D_p$ decoding $x_q$ in the SIC process when $a_p > 0.4$. Further, there exists a value $a_p = 0.25$ that enables NOMA systems to yield better outage performance. Consequently, to improve the outage performance and fairness of paired users, $a_p$ can be set between 0.17 and 0.33 when $\gamma = 30$ dB.
rate for $D_p$ of the proposed system. The curves of ergodic rate of the proposed system, $(PD-NOMA)$, is selected as a benchmark to evaluate the performance of the proposed system. From different perspectives, the spectral efficiency of the proposed system decreases significantly, and the transmit SNR that enables the considered system to obtain the maximum energy efficiency moves to the right. This happens because the ergodic rate achieved by the terrestrial users decreases under the influence of HIs, which is consistent with the analysis results in Figure 3. Therefore, the design of the effective compensation algorithm and calibration method is of great significance to alleviate the influences of HIs on the performance of NOMA-based satellite-terrestrial networks in practical application.

Figure 8 illustrates the ergodic rate versus the transmit SNR with different levels of residual interference from the SIC cycle. In this simulation, we set the number of REs $N = 2$ and the level of HIs $k^2 = 0$. Then, the cases of residual interference level $\kappa = 0.01$, $\kappa = 0.02$, and $\kappa = 0.03$ are considered for the proposed system. As can be observed, the ergodic rate of $D_p$ decreases significantly when the level of residual interference increases from 0 to 0.01, 0.02, and 0.03. Hence, it is necessary to design a reasonable multiuser reception algorithm to improve the performance of NOMA-based satellite-terrestrial networks in practical scenarios.

Figure 6 illustrates the ergodic sum rate versus the transmit SNR with different numbers of REs (i.e., $N = 1$, $N = 2$, and $N = 3$). In this simulation, we set $\kappa = 0$ and $k^2 = 0$, and all satellite links undergo FHS fading. A special case, $N = 1$ (PD-NOMA), is selected as a benchmark to evaluate the ergodic rate versus the transmit SNR are presented in three different levels of residual interference and power distribution factors on the energy efficiency, Figure 9 depicts the energy efficiency versus the transmit SNR in delay-tolerant transmission mode, where $k^2 = \{0.0.01,0.02,0.03\}$ and $\kappa = \{0.0.01,0.02,0.03\}$. The power amplifier and fixed energy loss are set as $\nu = 2$ and $P_c = 50$ W, respectively [6, 39]. As can be seen from Figure 9(a), when $k^2$ increases from 0 to 0.03, the energy efficiency of the proposed system decreases significantly, and the transmit SNR that enables the considered system to obtain the maximum energy efficiency moves to the right. This happens because the ergodic rate achieved by the terrestrial users decreases under the influence of HIs, which is consistent with the analysis results in this example, we set $\nu = k$, and $\kappa = 0.03$. Hence, it is necessary to design a reasonable multiuser reception algorithm to improve the performance of NOMA-based satellite-terrestrial networks in practical scenarios.

![Figure 8: Ergodic sum rate versus the transmit SNR.](image-url)
Figure 7. In Figure 9(b), the energy efficiency under perfect SIC condition is obviously better than that under imperfect SIC condition, which also corresponds to the analysis in Figure 8. In addition, it is observed that the energy efficiency of the considered system is improved with the increase in the power distribution factor for \( D_p \). Although the energy efficiency can be improved to a certain extent by increasing the power allocation factor of \( D_p \), it will greatly sacrifice the performance of \( D_q \), thus losing the fairness of users in the weak channel. This phenomenon indicates that optimizing the power distribution factor can further enhance the performance of the paired NOMA users. Moreover, as illustrated in Figures 9(a) and 9(b), energy efficiency increases with the number of REs.

7. Conclusion

This paper proposed a unified NOMA framework for the satellite-terrestrial network under Shadowed-Rician fading channels. Considering the realistic scenario with HIs and imperfect SIC, the performance of the considered system has been characterized in terms of outage probability, ergodic sum rate, and energy efficiency. More specifically, the closed-form expressions of exact outage probability for the paired users were derived. In order to gain more insights, the asymptotic outage probability was provided in the high SNR region. The analytical results showed that a pair of NOMA users was able to achieve better outage performance as the number of REs increased, and the CD-NOMA was capable of outperforming the PD-NOMA on the outage performance. Furthermore, the approximate expression of ergodic sum rate was obtained due to the integral being difficult to solve. We can observe that the HIs and residual interference had significant negative effects on ergodic sum rate, and the ergodic rate of \( D_p \) is more sensitive to HIs than that of \( D_q \). The energy efficiency of the considered system with imperfect SIC and HIs has been discussed in delay-tolerant transmission mode. Finally, the Monte Carlo simulation was presented to verify the theoretical derivation. The unified NOMA framework applied to the satellite-terrestrial network provides a novel design idea, which is a relatively promising research topic.
Appendix

A. Proof of Lemma 1

For mathematical manageability, we assume that $g_p$, $g_0$, and $g_r$ have the same column weights. That means that $\|h_0^k \odot g_p\|_2^2$, $\|h_0^k \odot g_0\|_2^2$, and $\|h_0^k \odot g_r\|_2^2$ undergo the same distribution. Let $Y_k = \|h_0^k \odot g_p\|_2^2$, and the corresponding PDF of $Y_k$ is given by

$$f_{Y_k}(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}},$$

where the symbol $\odot$ represents the statistical convolution between independent distributions. And then we can use mathematical induction to prove the PDF of $Y_k$. When $N = 2$, we can get $Y_k = \|h_1^k\|^2 + \|h_2^k\|^2$, and the corresponding PDF of $Y_k$ is given by

$$f_{Y_k}(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}.$$

Substituting (11) into (A.1) and utilizing Eq. (8.191.1) of [37], we can obtain

$$f_{Y_k}(y) = \sum_{i=0}^{m-1} \sum_{j=0}^{m-1} \zeta(i_1)\zeta(i_2)B(i_2 + 1, i_1 + 1) \times y^i(y + 1)^j e^{-\frac{(y-\mu)}{\sigma^2}}.$$  

(A.3)

When $N = 3$, the corresponding PDF of $Y_k$ is given by

$$f_{Y_k}(y) = f_{Y_k}(y) * f_{Y_k}(y).$$  

(A.4)

Similarly, by plugging (A.3) and (11) into (A.4), $f_{Y_k}(y)$ can be obtained as follows:

$$f_{Y_k}(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}.$$  

(A.5)

Assuming that $N = N$ and $Y_k = \sum_{j=1}^{N} |\hat{h}_k|^2$, the PDF of $Y_k$ is derived, as shown in (13).

The proof is completed.

B. Proof of Theorem 2

By assuming $Y_k = \|h_0^k \odot g_p\|_2^2 = \|h_0^k \odot g_0\|_2^2$ and $Z_k = \|h_0^k \odot g_r\|_2^2$, and substituting (2)–(5), (9), and (10) into (16), the outage probability of $D_k$ can be expressed as

$$P_{out_D} = \left[1 - P_r(Y_p > \epsilon_2)\right] \left[1 - P_r(Y_r > \epsilon_1, Z_p > \epsilon_3)\right].$$

By plugging (13) into (A.1), the closed-form expression of $P_{out_D}$ can be derived.

The proof is completed.

C. Proof of Theorem 4

By applying the assumptions in Appendix B and plugging (2), (6), and (8) into (19), the outage probability of $D_k$ can be expressed as
Using steps similar to Appendix B of Theorem 2, with the help of (13) and Eq. (3.51.2) of [37], $J_4$ and $J_5$ are evaluated as

$$J_4 = \alpha^N \sum_{l=0}^{m-1} \cdots \sum_{l=0}^{m-1} \prod_{i=0}^{N-1} \xi(i) \prod_{k=1}^{N-1} B \left( i_{k+1}, \sum_{i=1}^{k} i + k \right) \times \sum_{\lambda=0}^{\infty} \frac{\lambda^\lambda e^{(\beta, \alpha-\delta, \gamma)\lambda}}{\lambda! (\beta, \alpha-\delta, \gamma)^{\lambda+1}},$$

(D.2)

Plugging (C.2)–(D.4) into (C.1), we can derive (20) and complete the proof.

**D. Proof of Theorem 6**

By substituting (3), (5), and (11) into (31), the expression for the ergodic rate of $D_p$ for $N > 1$ can be expressed as

$$R_p^{CD} = \mathbb{E} \left( \frac{1}{2} \log_2 \left( 1 + \min \left\{ \frac{Y_r a_p p_r y}{Y_r p_r y (\kappa a_q + k^2_p) + 1}, \frac{Y_p a_p p_r y}{Y_p p_r y (\kappa a_q + k^2_p) + 1}, \frac{Z_p b_p p_r y}{Z_p p_r y (\kappa b_q + k^2_p) + 1} \right\} \right) \right).$$

(D.1)

Following the inequality ([21], Eq. (35)) \( \mathbb{E}\{\log_2(1 + (x/y))\} \approx \log_2(1 + (\mathbb{E}(x)/\mathbb{E}(y))) \), (D.1) can be approximated as

$$R_p^{CD} \approx \frac{1}{2} \log_2 \left( 1 + \min \left\{ \frac{\mathbb{E}(Y_r) a_p p_r y}{\mathbb{E}(Y_r) p_r y (\kappa a_q + k^2_p) + 1}, \frac{\mathbb{E}(Y_p) a_p p_r y}{\mathbb{E}(Y_p) p_r y (\kappa a_q + k^2_p) + 1}, \frac{\mathbb{E}(Z_p) b_p p_r y}{\mathbb{E}(Z_p) p_r y (\kappa b_q + k^2_p) + 1} \right\} \right).$$

(D.2)

As a further development, we assume that $X_1 = \mathbb{E}(Y_r)$ and $X_2 = \mathbb{E}(Y_p)$. With the aid of (13), $X_1$ can be calculated as

$$X_1 = \int_0^\infty x f_{Y_r}(x) dx = \alpha^N \sum_{l=0}^{m-1} \cdots \sum_{l=0}^{m-1} \prod_{i=0}^{N-1} \xi(i) \prod_{k=1}^{N-1} B \left( i_{k+1}, \sum_{i=1}^{k} i + k \right) \times \int_0^\infty x e^{x \beta} e^{(\beta, \alpha-\delta, \gamma)\lambda} dx \times (\lambda + 1)! (\beta - \delta)^{\lambda-2},$$

(D.3)

where step (i) is derived from Eq. (3.351.3) of [37].

Similar to the proof process of $X_1$, we can easily obtain the calculation results of $X_2$ and $\mathbb{E}(Z_p) = N$.

Finally, substituting $X_1$, $X_2$, and $\mathbb{E}(Z_p)$ into (D.2), we can obtain (32).

The proof is completed.

**Data Availability**

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also forms part of an ongoing study.

**Conflicts of Interest**

The authors declared that they have no conflicts of interest to this work.
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