Retraction

Retracted: Partial Differential Equation Noise Reduction Model and Fuzzy Image Processing in Optimal Application of Sports Dance Exercise Training Mode

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

1. Discrepancies in scope
2. Discrepancies in the description of the research reported
3. Discrepancies between the availability of data and the research described
4. Inappropriate citations
5. Incoherent, meaningless and/or irrelevant content included in the article
6. Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article’s content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

Research Article

Partial Differential Equation Noise Reduction Model and Fuzzy Image Processing in Optimal Application of Sports Dance Exercise Training Mode

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Ballroom dancing has been standardized in the United Kingdom since the beginning of the 20th century, and it has been competitive by WDC to form competitive ballroom dancing. Later, under the vigorous promotion of WDSF, it became a competitive sport recognized by the International Olympic Committee. In the process of teaching experiments, dancers’ wrong movements in sports dance training are inevitable. If they are not corrected in time, it will not only reduce the effect of dance expression, but also directly affect the improvement of sports performance. The dance images are often disturbed by many factors such as imaging equipment and the external environment during the process of acquisition, compression, transmission and storage, resulting in image distortion. Image distortion is mainly manifested in that the image structure information is submerged by noise, and the distorted image can no longer truly reflect the original image information, which brings difficulties to the post-processing of the image. In this paper, the image denoising method based on partial differential equation and its application in sports dance training are deeply studied.

1. Introduction

Dance sports is a collection of pas de deux based on sociality and competition, originally derived from ballroom dancing. Ballroom dancing was originally a social dance for the privileged class, while the lower class was called folk dance [1]. With the passage of time, these boundaries have become blurred. By 1920, with the popularity of modern ballroom dancing, the Dance Teachers Association was established in the United Kingdom to standardize ballroom dancing. At this time, seven ballroom dances with standardized requirements were formed, including slow waltz, quickstep, foxtrot, and tango. By 1950, the World Dance Council (WDC) was established in Edinburgh, with the main goal of hosting the world championships of competitive ballroom dancing, also known as competitive ballroom dancing [2, 3]. With the 1960 Latin dance becoming an official dance world championship event, competitive ballroom dancing can also be called Dancesport [4, 5]. Since then, through the vigorous promotion of WDC and WDSF, two world organizations, especially the outstanding performance at the closing ceremony of the Sydney 2000 Olympic Games, sports dance has swept the world. Today, it is a competitive sport recognized by the International Olympic Committee and is expected to be included in the Olympic Games.

Characteristics of dance sports include agility, coordination, physical fitness, stamina, grace, and musical interpretation. It perfectly combines artistry and sports competition, making people feel the joy of participating in social dance. The development of dance sports provides opportunities to study the physical discipline of physical and mental development and to interpret artistic creativity, costume design and choreography through music. In recent years, with the continuous development of international events and the
country’s strong support for sports dance, my country’s sports dance competition level has entered the world’s high-level ranks. In international competitions such as the Blackpool Dance Festival and WDSF, Chinese players have won many good results. With the further improvement of people’s requirements for the competitiveness and viewing of sports dance, how to better perform dance performance is a problem that sports dancers must face. At present, although there are many top-quality sports dance players in China, they participate in international competitions and are mostly trained by foreign coaches. There are still some problems in domestic sports dance teaching. First of all, the uncertainty of dance style does not show the expressive features of sports dance very well. Second, the irregularity of dance techniques, such as overemphasizing the twisting of the toes, undermines the overall center of gravity movement of the body. Also, dance training methods are unreasonable, such as overemphasizing routine training and ignoring physical ability training. Only by further studying the attributes of sports dance, strengthening the training of factors affecting expressiveness, and improving the awareness of training pertinence, can we truly improve the level of players and jump out of high-level sports dance. However, the discussion of training methods is not the focus of this paper. This paper will focus on the recognition of image movements and the correction of wrong movements in sports dance.

With the rapid development of computer intelligent vision technology, the correction of wrong movements through image recognition can not only correct the dancers’ dancing posture, but also have important value in the analysis of dance technology, thus promoting the development of sports dance. However, due to the complex movements of sports dance, the variety of movements and the difficulty of image recognition, the signal-to-noise ratio of images obtained by traditional image processing methods is low. Images with low signal-to-noise ratio lead to poor visual effects, which cannot meet the user’s requirements for the accuracy of wrong action recognition, and it is difficult to achieve comprehensive correction of dance movements.

In this paper, an image noise reduction model based on variable order variation is proposed. The model takes into account the advantages of the first-order variational model and the second-order variational model, and divides the image into flat regions and subdivision regions according to the gradient modulus value of the image and the feature detection factor of local entropy fusion. The model adaptively selects the variational order in different regions to realize different diffusion modes. In addition, we adopt the split Bregman algorithm based on fast Fourier transform to solve the proposed variable-order variational model to improve the operation speed of this model. The experimental results show that the variable-order variational model proposed in this paper has good denoising ability for natural images. In addition, the model significantly improves the quality of the image, and the obtained noise reduction effect is good, which is suitable for the detection and correction of sports dance movements, and has practical application value.

2. Related Work

In image processing and computer vision, image noise reduction methods based on variational and partial differential equations have the advantages of relatively complete theoretical support, flexible and diverse numerical solution formats, etc., and have been unique in the field of image noise reduction in the past 20 years. Image noise reduction is the most important and one of the most basic research contents in the field of image processing. In recent years, scholars at home and abroad have conducted in-depth research on image noise reduction technology and proposed a large number of high-performance noise reduction methods.

At present, the commonly used image noise reduction methods can be roughly divided into two categories, namely, transform domain noise reduction methods and spatial domain pixel feature noise reduction methods. The former first converts the image from the spatial domain to the transform domain space, then indirectly processes the transform domain spatial coefficients to complete the noise reduction process, and finally obtains the denoised spatial domain image through inverse transformation. The latter is to process the gray value of image pixels directly in the spatial domain.

2.1. Transform Domain Noise Reduction Methods. The representative methods of denoising methods based on transform domain include Fourier transform [6], wavelet transform [7], principal component analysis method [8] and sparse representation of overcomplete dictionary [9]. Among these transform domain image noise reduction methods, due to the localization, multi-resolution, low entropy, and de-correlation characteristics of wavelet transform, the application of wavelet transform in image noise reduction is more simple and effective, thereby breaking the It has become a hot spot in the study of transform domain noise reduction methods [10–12]. However, the non-translation-invariant wavelet transform method is prone to pseudo-Gibbs effect in discontinuous areas of denoised images. The formation of principal component analysis theory has laid a solid foundation for the research of image noise reduction methods based on principal component analysis at home and abroad. Many noise reduction methods based on principal component analysis theory have been proposed. Muresan and Parks et al. proposed another method of image denoising based on local adaptive basis principal component analysis. This method overcomes the difficulty that a single fixed wavelet base cannot fully describe the structural information of natural images in wavelet transform, and has good structure-preserving noise reduction performance [13]. Dai et al. proposed an improved image denoising method based on guided principal component analysis, which also has good texture preservation performance for images with high noise pollution [14]. Although the image denoising method based on principal component analysis can effectively remove noise, in view of the limitation of the actual meaning of principal components obtained by principal component analysis and the
limitations of data that cannot fully represent Gaussian distribution, it is necessary to combine principal component analysis with other theories are combined with each other [15]. At present, with the sparse representation theory, great success has been achieved in image processing fields such as image super-resolution, image compression and image fusion. The sparse representation based on overcomplete dictionary is also widely used in the field of image denoising, which makes it become the main research direction of image denoising in transform domain. The most typical one is the K-SVD (K-Singular Value Decomposition) image noise reduction method based on sparse representation proposed by Elad and Aharon, which achieves good noise reduction effect. The solution idea of this method is to alternately iterate the sparse representation, the dictionary of the current sample, and the update process of dictionary atoms. Since the dictionary trained by the K-SVD method usually contains noise atoms, the image denoising effect of this method is not good when there is strong noise pollution. In view of the limitations of K-SVD image noise reduction method, domestic and foreign scholars have conducted in-depth research on it and proposed a large number of improved methods to achieve better image noise reduction effect.

2.2. Spatial Domain Noise Reduction Methods. The noise reduction method based on the spatial domain has undergone decades of research and exploration, and many mature methods have been formed in the field of image noise reduction, such as Gaussian filtering, mean filtering, median filtering, bilateral filtering, and variational and partial differential equations. Among them, the traditional spatial filters such as Gaussian filtering, mean filtering and median filtering over-smooth some structural information of the image while filtering out the noise information, so that the image quality cannot meet the requirements of subsequent image processing. Therefore, it is imperative to study image noise reduction methods with good structure preservation. Compared with other spatial domain noise reduction methods, the non-local mean noise reduction method has created a new rule for the research and development of image noise reduction processing. Since then, domestic and foreign scholars have successively proposed a large number of optimized non-local mean noise reduction methods. However, these methods suffer from low efficiency and certain limitations in practical applications. Image noise reduction methods based on variational methods and partial differential equations have made remarkable achievements in the field of image noise reduction with their solid mathematical theoretical foundation, mature numerical analysis and computational solution systems, and become the most popular type of image noise reduction methods at present. Image noise reduction is an ill-posed inverse problem, so regularization theory is often used to transform the ill-posed problem into a well-posed problem to find the optimal solution. The image noise reduction method based on the variational method is to solve the problem of the optimal solution of the energy functional of the image. The variational method for image noise reduction originated from the TV (Total Variation) model proposed by Rudin et al., which can well preserve the edge details of the image while smoothing the noise. However, there is usually a staircase effect in the noise reduction results of the TV model. In order to reduce the staircase effect of the TV model, Deng et al. used the high-order difference as the regularization term of the total variation model, and proposed a high-order total variation HOTV model. This model can effectively suppress the staircase effect, but it over-smoothes the edge details of the image while smoothing the noise [16]. Kumar et al. proposed an adaptive controllable total variational regularization ASTV method based on geometric moments. The method can denoise the image according to the geometric direction of the edge, which effectively improves the denoising performance, and the whole denoising framework is optimized using the multivariate minimization technique based on split Bregman [17]. Ma et al. [18] optimized the data fidelity term of the TV model by using the long-term memory and non-locality of fractional differentiation. The new fidelity term couples the fractional fitting term and the global fitting term to measure the fidelity of image changes. Similarity, this method can effectively remove Gaussian noise while suppressing the staircase effect.

So far, scholars at home and abroad have continuously achieved new research results using variational thinking, which has not only been widely used in the field of image noise reduction, but also promoted the development of partial differential equation image noise reduction technology.

The most famous noise reduction method based on partial differential equation is the anisotropic diffusion model, namely PM model, proposed by Perona and Malik in 1990. This model is the seminal work of partial differential equations in the field of image restoration. The PM model qualitatively uses the gradient modulus value as the edge detection factor to achieve adaptive control of the diffusion intensity in the edge and non-edge regions of the image. The PM model has been concerned by many scholars, and people continue to conduct in-depth research on it, and have made outstanding contributions to many image processing applications such as image segmentation, image noise reduction, and edge detection.

The PM noise reduction model based on the second-order partial differential equation is the optimization of the heat conduction model and has good edge-preserving noise reduction performance. However, for noise points with similar gradient modulus values to edges, the PM model will misjudge such high-frequency noise information as edge information, resulting in a staircase effect in the process of image evolution. In addition, the PM model over-smoothed the texture details of the image, resulting in the loss of image information. Subsequently, many researchers proposed a series of improvement schemes. However, these improved schemes do not fully consider the directional information of the local structure of the image, and uniformly reduce the diffusion intensity on each method of the image edge. Therefore, these models cannot effectively reduce the noise and edge aliasing at the edges, and may lose some edge structure information.
In recent years, with the rapid development of computer technology, more and more noise reduction algorithms with superior performance have been proposed, but they still cannot meet people’s image quality requirements in the field of science and technology and engineering applications. Therefore, the research on image denoising technology is still a topic worthy of further study.

2.3. Image Quality Evaluation Criteria. In the research of image denoising, the effectiveness of image denoising methods is measured by analyzing and evaluating the quality of denoised images. At present, there is no unified standard for evaluating image quality, so establishing a complete image quality evaluation system is also an urgent task to be solved in image processing research. There are two main methods for evaluating image quality: subjective evaluation methods and objective evaluation methods. The subjective evaluation method mainly evaluates and analyzes the denoised image according to the visual effect of the denoised image. There are generally three cases in the subjective evaluation method. The first is to randomly select a group of observation values in a certain observation environment to subjectively evaluate the quality of the evaluation image to give their respective scores, and then use the average score opinion MOS (Mean Opinion Score) method to comprehensively evaluate their quality of images. The second is to use the theoretical knowledge of fuzzy mathematics to evaluate the image quality approximately quantitatively, such as the average index method, the method noise and the image index method, but some evaluation parameters in these methods still need to be manually set. The third is the subjective observation and comparison method based on artificial judgment, which evaluates the quality of the denoised image by observing the difference in the image structure information and the smoothness of the noise in the denoised image and the original image or the noise image. The subjective evaluation method is mainly based on the mathematical model to objectively and quantitatively calculate the image quality. At present, the commonly used objective evaluation indicators mainly include Mean Absolute Error (MAE, Mean Absolute Error), Peak Signal to Noise Ratio (PSNR, Peak Signal to Noise Ratio) and so on. Among them, MAE and PSNR are used to measure the noise reduction ability of the noise reduction algorithm by statistical analysis of the grayscale difference of each pixel between the noise reduction image and the original image. The specific definitions are as follows:

\[
MAE = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} |u(i, j) - f(i, j)|}{M \times N}
\]

\[
PSNR = 10 \log_{10} \left( \frac{255^2 \times M \times N}{\sum_{i=1}^{M} \sum_{j=1}^{N} |u(i, j) - f(i, j)|^2} \right)
\]

In the formula, \( M \times N \) represents the size of the image. \( u \) and \( f \) denote the original image and the denoised image, respectively. The larger the PSNR value and the smaller the MAE value, the better the image noise reduction effect. All experiments in this paper are simulated in MATLAB 2016 environment. In the simulation experiment, the advantages and disadvantages of each noise reduction method are comprehensively judged by subjective observation and comparison method combined with a series of objective evaluation indicators MAE, PSNR and MSSIM.

3. Image Noise Reduction Model Based on Variable Order Variation

3.1. Concept of Variable Order Variational Model. Image denoising based on variational method is the problem of solving the optimal solution of the energy functional of the image. Image noise reduction is an ill-posed inverse problem, so regularization theory is often used to transform the ill-posed problem into a well-posed problem to find the optimal solution.

First, the following convex associative variational regularization model is proposed:

\[
\min_{u} E(u) = \frac{1}{2} \int_{\Omega} (u - u_0)^2 \, dx \, dy + \lambda \int_{\Omega} \left[ \theta |\nabla u| + (1 - \theta) |\nabla^2 u| \right] \, dx \, dy
\]  

Among them, \( \Omega \) is the definition domain of the image, \( u \) is the denoised image, and \( u_0 \) is the noise image. \( \lambda \) is the regularization parameter. The first term on the right side of the equation is a fidelity term, and the second term is a regular term that has a smoothing effect on the image. The Hessian matrix \( (\nabla^2 u) \) in this model can be expressed as:

\[
\nabla^2 u = \begin{bmatrix}
\partial_{i} \partial_{i} u & \partial_{i} \partial_{j} u \\
\partial_{j} \partial_{i} u & \partial_{j} \partial_{j} u
\end{bmatrix}
\]

Obviously, we choose \( \theta \) reasonably. The model in this paper has the following situations.

(1) When \( \theta = 1 \), the model in this paper can be rewritten as:

\[
\min_{u} E(u) = \frac{1}{2} \int_{\Omega} (u - u_0)^2 \, dx \, dy + \lambda \int_{\Omega} \| \nabla u \| \, dx \, dy
\]

At this time, the model degenerates into a TV model whose regularization operator is a first-order variation. It can be seen from the previous analysis that the model has good edge-preserving performance, but there will be a staircase effect.

(2) When \( \theta = 0 \), the model in this paper can be rewritten as:

\[
\min_{u} E(u) = \frac{1}{2} \int_{\Omega} (u - u_0)^2 \, dx \, dy + \lambda \int_{\Omega} \| \nabla^2 u \| \, dx \, dy
\]

At this time, the model degenerates into a bounded...
Hessian model whose regularization operator is a second-order variation. It can be seen from the previous analysis that the model has good smoothing performance, but the edge preservation ability is not good.

(3) When $\theta \in (0, 1)$, the model in this paper is similar to the TVBH model, taking into account the first-order variation and the second-order variation, which is a fusion of the TV model and the BH model.

In summary, the choice of parameter $\theta$ determines the filtering form and filtering performance of the new model. Since the parameter constant $\theta$ is usually obtained by trial and error through a large number of experiments or empirically to obtain the best value, it is a rough evaluation of the global content, ignoring the local characteristics of the image.

Next, considering the local features of the image, the edge spread function $\theta(z)$ is used to replace the constant $\theta$ to improve the adaptability of the model in this paper. We propose an edge-order variational model. The optimized new model is as follows:

$$
\min_u E(u) = \frac{1}{2} \int_{\Omega} (u - u_0)^2 dx dy + \lambda \left[ \theta(z) |\nabla u| + (1 - \theta(z)|\nabla^2 u|) \right] dx dy
$$

$$
\theta(z) = \exp \left( -\frac{3.31488}{(z/k)^8} \right)
$$

In equation (7), $k$ is the edge threshold parameter. In order to better detect the detailed information such as edge texture contained in the image, two feature detection operators, gradient and local entropy, are fused for the construction of the feature detection factor $z$ of the edge spread function $\theta(z)$. 

---

**Figure 1:** Comparison of denoising results on Lena image.
3.1.1. Gradient of the Image. The gradient characterizes the change size and direction of the gray value of the image, so the gradient modulus value is often used to distinguish the edge area and the non-edge area of the image. The gradient of the edge area is larger, and the gradient of the flat area is smaller, but the gradient of some detail information is not much different from that of the flat area. Sometimes the gradient at the noise point is even larger than the gradient at the edge. In this way, the gradient edge detection operator may cause misjudgment of weak edge regions and strong noise points with rich details in the image, resulting in loss of detailed information or incomplete noise reduction in the processed image.

3.1.2. Local Entropy of the Image. Local entropy characterizes the intensity of pixel gray value changes in local areas in the image. The local entropy is used in image processing to distinguish different regions and to enhance the image. The local entropy can be defined as the entropy of the gray level distribution in a local region of the image. The local entropy is calculated using the image gradient magnitude and the grayscale value of the pixel.

![Figure 2: Comparison of denoising results on Boat image.](image)

**Table 1: Quantitative comparison of denoising results for various models.**

<table>
<thead>
<tr>
<th>Noise reduction model</th>
<th>MAE</th>
<th>PSNR</th>
<th>MSSIM</th>
<th>MAE</th>
<th>PSNR</th>
<th>MSSIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV model</td>
<td>4.6587</td>
<td>31.7500</td>
<td>0.8957</td>
<td>5.0460</td>
<td>31.2904</td>
<td>0.8849</td>
</tr>
<tr>
<td>BH model</td>
<td>4.6329</td>
<td>31.3813</td>
<td>0.8976</td>
<td>5.2014</td>
<td>31.0289</td>
<td>0.8794</td>
</tr>
<tr>
<td>TVBH model</td>
<td>4.3988</td>
<td>31.7966</td>
<td>0.8993</td>
<td>5.2217</td>
<td>30.7591</td>
<td>0.8709</td>
</tr>
<tr>
<td>EWSO model</td>
<td>4.9060</td>
<td>30.2603</td>
<td>0.8873</td>
<td>5.4695</td>
<td>29.8877</td>
<td>0.8684</td>
</tr>
<tr>
<td>MBH model</td>
<td>4.4117</td>
<td>31.7782</td>
<td>0.8989</td>
<td>5.1898</td>
<td>30.9014</td>
<td>0.8701</td>
</tr>
<tr>
<td>VOV model</td>
<td>4.2180</td>
<td>32.7156</td>
<td>0.9125</td>
<td>4.6739</td>
<td>32.0090</td>
<td>0.8990</td>
</tr>
</tbody>
</table>
image, so that it can reflect the richness of the information contained in the image. The entropy value of a grayscale image $f$ of size $M \times N$ is defined as:

$$H = - \sum_{i,j=1}^{M,N} p_{i,j} \log_2(p_{i,j})$$

$$p_{i,j} = \frac{f(i,j)}{\sum_{i=1}^{M} \sum_{j=1}^{N} f(i,j)}$$

where $f(i,j)$ represents the gray value of the pixel located at point $(i,j)$ of the image. $p_{i,j}$ represents the distribution probability of the gray value of the pixel at point $(i,j)$ in the local neighborhood of size $M \times N$. $H$ represents the local entropy of the image. According to the local entropy, the local features of the image can be effectively determined. In the edge detail area with complex gray distribution, the local entropy value is larger. In a flat area with uniform gray distribution, the local entropy value is small. In addition, local entropy has strong anti-noise ability, and independent noise points have little effect on it. Therefore, local entropy can be widely used in image processing.

3.2. Numerical Solutions of Variable-Order Variational Models. In order to improve the computational speed of the proposed VOV model, we adopt the FFT-based split Bregman method to solve the model. First, the continuous first-, second-, and fourth-order differential operators and divergence operators are discretized. In addition, in order to further improve the calculation speed of the split Bregman algorithm, periodic boundary conditions are used to make the FFT suitable for the split Bregman algorithm. Let $\Omega$ be a two-dimensional grayscale image area with a size of $M \times N$, and the coordinates in the column and row directions of the image are denoted by $x$ and $y$, respectively. The first-order forward difference along the coordinate $x$ and $y$ directions at the pixel point $(i,j)$ is denoted as:

$$\nabla_x u_{i,j} = \begin{cases} u_{i,j+1} - u_{i,j} & 1 \leq i \leq M, 1 \leq j < N \\ u_{i,j} - u_{i,j+1} & 1 \leq i \leq M, j = N \end{cases}$$

$$\nabla_y u_{i,j} = \begin{cases} u_{i+1,j} - u_{i,j} & 1 \leq i \leq M, 1 \leq j \leq N \\ u_{i+1,j} - u_{i,j} & 1 \leq i \leq M, j = N \end{cases}$$

The discretized gradient, Hessian matrix is as follows:

$$\nabla u = \begin{bmatrix} \nabla_x u \\ \nabla_y u \end{bmatrix}$$

$$\nabla^2 u = \begin{bmatrix} \nabla_x^2 u & \nabla_x \nabla_y u \\ \nabla_y \nabla_x u & \nabla_y^2 u \end{bmatrix}$$

Finally, for any first-order divergence $\text{div}$ of $p = (p_1, p_2)$ and the discrete forms of any second-order divergence $\text{div}^2$ of $q = \begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix}$ are, respectively, recorded as:

$$\text{div}(p)_{i,j} = \partial_x (p_1)_{i,j} + \partial_y (p_2)_{i,j}$$

$$\text{div}^2(q)_{i,j} = \partial_x^2 (q_1)_{i,j} + \partial_x \partial_y (q_2)_{i,j} + \partial_y^2 (q_3)_{i,j} + \partial_y \partial_x (q_4)_{i,j}$$

To obtain the optimal solution of the energy functional of the proposed VOV model, auxiliary variables $w, v$, Bregman iteration parameters $b, d$ and penalty parameters $\theta_1$ and $\theta_2$ are first introduced. We transform the energy functional of the proposed VOV model into the following form.

$$E(u, w, v; b, d) = \frac{1}{2} \int_{\Omega} (u - u_0)^2 + \lambda \int_{\Omega} \theta(z)|\nabla z| + |1 - \theta(z)||\nabla z|$$

$$+ \frac{\theta_1}{2} \int_{\Omega} (v - \nabla u - b)^2 + \frac{\theta_2}{2} \int_{\Omega} (v - \nabla^2 u - d)^2$$

Next, fixing the variables $(w, v; b, d)$, the Euler equation for $u$ is obtained as follows:

$$u - \theta_1 \text{div} (\nabla u) + \theta_2 \text{div}^2 (\nabla^2 u) = u_0 - \theta_1 \text{div} (w - b) + \theta_2 \text{div} (v - d)$$

Regarding the update of the variable $w$, by fixing $(u, w; b, d)$, the Euler equation related to $w$ is obtained:

$$\lambda \theta(z) \frac{w}{|w|} + \theta_1 (w - \nabla u - b) = 0$$

In the above formula, the analytical solution for $w$ can be obtained by the following two-dimensional generalized soft threshold formula.

$$w_{i,j} = \max \left( \left| \nabla u_{i,j} + b_{i,j} \right| - \frac{\lambda}{\theta_1} \theta(z), 0 \right) \frac{\nabla u_{i,j} + b_{i,j}}{|\nabla u_{i,j} + b_{i,j}|}$$

Using the same method to update the variable $v$, the analytical solution can be expressed as follows:

$$v_{i,j} = \max \left( \left| \nabla^2 u_{i,j} + d_{i,j} \right| - \frac{\lambda}{\theta_1} (1 - \theta(z)), 0 \right) \frac{\nabla^2 u_{i,j} + d_{i,j}}{|\nabla^2 u_{i,j} + d_{i,j}|}$$

Finally, the Bregman iteration parameters $b$ and $d$ can be updated. In summary, the solution process of the VOV model proposed in this paper based on the FFT split Bregman algorithm can be summarized as follows.

Step1: Initialize $u_0$ ($u_0$ is set as the noise image to be improved).

Step2: Initialize $(w, v; b, d) = 0$.

Step3: Set parameters $\lambda, \theta_1$ and $\theta_2$. 
Step 4: The variables u, w, and v are updated by \( (u, w, v) = \arg \min_{n \in N} E(u, w, v) \).

Step 5: Update the Bregman iteration parameter b.

Step 6: Update the Bregman iteration parameter d.

Step 7: If the iteration termination condition \( \|u^k - u^{k-1}\|_1 < 10^{-6} \) is satisfied, the iteration is stopped. Otherwise, jump to Step 4 until the termination condition is met.

4. Experimental Results and Analysis

In this section, we verify the effectiveness of our proposed variable-order variational VOV model by numerical experimental comparison and analysis on natural images. In the simulation experiment, the noise reduction effects of the VOV model in this chapter and the classic TV model, BH model, TVBH model, EWSO model and MBH model are compared and analyzed from both subjective and objective aspects.

In the simulation experiments of natural images, standard images of Lena and Boat with a size of 256 x 256 are used as test images, and Gaussian noises with a mean of 0 and a variance of 0.002 are added to the test images, respectively. After the noise image is acquired, the noise reduction experiment is performed on the noise image. For the noise reduction performance of each model, we use subjective visual evaluation and objective evaluation indicators MAE, PSNR and MSSIM to evaluate and analyze the noise reduction image quality. The size of the parameters in the model affects the effect of noise reduction, and the parameters of each comparative model in the experiment are set according to the optimal experimental results. In the edge diffusion function \( \theta(z) \) of the VOV model, \( k \) is an important parameter, which determines that the contrast of the edge should be maintained and affects the diffusion process. More specifically, if the value of \( k \) is too large, the edges of the image are over-smoothed. If the value of \( k \) is too small, some noise in the image is difficult to filter out. It is verified by experiments that when the parameters are set to \( k = 10, \lambda = 30, \) and \( \theta 1 = \theta 2 = 5 \), the proposed VOV model has the best noise reduction effect. After filtering, the subjective visual effect of the image is shown in Figures 1 and 2, and the comparison of objective evaluation performance indicators corresponding to various models is shown in Table 1.

It can be seen from Figures 1 and 2 that the edge information of the image in the processing result of the TV model is well preserved, but the resulting staircase effect brings bad visual effects. Although the BH model can effectively suppress the staircase effect of TV, it has poor protection ability for edge details. The processing results of TVBH model, EWSO model and MBH model have good visual effects, image noise is effectively suppressed, and edge details are well preserved, but some weak edges and texture information in the image are over-smoothed. The processing results of the VOV model proposed in this paper have better visual effects than other models, and the edge details of the image are well preserved. Details such as the eye and hat decorations in the Lena image, and numerous subtle line outlines in the Boat image. From the objective evaluation criteria in Table 1, it can be seen that under the same noise conditions, the VOV model proposed in this paper has stronger noise reduction and edge detail protection capabilities than the TV model, BH model, TVBH model, EWSO model and MBH model.

5. Conclusion

This paper focuses on the image noise reduction method based on variational method, and analyzes the advantages and disadvantages of TV model, BH model and TVBH model. Aiming at the shortcomings of TV model, BH model and TVBH model, a variable-order variational model is proposed. The model proposed in this paper takes into account the advantages of the first-order variational model and the second-order variational model, and divides the image into flat area and detail area according to the gradient modulus value of the image and the feature detection factor of local entropy fusion. Then the variational order is adaptively selected in different regions to achieve different diffusion modes. At the same time, we use the split Bregman algorithm based on fast Fourier transform to solve the proposed variational model, which improves the operation speed. The experimental results show that the model proposed in this paper has a good visual effect for natural image noise reduction, so it is very suitable for movement correction and online teaching of sports dance training.

Data Availability

The experimental data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest to report regarding the present study.

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